Fermionic Higher-form Symmetries

2303.12633 w/ Zhang

Ongoing work w/ Ambrosino, Luo, Zhang

Yi-Nan Wang

Peking University

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Motivations and Summary

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• No global symmetry swampland conjecture: any UV complete QG theory has no exact global symmetry, including the generalized symmetries

• The past research are almost all about bosonic symmetries
• In this talk, discuss fermionic (higher-form) symmetries, the examples and other properties: gauging, curved space-time ...

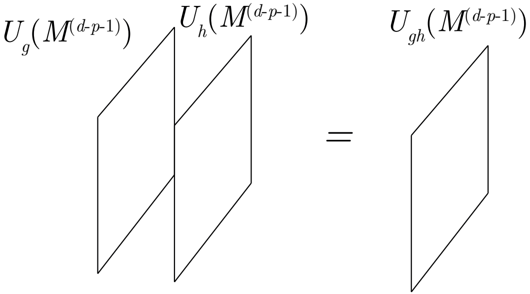
p-form symmetry

- We consider a QFT in d-dimensional space-time $\mathbb{R}^{1,d-1}$
- An invertible p-form symmetry with symmetry group G is generated by $(d-p-1)$-dimensional topological operators $U_g(M^{(d-p-1)})$, $g \in G$:

(Gaiotto, Kapustin, Seiberg, Willett 14')

$$U_g(M^{(d-p-1)})U_h(M^{(d-p-1)}) = U_{gh}(M^{(d-p-1)})\,. \tag{1}$$

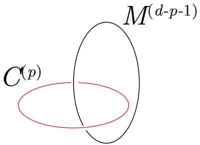
and acts on p-dimensional objects(operators) $V(\mathcal{C}^{(p)})$.



p-form symmetry

- $U_g(M^{(d-p-1)})$ has non-trivial action on $V(\mathcal{C}^{(p)})$ when $M^{(d-p-1)}$ and $\mathcal{C}^{(p)}$ are non-trivially linked. $\langle C^{(p)}, M^{(d-p-1)} \rangle$ is the linking number. $R(g)$ is a representation of G.

$$U_g(M^{(d-p-1)})V(\mathcal{C}^{(p)}) = R(g)^{\langle C^{(p)}, M^{(d-p-1)} \rangle} V(\mathcal{C}^{(p)}) \,. \tag{2}$$



- Example: pure 4d $U(1)$ gauge theory

(1) $U(1)_e^{(1)}$ 1-form symmetry acting on Wilson loops

(2) $U(1)_m^{(1)}$ 1-form symmetry acting on 't Hooft loops

Fermionic p-form symmetry

- An invertible fermionic p-form symmetry is generated by $(d-p-1)$-dimensional topological operators $U_\epsilon(M^{(d-p-1)})$, ϵ is a fermionic spinor with Grassmannian components

$$U_{\epsilon_1}(M^{(d-p-1)})U_{\epsilon_2}(M^{(d-p-1)}) = U_{\epsilon_1+\epsilon_2}(M^{(d-p-1)})\,. \tag{3}$$

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- What is the symmetry group G? In this talk assume to be non-compact $G=\mathbb{R}^s$, s is the number of spinor components

Fermionic 0-form symmetry

(1) Global supersymmetry acts on local operators, with supercurrent $j_{(d-1)}$ and supercharge $Q = \int_{M^{(d-1)}} j_{(d-1)}$ (and $\bar{Q}$).

$$U_\epsilon(M^{(d-1)}) = e^{i(\bar{\epsilon}Q + \bar{Q}\epsilon)} . \quad (5)$$

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(2) Shifting symmetry of a free Dirac spinor

$$S = \int -\bar{\psi}\gamma^\mu \partial_\mu \psi d^d x \,. \tag{6}$$

- Invariant under the shift $\psi \to \psi + \epsilon$, $\partial_\mu \epsilon = 0$.
- Generated by the topological operator

$$U_\epsilon(M^{(d-1)}) = \exp\left(i \int_{M^{(d-1)}} \star \left[\bar{\epsilon}\gamma_{(1)}\psi - \bar{\psi}\gamma_{(1)}\epsilon \right] \right) \,, \tag{7}$$

- $\gamma_{(p)}$ is the p-form γ-matrix $\gamma_{(p)} = \frac{1}{p!}\gamma_{\mu_1 \dots \mu_p} dx^{\mu_1} \dots dx^{\mu_p}$.

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- However there is no $(d-2)$-dim. topological operator in the free fermion theory that links $\mathcal{C}$.
- One should consider theories with Rarita-Schwinger field ψ_μ!

Fermionic 1-form symmetry

- Consider a free Rarita-Schwinger field ψ_μ ($d \geq 3$)

$$S = \int -\bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho d^d x \,, \tag{9}$$

with gauge symmetry $\delta_\lambda \psi_\mu = \partial_\mu \lambda$.

- There is a gauge invariant operator (fermionic analog of the bosonic Wilson loop)

$$V_\eta(\mathcal{C}) = \exp\left(i \int_{\mathcal{C}} (\bar{\eta}\psi_\mu + \bar{\psi}_\mu \eta) dx^\mu \right) . \tag{10}$$

- The "charge" η is unquantized, since the gauge group is non-compact.

Fermionic 1-form symmetry

- From the conserved 2-form current

$$\mathcal{J}_{\mu\nu} = -\gamma_{\mu\nu\rho}\psi^{\rho}\,, \tag{11}$$

one can construct $(d-2)$-dim. topological operator

$$U_{\epsilon}(M^{(d-2)}) = \exp\left(i\int_{M^{(d-2)}}\left[\bar{\epsilon}\,(\star\mathcal{J})_{(d-2)} + (\star\bar{\mathcal{J}})_{(d-2)}\,\epsilon\right]\right) \tag{12}$$

- Acts on $V_{\eta}(\mathcal{C})$ as

$$\langle U_{\epsilon}(M^{(d-2)})V_{\eta}(\mathcal{C})\rangle = \exp\left(i(\bar{\epsilon}\eta + \bar{\eta}\epsilon)\langle\mathcal{C}, M^{(d-2)}\rangle\right)\langle V_{\eta}(\mathcal{C})\rangle\,. \tag{13}$$

- The phase factor is a Grassmannian even element.

Fermionic p-form symmetry

- Generalize to a free fermionic p-form tensor gauge field

$\psi_{(p)} = \frac{1}{p!}\psi_{\mu_1\mu_2\ldots\mu_p} dx^{\mu_1}\ldots dx^{\mu_p}$, with fermionic p-form symmetry ($d \geq 2p+1$)

$$S = \int -\bar{\psi}_{(p)} \wedge \gamma_{(d-2p-1)} \wedge d\psi_{(p)} \tag{14}$$

with gauge symmetry $\delta_\lambda \psi_{(p)} = d\lambda_{(p-1)}$. Quantization: (Lekeu, Zhang 21')

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- Similarly, one can construct the p-dimensional gauge invariant operator

$$V_\eta(\mathcal{C}^{(p)}) = \exp\left(i \int_{\mathcal{C}^{(p)}} (\bar{\eta}\psi_{(p)} + \bar{\psi}_{(p)}\eta)\right) , \tag{15}$$

- $(d-p-1)$-dim. topological operator acting on $V_\eta(\mathcal{C}^{(p)})$

$$U_\epsilon(M^{(d-p-1)}) = \exp\left(i \int_{M^{(d-p-1)}} \left[\bar{\epsilon}\,(\star\mathcal{J})_{(d-p-1)} + (\star\bar{\mathcal{J}})_{(d-p-1)}\epsilon\right]\right) \tag{16}$$

$$\mathcal{J}_{\mu_1\mu_2\ldots\mu_{p+1}} \equiv \gamma_{\mu_1\ldots\mu_{p+1}\nu_1\ldots\nu_p}\psi^{\nu_1\ldots\nu_p} , \tag{17}$$

Examples with fermionic higher-form symmetry

- A general class of examples: Lagrangian with $\psi_{(p)}$ fields, γ-tensor and one derivative in each term.
- For example the "fermionic BF theory" in 4d

$$S = \int d^4x \left(-\bar{\chi}_{\mu\nu}\gamma^{\mu\nu\rho\sigma}\gamma_5\partial_\rho\psi_\sigma - \bar{\psi}_\mu\gamma^{\mu\nu\rho\sigma}\gamma_5\partial_\nu\chi_{\rho\sigma}\right). \tag{18}$$

- Gauge transformation $\delta_\epsilon\psi_\mu = \partial_\mu\epsilon$, $\delta_\lambda\chi_{\mu\nu} = 2\partial_{[\mu}\lambda_{\nu]}$
- The gauge invariant operators

$$V_\eta(\mathcal{C}) = \exp\left(i\oint_{\mathcal{C}}(\bar{\eta}\psi_\mu + \bar{\psi}_\mu\eta)\, dx^\mu\right) \tag{19}$$

and

$$W_\xi(\mathcal{S}) = \exp\left(i\oint_{\mathcal{S}}\frac{1}{2}(\bar{\xi}\chi_{\mu\nu} + \bar{\chi}_{\mu\nu}\xi)\, dS^{\mu\nu}\right) \tag{20}$$

are the charged objects under fermionic 1-form and 2-form symmetries.

Examples with fermionic higher-form symmetry

- $V_\eta(\mathcal{C})$ and $W_\xi(\mathcal{S})$ has non-trivial action on each other

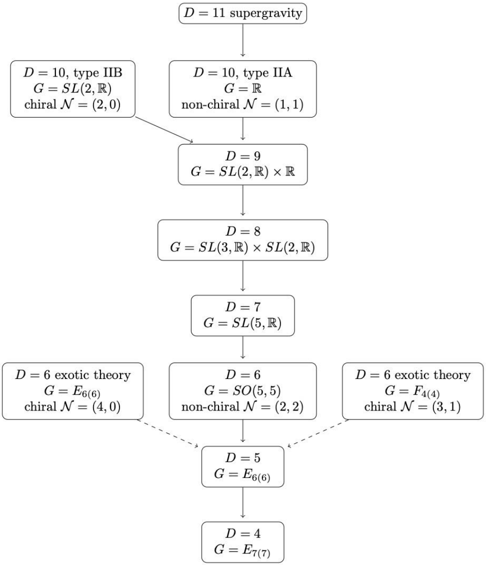
$$\begin{aligned} \langle V_\eta(\mathcal{C}) W_\xi(\mathcal{S})\rangle &= \exp\left(\frac{1}{2}(\bar{\xi}\eta + \bar{\eta}\xi)\langle \mathcal{S}, \mathcal{C}\rangle\right)\langle W_\xi(\mathcal{S})\rangle\,, \\ \langle W_\xi(\mathcal{S}) V_\eta(\mathcal{C})\rangle &= \exp\left(\frac{1}{2}(\bar{\xi}\eta + \bar{\eta}\xi)\langle \mathcal{C}, \mathcal{S}\rangle\right)\langle V_\eta(\mathcal{C})\rangle\,, \\ &= \exp\left(\frac{1}{2}(\bar{\xi}\eta + \bar{\eta}\xi)\langle \mathcal{S}, \mathcal{C}\rangle\right)\langle V_\eta(\mathcal{C})\rangle\,. \end{aligned} \tag{21}$$

- Hence in $d = 4$, the theory is a fermionic TQFT on $\mathbb{R}^{1,3}$, in the sense that correlation functions are given by the linking number.
- Analogous to the bosonic BF theory, it is also gapped.

6d (4,0) theory

- Another example is 6d (4,0) theory with 32 supercharges, which is conjectured to be the strongly coupled limit of 5d maximal supergravity (Hull 00')(Bertrand, Borsten, Cederwall, Gunaydin, Henneaux, Hohenegger, Hohm, Hull, Minasian, Lekeu, Leonard, Samtleben, Strickland-Constable, Zhang. . .)
- Analogue of strongly coupled 5d $\mathcal{N} = 2$ SYM $\rightarrow$ 6d (2,0) theory

6d (4,0) theory


$D = 11$ supergravity
$D = 10$, type IIB
$G = SL(2, \mathbb{R})$
chiral $\mathcal{N} = (2, 0)$
$D = 10$, type IIA
$G = \mathbb{R}$
non-chiral $\mathcal{N} = (1, 1)$
$D = 9$
$G = SL(2, \mathbb{R}) \times \mathbb{R}$
$D = 8$
$G = SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$
$D = 7$
$G = SL(5, \mathbb{R})$
$D = 6$ exotic theory
$G = E_{6(6)}$
chiral $\mathcal{N} = (4, 0)$
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$G = SO(5, 5)$
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$D = 5$
$G = E_{6(6)}$
$D = 4$
$G = E_{7(7)}$

6d (4,0) theory

• 6d (4,0) supermultiplet, fields organized under reps of little group $SU(2) \times SU(2)$ and the R-symmetry group $USp(8)$:

(1) C_{MNPQ}: $(\mathbf{5}, \mathbf{1}; \mathbf{1})$, a 4-index object with the same symmetry as the Riemann tensor

(2) Ψ^a_{MN}: $(\mathbf{4}, \mathbf{1}; \mathbf{8})$, a fermionic 2-form gauge field, in the fundamental rep. of $USp(8)$

(3) B^-_{MN}: $(\mathbf{3}, \mathbf{1}; \mathbf{27})$

(4) ϕ: $(\mathbf{1}, \mathbf{1}; \mathbf{42})$

(5) λ: $(\mathbf{2}, \mathbf{1}; \mathbf{48})$

• Even a 6d Lorentz covariant free action is not found yet

• Nonetheless, in the free limit, Ψ_{MN} fields have a fermionic 2-form symmetry!

6d (4,0) theory

- In the free limit, the kinetic term for Ψ_{MN} fields is

$$\mathcal{L}_{\text{6d}} = \bar{\Psi}_{MN}\Gamma^{MNPQR}\partial_P\Psi_{QR}\,, \tag{22}$$

One can construct gauge invariant surface operator charged under fermionic 2-form symmetry

$$V_\eta(\mathcal{S}) = \exp\left(i\oint_{\mathcal{S}}\frac{1}{2}(\bar{\eta}\Psi_{MN} + \bar{\Psi}_{MN}\eta)\,dS^{MN}\right)\,. \tag{23}$$

6d (4,0) theory

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- After dimensional reduction to 5d on S^1, decompose $\{M\} = \{\mu, 5\}$, write $\Psi_{MN} = (\hat{\psi}_{MN}, 0)^T$

$$\psi_{\mu\nu} = \hat{\psi}_{\mu\nu}\ ,\ \psi_\mu = \hat{\psi}_{\mu 5}\,. \tag{24}$$

- 5d Lagrangian

$$\mathcal{L}_{5d} = \bar{\psi}_{\mu\nu}\gamma^{\mu\nu\rho\sigma\tau}\partial_\rho\psi_{\sigma\tau} + 2i\bar{\psi}_{\mu\nu}\gamma^{\mu\nu\rho\sigma}\partial_\rho\psi_\sigma - 2i\bar{\psi}_\mu\gamma^{\mu\nu\rho\sigma}\partial_\nu\psi_{\rho\sigma}\,. \tag{25}$$

- A fermionic 1-form symmetry and a fermionic 2-form symmetry in 5d

Gauging of fermionic p-form symmetries

- How do we gauge such symmetries? Take the 0-form example

$$S_{\text{free}} = \int_{M_d} -\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi d^d x\,. \tag{26}$$

- Invariant under the shift $\psi \to \psi + \epsilon$, global symmetry $\longrightarrow \partial_{\mu}\epsilon = 0$.
- After ϵ becomes local, the way to write down a gauge invariant action is to introduce the dynamical gauge field ψ_{μ},

$$\delta_{\epsilon}\psi_{\mu} = \partial_{\mu}\epsilon\,. \tag{27}$$

- The combination $\psi_{\mu} - \partial_{\mu}\psi$ is gauge invariant.
- The gauged action is

$$S_{\text{gauged}} = \int d^d x \bar{\psi}\gamma^{\mu}(\psi_{\mu} - \partial_{\mu}\psi)\,. \tag{28}$$

- Analogous to gauging shifting symmetry of a free scalar??

Gauging of fermionic p-form symmetries

- However, the action is still not gauge invariant!

$$S_{\text{gauged}} = \int d^d x \bar{\psi} \gamma^\mu (\psi_\mu - \partial_\mu \psi) \,. \tag{29}$$

$$\delta_\epsilon S_{\text{gauged}} = \int d^d x \bar{\epsilon} \gamma^\mu (\psi_\mu - \partial_\mu \psi) \tag{30}$$

- 't Hooft anomaly: cannot cancel it by adding local counter terms.

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- 't Hooft anomaly: cannot cancel it by adding local counter terms.
- Uplift to $(d+1)$-dim., by rewriting

$$S_{\text{free}} \sim \int_{M_{d+1}} \partial_M \bar{\Psi} \Gamma^{MN} \partial_N \Psi \tag{31}$$

- M_d is the boundary of M_{d+1}
- Introduce the $(d+1)$-dim. gauge field Ψ_M, one can write the gauge invariant $(d+1)$-dim. action

$$S_{\text{gauge invariant}} \sim \int_{M_{d+1}} (\bar{\Psi}_M - \partial_M \bar{\Psi}) \Gamma^{MN} (\Psi_N - \partial_N \Psi) \tag{32}$$

Gauging of fermionic p-form symmetries

- Nonetheless, one can gauge it by introducing a more exotic gauge field ψ_μ, with gauge transformation

$$\delta\psi_\mu = \partial_\mu \epsilon + \frac{1}{f}\gamma_\mu \epsilon\,, \tag{33}$$

f is a constant with the unit $[M]^{-1}$. (Love 03')

- The action

$$S_{\text{gauged}} = -\frac{1}{2}\int d^d x(\bar{\psi}\gamma^\mu D_\mu \psi + \bar{\psi}\overleftarrow{D}_\mu \gamma^\mu \psi + \frac{1}{f}\bar{\psi}\psi - f\overline{D_\mu\psi}D^\mu\psi - f\overline{D_\mu\psi}\gamma^{\mu\nu}D_\nu\psi) \tag{34}$$

is gauge invariant! ($D_\mu\psi \equiv \partial_\mu\psi - \psi_\mu$)

- Similar phenomenon happens for fermionic p-form symmetry as well, in the cases of a free fermion p-form gauge field. (Ongoing work)
- Nonetheless, such gauge fields do not have a gauge invariant kinetic term, geometric and physical interpretation?

Breaking of fermionic p-form symmetries and swampland

- What about theories in curved space-time
- Expectation: fermionic p-form global symmetries should be broken in a quantum gravity theory (no global symmetry swampland conjectures)

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(1) 0-form symmetry case, a Dirac spinor in curved space-time

$$S[\psi]_{\text{sugra}} = -\int d^d x \sqrt{|\det g|}\,\bar{\psi}\gamma^\mu \nabla_\mu \psi\,, \tag{35}$$

$$\nabla_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\,. \tag{36}$$

- The action is invariant under $\psi \to \psi + \epsilon$, where $\nabla_\mu \epsilon = 0$, ϵ is a covariantly constant spinor.
- The symmetry is not present on a general space-time background, hence such fermionic symmetry is broken in quantum gravity theory

Breaking of fermionic p-form symmetries and swampland

(2) fermionic 1-form symmetry

$$S[\psi_\mu]_{\mathrm{sugra}} = -\int d^d x \sqrt{|\det g|}\, \bar{\psi}_\mu \gamma^{\mu\nu\rho} \nabla_\nu \psi_\rho \,. \tag{37}$$

The symmetry generator

$$U_\epsilon(M_{(d-2)}) = \exp\left(\int_{M_{(d-2)}} \bar{\epsilon} \star (\gamma^{\mu\nu\rho} \psi_\rho) + c.c. \right) \tag{38}$$

is only topological when $\nabla_\mu \epsilon = 0$.

• Hence the fermionic higher-form symmetries can also be present on a particular fixed background M with covariantly constant spinors, but broken on a general background.

Breaking of fermionic *p*-form symmetries and swampland

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• Hence the fermionic higher-form symmetries can also be present on a particular fixed background M with covariantly constant spinors, but broken on a general background.

• As another comment, on such background M, one can still define the fermionic TQFTs and gauge invariant extended operators!

Summary

• We discussed fermionic p-form symmetries, as non-compact shifting symmetry of fermionic p-form fields

• Example of fermionic higher-form symmetry:

(1) Free fermionic p-form fields

(2) fermionic TQFTs constructed with fermionic p-form fields

$$S = \int d^4x \left(-\bar{\chi}_{\mu\nu}\gamma^{\mu\nu\rho\sigma}\gamma_5\partial_\rho\psi_\sigma - \bar{\psi}_\mu\gamma^{\mu\nu\rho\sigma}\gamma_5\partial_\nu\chi_{\rho\sigma}\right). \tag{39}$$

• Relation to known spin TQFTs?

• The free limit of 6d (4,0) theory also possesses fermionic 2-form symmetry. Breaking the symmetry with interaction terms?

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• Fermionic higher-form symmetries in string theory context? Seems hard as the only massless fermionic tensor field is the gravitino in supergravity.

• Higher version of supersymmetry?

Summary

- Constructing more interesting physical examples with fermionic higher-form symmetries, e. g. gaugino condensation? (Farakos's talk)
- Non-invertible fermionic symmetries?
- Fermionic higher-form symmetries in string theory context? Seems hard as the only massless fermionic tensor field is the gravitino in supergravity.
- Higher version of supersymmetry?
- Relation to condensed matter physics models, bosonization etc..
- Thanks!