Infinite distance limits with defects

work to appear with Rafael Alvarez-García and Seung-Joo Lee

Timo Weigand, String Phenomenology 2023, IBS Daejeon

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Infinite Distance Limits

[Ooguri,Vafa'06]

Asymptotic Tower Weak Gravity Conjecture

motivates

asymptotic version of de Sitter conjecture

implies

Swampland Distance Conjecture:

∃ tower: $m_P(\phi) = m_O \, e^{-c \, d_{0,P}}$

inspires

AdS Distance Conjecture

is dual to

CFT Distance Conjecture

contrains

Cosmological field excursions

Emergent String Conjecture

Lee, Lerche,TW'19

All infinite distance limits in Quantum Gravity are either

✳ *decompactification limits or*

✳ *emergent string limits (weakly coupled fundamental string limits if c.c. is zero)*

➡ Makes strong claims about the nature of the asymptotic theory.

➡ If correct, it is very constraining.

Emergent String Conjecture

Lee, Lerche,TW'19

All infinite distance limits in Quantum Gravity are either

✳ *decompactification limits or*

✳ *emergent string limits (weakly coupled fundamental string limits if c.c. is zero)*

- bounds on exponential decay rates

Etheredge, Heidenreich,Kaya,Qiu,Rudelius'22

Gendler Valenzuela'20

- information on species scale of limit: $\Lambda_{\text{sp,KK}}$ vs $\Lambda_{\text{sp,string}}$

Heisteeg,Vafa,Wiesner,Wu'22/23
Cribiori, Lüst,Staudt'22
Cribiori,Lüst,Montella'23
Blumenhagen,Gligovic,Paraskevopoulou'23
Castellano,Herraez,Ibanez'22
Marchesano,Melotti'22

- nature of asymptotically weakly coupled gauge groups and WGC towers

Lee,Lerche,TW'18-20
Cota,Mininno,Wiesner,TW'22

see talk by Alessandro Mininno

ESC: Potential Caveats

1. **Stringy uniqueness:** Could there not be competing higher-dimensional objects?

[Alvarez-García,Kläwer,TW'21]

- *emergent membrane limits* excluded by consistency under dimensional reduction
- ensured explicitly in string/M-theory

2. **What is the nature of the asymptotic KK theories?**

Message of this talk: Have to allow for limits breaking higher dim. Lorentz invariance ***(defects)***

cf different setup: Etheredge,Heidenreich,McNamara,Rudelius,Ruiz,Valenzuela'23

Open String Limits in F-theory

Consider certain **complex structure degenerations in F-theory**

Interpretation as **(non-perturbative) open string moduli limits** (possibly along with dilaton/other complex structure moduli)

Goal:

Understanding of the geometry and the asymptotic physics of the infinite distance limit

Complementary to analysis of complex structure analysis via asymptotic Hodge theory as initiated in *[Grimm,Palti,Valenzuela'18]*

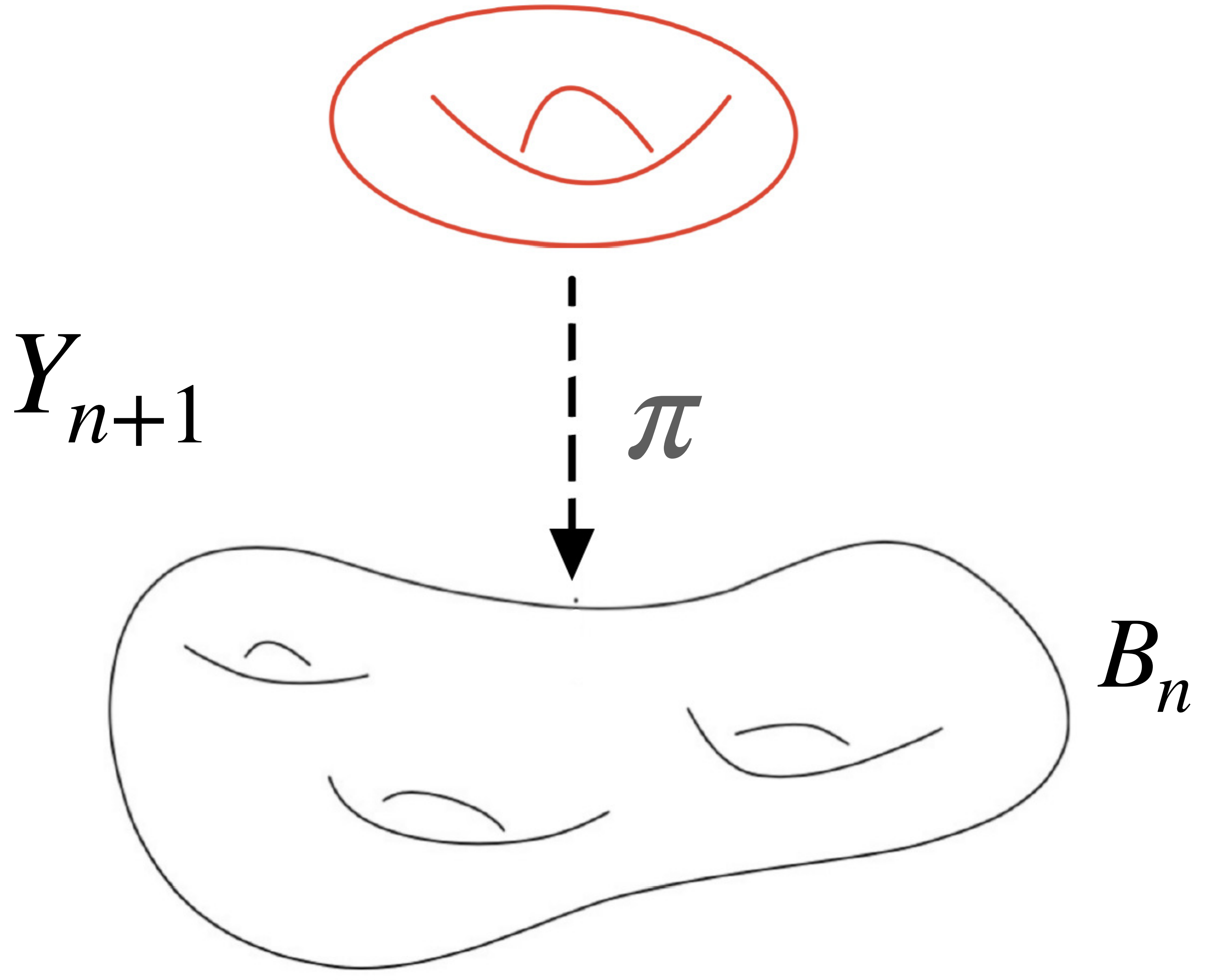
F-theory - Reminder

- defined for elliptic fibration Y_{n+1}
- **Weierstrass model**

$$y^2 = x^3 + f(w_i)\, x\, z^4 + g(w_i)\, z^6$$

local coordinates on B_n

$[x : y : z]$: fiber



- position of **7-branes**

$\longleftrightarrow$ vanishing of discriminant $\Delta = 4f^3 + 27g^2$

- **non-minimal singularities** require special treatment

Algebra	Kodaira-type	ord(f)	ord(g)	ord(Δ)
A_n	I_{n+1}	0	0	n+1
D_n	I^*_{n-4}	2	3	n+2
E_6	IV*	≥ 3	4	8
E_7	III*	3	≥ 5	9
E_8	II*	≥ 4	5	10
	non-min	≥ 4	≥ 6	≥ 12

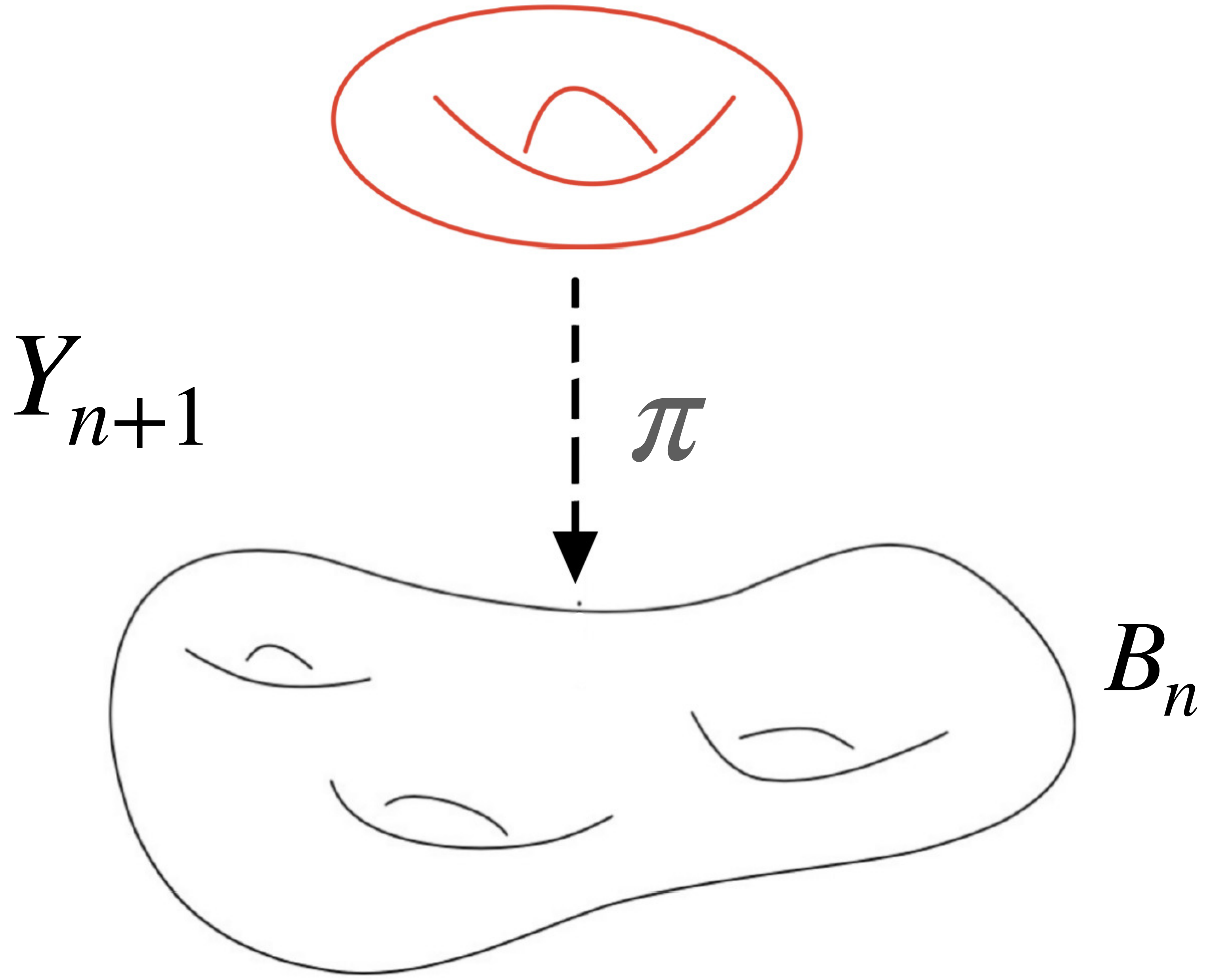
F-theory - Reminder

- defined for elliptic fibration Y_{n+1}
- **Weierstrass model**

$$y^2 = x^3 + f(w_i)\, x\, z^4 + g(w_i)\, z^6$$

local coordinates on B_n

$[x : y : z]$: fiber



Codimension-one singularities with

$$\mathrm{ord}(f, g, \Delta) = (4+\alpha, 6+\beta, 12+\gamma)\,, \qquad \alpha, \beta, \gamma \geq 0$$

potentially at ∞ distance in complex structure

Algebra	Kodaira-type	ord(f)	ord(g)	ord(Δ)
A_n	I_{n+1}	0	0	n+1
D_n	I^*_{n-4}	2	3	n+2
E_6	IV*	≥ 3	4	8
E_7	III*	3	≥ 5	9
E_8	II*	≥ 4	5	10
	non-min	≥ 4	≥ 6	≥ 12

F-theory - Non-minimal singularities

Weierstrass model

$$y^2 = x^3 + f(w_i)\,x\,z^4 + g(w_i)\,z^6$$

$[x : y : z]$: fiber

A_n	I_{n+1}	0	0	n+1
D_n	I^*_{n-4}	2	3	n+2
E_6	IV*	≥ 3	4	8
E_7	III*	3	≥ 5	9
E_8	II*	≥ 4	5	10
	non-min	≥ 4	≥ 6	≥ 12

Codimension-one singularities with

$$\mathrm{ord}(f, g, \Delta) = (4 + \alpha, 6 + \beta, 12 + \gamma)\,, \qquad \alpha, \beta, \gamma \geq 0$$

Analysis has **two parts:**

1) associated with **fibration** as such:

➡ present already for F-theory on K3 *[Lee,TW'21; Lee,Lerche,TW'21]*

2) different effects **dependent on base** B_n

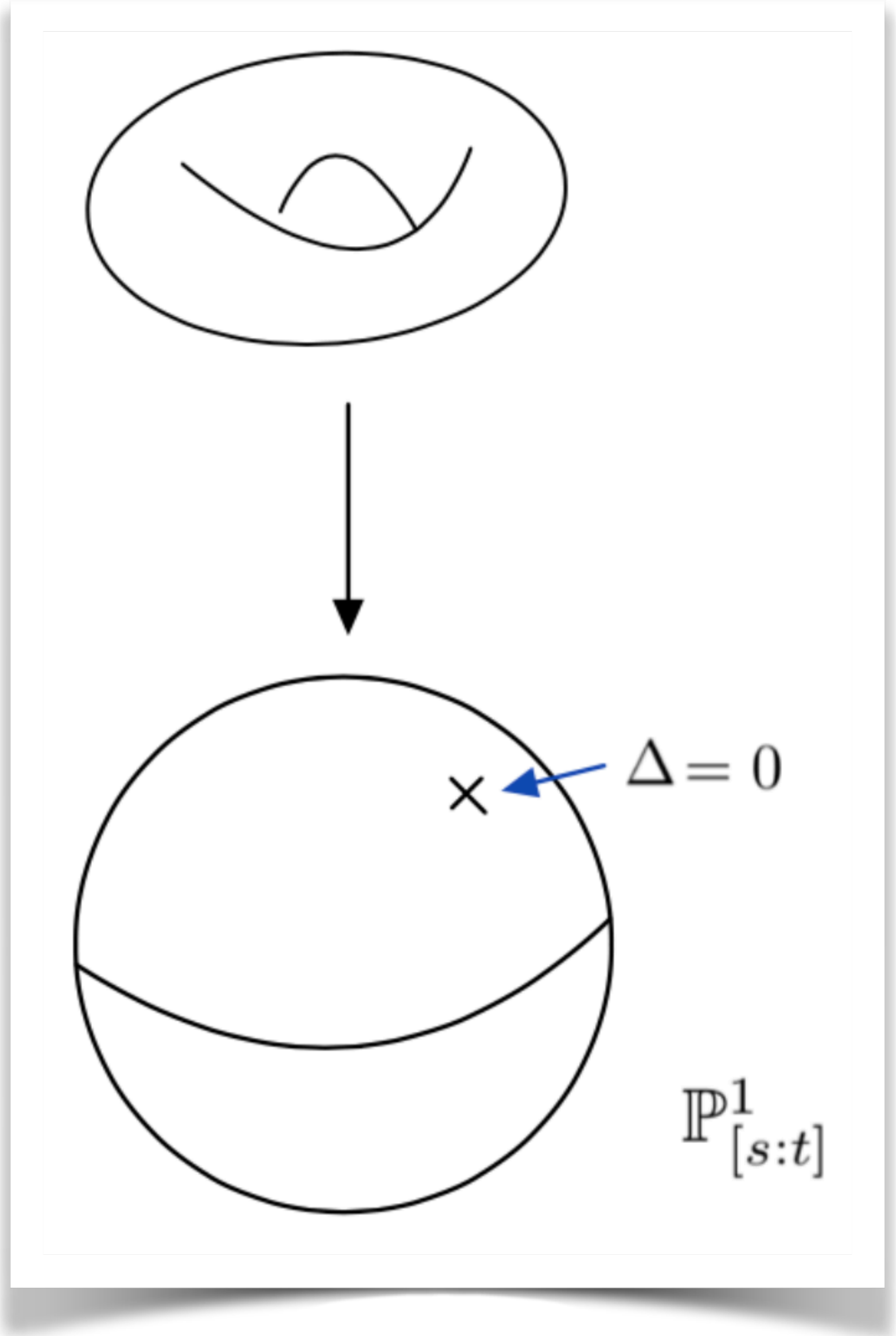
➡ start playing a role on elliptic Calabi-Yau 3-folds *[Alvarez-García,Lee,TW to appear]*

F-theory on K3: Beyond Kodaira

Explicit analysis for elliptic K3 surfaces (F-theory in 8d)

$$y^2 = x^3 + f_u(s,t)\, x\, z^4 + g_u(s,t)\, z^6 \qquad u \in \mathbb{C}$$

$$\mathrm{ord}(f, g, \Delta)\,|_{u=s=0} = (4+\alpha, 6+\beta, 12+\gamma)$$



[Lee,TW'21; Lee,Lerche,TW'21]

$\alpha = 0$ or $\beta = 0$:

Infinite distance: **Kulikov models** of Type II/III

$\alpha > 0$ and $\beta > 0$:

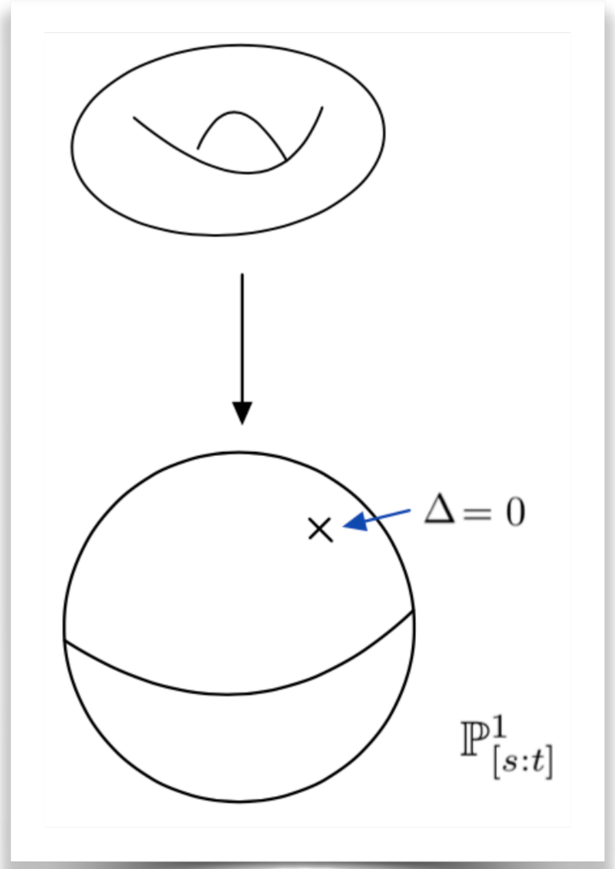
Infinite distance (II/III) or finite distance (I)

F-theory on K3: Beyond Kodaira

Explicit analysis for elliptic K3 surfaces (F-theory in 8d)

$$y^2 = x^3 + f_u(s,t)\, x\, z^4 + g_u(s,t)\, z^6 \qquad u \in \mathbb{C}$$

$$\mathrm{ord}(f, g, \Delta)\,|_{u=s=0} = (4+\alpha, 6+\beta, 12+\gamma)$$



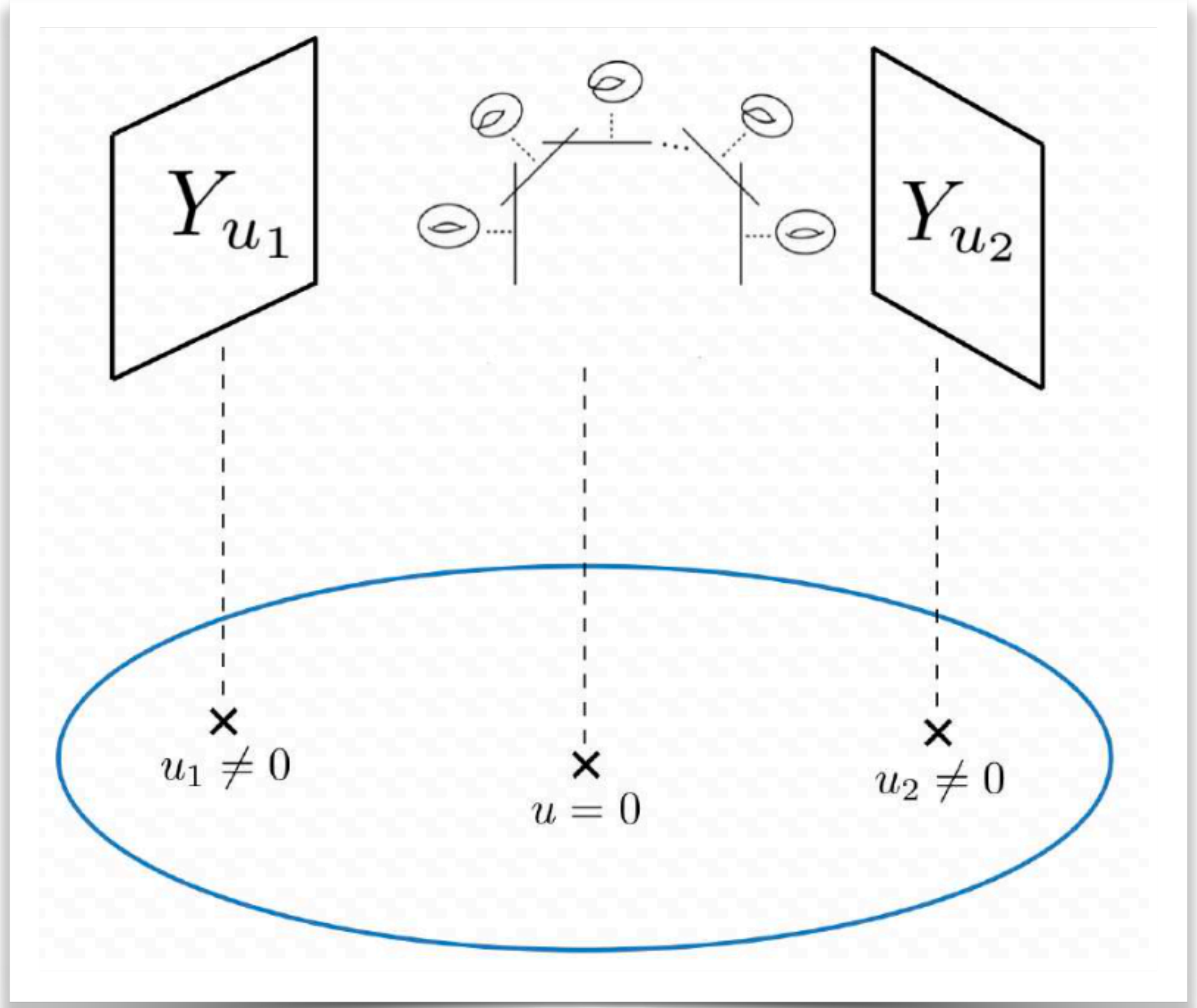
[Lee,TW'21; Lee,Lerche,TW'21]

Perform a **sequence of blowups in the base**:

$$u = s = 0 : (u, s) \to (e_0 e_1, s e_1)\,, \quad \ldots$$

After P steps: $Y_0 = \cup_p Y_0^p$ $\qquad$ Y_0^p non-CY component

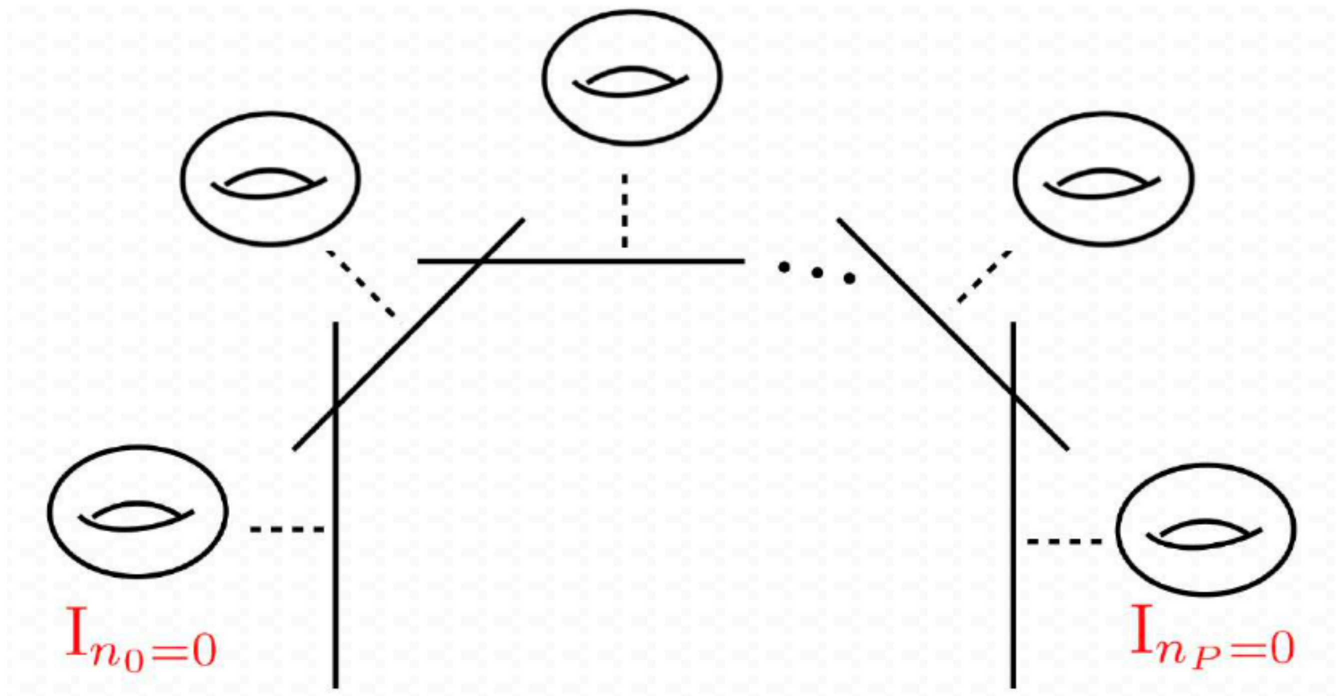
Y_0^p is elliptically fibered over base $B^p : \{e_p = 0\}$

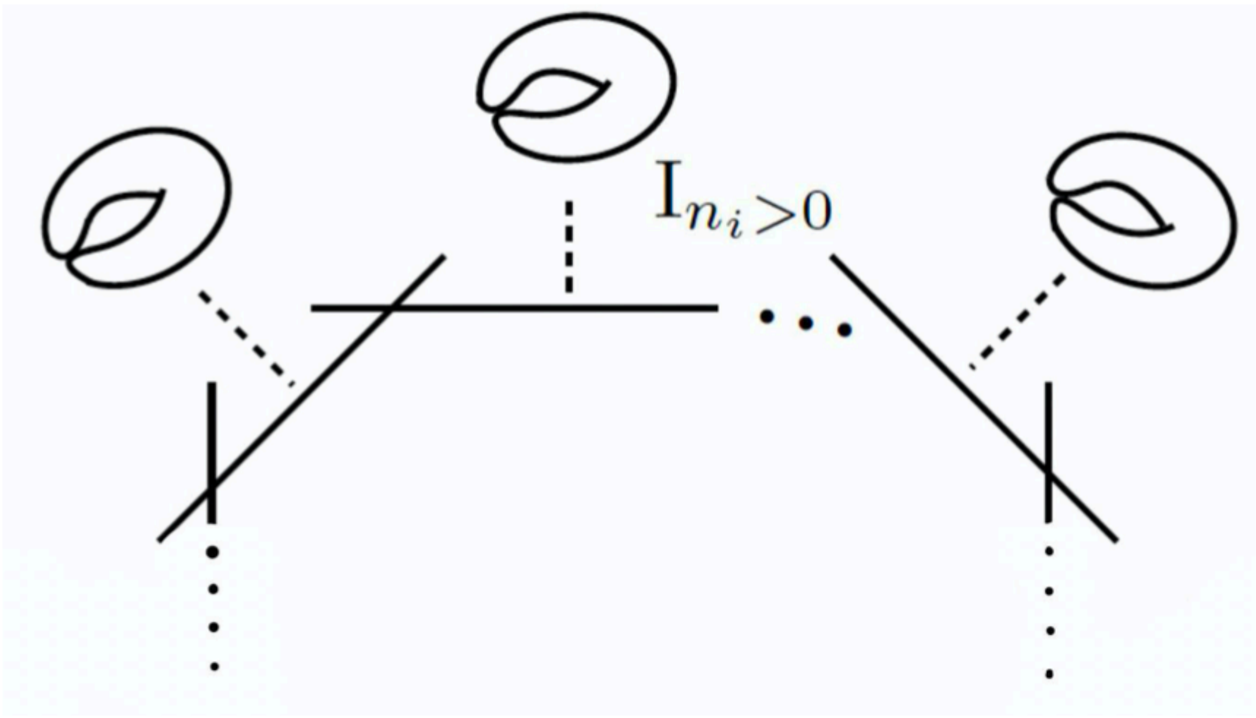


F-theory on K3: Beyond Kodaira

$y^2 = x^3 + f_u(s,t)\, x\, z^4 + g_u(s,t)\, z^6$

$\mathrm{ord}(f, g, \Delta)\,|_{u=s=0} = (4+\alpha, 6+\beta, 12+\gamma)$





$\gamma = 0$: **II.a**

- middle components trivial I_0
- end components dP_9 with 12 branes

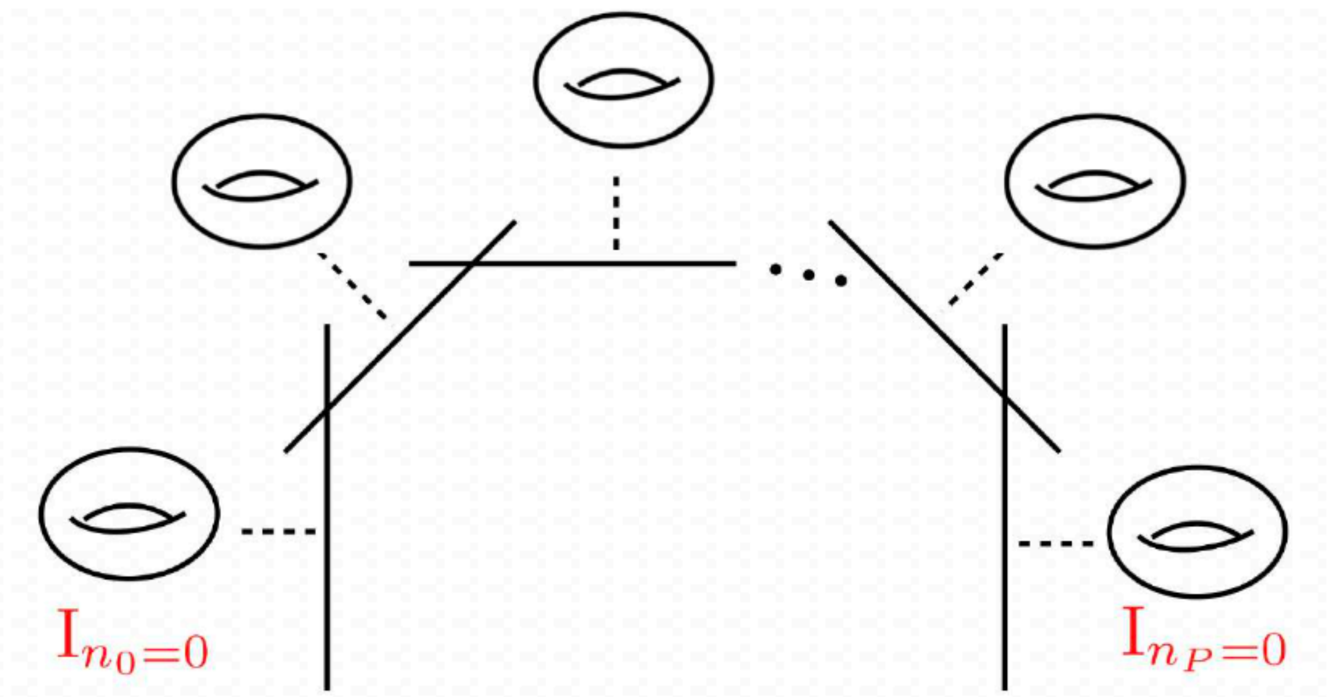
$\gamma > 0,\ \alpha = 0 = \beta$: **III.a/b**

- middle components $I_{n_i>0}$: local weak coupling
- III.a: at least one end I_0 — III.b: both ends $I_{n_{0/P}>0}$: global weak coupling
- patterns of brane on components understood

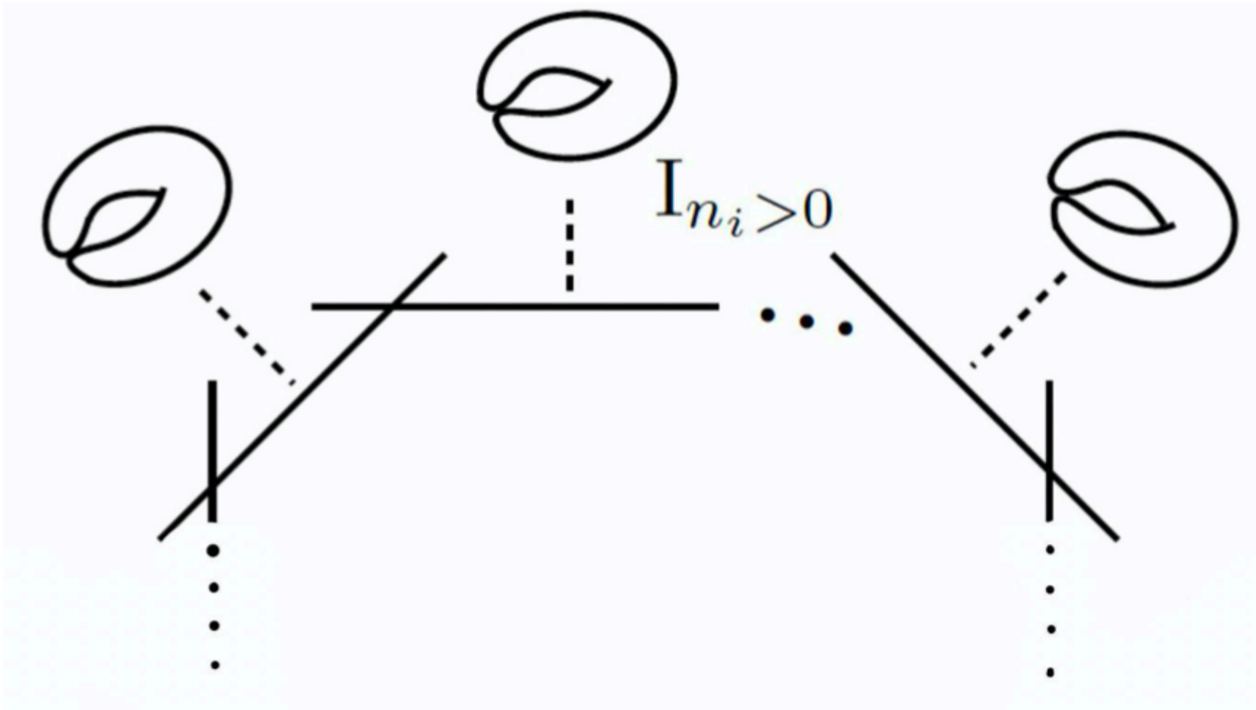
F-theory on K3: Beyond Kodaira

$$y^2 = x^3 + f_u(s,t)\, x\, z^4 + g_u(s,t)\, z^6$$

$$\mathrm{ord}(f, g, \Delta)\,|_{u=s=0} = (4+\alpha, 6+\beta, 12+\gamma)$$



$\gamma = 0$: **II.a**



$\gamma > 0,\ \alpha = 0 = \beta$: **III.a/b**

Can be brought into

- either: Kodaira type (finite distance)
- or: into form II.a or III.a/b (infinite distance)

$\gamma > 0,\ \alpha > 0,\ \beta > 0$: **I or II/III**

F-theory on K3: Beyond Kodaira

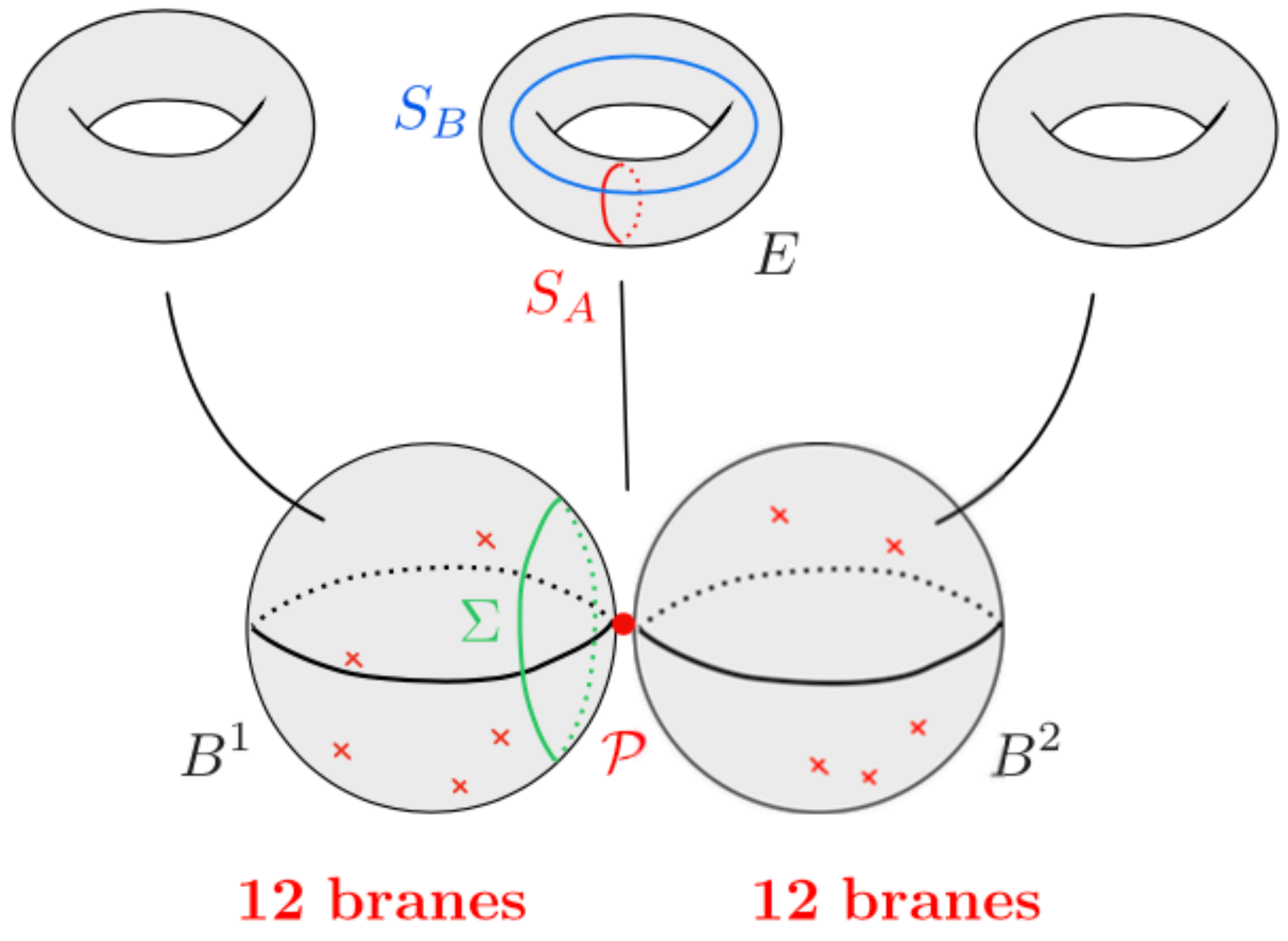
Type II.a

2 vanishing T^2 $\quad \gamma_1 = S_A \times \Sigma\,, \quad \gamma_2 = S_B \times \Sigma$

$\Longrightarrow$ 2 particle towers:

- M2-branes on *two* 2-tori γ_i
- Interpretation as 2 KK towers

$\Longrightarrow$ decompactification $8d \to 10d$



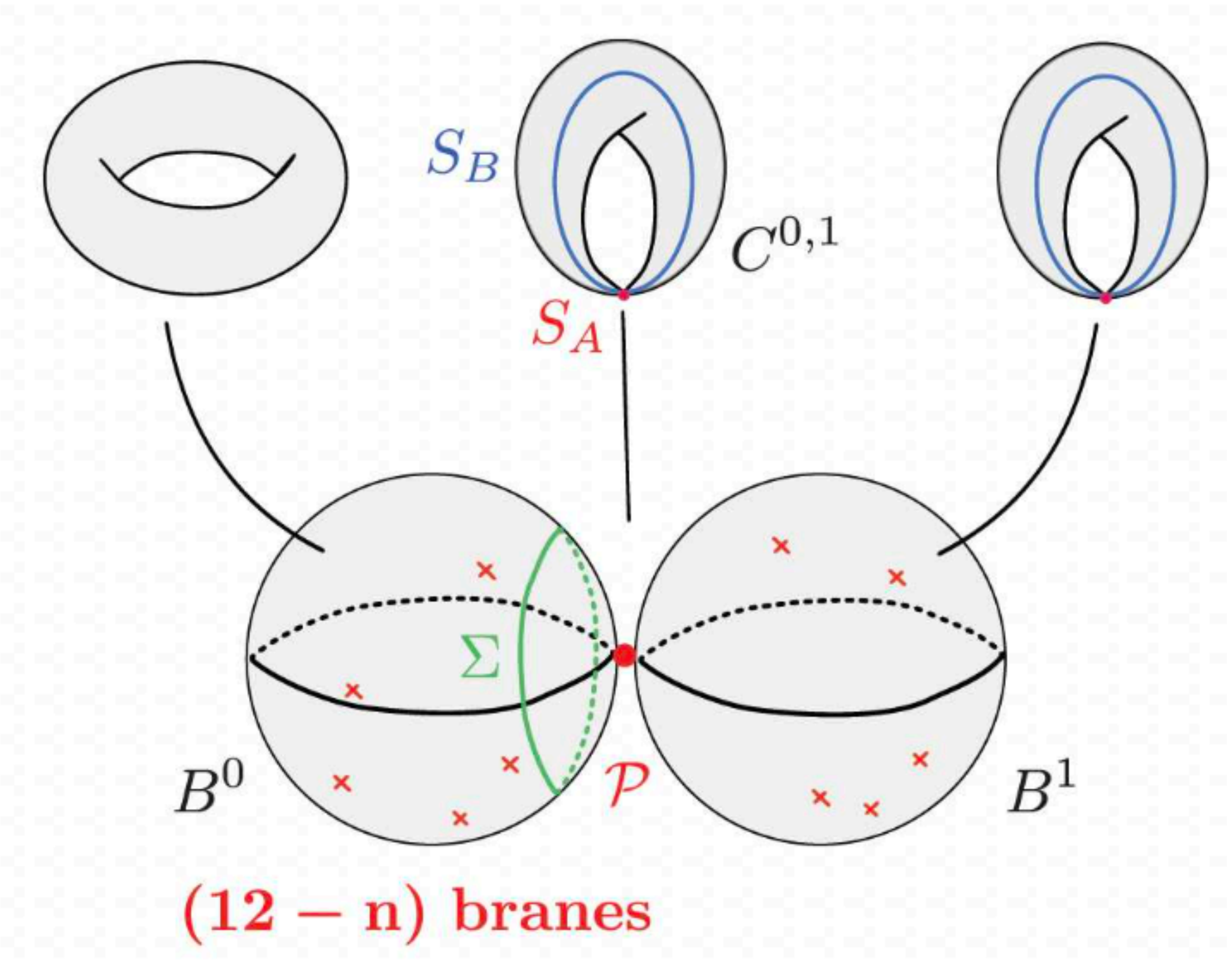
Type III.a

1 vanishing T^2 $\quad \gamma_1 = S_A \times \Sigma$

$\Longrightarrow$ 1 particle tower:

- M2-branes on *single* 2-torus γ_1
- Interpretation as 1 KK tower

$\Longrightarrow$ partial decompactification $8d \to 9d$



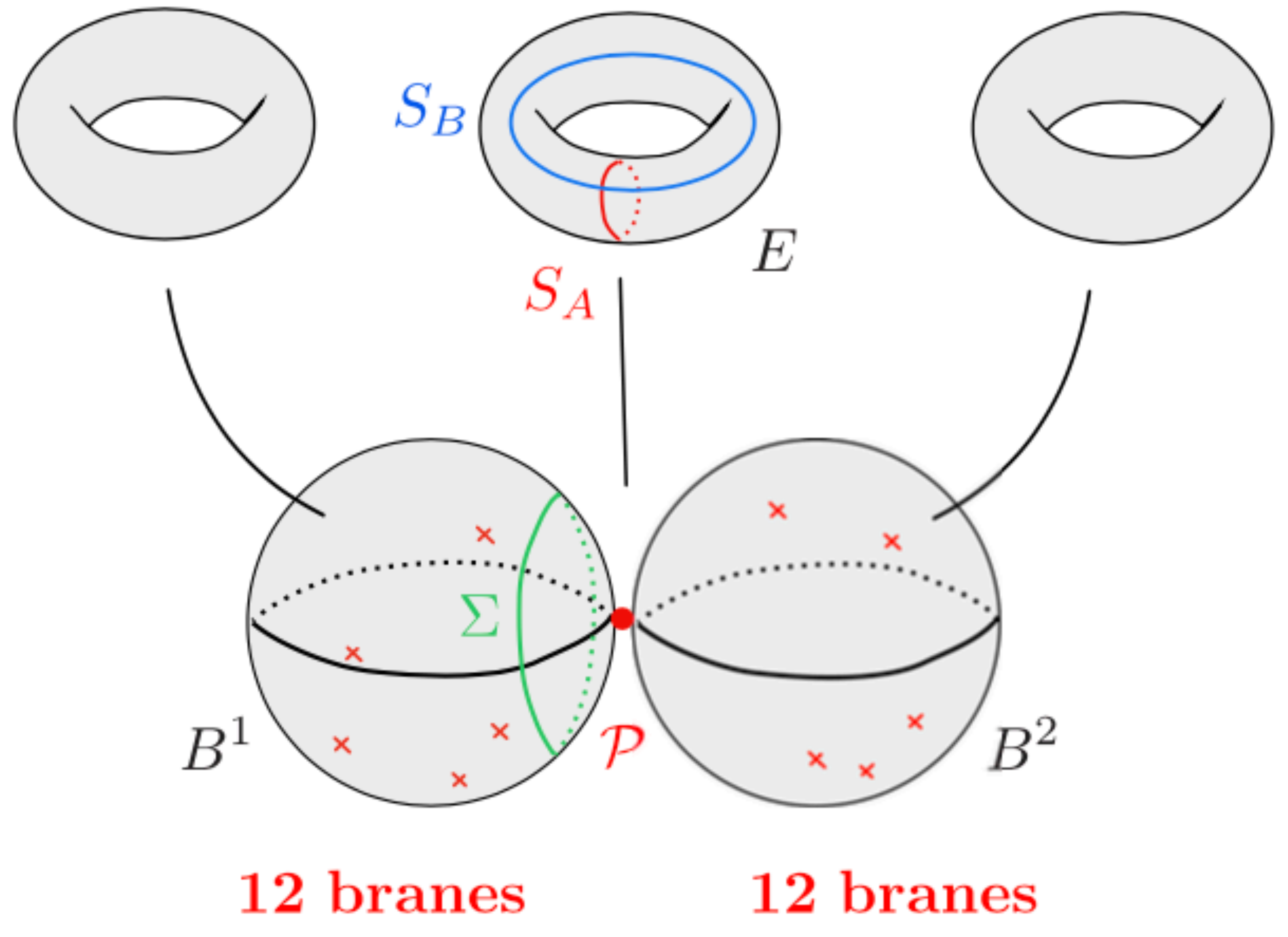
Non-Kodaira on K3: Heterotic dual

Type II.a

Stable degeneration limit:

heterotic $\mathrm{vol}(T^2_{\mathrm{het}}) \to \infty$

asymptotic gauge algebra $E_8 \times E_8$ in 10d



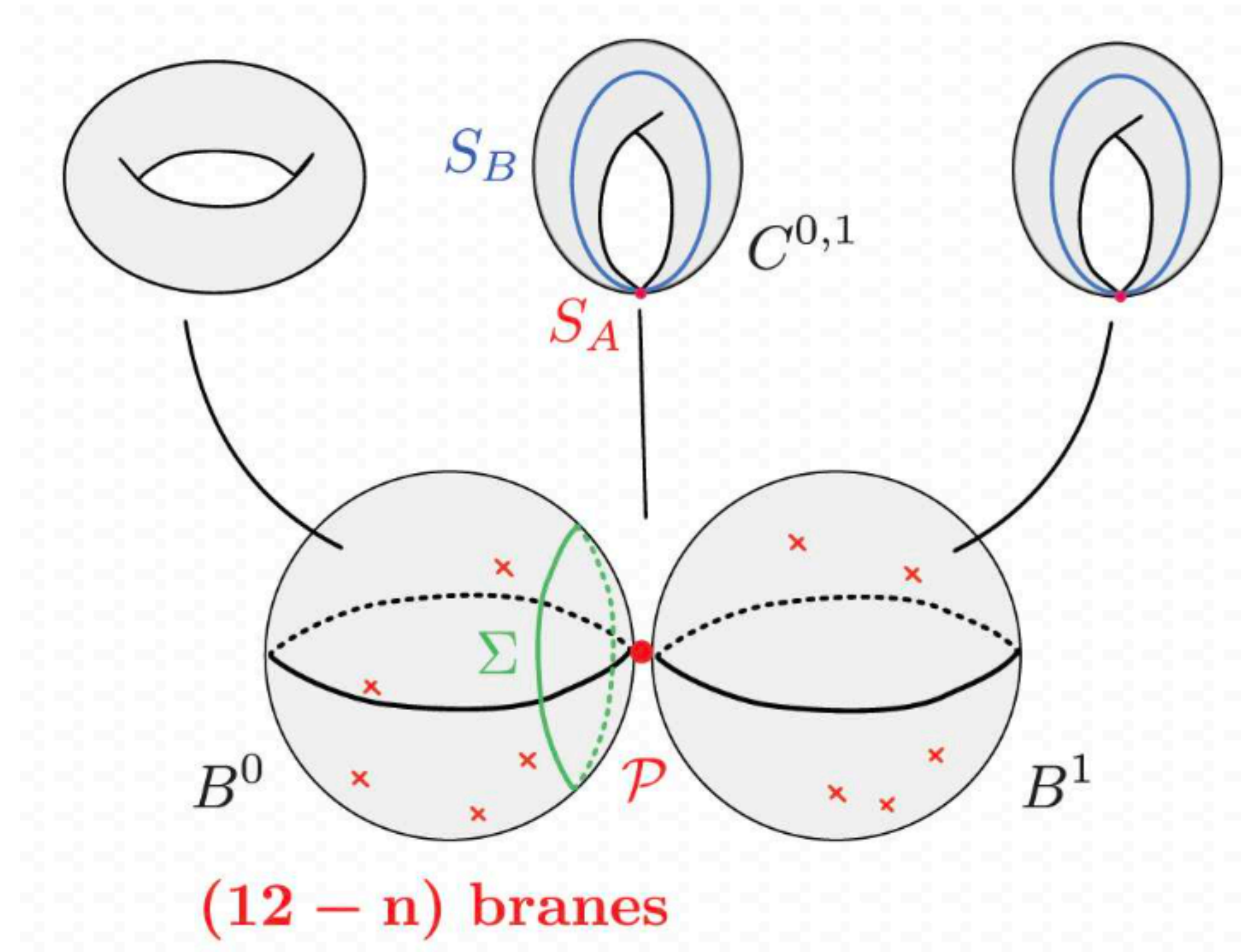
Type III.a

Heterotic:

$\mathrm{vol}(T^2_{\mathrm{het}}) \to \infty$ and $\mathrm{U}(T^2_{\mathrm{het}}) \to \infty$

asymptotic gauge algebra in 9d classified and in agreement with het. analysis

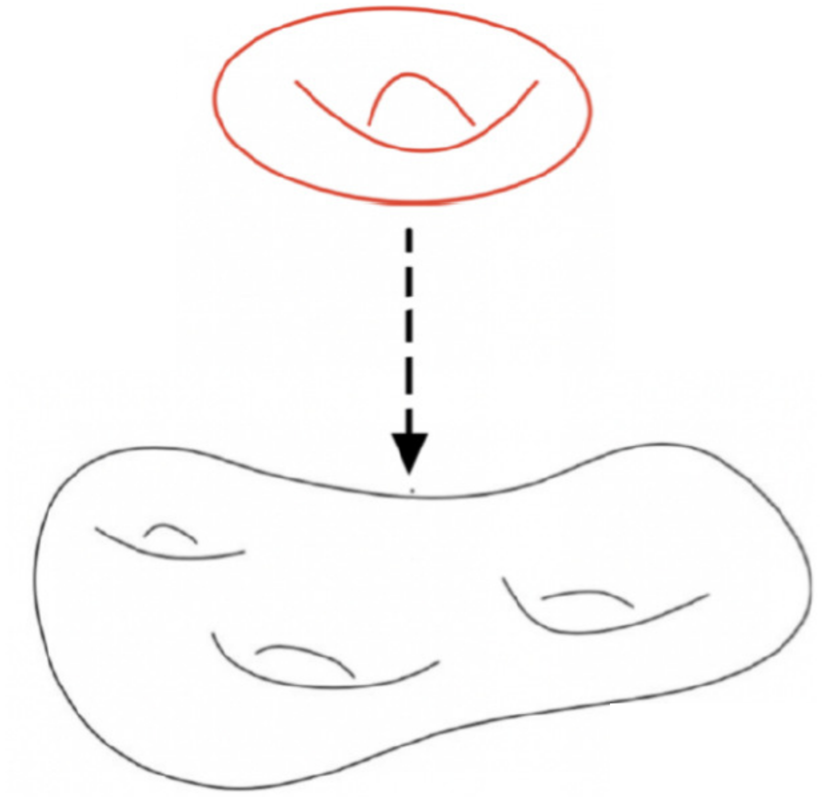
[Cachazo,Vafa'00] [Font,Fraiman,Grana,Nunez,Freitas'20/'21] [Bedroya,Hamada,Montero,Vafa'21] [Collazuol,Grana,Herraez,Freitas'22] [Cvetič,Dierigl,Lin,Zhang'22]



Non-Kodaira in F-theory on Calabi-Yau 3-fold

Potential infinite distance enhancements:

- $\mathrm{ord}(f, g, \Delta) \geq (4, 6, 12)$ over curve $\mathscr{C}$ on base B_2
- $\mathrm{ord}(f, g, \Delta) = (8, 12, 24)$ over point on base B_2



This work: *[Alvarez-García,Lee,TW to appear]*

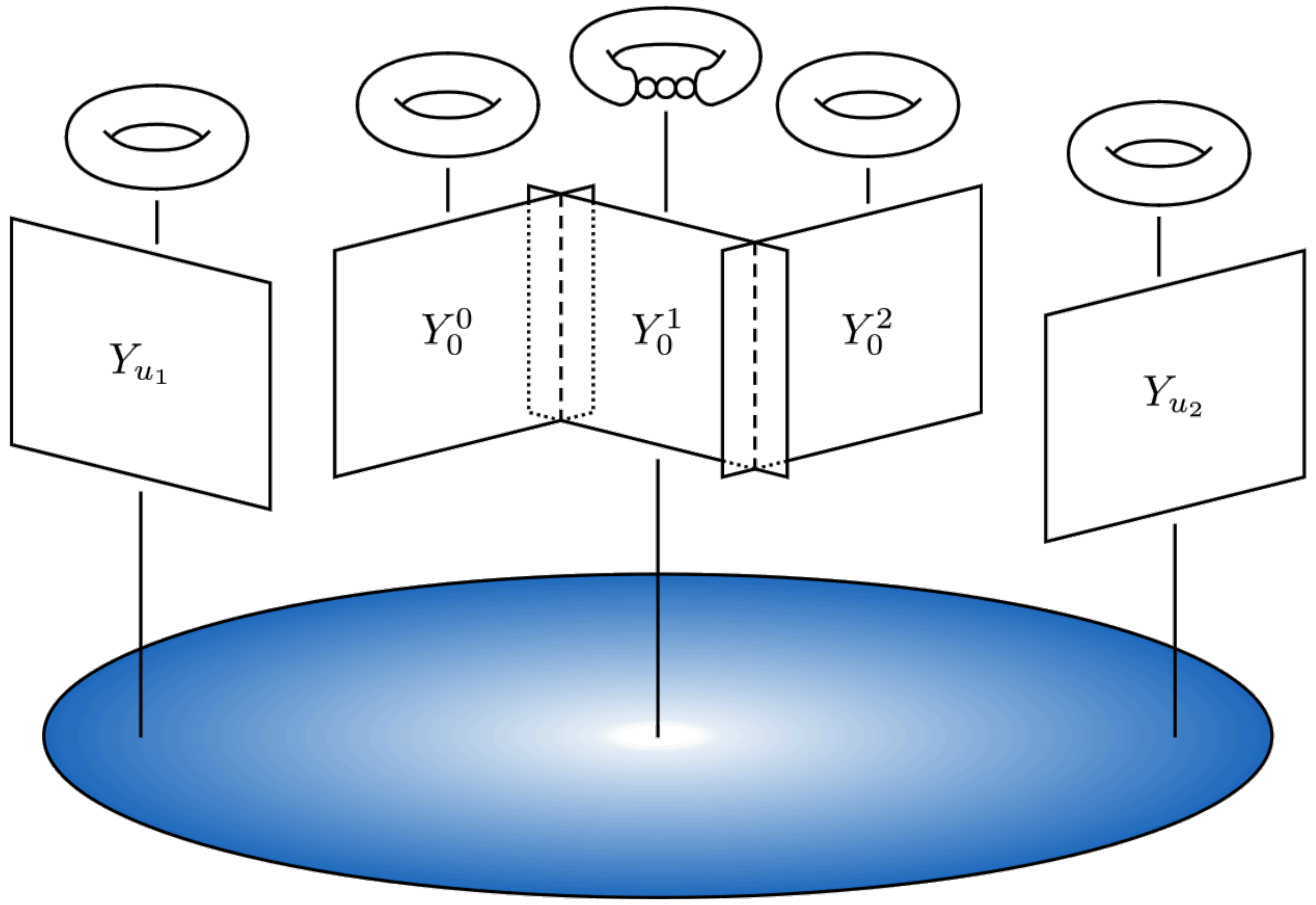
Non-Kodaira singularities over smooth irreducible non-intersecting $\mathscr{C}$

$\Longrightarrow$ succession of **single infinite distance limits**

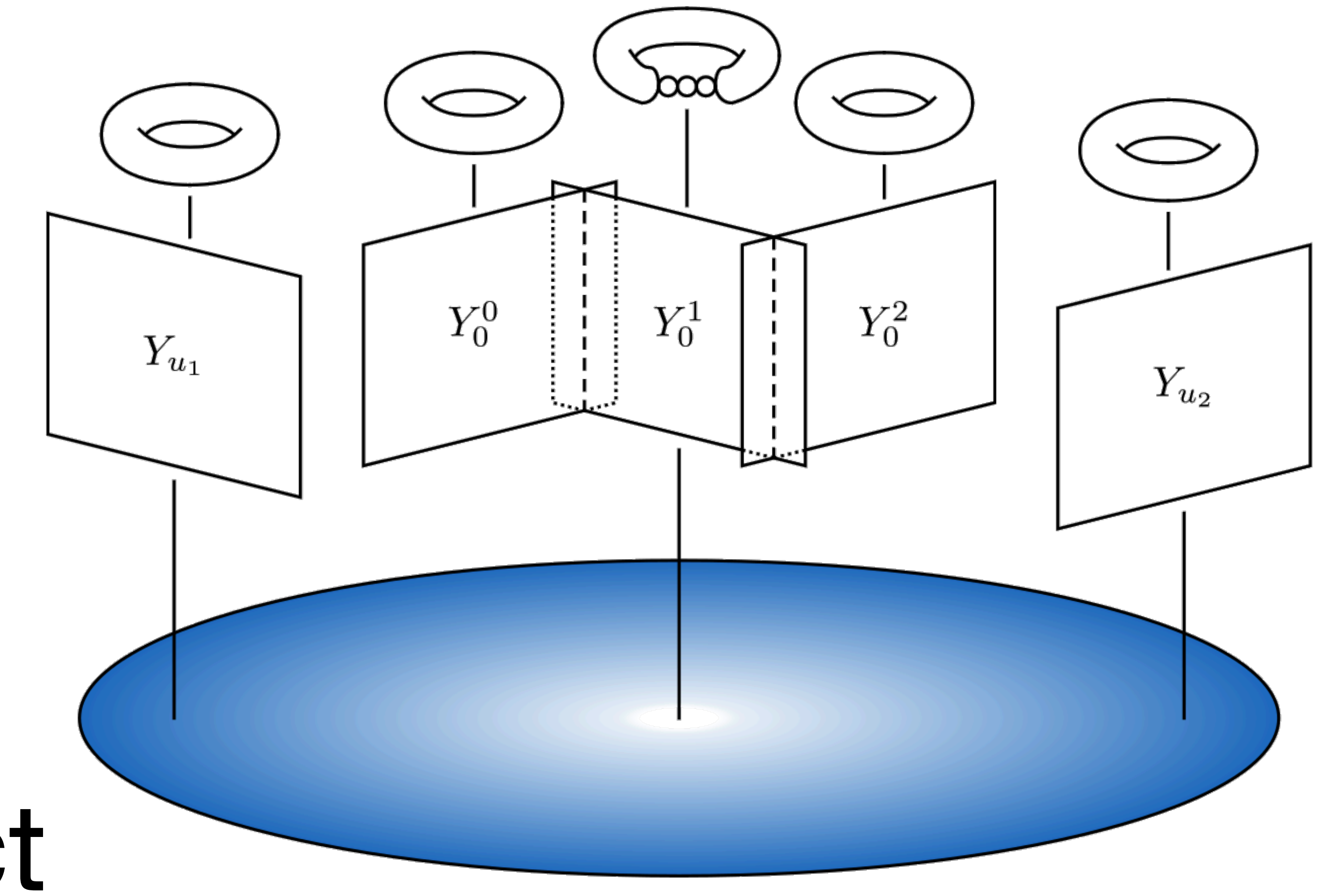
(over separate curves)

$\Longrightarrow$ **chain of components**

$$Y_0 = \cup_p Y_0^p \,, \qquad Y_0^p \cap Y_0^r \cap Y_0^s = 0 \quad \text{if} \quad p, q, r \ \text{distinct}$$



Non-Kodaira in F-theory on Calabi-Yau 3-fold



Non-Kodaira singularities over smooth irreducible non-intersecting $\mathscr{C}$

$\Longrightarrow$ **chain of components**

$$Y_0 = \cup_p Y_0^p\,, \qquad Y_0^p \cap Y_0^r \cap Y_0^s = 0 \quad \text{if} \quad p, q, r \ \text{distinct}$$

Detailed analysis gives:

- non-Kodaira over $\mathscr{C}$ **smooth irreducible requires:** genus $g(\mathscr{C})$= 0, 1
- $g(\mathscr{C})$ = 1 only for $\mathscr{C}$ the anti-canonical divisor

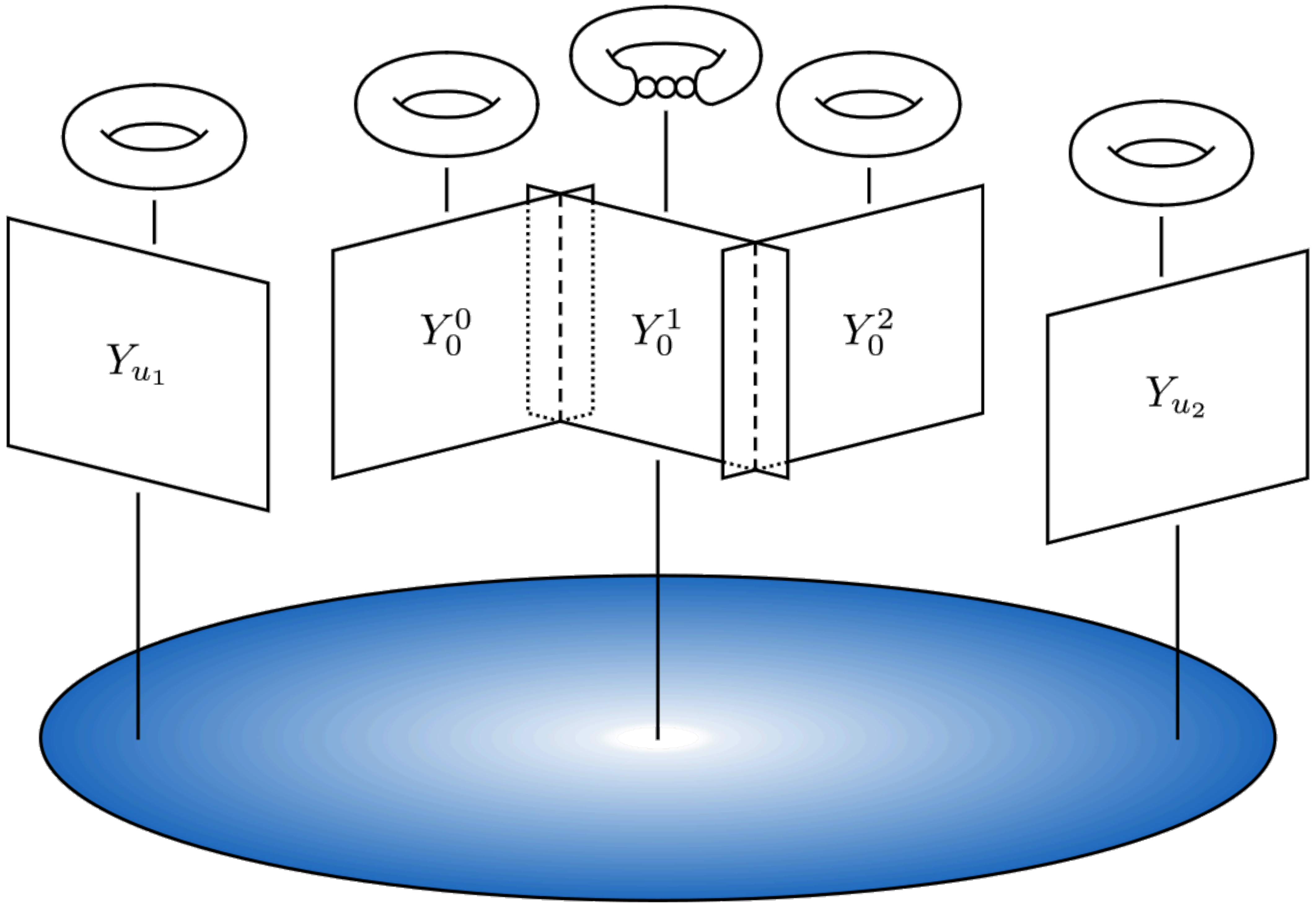
Non-Kodaira in F-theory on Calabi-Yau 3-fold

Remaining analysis:

1) Details of central fiber after blowup - purely geometric classification problem

2) Brane content in F-theory

3) Interpretation of infinite distance limit



Base spaces for elliptic 3-folds

$$\begin{array}{ccc} \mathbb{P}^1_{[s:t]} & \longrightarrow & \mathbb{F}_n \\ & & \big\downarrow \pi_{\mathbb{F}_n} \\ & & \mathbb{P}^1_{[v:w]} \end{array}$$

Base of elliptic 3-fold (with 7-branes):

$\mathbb{P}^2$ or (blowups of) **Hirzebruch** $\mathbb{F}_n$

Focus on $\mathbb{F}_n$ (all others only a few blowups away)

- **2 sections:** $C_0 = h\,,\qquad C_\infty = h + nf$ $\qquad f$: rational **fiber class**
- $C_0 \cdot C_0 = -\,n\,,\qquad C_\infty \cdot C_\infty = +\,n\,,\qquad C_i \cdot f = 1\,,\qquad f \cdot f = 0$
- elliptic 3-fold has **compatible elliptic and K3 fibration**

Non-Kodaira singularities - Systematics

Non-Kodaira singularities over divisors = curves $\mathscr{C}$ on base

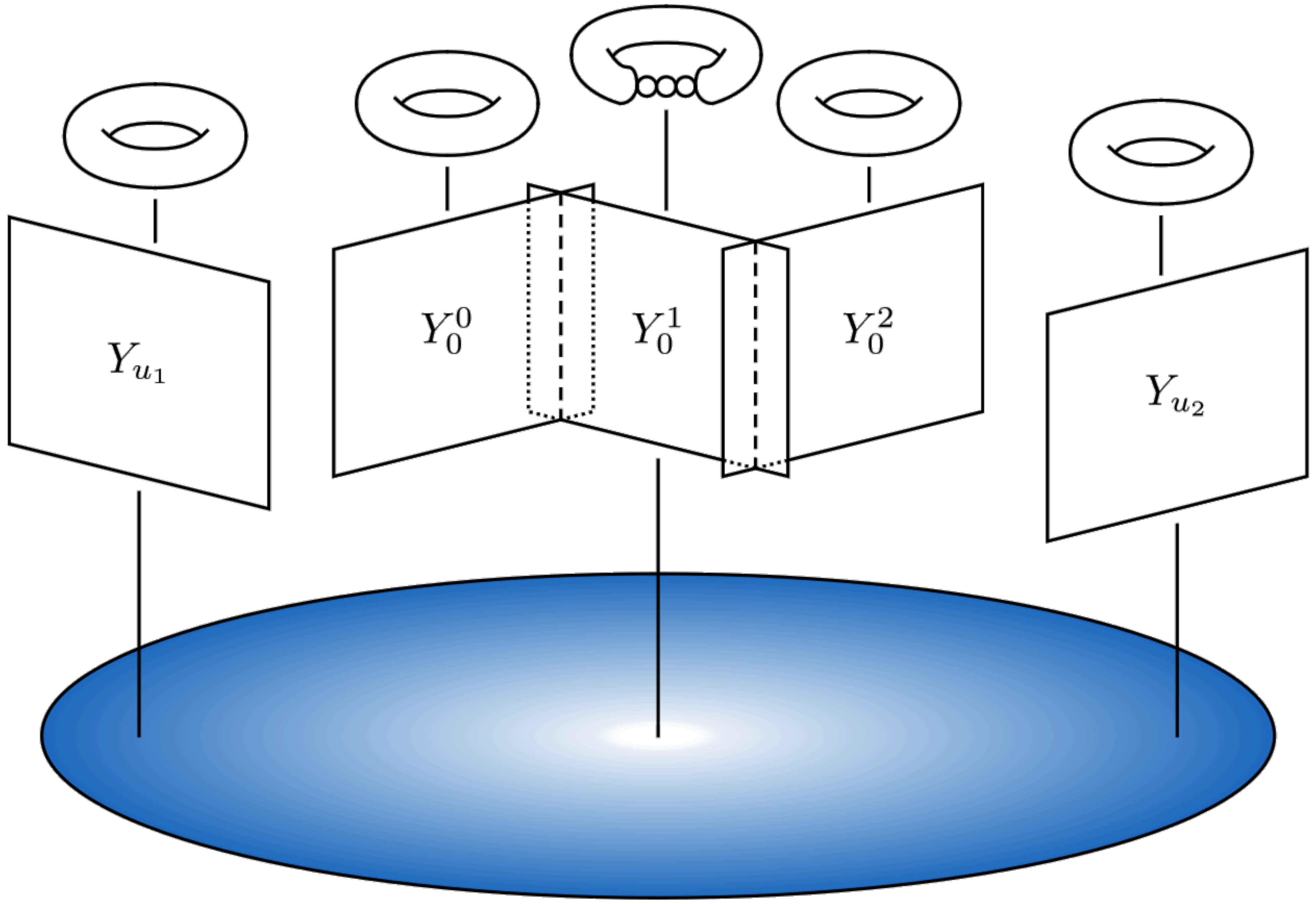
Possible types of smooth irreducible curves with $g(\mathscr{C}) = 0$ on $\mathbb{F}_n$:

- Class A: $\mathscr{C} = h$ or $\mathscr{C} = h + nf$ (one of the sections) — **horizontal**
- Class B: $\mathscr{C} = f$ (fiber) — **vertical**
- Class C: $\mathscr{C} = h + (n+1)f \quad n \leq 6$ or $\mathscr{C} = h + 2f \quad n = 0$
- Class D: $\mathscr{C} = 2h + bf \qquad (n, b) = (0,1), (1,2)$

mixed

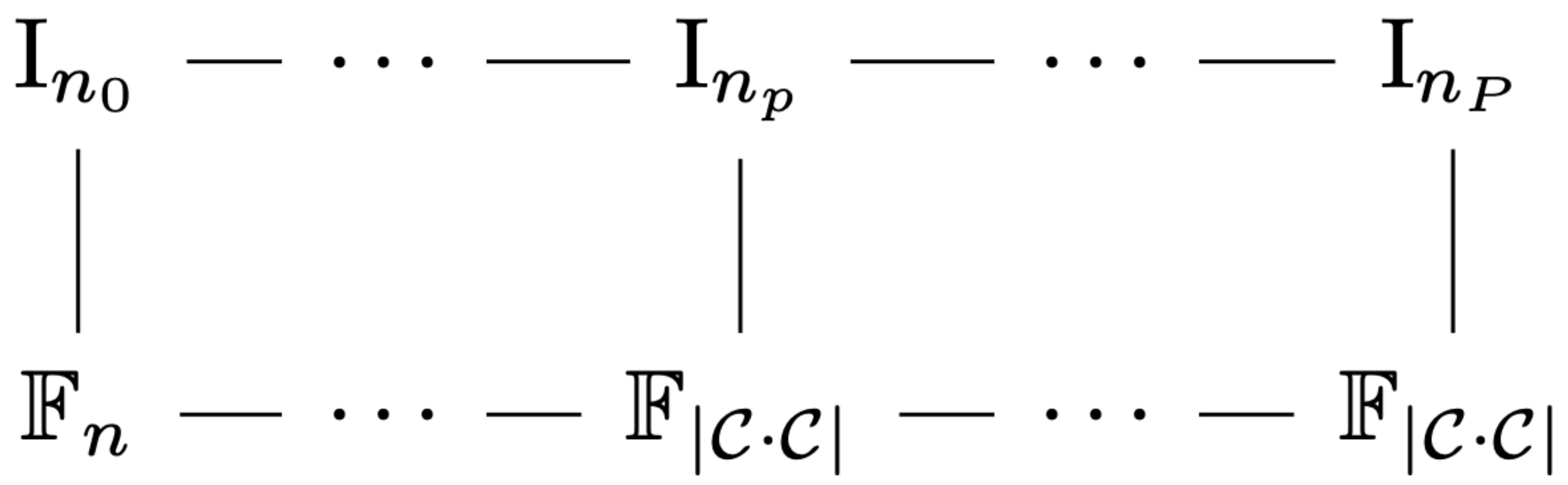
Geometry of central fiber

$\mathscr{C}$**: genus 0 curve of non-minimality**
$\Longrightarrow$ blow-up over curve $\mathscr{C}$



- **Bases of new components** Y_0^i are $\mathbb{F}_k$ for $k = |\mathscr{C} \cdot \mathscr{C}|$ **intersecting over** $\pm k$ **curves**

- **Codimension-zero fibers all of type** I_{n_i} :
  - $n_i > 0$: local weak coupling region
  - $n_i = 0$: local strong coupling region



- **Allowed patterns** $I_{n_0} - I_{n_1} - \ldots - I_{n_p}$ **constrained**
  - II.a - (II.b) - III.a - III.b as on K3

Focus: Horizontal models

Non-minimality over curve $\mathscr{C} = h$ (-n section) or $\mathscr{C} = h + nf$ (+n section)

- **fibered versions of 8d Kulikov models - with extra complications!**
- allow for **dual heterotic interpretation to gain intuition**

Brane Types

✳ **horizontal branes:**

- fully localised in one of the components

Direct analogue of 8d branes

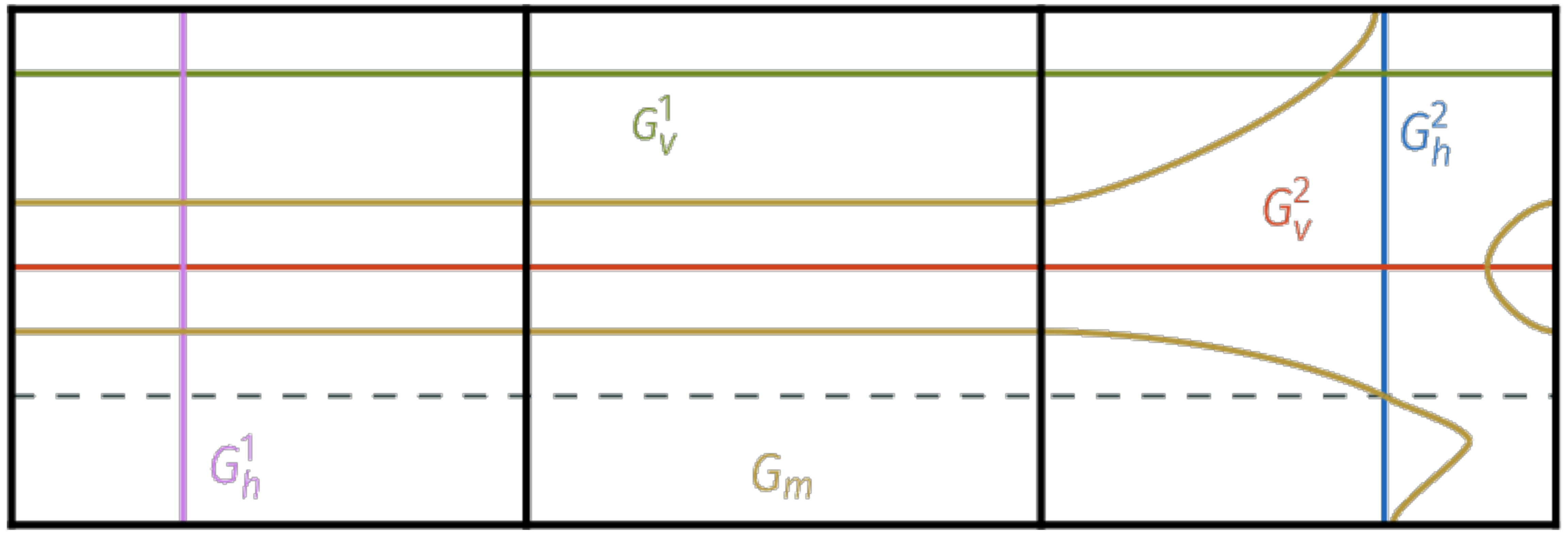
✳ **vertical branes:**

- lie in full fiber over points on base - project to points there

No analogue in 8d

✳ **mixed branes:**

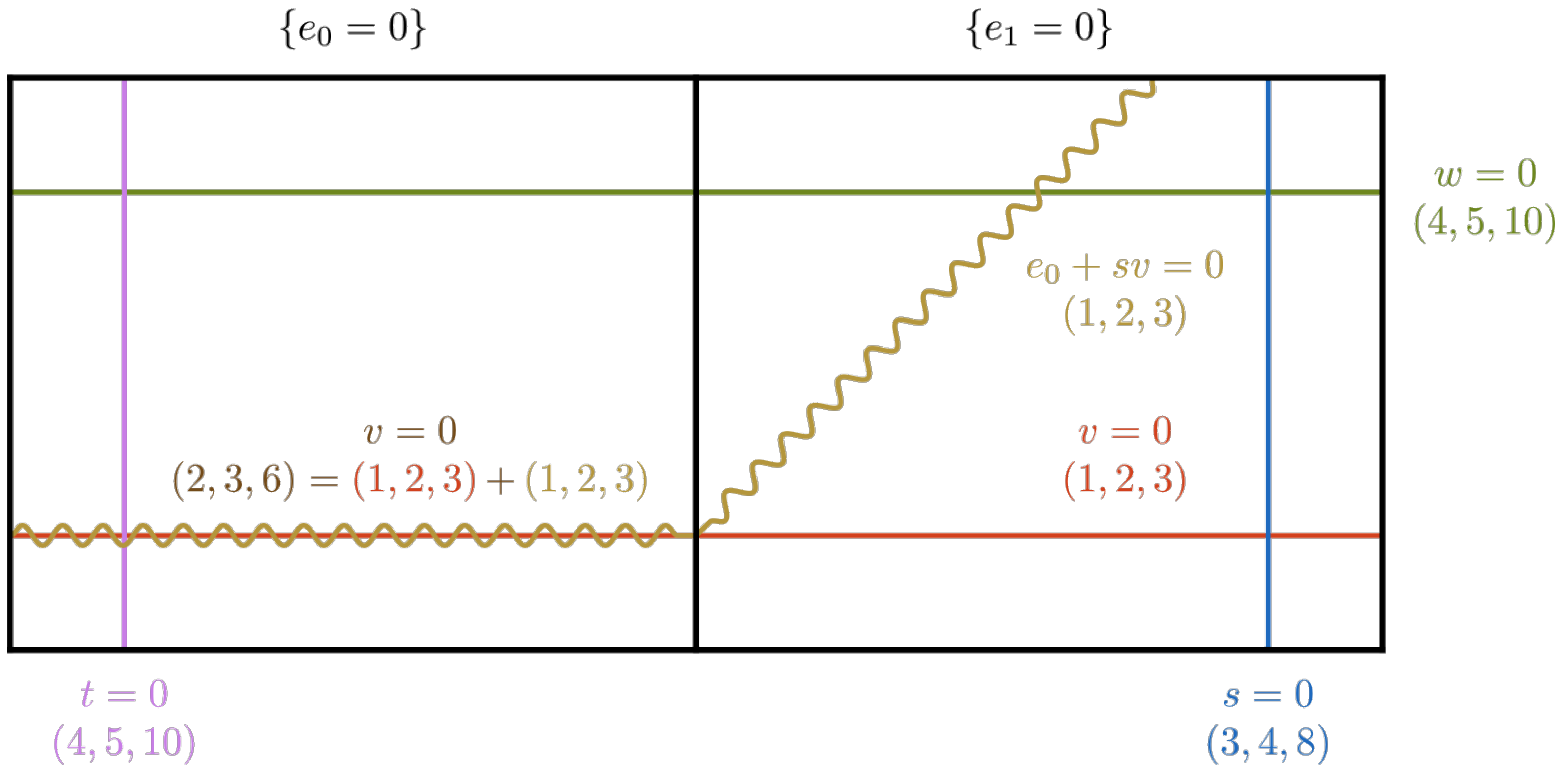
- linear combinations of horizontal and vertical

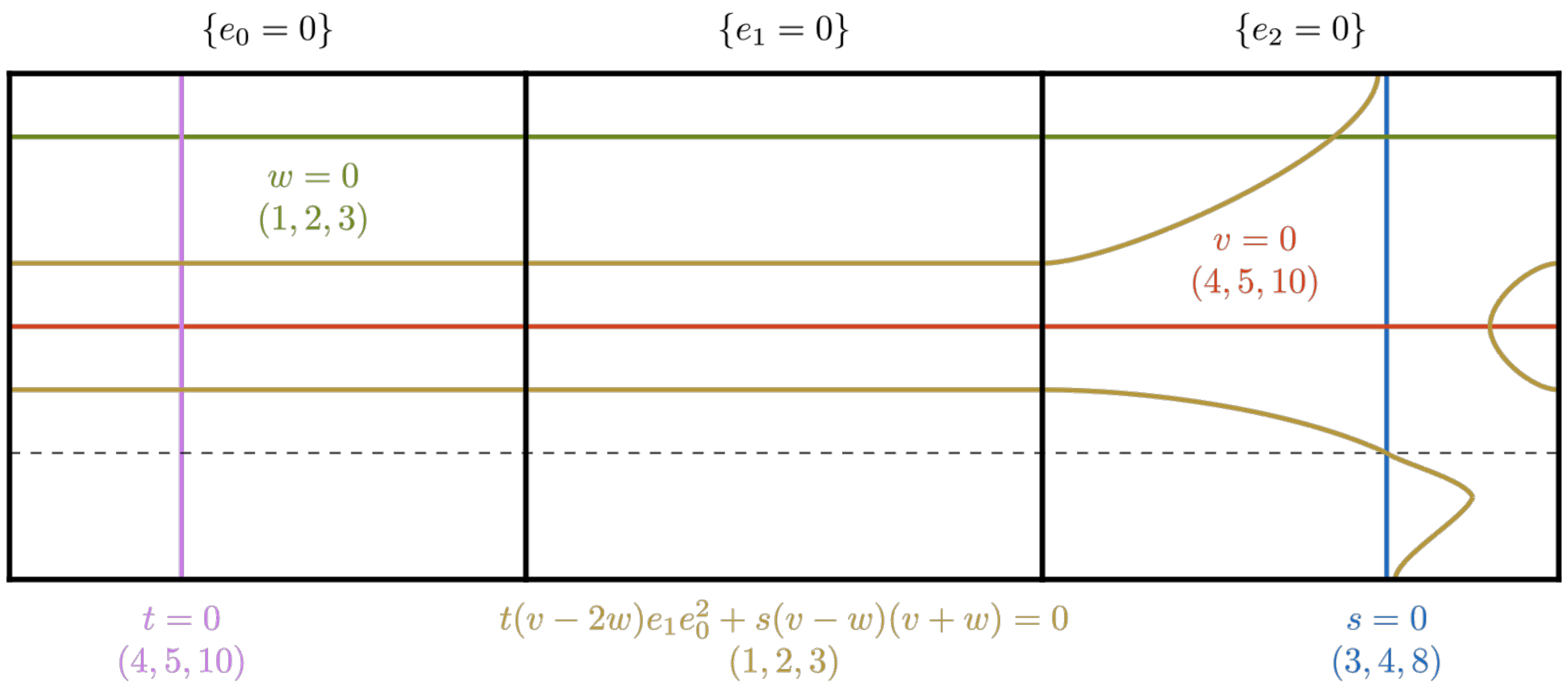


Brane Types

Global analysis required to identify brane content

- Branes coincident in one component may be globally distinct
- Irreducible curves in one component may belong to a single 7-brane globally





Interpretation of infinite distance degenerations

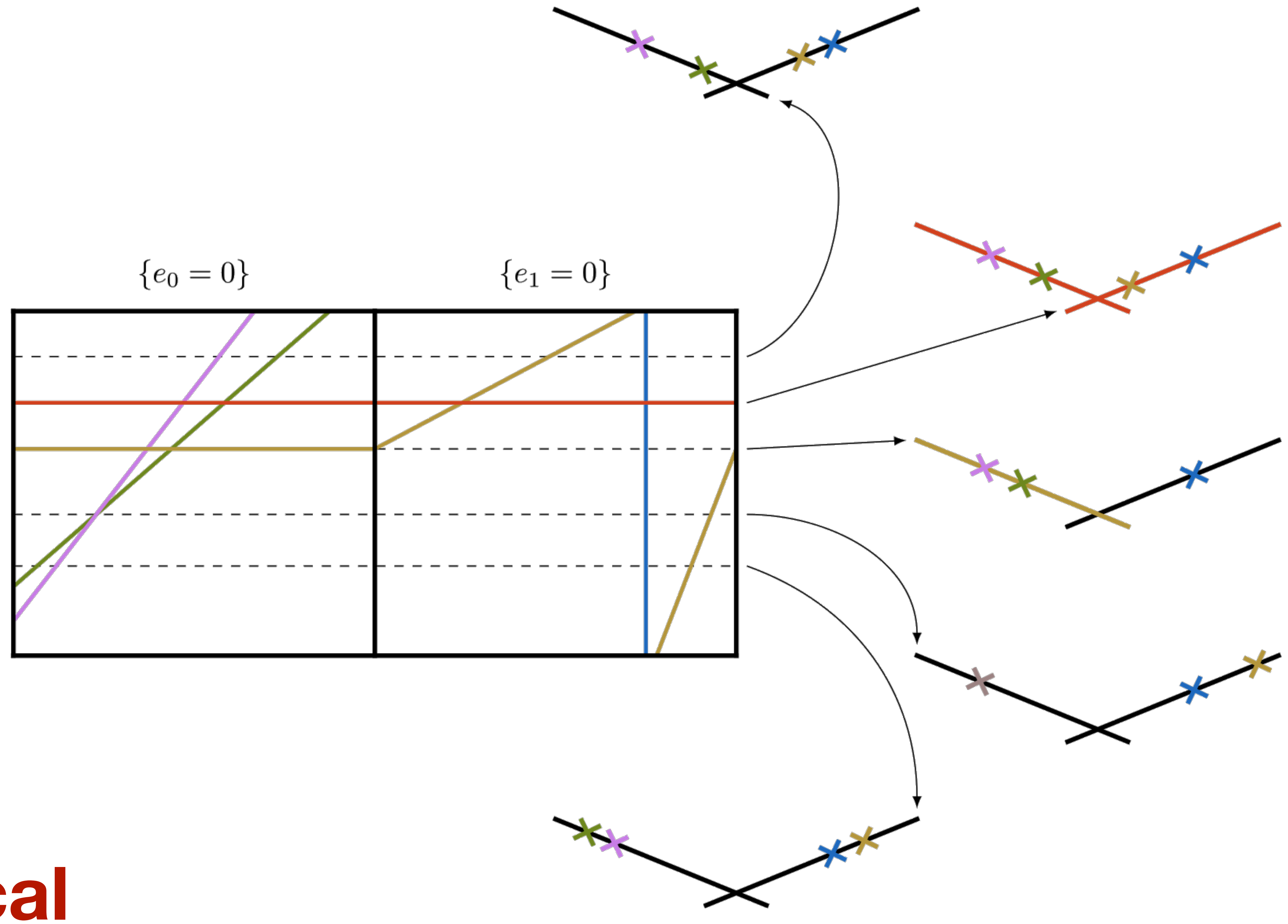
Example: II.a models $(I_0 - I_0)$

✴ **Fibered versions of 8d II.a models**

✴ Different points on $\mathbb{P}^1_b$:

different position of analogue of 8d branes

✴ Over special points: **fiber** is filled with **vertical brane**



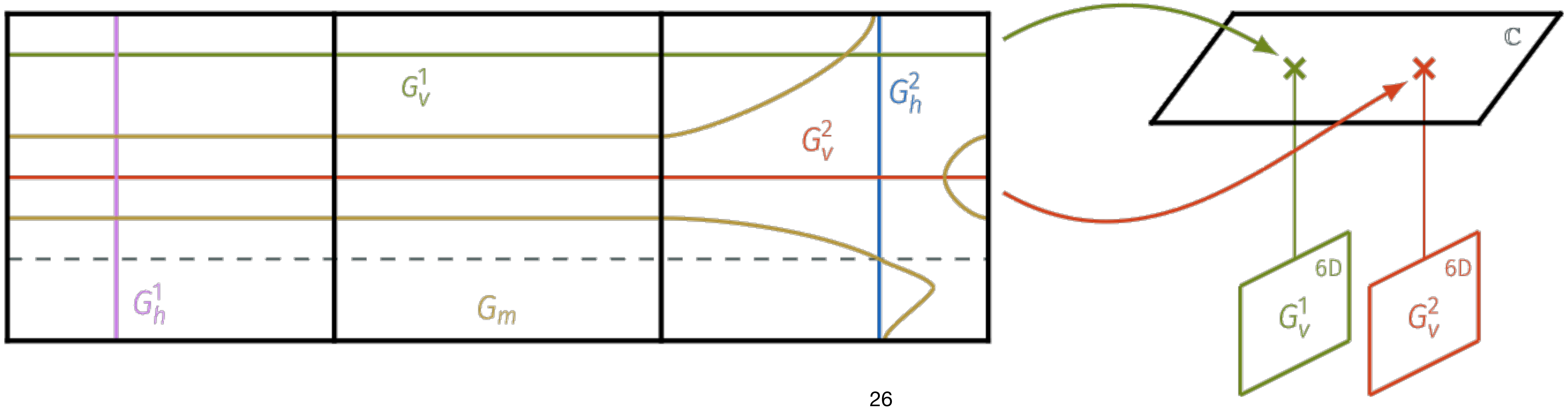
Interpretation: II.a models

✳ locally over each point on base:

2 degenerating $T^2 \Longrightarrow$ 2 towers of asymptotically massless states
(M2 branes in dual M-theory - **2 dual KK towers**)

✳ This fails over the points corresponding to vertical branes

✳ Modes localised on **vertical branes cannot combine with KK towers**



Interpretation of infinite distance degenerations

✳ locally over each point on base:

2 degenerating $T^2 \Longrightarrow$ 2 towers of asymptotically massless states

(M2 branes in dual M-theory - **2 dual KK towers**)

✳ This fails over the points corresponding to vertical branes

✳ Modes localised on **vertical branes cannot combine with KK towers**

Asymptotic physics (horizontal II.a model):

Decompactification (at least) **6d → 8d but with 6d defects associated with vertical branes**

Asymptotic gauge algebra: $(E_8 \times E_8)_{8d} \times G_{\text{vert,6d}}$

$G_{\text{vert,6d}}$ classified

talk by Alvarez-Garcia

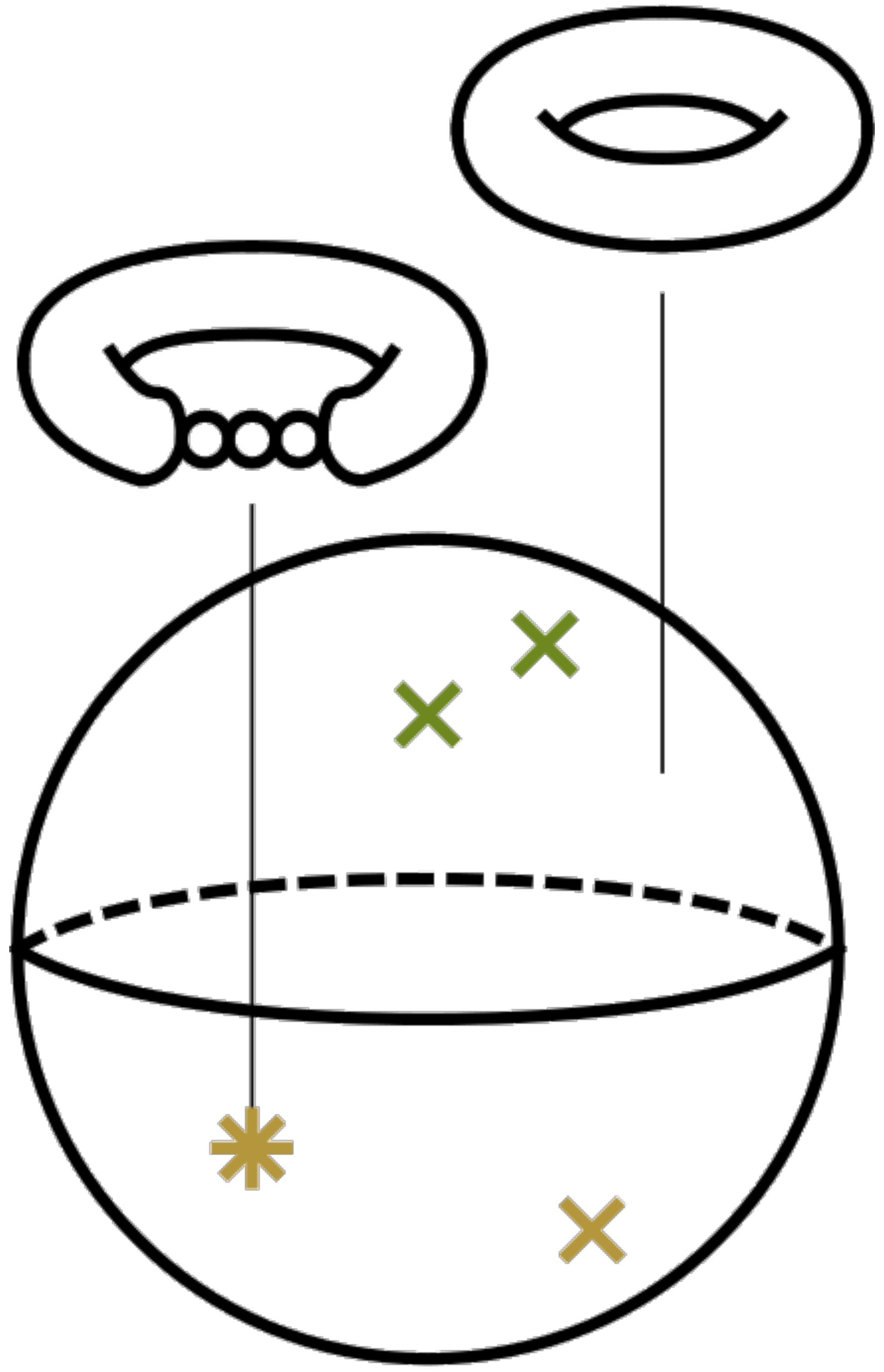
Heterotic dual interpretation

First assume adiabatic regime:

$$\mathscr{V}_{T^2_{\text{het}}} \to \infty\,, \qquad \mathscr{V}_{\mathbb{P}^1} \gg \mathscr{V}_{T^2_{\text{het}}}$$

$\implies$ decompactification 6d → 10d with **6d defects** from

certain fractional **small instantons probing K3 singular fibers**



Making base smaller (if possible) gives 6d → 8d limit

Summary

- ✴ Non-Kodaira singularities give rich pattern of **infinite distance limits in non-perturbative open string moduli space of F-theory**
- ✴ Systematic geometric analysis begun for elliptic 3-folds
- ✴ Proposed interpretation:
  - **Partial decompactification limits - but with lower-dimensional defects**
  - **Weak coupling limits (string limits)** — see **talk by Alvarez-García**
- ✴ Interpretation expected to generalize also to non-horizontal models despite absence of simple heterotic dual frame