Heterotic/F theory dual SU(5) model

H. Clemens & Stuart Raby

arXiv:1906.07238, 1908.01110, 1908.01913, 1912.06902, 2001.10047

String Pheno 2023
CTPU Daejeon, Korea
July 7, 2023

The Ohio State University Department of Physics

Outline

- Heterotic – F theory duality
- SU(5) GUT w/Wilson line breaking
- 4 + 1 split $(SU(4)xU(1)_X)_{Higgs}$
- Vector-like exotics / bisection
- GUT breaking & Gauge coupling unification
- R parity / $\mathbb{Z}_4^R$ symmetry
- Conclusions

Conclusions

- Constructed Global SU(5) F theory model with Wilson line breaking
- 3 SO(10) families and one pair of Higgs doublets + NO vector-like exotics !
- $U(1)_X$ and $\mathbb{Z}_4^R$ symmetry
- $10_M 10_M 5_H,\ 10_M \overline{5}_M \overline{5}_H,\ \Gamma_M \overline{5}_M 5_H,\ \Gamma_M \Gamma_M \Lambda$
- Complete twin sector
- Different scales !

Heterotic - F theory duality

Heterotic side

- E8 x E8 on elliptically fibered CY_3
  - Torus fibered over base B_2
- E8 broken to $(SU(5) \times U(1)_X)_{gauge}$ by $(SU(4) \times U(1)_X)_{Higgs}$ vector bundle
- Freely acting Z_2 involution (preserving the gauge symmetry) $\pi_1(CY_3) = Z_2$
- Wilson line wraps non-contractible cycle, breaks $SU(5)_{gauge}$ to SM
- Higgs data in semi-stable degeneration limit $dP_9 \cup dP_9$ connected along elliptic fiber
  - Defines the spectral cover

Heterotic side

- E8 x E8 on elliptically fibered CY_3
  - Torus fibered over base B_2
- E8 broken to $(SU(5) \times U(1)_X)_{gauge}$ by $(SU(4) \times U(1)_X)_{Higgs}$ vector bundle
- Freely acting Z_2 involution (preserving the gauge symmetry) $\pi_1(CY_3) = Z_2$

$$\frac{dx}{y} \to \frac{dx}{y}$$

- Wilson line wraps non-contractible cycle, breaks $SU(5)_{gauge}$ to SM
- Higgs data in semi-stable degeneration limit $dP_9 \cup dP_9$ connected along elliptic fiber
  - Defines the spectral cover

F theory side

- Elliptically fibered CY_4 over base B_3
- $B_3 = P^1$ fibered over same base B_2
- dP_9 s determine the Weierstrass function/Tate form

$$y^2 = x^3 + a_5 xy + a_4 z x^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

$$a_i\,, z,\, t = y/x \quad \in\; H^0(K_{B_3}^{-1})^{[-1]}$$

Z_2 involution (freely acting on $S^{\wedge}_{GUT} = K3$) defines
$S^{\vee}_{GUT}$ = Enriques surface with $\pi_1(S^{\vee}_{GUT}) = \mathbb{Z}_2$

F theory side

- Elliptically fibered CY_4 over base B_3
- $B_3 = P^1$ fibered over same base B_2
- dP_9 s determine the Weierstrass function/Tate form

$$y^2 = x^3 + a_5 xy + a_4 z x^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

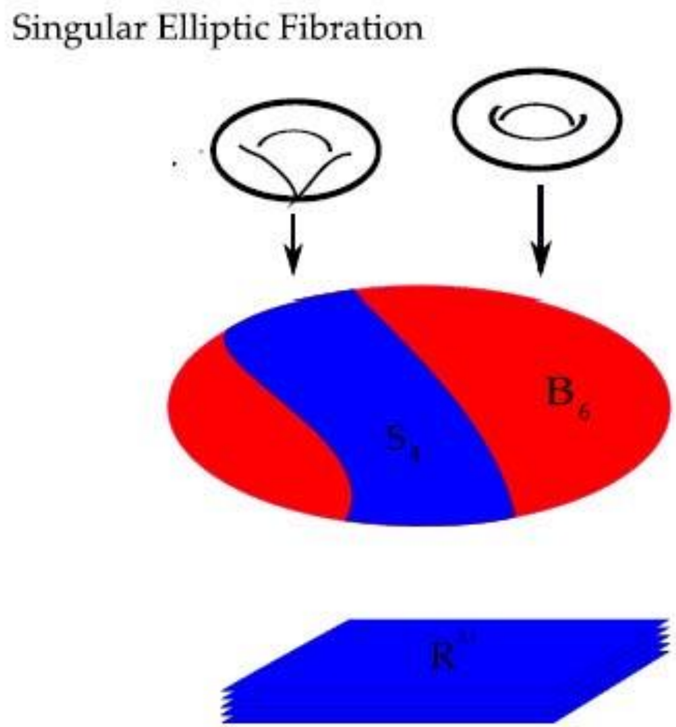
$$a_i\,, z,\, t = y/x \quad \in\; H^0(K_{B_3}^{-1})^{[-1]}$$

Z_2 involution (freely acting on $S^{\wedge}_{GUT} = K3$) defines
$S^{\vee}_{GUT}$ = Enriques surface with $\pi_1(S^{\vee}_{GUT}) = \mathbb{Z}_2$

$$\text{Preserving } CY_4 \Rightarrow \frac{dx}{y} \rightarrow -\frac{dx}{y}$$

Singular elliptic fibration

Gauge degrees of freedom on 7-branes realized in terms of ADE singularities, in codim 1 in the base B_3: divisor $S_{\rm GUT}$



Geometrically: elliptically fibered CY**4** with Weierstrass form

$$y^2 = x^3 + fx + g$$

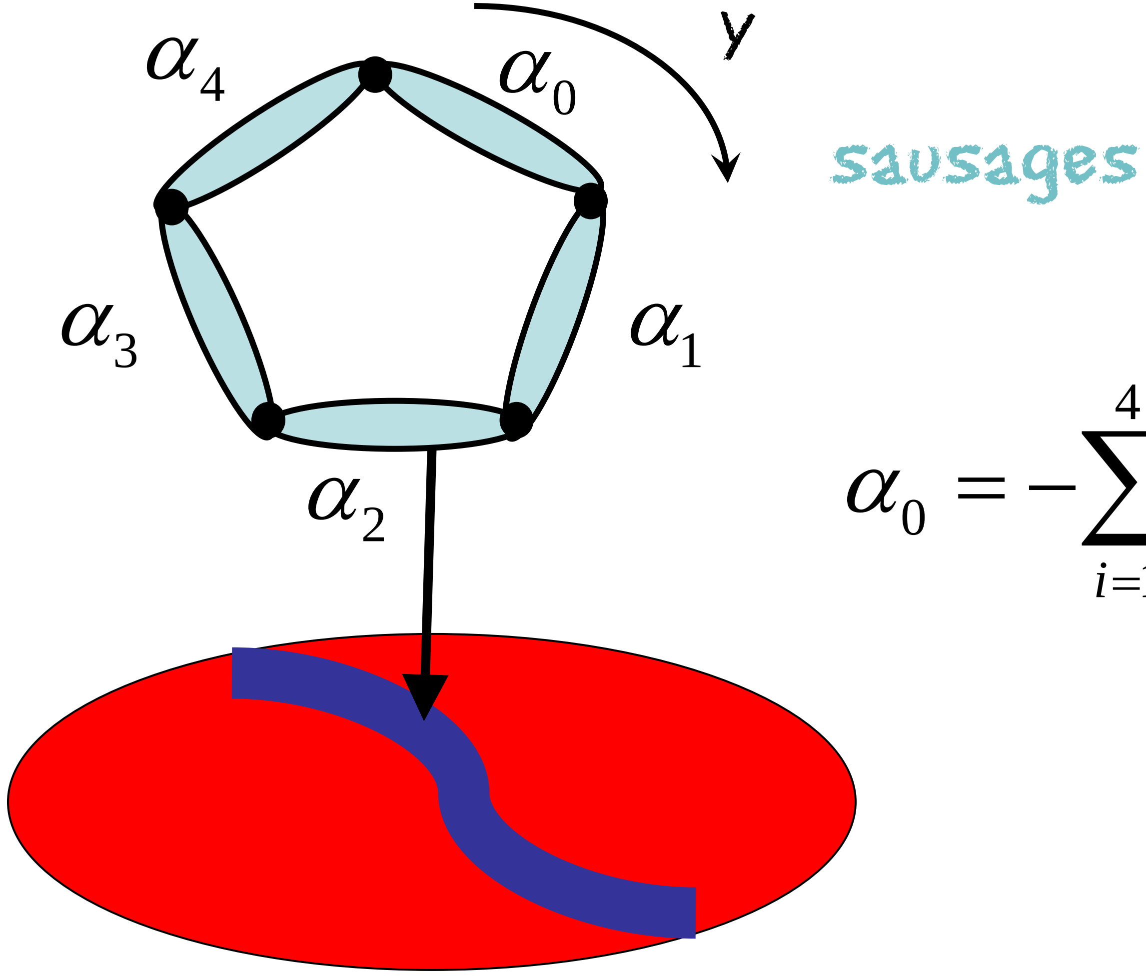
f and g are global sections of $\mathcal{O}(-4K_B)$ and $\mathcal{O}(-6K_B)$, resp.

Gauge degrees of freedom: discriminant locus

$$\Delta = 4f^3 + 27g^2 = 0 \quad \supset \quad S_{\rm GUT}$$

Kodaira classification ADE

SU(5) extended Dynkin diagram



$$\alpha_0 = -\sum_{i=1}^{4} \alpha_i$$

Wilson line breaking : New Problems

(1) $y \to -y$ breaks gauge symmetry

(2) vector-like exotics

Follow the roots

$$y^2 = x^3 + a_5 xy + a_4 z x^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

Divide by a_0^6 and $\frac{y}{a_0^3} \to y, \frac{x}{a_0^2} \to x, \frac{z}{a_0} \to z, \frac{a_j}{a_0} \to c_j$

$$y^2 = x^3 + c_5 xy + c_4 z x^2 + c_3 z^2 y + c_2 z^3 x + z^5$$

Equivariant crepant resolution => sausages

Under involution 'breaks the gauge group'

$y \to -y \qquad c_j = 0 \qquad h_{E_8} \to -h_{E_8}$ i.e. roots $\to$ – roots

$y \to -y \qquad c_j \to (-1)^j c_j \qquad h_{SU(5)} \to -h_{SU(5)}$

Follow the roots

Narasimhan – Seshadri Theorem:

A holomorphic vector bundle of degree zero on a Riemann surface is stable IFF it comes from an irreducible unitary representation of the fundamental group of the surface.

Freedman, Morgan & Witten use this to define dP_9 *in terms of the data of the non – flat* $SU(5)_{Higgs}$ *bundle on the Heterotic side.*

But this requires the complex gauge group with a choice of $\pm i$!!

Summary: $\mathbb{Z}_2$ involution on elliptic curve : $y \to -y$

Problem 1 - involution breaks GUT

Involution takes GUT roots $\rho^c \to -\rho^c$

Note: roots are defined in terms of the pure imaginary part of the complexified group

$$\rho^c = i\,\rho \to -\rho^c$$

where ρ are physicist's roots

Solution to problem 1

Add to involution action by conjugation

$$\rho^c = i\,\rho \rightarrow -\left(\rho^c\right)^* = i\,\rho$$

Hence $i\ \rho$ and thus ρ are unchanged !!!

Clemens & Raby arXiv:1906.07238

Problem 2

Representation	Type of multiplet	Cohomology group dimension
$(\mathbf{8},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)=1$
$(\mathbf{1},\mathbf{3})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)=1$
$(\mathbf{1},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)=1$
$(\mathbf{8},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)\oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)=0$
$(\mathbf{1},\mathbf{3})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)\oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)=0$
$(\mathbf{1},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)\oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right)=0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Vector	$h^0\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right)=0$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Vector	$h^0\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right)=0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Chiral	$h^1\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right)\oplus h^2\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right)=1$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Chiral	$h^1\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right)\oplus h^2\left(S^\vee_{\mathrm{GUT}}, \mathcal{O}_{S^\vee_{\mathrm{GUT}}}(\varepsilon_{u,v})\right)=1$

Vector-like exotics

Beasley, Heckman & Vafa arXiv:0806.0102 section 7

Donagi & Wijnholt arXiv:0802.2969

Marsano, Clemens, Pantev, Raby & Tseng arXiv: 1206.6132

Theorem says always occurs on elliptic fiber w/section

Solution to problem 2

Build CY_4 = elliptic fiber over base B_3 with 2 sections

$$\begin{array}{ccc} B_3 & = & \mathbb{P}^1 \\ & & \downarrow \\ & & B_2 \end{array}$$

Given by **Tate form** of Weierstrass function

$$\omega y^2 = x^3 + a_5\, \omega\, x\, y + a_4\, z\, \omega\, x^2 + a_3\, z^2 \omega^2 y + a_2\, z^3 \omega^2 x + a_0\, z^5 \omega^3$$

$\varsigma(b_3) = \{[\omega, x, y] = [0,0,1]\}$ first section

Tate form

$\omega, y, x =$ elliptic fiber (torus)

$z = 0 \;\Rightarrow\; S_{GUT} \;\Rightarrow$ descriminant vanishes

z, a_j functions on B_3

Choose $\quad a_5 + a_4 + a_3 + a_2 + a_0 = 0$ $\quad a_5\, s^5 +$

$\tau(b_3) = \{[\omega, x, y] = [1, z^2, z^3]\}$ second section

Tate form

$\omega, y, x =$ elliptic fiber (torus)

$z = 0 \;\Rightarrow\; S_{GUT} \;\Rightarrow$ descriminant vanishes

z, a_j functions on B_3

Choose $\quad a_5 + a_4 + a_3 + a_2 + a_0 = 0$

$\tau(b_3) = \{[\omega, x, y] = [1, z^2, z^3]\}$ second section

Let $\quad \omega = 1,\; y = t^3, x = t^2,\; s = z/t$

$$C \equiv a_5 + a_4 s + a_3 s^2 + a_2 s^3 + a_0 s^5 \quad - \text{spectral cover}$$

$$= \left(a_5 + a_{54} s - a_{20} s^2 - a_0\, s^3 - a_0 s^4\right)\left(1 - s\right) \quad 4+1 \text{ split}$$

Now our elliptic fibration has two sections which are invariant under the initial Z_2 involution

In the final def. of the Involution we include a translation by $\xi(b_3) - \tau(b_3)$

No Vector-like exotics !!

Clemens & Raby 1908.01913

Representation	Type of multiplet	Cohomology group dimension
$(\mathbf{8},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = h^0\left(\mathcal{O}_{S^\vee_{\mathrm{GUT}}}\right) = 1$
$(\mathbf{1},\mathbf{3})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = h^0\left(\mathcal{O}_{S^\vee_{\mathrm{GUT}}}\right) = 1$
$(\mathbf{1},\mathbf{1})_0$	Vector	$h^2\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = h^0\left(\mathcal{O}_{S^\vee_{\mathrm{GUT}}}\right) = 1$
$(\mathbf{8},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) \oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 0$
$(\mathbf{1},\mathbf{3})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) \oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 0$
$(\mathbf{1},\mathbf{1})_0$	Chiral	$h^0\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) \oplus h^1\left(S^\vee_{\mathrm{GUT}}, K_{S^\vee_{\mathrm{GUT}}}\right) = 0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Vector	$h^0\left(\mathcal{O}_{S^\vee_{\mathrm{GUT}} \times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) = 0$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Vector	$h^0\left(\mathcal{O}_{S^\vee_{\mathrm{GUT}} \times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) = 0$
$(\mathbf{3},\mathbf{2})_{-5/6}$	Chiral	$h^1\left(\mathcal{O}_{S^\vee_{\mathrm{GUT}} \times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) \oplus h^2(\ldots) = 0$
$(\bar{\mathbf{3}},\mathbf{2})_{5/6}$	Chiral	$h^1\left(\mathcal{O}_{S^\vee_{\mathrm{GUT}} \times_{B_3^\vee} W_4^\vee}\left(\varepsilon_{u,v}\cdot\tilde{\tau} - \varepsilon_{u,v}\cdot\tilde{\zeta}\right)\right) \oplus h^2(\ldots) = 0$

4+1 split of the spectral divisor

$(SU(4) \times U(1)_X)_{Higgs}$ breaking

$E_8 \rightarrow (SU(5) \times U(1)_X)_{guage}$ on GUT surface

Orbifold has fixed points on B_3 and B_2 -

But freely acting on S_{GUT}

4+1 split of the spectral divisor

$(SU(4) \times U(1)_X)_{Higgs}$ breaking

$E_8 \to (SU(5) \times U(1)_X)_{guage}$ on GUT surface

Orbifold has fixed points on B_3 and B_2 -

But freely acting on S_{GUT}

NOTE, these fixed points are NOT crepant !

AND S_{GUT} is in B_3 and avoids all fixed pts

We choose the base $B_3 = \mathbb{P}^1 \times B_2$
and $B_2 = dP_7$

This choice has several nice properties

1. It is the unique del Pezzo surface with a finite discrete set of orbifold fixed points under the $\mathbb{Z}_2$ involution (see Blumenhagen et al. 0811.2936) and has the following properties -

a. There are 3 families of quarks and leptons

b. dP_7 has the global symmetry $S_4 \times S_2$

This results in a $\mathbb{Z}_4^R$ symmetry!

$\mathbb{Z}_4^R$ symmetry explains low energy MSSM

SU(5)

$q_{\mathbf{10}}$	$q_{\bar{\mathbf{5}}}$	q_{H_u}	q_{H_d}
1	1	0	0

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg & Vaudrevange
arXiv:1009.0905, 1102.3595

$$\mathcal{W}_p = Y_e^{ij} H_d L_i \bar{E}_j + Y_d^{ij} H_d Q_i \bar{D}_j + Y_u^{ij} H_u Q_i \bar{U}_j + \kappa_{ij}^{(0)} H_u L_i H_u L_j$$

$$\mathcal{W} = \mathcal{W}_p + \Delta \mathcal{W}_{non-perturbative}$$

$$\left\langle W \right\rangle_0 \Big/ M_{Pl}^2 \sim m_{3/2}$$

$$\Delta W_{np} \propto B_0\, m_{3/2} M_{Pl}^2 + m_{3/2}\, H_u H_d + \frac{m_{3/2}}{M_{Pl}^2}\left(QQQL + \overline{U}\,\overline{U}\,\overline{D}\,\overline{E} \right)$$

Sufficiently suppressed to make dim 5 proton decay unobservable !

Matter Curves

$$z = a_5 = 0$$

$$z = a_{420} = 0$$

$$z = (\text{quadratic in } a_j) = 0$$

$$h^0\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(4)}_{\mathbf{10}}}\right) - h^1\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(4)}_{\mathbf{10}}}\right) = 7 - 1 = 6$$

$$h^0\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(41)}_{\bar{\mathbf{5}}}}\right) - h^1\left(\mathcal{L}^{(0)}_{Higgs}\Big|_{\Sigma^{(41)}_{\bar{\mathbf{5}}}}\right) = 7 - 1 = 6$$

$$h^0\left(\mathcal{L}^{(0,-)}_{Higgs}\Big|_{\Sigma^{(44)}_{\bar{\mathbf{5}}}}\right) - h^1\left(\mathcal{L}^{(0,-)}_{Higgs}\Big|_{\Sigma^{(44)}_{\bar{\mathbf{5}}}}\right) = 0.$$

Involution -

$y \to -y$, conjugation of roots,

translation by $\xi(\mathrm{b}_3) - \tau(b_3)$, and Wilson line ~Y

Downstairs (after the involution) Keep only symmetric subspace

$$\Rightarrow\ 6 \to 3$$

3 families

$\Sigma^{(4)}_{\mathbf{10}} = \{a_5 = z = 0\}$	$C_{u,v}$	L_Y	$\mathcal{L}_{Higgs}$	$SU(3) \times SU(2) \times U(1)_Y$
$h^0\left(\check{\mathcal{L}}^{(4)[\pm 1]}_{\mathbf{10}}\right)$	$+1$	$+1$	3	$(\mathbf{1},\mathbf{1})_{+1}$
	-1	-1		$(\mathbf{3},\mathbf{2})_{+1/6}$
	$+1$	$+1$		$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$
$h^1\left(\check{\mathcal{L}}^{(4)[\pm 1]}_{\mathbf{10}}\right)$	$+1$	$+1$	0	$(\mathbf{1},\mathbf{1})_{+1}$
	-1	-1		$(\bar{\mathbf{3}},\mathbf{2})_{+1/6}$
	$+1$	$+1$		$(\mathbf{3},\mathbf{1})_{+2/3}$

$\Sigma^{(41)}_{\bar{\mathbf{5}}} = \{a_{420} = z = 0\}$	$C_{u,v}$	L_Y	$\mathcal{L}_{Higgs}$	$SU(3) \times SU(2) \times U(1)_Y$
$h^0\left(\check{\mathcal{L}}^{(41)[\pm 1]}_{\bar{\mathbf{5}}}\right)$	$+1$	$+1$	3	$(\bar{\mathbf{3}},\mathbf{1})_{+1/3}$
	-1	-1		$(\mathbf{1},\mathbf{2})_{-1/2}$
$h^1\left(\check{\mathcal{L}}^{(41)[\pm 1]}_{\bar{\mathbf{5}}}\right)$	$+1$	$+1$	0	$(\mathbf{3},\mathbf{1})_{-1/3}$
	-1	-1		$(\mathbf{1},\mathbf{2})_{+1/2}$

Clemens & Raby 1908.01913

1 pair of Higgs doublets
And No Higgs triplets !

$\Sigma_{\bar{\mathbf{5}}}^{(44)} = \{a_4 a_3 + a_5 (a_0 - a_3) = z = 0\}$	$C_{u,v}$	L_Y	$\mathcal{L}_{Higgs}$	$SU(3) \times SU(2) \times U(1)_Y$
$h^0 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[+1]}\right)$	$+1$	$+1$	0	$(\bar{\mathbf{3}}, \mathbf{1})_{+1/3}$
$h^0 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[-1]}\right)$	-1	-1	1	$(\mathbf{1}, \mathbf{2})_{-1/2}$
$h^1 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[+1]}\right)$	$+1$	$+1$	0	$(\mathbf{3}, \mathbf{1})_{-1/3}$
$h^1 \left(\check{\mathcal{L}}_{\bar{\mathbf{5}}}^{(44)[-1]}\right)$	-1	-1	1	$(\mathbf{1}, \mathbf{2})_{+1/2}$

Clemens & Raby   1908.01913

$U(1)_X$ due to 4 + 1 split

$$\omega = 1,\ y = t^3, x = t^2,\ s = t/z$$

$$C = \left(a_5 s^4 + a_{54} s^3 - a_{20} s^2 - a_0 (s+1)\right)(s-1) \quad 4+1 \text{ split}$$

Intersection of 3 matter curves = cubic coupling

$$10^{-1}{}_m\ ,\ \bar{5}^{+3}{}_m\ ,\ 5^{+2}{}_h + \bar{5}^{-2}{}_h$$

$$10_m\ \bar{5}_m\ \bar{5}_h\ ,\ 10_m\, 10_m\, 5_h$$

but NOT $10_m\ \bar{5}_m\ \bar{5}_m$

Right-handed Neutrinos

Clemens & Raby 2001.10047

$$\Gamma^{-5} \equiv 1^{-5}{}_m, \quad \Lambda^{+10}$$

$\Gamma^{-5}{}_m \ \bar{5}^{+3}{}_m \ 5_h{}^{+2}$ Dirac neutrino mass allowed

$\Gamma^{-5}{}_m \ \Gamma^{-5}{}_m \ \Lambda^{+10}$ also allowed

$U(1)_X \rightarrow \mathbb{Z}_2$ matter parity by involution

Defining equations for fermionic states (then SUSY gives bosons)

$10,\ \bar{5}\ :\quad a_j, z = 0$

$(5,\bar{5})_{Higgs}:\quad a_j\, a_k,\ z = 0$

$gauginos:\quad z = 0$

$T_{u,v} : \{a_i, z\} = -i\ \{a_i, z\}$

$T_{u,v} :\ \{w, x, y\} = \{w, -x, i\,y\}$

$T_{u,v} :\ Tate\ form = -Tate\ form$

TABLE 3: $T_{u,v}$	$T_{u,v}$-charge	space
matter fields on $\frac{\Sigma_{\mathbf{10}}^{(4)}}{\{C_{u,v}\}}$	-1	$H^0\left(\frac{\Sigma_{\mathbf{10}}^{(4)}}{\{C_{u,v}\}}; \mathcal{L}_{Higgs}^{\vee,[\pm 1]}\right)$
matter fields on $\frac{\Sigma_{\bar{\mathbf{5}}}^{(41)}}{\{C_{u,v}\}}$	-1	$H^0\left(\frac{\Sigma_{\bar{\mathbf{5}}}^{(41)}}{\{C_{u,v}\}}; \mathcal{L}_{Higgs}^{\vee,[\pm 1]}\right)$
Higgs fields on $\frac{\Sigma_{\bar{\mathbf{5}}}^{(44)}}{\{C_{u,v}\}}$	$+i$	$H^0\left(\frac{\Sigma_{\bar{\mathbf{5}}}^{(44)}}{\{C_{u,v}\}}; \mathcal{L}_{Higgs}^{\vee,[-1]}\right) \ /\ H^1\left(\frac{\Sigma_{\bar{\mathbf{5}}}^{(44)}}{\{C_{u,v}\}}; \mathcal{L}_{Higgs}^{\vee,[-1]}\right)$
bulk matter on $\frac{S_{\mathrm{GUT}}}{\{C_{u,v}\}}$	$-i$	$H^2\left(K_{\frac{S_{\mathrm{GUT}}}{\{C_{u,v}\}}}\right)$

*Given the $\mathbb{Z}_4^R$ charges, i^{q+1} for the fermionic components, then bosonic components have charge i^q with $\theta' = -i\,\theta$

$\mathbb{Z}_4^R$ – Lee, Raby, Ratz, Ross, & Schieren 1009.0905

Relative Scales - Visible vs. Hidden sector

Clemens & Raby 2001.10047

$$S_{EH} \sim M_*^8 \int_{\mathbb{R}^{3,1}\times B_3} R\sqrt{-g_\delta}\, d^{10}x$$

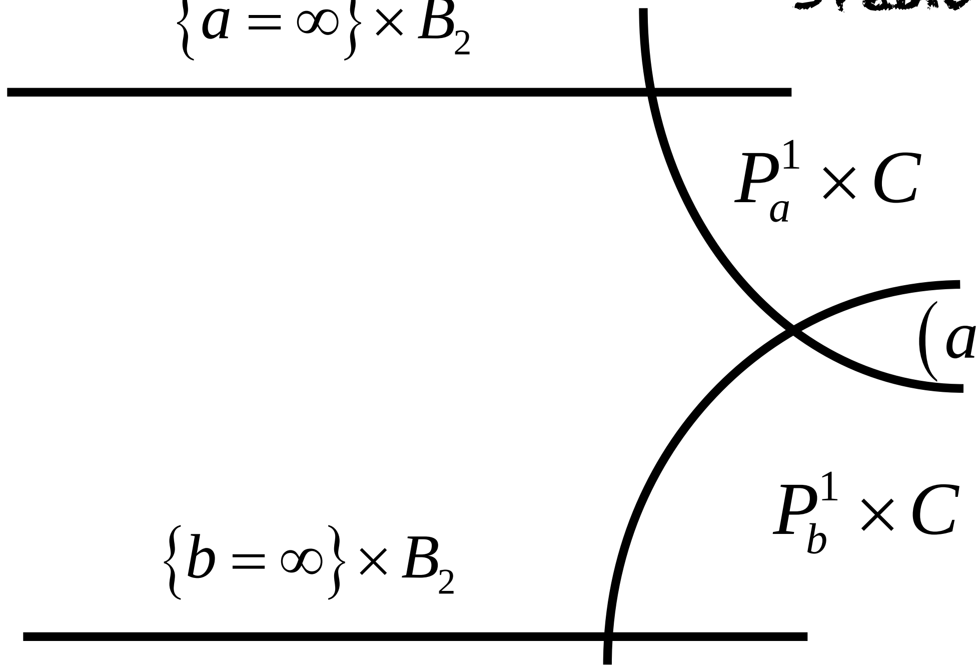
$$M_{Pl}^2 \simeq M_*^8 \cdot Vol\,(B_{3,\delta})$$

$$S_{guage} \sim -M_*^4 \int_{R^{3,1}\times S_i} \left(Tr(F_1^2)\sqrt{-g_1} + Tr(F_2^2)\sqrt{-g_2}\right) \delta^2(z_0)\; d^{10}x$$

$$\alpha_G^{-1} \sim M_*^4\; Vol(S_i)$$

$$M_G(i)^{-4} \sim Vol(S_i)$$

Stable degeneration limit



$$B_{3,0} = B_3^{(1)} \bigcup B_3^{(2)} = P_a^1 \times B_2 \bigcup P_b^1 \times B_2$$

$$S_1 = \left(\{a=\infty\}\times B_2\right) \bigcup (P_a^1 \times C)$$

$$S_2 = \left(\{b=\infty\}\times B_2\right) \bigcup (P_b^1 \times C)$$

$$m_i = Vol(P_i^1),\ \ i = a,b, \quad Vol(C) = \int_{B_2} |q|^2$$

$$Vol(S_i) = Vol(B_2) + m_i Vol(C), \quad m_i = Vol(P_i^1), \; i = a, b$$

$$\alpha_G(i) M_{Pl} \sim \frac{\sqrt{(m_1 + m_2)\, Vol(B_2)}}{Vol(B_2)(1 + K\, m_i)}$$

Eg.

Visible sector $\alpha_G(1)^{-1} = 24, \; M_G(1) = 3 \times 10^{16}$ GeV

$$\alpha_G(2) \big/ \alpha_G(1) = \frac{1 + Km_1}{1 + K\, m_2}, \quad M_G(2) \big/ M_G(1) = \left(\frac{1 + Km_1}{1 + K\, m_2} \right)^{1/4}$$

Twin sector, take $M_G(2) = 3.9 \times 10^{16}$ GeV, $\alpha_G(2)^{-1} = 8.7$

or $$\frac{1 + Km_1}{1 + K\, m_2} = 2.8$$

There are at least two possible effective low energy theories

1) Twin Higgs VEV = 0. $M_{tGUT} > M_{GUT}$ and $\alpha_{tGUT} > \alpha_{GUT}$.
Thus the LE theory has a complete twin sector with twin baryons and leptons heavier then the SM baryons and leptons.*

*This assumes twin Yukawa couplings are equal to SM Yukawas.

2) Effective twin theory has $N_{QCD} = 3,\ N_F = 6$ and described by Seiberg electric dual - i,j = 1,2,3, a=1,2 weak isospin

$$W \propto \lambda_{ij}^{u} Q^{ia} H_{ua} \overline{Q_{j1}} + \lambda_{ij}^{d} Q^{ia} H_{da} \overline{Q_{j2}}$$

$$If\ \langle H_u \rangle = \langle H_d \rangle \simeq M_{GUT}$$ Flat direction

All twin quarks and charged leptons obtain mass at M_{GUT} and $SU(2) \times U(1)_Y \to U(1)_{tEM}$.
LE theory includes tHiggs & tHiggsinos, tphoton & tphotino

$$\Lambda_{tQCD} \approx M_{GUT} \exp\left(-\frac{2\pi}{9\alpha_G(2)} \right) \sim 9 \times 10^{13}\ \text{GeV},$$

$$\Lambda_{tQCD} \approx T \exp\left(-\frac{2\pi}{9\alpha_G(2)} \right) \sim 9 \times 10^{13} \text{ GeV},$$

$$m_{3/2} \sim \frac{\Lambda_{tQCD}{}^3}{M_{Pl}^2} \sim 130 \text{ TeV}$$

Gluino condensate

Possible μ term due to Giudice-Masiero mechanism produces a negative mass squared for the Higgs which keeps the Higgs VEV at the GUT scale.

Wilson line and the GUT scale

- The Wilson line wraps the GUT surface breaking SU(5) -> SM gauge group
- $M_{GUT} = M_C \sim 1/R_{cycle}$
- Non-local GUT breaking - Precise Gauge Coupling Unification
- Complete twin world with scales fixed by the size of the twin manifold
- Mirror matter - Dark Matter candidate

Conclusions

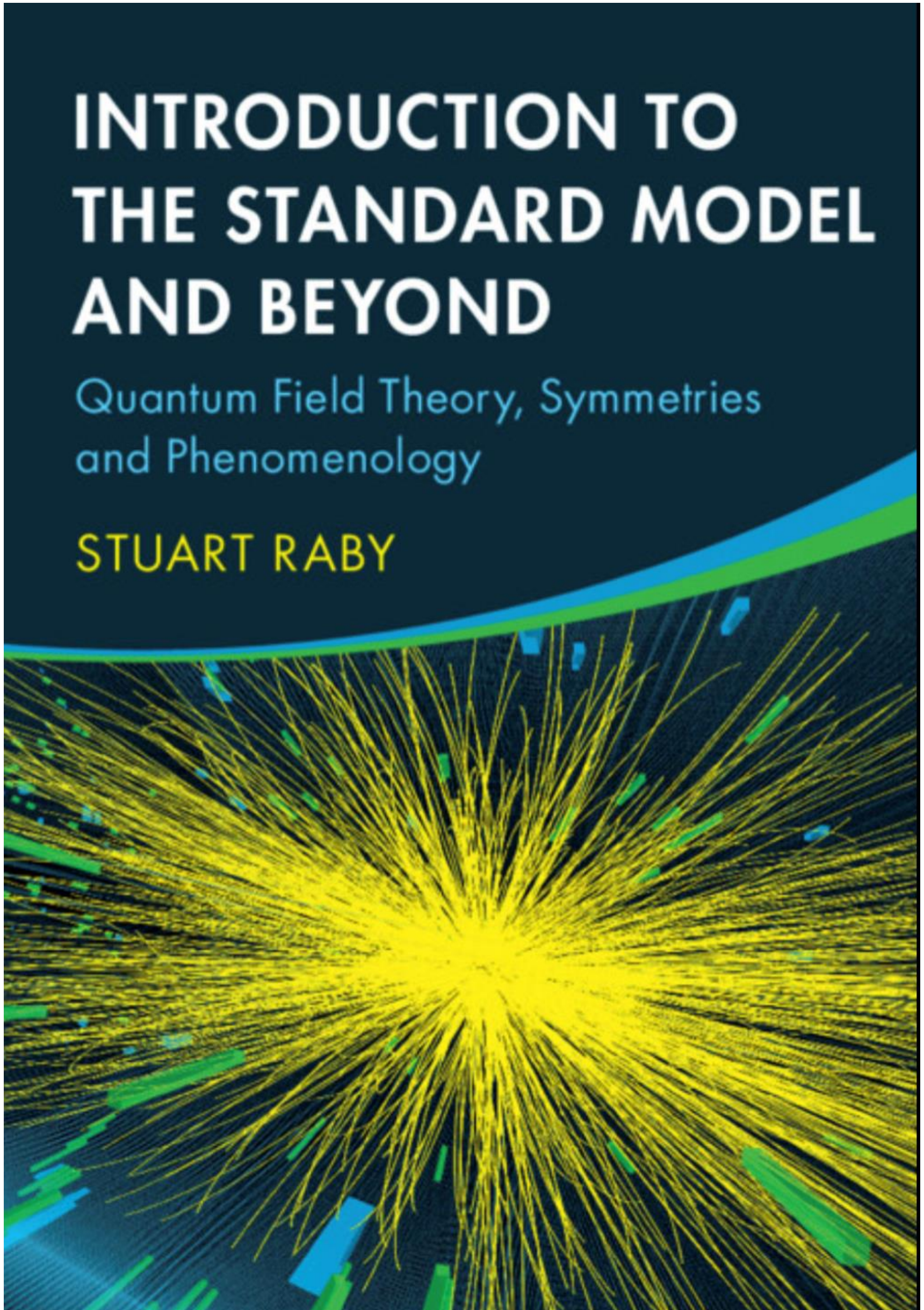
- Constructed Heterotic/F theory dual SU(5) model with Wilson line breaking
- 3 SO(10) families and one pair of Higgs doublets + NO vector-like exotics !
- $\mathbb{Z}_2$ matter parity and $\mathbb{Z}_4^R$ symmetry
- Complete twin sector
- There are many, many more open questions

For the Future

- $U(1)_X$ anomly ?
- Right-handed neutrino masses
- Yukawa couplings
- Stabilizing moduli and SUSY breaking
- Do orbifold fixed points in B_3 break SUSY?
- Low energy theory ??
- Inflation ??
- Dark matter & possible portal to the visible sector (see Kawamura & Raby 2212.00840)

This model provides a virgin theoretical laboratory with many hidden possibilities !!

Thank you



Cambridge University Press
2021