Modular weights of matter fields in LEEFT from type II string theory

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Collaboration with Kikuchi, Nasu, Otsuka, Takada, Uchida, arXiv:2301.10356

Introduction

One of important purposes in string phenomenology
is
to derive 4D low energy effective field theory
from string theory,
and to discuss its aspects
in particle physics and cosmology.

Introduction

Properties of 4D low energy effective field theory
are related with geometrical aspects
of string compactification
and

4D LEEFT is controlled by
geometrical symmetries of compact space.

Introduction

Recently,

Modular symmetries of compact spaces such as torus, torodidal orbifolds, and Calabi-Yau manifolds are studied very actively from both top-down and bottom-up approaches.

SL(2,Z) and Sp(2g,Z) and their finite groups, S3, A4, S4, A5 as well as covering groups.

Nilles's talk, Kawamura's talk.

Introduction

In modular symmetric LEEFT,
modular weights of matter fields are
important properties.

$$K = (T_i + \bar{T}_i)^{k^i} |C|^2$$

Here, we study modular weights.

Heterotic string on CY manifolds

Witten, 1985

dimensional reduction from 10D theory

$$K = -3\ln(T + \bar{T} - 2|C|^2)$$

Untwisted matter fields (of orbifolds)

$$K \sim \frac{1}{T + \bar{T}}|C|^2$$

modular weight -1

Heterotic string on orbifolds

Dixon, Kaplunovsky, Louis, 1990

Ibanez, Lust, 1992

String amplitude computation by CFT

twisted matter fields on ZN orbifolds

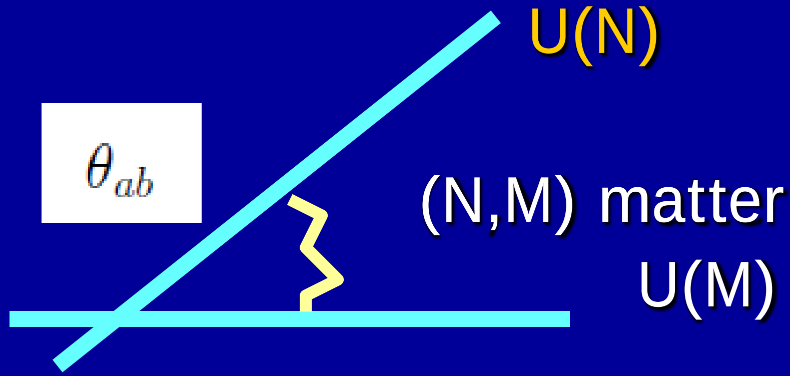
$$K = (T_i + \bar{T}_i)^{k^i} |C|^2$$

$$k^i = -(1 - \theta^i), \qquad \theta^i = \text{twist angle on } i - \text{plane}$$

weights decrease/increase in oscillators by one.

Intersecting D-brane models

matter fields



Modular weights depend on intersecting angles.

Kors, Nath, 03, Lust, Mayr, Richter, Stieberger, '04
Akerblom, Blumenhagen, Lust, Schmidt-Sommerfeld, '07
Blumenhagen, Schmidt-Sommerfeld, '07

Intersecting D-brane models

Akerblom,Blumenhagen,Lust,Schmidt-Sommerfeld,'07

Blumenhagnen,Schmidt-Sommerfeld, '07

Reasonable Ansatz from discussion on gauge threshold corrections

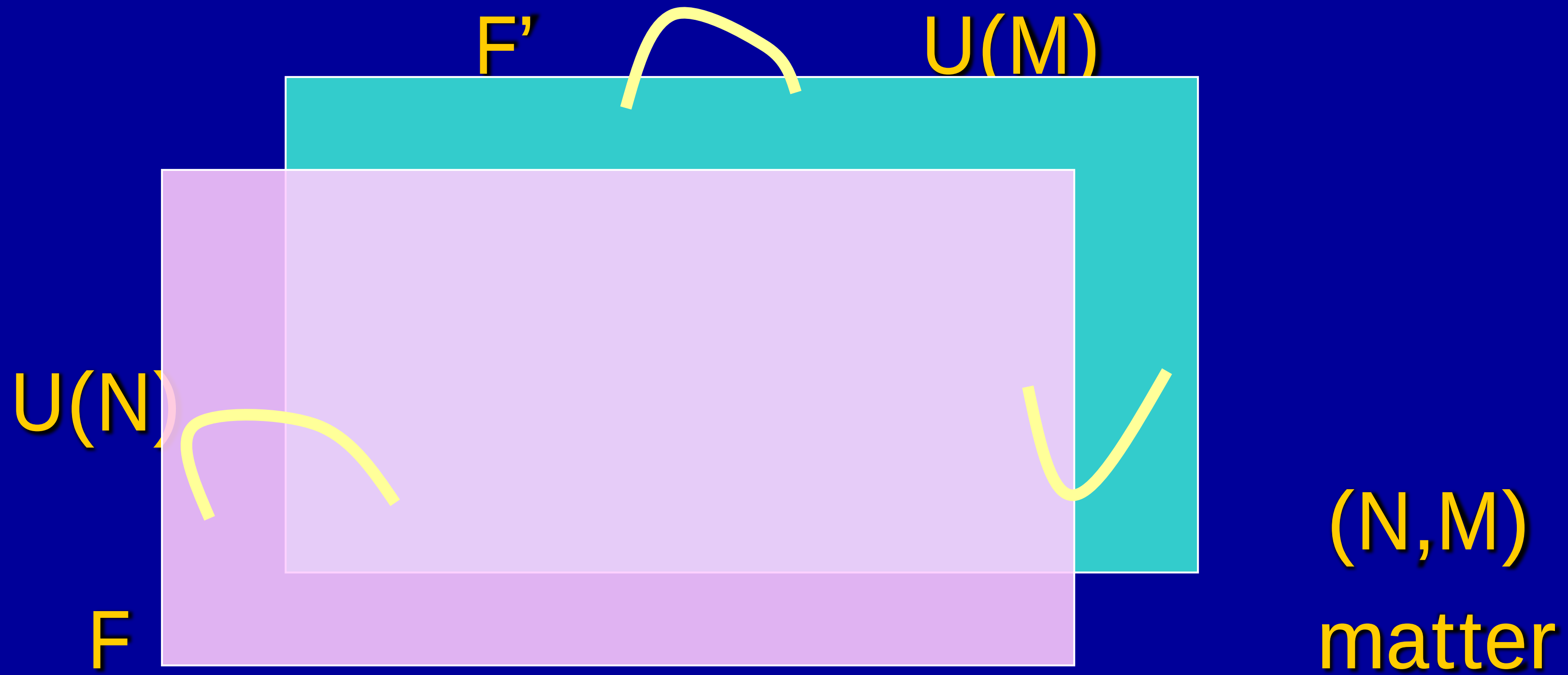
modular weights

$$k = -\frac{1}{2} \pm \frac{\theta_{ab}}{2}$$

Magnetized D-branes models

We consider D-brane models on
torus/orbifold compactifications

with magnetic flux background, F_{45}, etc.



Magnetized D-brane models

Magnetized D-brane models are T-dual of intersecting D-brane models.

Boundary conditions of open string

Intersecting D-brane models $\partial_\tau X^4 + \cot\theta\, \partial_\tau X^5 = 0$

Magnetized D-brane models $\partial_\sigma X^4 + \tilde{F}_5^4\, \partial_\tau X^5 = 0$

T-dual along 4-th direction $\cot\theta \leftrightarrow \tilde{F}_5^4$

Magnetized D-brane models

Magnetized D-brane models are T-dual of intersecting D-brane models.

We may have the same 4D LEEFT.

Its 4D LEFFT has been studied by dimensional reduction of higher dimensional super YM theory,

e.g. Yukawa couplings.

Cremades, Ibanez, Marchesano, '04

Magnetized D-brane models

We solve Dirac equation on T2
with magnetic background.

$$i\,\not{D} = i\Gamma^z\nabla_z + i\Gamma^{\bar{z}}\nabla_{\bar{z}} = i\begin{pmatrix} 0 & \frac{2}{e_1}\left(\partial - \frac{\pi M}{2\mathrm{Im}\tau}\bar{z}\right) \\ \frac{2}{\bar{e}_1}\left(\bar{\partial} + \frac{\pi M}{2\mathrm{Im}\tau}z\right) & 0 \end{pmatrix} \equiv i\begin{pmatrix} 0 & -D^\dagger \\ D & 0 \end{pmatrix}$$

$$\nabla_z = \partial - iA_z, \quad \nabla_{\bar{z}} = \bar{\partial} - iA_{\bar{z}}.$$

$$D\psi_+(z,\tau) = 0,$$
$$D^\dagger\psi_-(z,\tau) = 0.$$

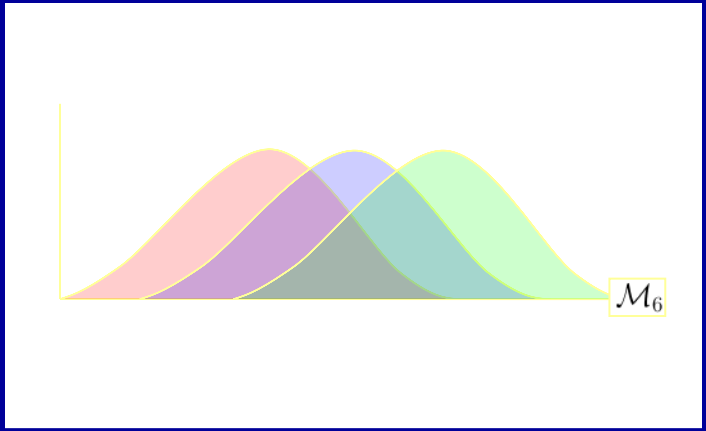
$$\psi_+^{j,M}(z,\tau) = \frac{M^{1/4}}{\mathcal{A}^{1/2}} e^{\pi i M z \frac{\mathrm{Im} z}{\mathrm{Im}\tau}}\ \vartheta\begin{bmatrix}\frac{j}{M}\\ 0\end{bmatrix}(Mz, M\tau).$$

Cremades, Ibanez, Marchesano, ‘04

Yukawa couplings

Yukawa couplings are obtained by
Overlap integral of wavefunctions.

$$Y_{ijk} = \int dy^{D-4} \psi_L^{i,M_1}(y) \psi_R^{j,M_2}(y) (\psi_H^{k,M_3}(y))^*$$



Cremades, Ibanez, Marchesano, '04

Normalization

We normalize wavefunctions such that Yukawa couplings are holomorhic functions of moduli.

We use the universal normalization for all of chiral matter fields.

$$\int d^2z \ \psi^{i,M}(z,\tau) \left(\psi^{j,M}(z,\tau)\right)^* = (2\mathrm{Im}\tau)^{-1/2}\delta_{i,j}.$$

wavefunction has modular weight 1/2

Kahler metric

Normalization

$$\int d^2z \, \psi^{i,M}(z,\tau) \left(\psi^{j,M}(z,\tau)\right)^* = (2\mathrm{Im}\tau)^{-1/2}\delta_{i,j}.$$

Dimensional reduction

$$\partial_{\mathcal{M}}\Phi^*\partial^{\mathcal{M}}\Phi,$$

$$\Phi(x,z) = \sum_j \phi_j(x)\psi^{j,M}(z,\tau) + \cdots,$$

$$K = K_{i\bar{i}}|\phi_i(x)|^2 = \frac{1}{(2\mathrm{Im}\tau)^{1/2}}|\phi_i(x)|^2.$$

modular weight $k = -\frac{1}{2}$ (opposite sign for wf.)

Similarly, we study models on orbifolds.

Weights of w.f. and 4D fields

Dimensional reduction

$$\Phi(x,z) = \sum_j \phi_j(x)\psi^{j,M}(z,\tau) + \cdots ,$$

Modular transformation is the change of w.f., i.e., the basis change of (KK)mode expansions.

The full field is invariant.

modular weights of w.f. in compact space and 4D matter fields have opposite signs.

Modular weight

Heterotic orbifold models (without oscillation)

$$k^i = -(1-\theta^i), \qquad \theta^i = \text{twist angle on } i-\text{plane}$$

Intersecting D-brane models

$$k = -\frac{1}{2} \pm \frac{\theta_{ab}}{2}$$

magnetized D-brane models (massless mode)

$$k = -\frac{1}{2}$$

We see difference.

Normalization

We may have ambiguity in normalization of wave functions.

We do not normalize them in a universal way but in a non-universal way

$$k = -\frac{1}{2}$$

$$k_i = -\frac{1}{2} + \Delta k_i, \quad k_j = -\frac{1}{2} + \Delta k_j, \quad k_k = -\frac{1}{2} + \Delta k_k,$$

However, we keep

$$\Delta k_i + \Delta k_j + \Delta k_k = 0,$$

$$K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}} = (2\mathrm{Im}\tau)^{-3/2},$$

such that the physical Yukawa couplings Y_{ijk} do not change.

Correction

$$k = -\frac{1}{2}$$

Also, we may have some corrections, which keeps

$$K_{i\bar{i}}K_{j\bar{j}}K_{k\bar{k}} = (2\mathrm{Im}\tau)^{-3/2},$$

and do not affect Yukawa Y_{ijk}

$$k_i = -\frac{1}{2} + \Delta k_i, \quad k_j = -\frac{1}{2} + \Delta k_j, \quad k_k = -\frac{1}{2} + \Delta k_k,$$

$$\Delta k_i + \Delta k_j + \Delta k_k = 0,$$

4D LEEFT is the same (except SUSY breaking).

That suggests the corrections are proportional to some charges of symmetries.

Intersecting D-brane models

modular weights

$$K = (T + \bar{T})^{k_{ab}} |\phi_{ab}|^2, \qquad k_{ab} = -\frac{1}{2} \pm \text{sign}(I_{ab}) \frac{\theta_{ab}}{2}.$$

Coupling selection rule of 3-point couplings

$$\text{sign}(I_{ab})\theta_{ab} + \text{sign}(I_{bc})\theta_{bc} + \text{sign}(I_{ca})\theta_{ca} = 0,$$

$$\Delta k_i + \Delta k_j + \Delta k_k = 0,$$

Magnetized D-brane models

We study localized flux = FI term
at fixed points of orbifold in magnetized
D-brane models.

Even if no such flux or FI term,
that may be induced by loop effects.

Lee, Nilles, Zucker, '03

Magnetized D-brane models

Singular gauge transformation

Dolan, Hunter-McCabe, '01,

Buchmuller, Dierigl, Ruehle, Schweizer, '15,

Buchmuller, Dierigl, Tatsuta, '18.

$$A \to A + \delta A, \quad \delta A = i U_{\xi^F} d U_{\xi^F}^{-1},$$

$$U_{\xi^F} = g(z)^{\xi^F/2} \overline{g}(z)^{-\xi^F/2}, \quad g(z) = e^{\frac{\pi}{2\mathrm{Im}\tau} z^2} \vartheta \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} (z, \tau).$$

At z=0

$$\delta A \simeq -i \frac{1}{2} \frac{\xi^F}{z} dz + i \frac{1}{2} \frac{\xi^F}{\bar{z}} d\bar{z}.$$

$$\frac{\delta F}{2\pi} = i \xi^F \delta(z) \delta(\bar{z}) dz \wedge d\bar{z},$$

Magnetized D-brane models

Singular gauge transformation

$$A \to A + \delta A, \quad \delta A = iU_{\xi^F} dU_{\xi^F}^{-1},$$

$$\frac{\delta F}{2\pi} = i\xi^F \delta(z)\delta(\bar{z}) dz \wedge d\bar{z},$$

induces or removes localized flux

wavefunction

$$\tilde{\psi}(z,\tau) = U_{\xi^F}^q \psi(z,\tau) = g(z)^{q\xi^F/2} \bar{g}(z)^{-q\xi^F/2} \psi(z,\tau)$$

the boundary condition around z=0 and modular transformation behavior change.

Magnetized D-brane models

Boundary condition around z=0 and
Modular transformation behavior change.

$$\tilde{\psi}(z,\tau) = U^q_{\xi^F}\psi(z,\tau) = g(z)^{q\xi^F/2}\bar{g}(z)^{-q\xi^F/2}\psi(z,\tau)$$

$$g(z) \to (-\tau)^{1/2}e^{3\pi i/4}g(z),$$

$$\Delta k = q\xi^F$$

We normalize

$$\int d^2z\, \tilde{\psi}^{i,M}_m(z,\tau)\left(\tilde{\psi}^{j,M}_m(z,\tau)\right)^* = (2\mathrm{Im}\tau)^{-1/2-\Delta k}\delta_{i,j},$$

Magnetized D-brane models

S-transformation of new wavefunction

$$\tilde{\psi}^{i,M}(z,\tau) \to (-\tau)^{1/2+\Delta k} \sum_j \tilde{\rho}(S)_{ij} \tilde{\psi}^{j,M}(z,\tau),$$

For 3-point couplings

$$\Delta k_i = q_i \frac{\xi^F}{2}, \quad \Delta k_j = q_j \frac{\xi^F}{2}, \quad \Delta k_k = q_k \frac{\xi^F}{2}.$$

gauge invariance

$$q_i + q_j + q_k = 0,$$

$$\Delta k_i + \Delta k_j + \Delta k_k = 0,$$

Magnetized D-brane models

Localized flux is one of possible corrections
on modular weights in magnetized D-brane models

correspondence between
magnetized models and intersecting models

$$\Delta k_i = q_i \frac{\xi^F}{2} \to \pm \mathrm{sign}(I_{ab}) \frac{\theta_{ab}}{2}.$$

Higher modes in Het. orbifold

Twisted ground states

$$k^i = -(1-\theta^i), \qquad \theta^i = \text{twist angle on } i-\text{plane}$$

Oscillator decreases modular weights by one.

$$\alpha_{-1}$$

We have a similar behavior in intersecting D-brane models, because its CFT behavior is very similar to heterotic string on orbifolds.

Higher modes in Het. orbifold

Twisted ground states

$$k^i = -(1-\theta^i), \qquad \theta^i = \text{twist angle on } i-\text{plane}$$

Oscillator decreases modular weights by one.

$$\alpha_{-1}$$

Is there a similar behavior in magnetized D-brane models ?

Yes.

Magnetized D-brane models

We solve Dirac equation on T2
with magnetic background.

$$i\,\not{D} = i\Gamma^z\nabla_z + i\Gamma^{\bar{z}}\nabla_{\bar{z}} = i\begin{pmatrix} 0 & \frac{2}{e_1}\left(\partial - \frac{\pi M}{2\mathrm{Im}\tau}\bar{z}\right) \\ \frac{2}{\bar{e}_1}\left(\bar{\partial} + \frac{\pi M}{2\mathrm{Im}\tau}z\right) & 0 \end{pmatrix} \equiv i\begin{pmatrix} 0 & -D^\dagger \\ D & 0 \end{pmatrix}$$

$$\nabla_z = \partial - iA_z, \quad \nabla_{\bar{z}} = \bar{\partial} - iA_{\bar{z}}.$$

$$i\,\not{D}\Psi_n(z,\tau) = m_n\Psi_n(z,\tau),$$

Cremades, Ibanez, Marchesano, '04

Magnetized D-brane models

Massive modes

$$(i \not{D})^2 \Psi_n = \begin{pmatrix} D^\dagger D & 0 \\ 0 & D D^\dagger \end{pmatrix} \begin{pmatrix} \psi_{+,n} \\ \psi_{-,n} \end{pmatrix} = m_n^2 \begin{pmatrix} \psi_{+,n} \\ \psi_{-,n} \end{pmatrix} .$$

$$\Delta = \frac{1}{2}\{D, D^\dagger\}.$$

$$\Delta = D^\dagger D + \frac{2\pi M}{\mathcal{A}}, \quad [D, D^\dagger] = \frac{4\pi M}{\mathcal{A}},$$

$$[\Delta, D^\dagger] = \frac{4\pi M}{\mathcal{A}} D^\dagger, \quad [\Delta, D] = -\frac{4\pi M}{\mathcal{A}} D.$$

$$a^\dagger = \sqrt{\frac{\mathcal{A}}{4\pi|M|}} D^\dagger, \quad a = \sqrt{\frac{\mathcal{A}}{4\pi|M|}} D,$$

looks like harmonic oscillator

Cremades, Ibanez, Marchesano, '04

Magnetized D-brane models

Massive modes

$$\psi_{n,+}^{j,M} \propto (\nabla_z)^n \psi_+^{j,M},$$

Eigenvalues of $\Delta = \frac{1}{2}\{D, D^\dagger\}.$

$$\lambda_n = \frac{2\pi|M|}{A}(2n+1).$$

mass spectrum of spinor $m_n^2 = \frac{4\pi|M|}{A}n.$

Magnetized D-brane models

We solve Dirac equation on T2
with magnetic background.

$$\psi_{n,+}^{j,M} \propto (\nabla_z)^n \psi_+^{j,M},$$

$$\nabla_z \to (-\tau)\nabla_z,$$

Modular weight of w.f. increases by one

modular weight of higher mode wavefunction

$$k^{(n)} = \frac{\mathcal{A}}{4\pi M}\lambda_n = n + \frac{1}{2},$$

that is nothing but energy spectrum of Laplacian=harmonic oscillator.

½ is the zero-point energy.

Heterotic orbifold models

Oscillator $i\partial X$ corresponding to α_{-1}

decrease the modular weight by one,

and increase the conformal dimension by one.

Correspondence between
conformal dimension and modular weight
in heterotic string,
and Laplacian spectrum and modular weight
in magnetized D-brane models.

Correspondence between modes of heterotic string
coordinates and KK modes
in magnetized D-brane models

OSp(1|2) in magnetized models

Subalgebra of super-Virasoro algebra

$$\ell_0 = \frac{\mathcal{A}}{8\pi M}\Delta, \qquad g_{-1/2} = \frac{1}{\sqrt{2}}\nabla_z, \qquad g_{1/2} = \frac{1}{\sqrt{2}}\nabla_{\bar{z}}, \qquad \ell_{\pm 1} = (g_{\pm 1/2})^2.$$

$$[\ell_m, \ell_n] = (m-n)\ell_{m+n}, \qquad [\ell_m, g_r] = \left(\frac{m}{2} - r\right) g_r, \qquad \{g_r, g_s\} = 2\ell_{r+s},$$

Osp(1|2) algebra

Correction or redefinition

$$k = -\frac{1}{2}$$

We may redefine modular weights
in non-universal way or have some corrections.

We keep $K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}} = (2\mathrm{Im}\tau)^{-3/2},$,
which is relevant to Yukawa

$$k_i = -\frac{1}{2} + \Delta k_i, \quad k_j = -\frac{1}{2} + \Delta k_j, \quad k_k = -\frac{1}{2} + \Delta k_k,$$

$$\Delta k_i + \Delta k_j + \Delta k_k = 0,$$

4D LEEFT is the same (except SUSY breaking).
That suggests the corrections are proportional
to some charges of symmetries.

SUSY breaking terms

Moduli-dominant SUSY breaking

$$m_i^2 = m_{3/2}^2 + k_i \frac{|F^U|^2}{(U+\bar{U})^2},$$

soft scalar mass depends on modular weight.

$$\begin{aligned} A_{ijk} &= A_{ijk}^0 + A'_{ijk}, \\ A_{ijk}^0 &= (k_i + k_j + k_k - 1)\frac{F^U}{(U+\bar{U})}, \qquad A'_{ijk} = -\frac{F^U}{Y_{ijk}}\frac{dY_{ijk}}{dU}. \end{aligned}$$

A-term depends only the sum.

$$m_i^2 + m_j^2 + m_k^2 = 3m_{3/2}^2 + (k_i + k_j + k_k)\frac{|F^U|^2}{(U+\bar{U})^2} = 3m_{3/2}^2 - \frac{3}{2}\frac{|F^U|^2}{(U+\bar{U})^2}.$$

The sum of soft masses also depends only the sum.

SUSY breaking terms

Moduli-dominant SUSY breaking

$$m_i^2 = m_{3/2}^2 + k_i \frac{|F^U|^2}{(U+\bar{U})^2} ,$$

soft scalar mass depends on modular weight.

Is this point important or not ?

Summary

We have studied the modular weight
in magnetized D-brane models.

They have corrections due to localized flux.
Its behavior is similar to intersecting D-brane models.

Modular weights of massive modes shift by one.
This is similar to oscillated modes of heterotic string.

Magnetized D-brane modes have
Osp(1|2) algebra, subalgebra of super-Virasoro.

Summary

Magnetized D-brane modes have
Osp(1|2) algebra, subalgebra of super-Virasoro.

We have correspondence between conformal dimension (modular weights) in het. String and Laplacian (harmonic oscillator) spectrum in magnetized D-brane model.

Its deep reason is not clear.
We just study effective field theory, but
Some aspects of CFT appears.
It would be important to study more.