CARVING OUT THE OPEN-STRING EFT

Yu-tin Huang
National Taiwan University

Based on, Jin-Yu Liu, Y-t H , Laurentiu Rodina (QMU), Yihong Wang 2012.15849
Li-Yuan Chiang (NTU), Y-t H, Wei Li (Boston Univ) Laurentiu Rodina (QMU), He-Chen Weng (NTU) 2105.02862
Li-Yuan Chiang, Y-t H, He-Chen Weng (NTU) in progress

2023 String Pheno

The Gravitation EFT

In recent years there's been tremendous progress in studying quantum gravity via bootstrapping the S-matrix of gravitational EFT amplitudes

. arXiv:2102.08951 [pdf, other] hep-th doi 10.1007/JHEP07(2021)110

Sharp Boundaries for the Swampland

Authors: Simon Caron-Huot, Dalimil Mazac, Leonardo Rastelli, David Simmons-Duffin

5. arXiv:2205.01495 [pdf, other] hep-th gr-qc

Graviton partial waves and causality in higher dimensions

Authors: Simon Caron-Huot, Yue-Zhou Li, Julio Parra-Martinez, David Simmons-Duffin

. arXiv:2103.12728 [pdf, other] hep-th gr-qc hep-ph doi 10.1088/1751-8121/ac0e51

Gravitational Effective Field Theory Islands, Low-Spin Dominance, and the Four-Graviton Amplitude

Authors: Zvi Bern, Dimitrios Kosmopoulos, Alexander Zhiboedov

. arXiv:2102.02847 [pdf, other] hep-th doi 10.1103/PhysRevLett.127.081601

Where is String Theory?

Authors: Andrea Guerrieri, Joao Penedones, Pedro Vieira

arXiv:2201.07177 [pdf, other] hep-th

(Non)-projective bounds on gravitational EFT

Authors: Li-Yuan Chiang, Yu-tin Huang, Wei Li, Laurentiu Rodina, He-Chen Weng

. arXiv:2104.09682 [pdf, other] hep-th gr-qc hep-ph doi 10.1103/PhysRevLett.127.091602

Is the Standard Model in the Swampland? Consistency Requirements from Gravitational Scattering

Authors: Katsuki Aoki, Tran Quang Loc, Toshifumi Noumi, Junsei Tokuda

The Gravitation EFT

Essentially a ``bottom up" approach where the EFT operators serve as IR parameterization of UV completions

$$\mathcal{L} = \int dx^D \sqrt{-g}\left(M_{\rm pl}^{D-2} R + \alpha_1 R^2 + \alpha_2 R^3 + \alpha_4 R^4 \cdots\right)$$

Arising from perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R + \hat{\alpha}_1 \frac{R^2}{M^2} + \hat{\alpha}_2 \frac{R^3}{M^4} + \hat{\alpha}_4 \frac{R^4}{M^6} \cdots\right)$$

$$M \ll M_{\rm pl}$$

From non-perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R + \hat{\alpha}_1 \frac{R^2}{M_{\rm pl}^2} + \hat{\alpha}_2 \frac{R^3}{M_{\rm pl}^4} + \hat{\alpha}_4 \frac{R^4}{M_{\rm pl}^6} \cdots\right)$$

String theory provides solutions to both scenarios

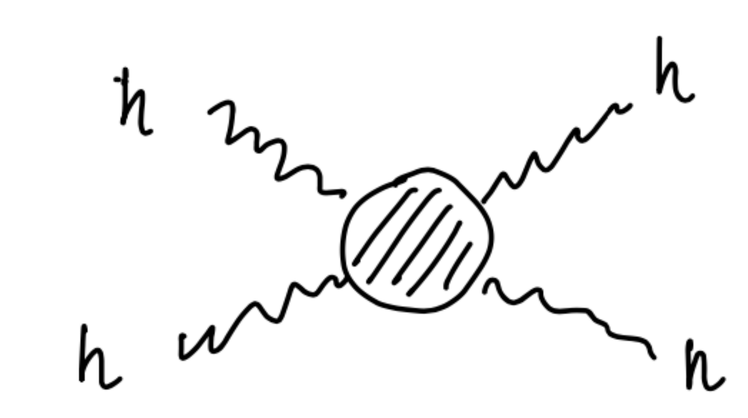
$$M_{\rm s} \propto (g_s)^{\frac{1}{4}} M_{\rm p}$$

Can we confine the space for allowed $\hat{\alpha}_i$ → Can we carve out the landscape of perturbative strings ?

The Gravitation S-matrix

The EFT operators are encoded in the four-graviton S-matrix which is subject to it's own consistency

$$M(s,t)$$



$$s = (p_1+p_2)^2 = E_c^2$$

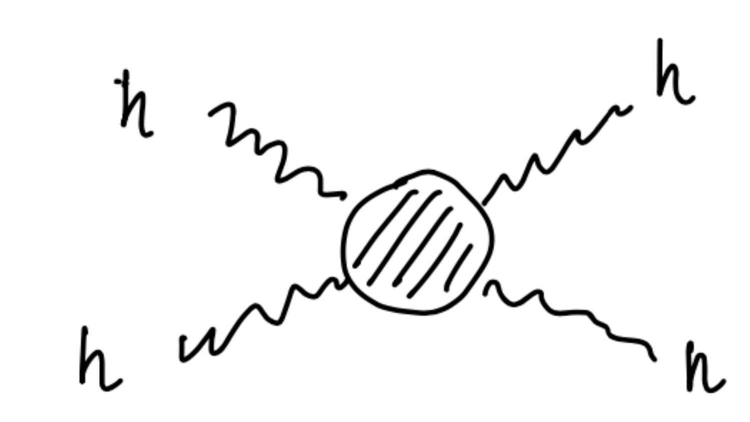
$$t = (p_1-p_4)^2 = -\frac{E_c^2}{2}(1-\cos\theta)$$

It is well defined (infrared finite) for D>4 but divergent in D=4

→ work with regulated observables, their axiomatic properties are less understood

→ restrict ourselves to perturbative (tree)-limit

The Gravitation S-matrix

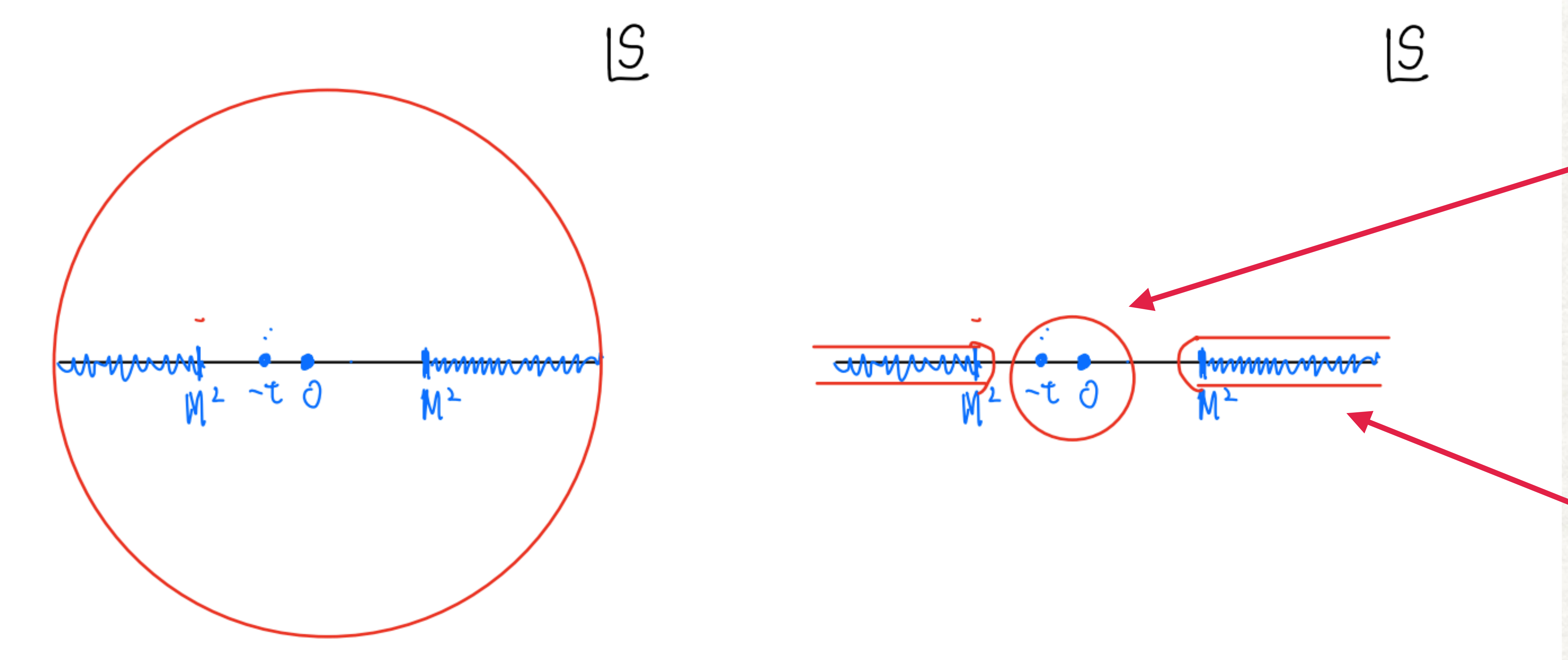


EFT information is embedded in the low-energy limit of $M(s,t)$

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R+\hat{\alpha}_1\frac{R^2}{M^2}+\hat{\alpha}_2\frac{R^3}{M^4}+\hat{\alpha}_4\frac{R^4}{M^6}\cdots\right)$$

For perturbative completion we can keep M_{pl} large, loops are suppressed

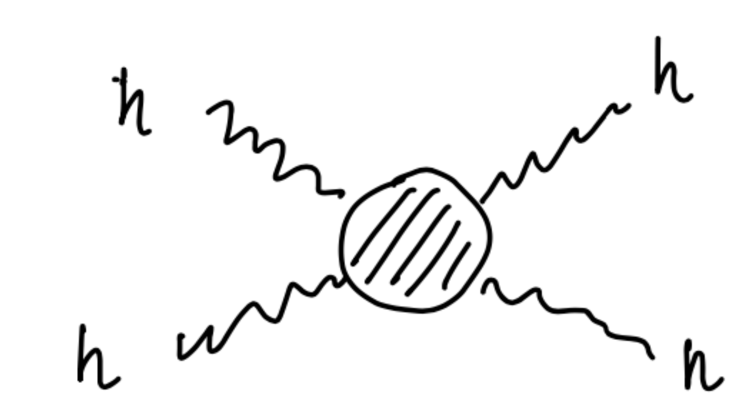
$$M^{\rm IR}(s,t)=R^4\left(\frac{1}{stu}+\{\text{massless poles from } R^2, R^3\}+\sum_{k,q} b_{k,q}s^{k-q}t^q\right)$$



$$b_{k,q}=\frac{1}{2\pi i}\frac{\partial^q}{\partial t^q}\oint\frac{ds}{s^{k-q+1}}M(s,t)$$

$$\frac{1}{\pi}\frac{\partial^q}{\partial t^q}\int_{m^2}^{\infty}\frac{ds}{s^{k-q+1}}\,{\rm Im}[M(s,t)]$$

The Gravitation S-matrix

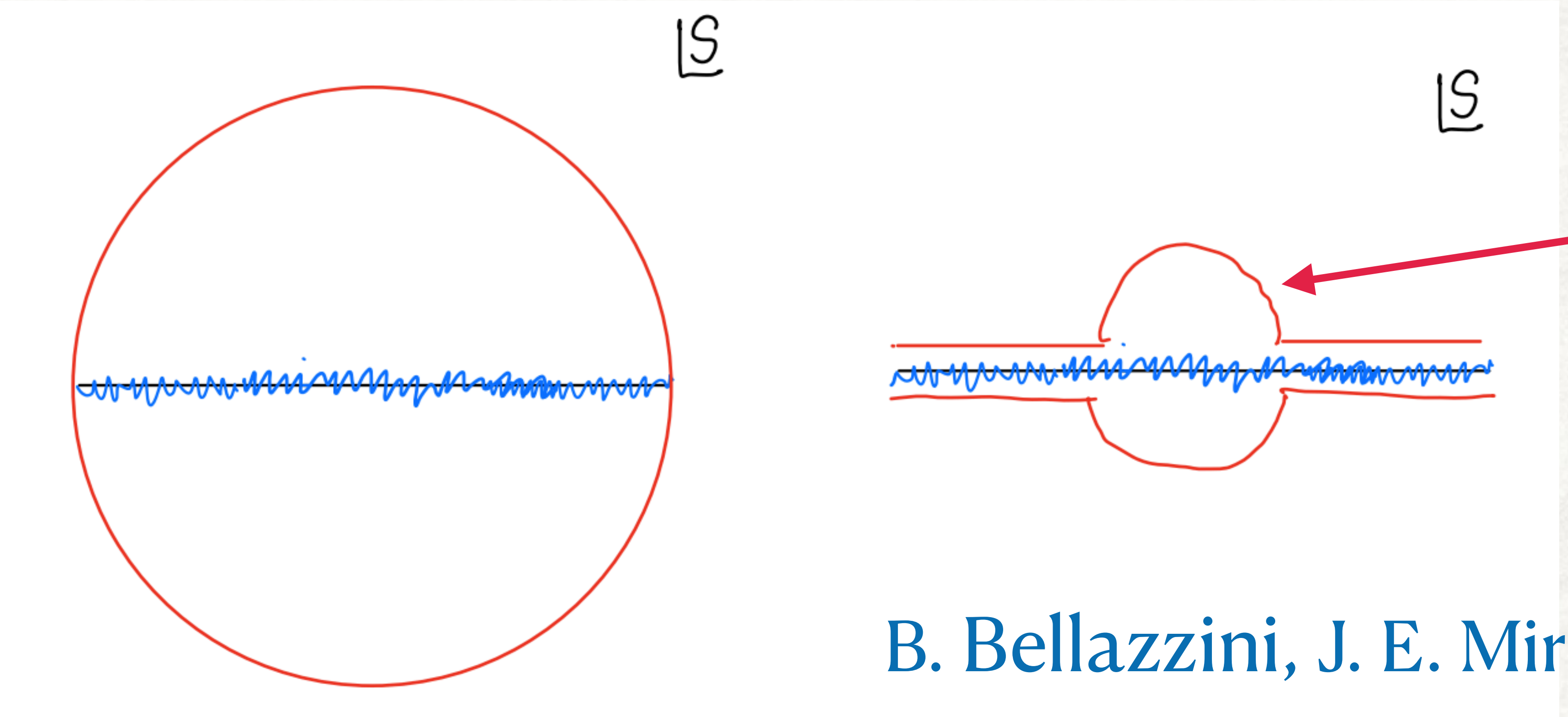


EFT information is embedded in the low-energy limit of $M(s,t)$

From non-perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2} \left(R + \hat{\alpha}_1 \frac{R^2}{M_{\rm pl}^2} + \hat{\alpha}_2 \frac{R^3}{M_{\rm pl}^4} + \hat{\alpha}_4 \frac{R^4}{M_{\rm pl}^6} \cdots \right)$$

$$M^{\rm IR}(s,t) = \{\text{massless poles from } R,\ R^2,\ R^3\} + \text{polynomials} + \text{massless branch cuts}$$



calculable from EFT

$$b^{\mathcal{C}}_{k,q} = \frac{1}{2\pi i} \frac{\partial^q}{\partial t^q} \int_{\mathcal{C}} \frac{ds}{s^{k-q+1}} M(s,t)$$

$$\frac{1}{\pi} \frac{\partial^q}{\partial t^q} \int_{m^2}^{\infty} \frac{ds}{s^{k-q+1}} \text{Im}[M(s,t)]$$

B. Bellazzini, J. E. Miró, R. Rattazzi, M. Riembau, F. Riva 2011.00037

The Gravitation S-matrix

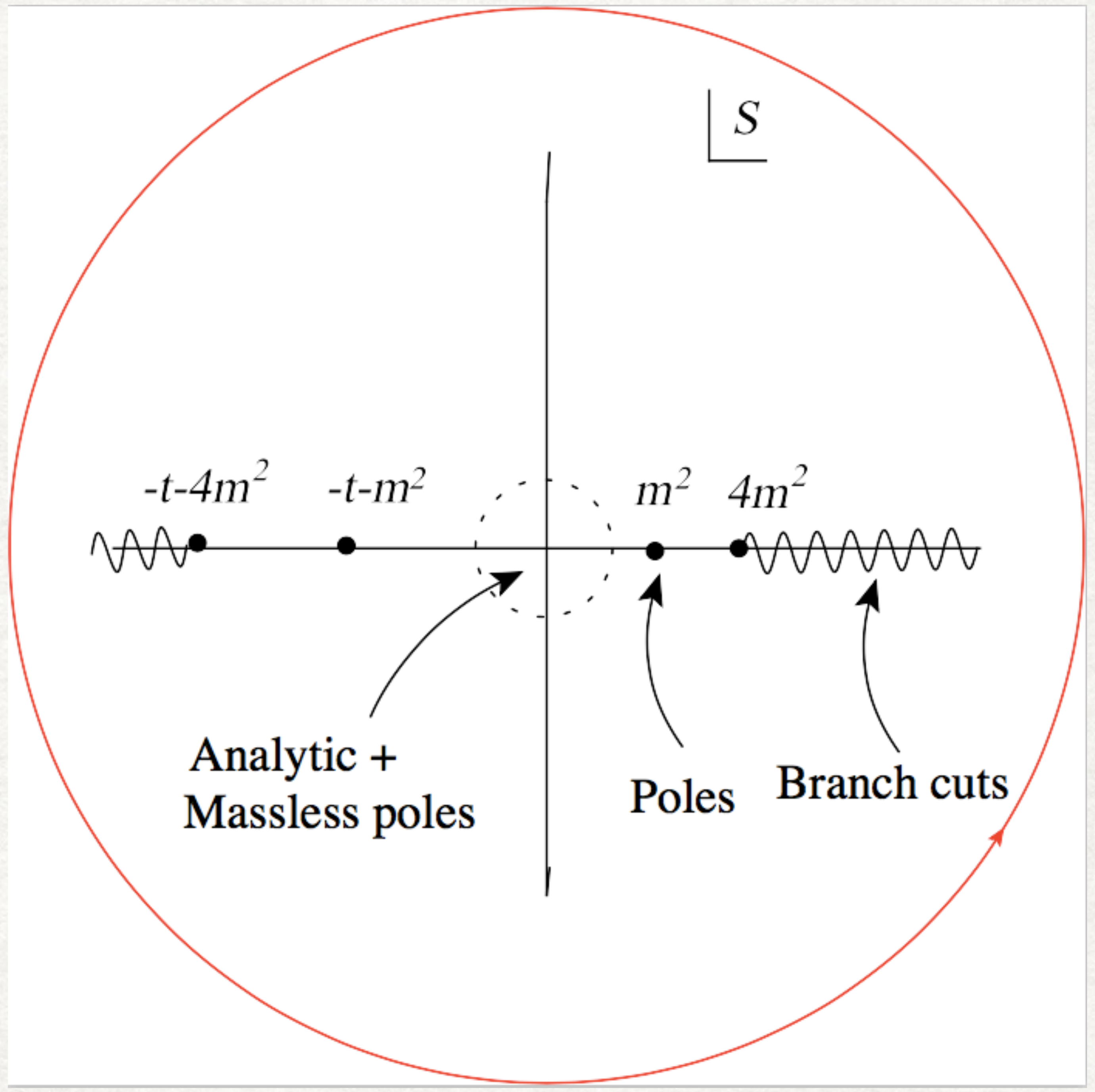
Arising from perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R+\hat{\alpha}_1\frac{R^2}{M^2}+\hat{\alpha}_2\frac{R^3}{M^4}+\hat{\alpha}_4\frac{R^4}{M^6}\cdots\right)$$

The coefficients can be derived from a contour integral of $M(s,t)$

$$b_{n+q,q} = \frac{\partial^q}{\partial t^q}\oint\frac{ds}{s^{n+1}}M(s,t)$$

$M^{\rm sub}$



- Analyticity: M(s,t) is analytic away from the real s-axes for fixed t

$$M(s,t^*) = M^{\rm sub} + \int_{M^2}^{\infty}\frac{{\rm Im}[M(s,t^*)]}{s-m^2} + \int_{M^2}^{\infty}\frac{{\rm Im}[M(s,t^*)]}{u-m^2}$$

$$M(s,t^*) = M^{\rm sub} + \int_{M^2}^{\infty}\frac{{\rm Im}[\rho_j(s)]P_j^s\left(1+\frac{2t^*}{m^2}\right)}{s-m^2} + \int_{M^2}^{\infty}\frac{{\rm Im}[\rho_j(s)]P_j^u\left(1+\frac{2t^*}{m^2}\right)}{u-m^2}$$

- Unitarity: $0 \leq {\rm Im}[\rho_j(s)]$

positivity
optical theorem

The Gravitation S-matrix

At large s the amplitude satisfies twice subtraction (at fixed t<0)

Haring, Zhiboedov 2202.08280

$$\lim_{|s|\to\infty} \frac{M(s,t)}{|s|^2} = 0$$

At fixed large impact parameter b scattering is well described by GR

Since fixed b at large energy corresponds to large spin $b \equiv \frac{2J}{\sqrt{s}}$

$$M(s,t) = \frac{1}{2}\sum_{J=0}^{\infty} n_J^d f_J(s) P_J^d(1+2t/s) = -\frac{8\pi G_N s^2}{t}\left(1 + O(G_N s|t|^{\frac{d-4}{2}})\right) \rightarrow f_J(s) \simeq \frac{\Gamma\left(\frac{d-4}{2}\right)}{(4\pi)^{\frac{d-4}{2}}} \frac{G_N s}{J^{d-4}}, \quad J \to \infty$$

The Gravitation S-matrix

- Since the amplitude is bounded by s^2 at $s \to \infty$

for $n \geq 2$
$$b_{n+q,q} = \frac{1}{\pi}\frac{\partial^q}{\partial t^q}\int_{m^2}^{\infty}\frac{ds}{s^{k-q+1}}\,\mathrm{Im}[M(s,t)]$$

- For $n=2$ the low energy graviton pole contributes

$$M^{\mathrm{IR}}(s,t) = R^4\left(\frac{1}{stu}+\{\text{massless poles from } R^2, R^3\} + \sum_{k,q} b_{k,q}s^{k-q}t^q\right)\Bigg|_{t\to 0} = \frac{s^2}{t}+\cdots$$

The fact that the subtraction term is absent means that the the imaginary part must reproduce the t-pole, i.e. it Reggeizes

$$-\frac{8\pi G_N s^2}{t}\left(1+O(G_N s|t|^{\frac{d-4}{2}})\right) = \int_{M^2}^{\infty}\frac{\mathrm{Im}[\rho_j(s)]P_j^s\left(1+\frac{2t^*}{m^2}\right)}{s-m^2}+\int_{M^2}^{\infty}\frac{\mathrm{Im}[\rho_j(s)]P_j^u\left(1+\frac{2t^*}{m^2}\right)}{u-m^2}$$

the sum will not converge as t->0

- Consider smeared amplitude Caron-Huot, Mazac, Rastelli, Simmons-Duffin 2102.08951

$$\int_0^\infty dp\, f(p) \frac{8\pi G}{p^2}$$

but the EFT coefficients can no longer be identified via forward limit expansion

$$\mathcal{C}_{2,u}|_{\text{EFT}} = \frac{8\pi G}{-u} + 2g_2 - g_3 u + 8g_4 u^2 - 2g_5 u^3 + 24 g_6 u^4 - 4g_7 u^5 \dots ,$$
$$\mathcal{C}_{4,u}|_{\text{EFT}} = 4g_4 - 2g_5 u + (24 g_6 + g_6')u^2 - 8g_7 u^3 + \dots ,$$
$$\mathcal{C}_{6,u}|_{\text{EFT}} = 8g_6 - 4g_7 u + \dots .$$

$$\longrightarrow \quad \mathcal{C}_{2,u}^{\text{improved}}\Big|_{\text{EFT}} = \frac{8\pi G}{-u} + 2g_2 - g_3 u$$

- Regge subtractions J. Tokuda, K. Aoki, S. Hirano, 2007.15009

K. Aoki, T-Q Loc, T. Noumi, J. Tokuda, 2104.09682

See Junsei Tokuda's & Sota Sato's talk

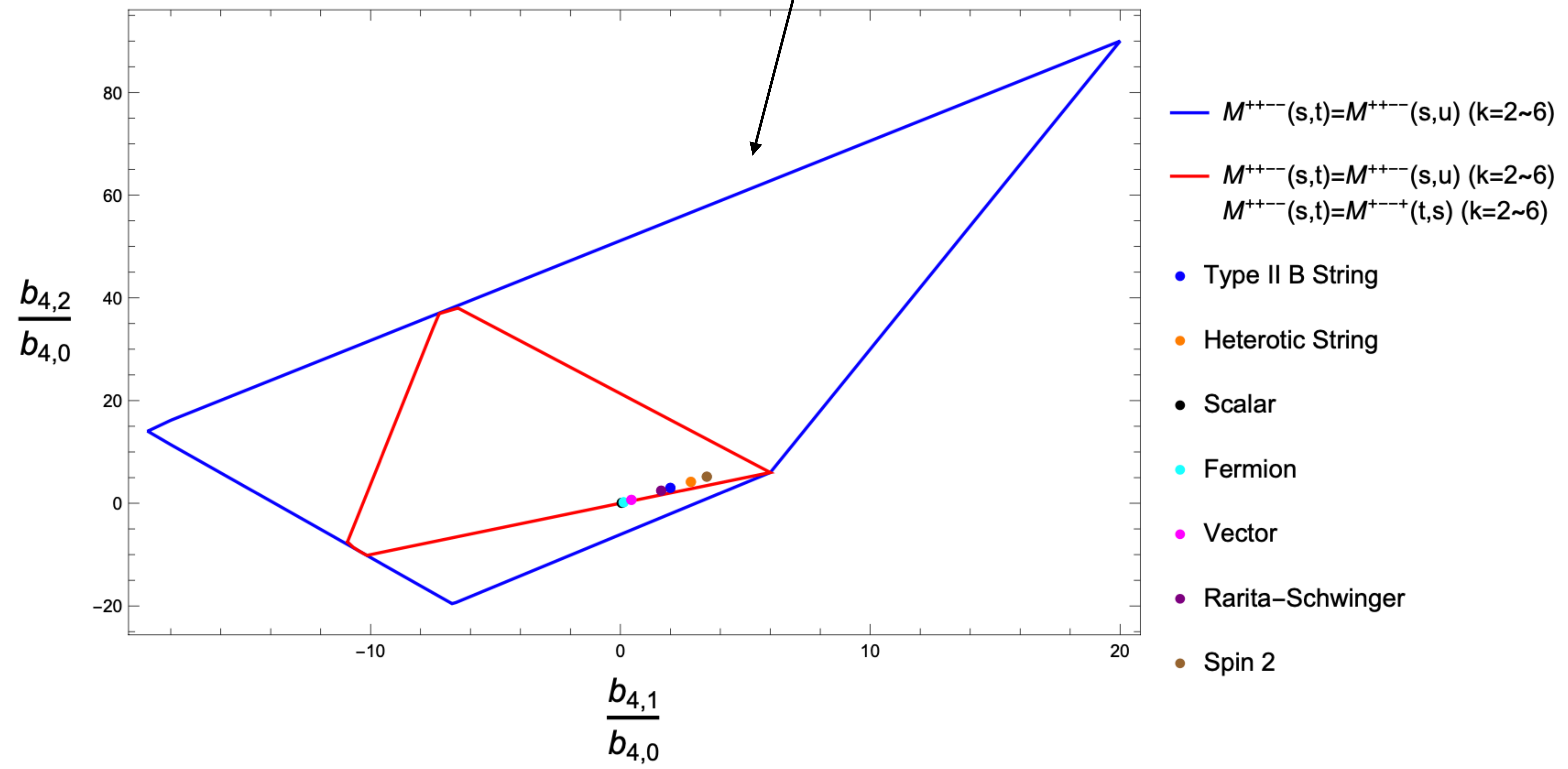
$$c_2(t) = \frac{4}{\pi}\int_{M_s^2}^\infty \mathrm{d}s \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{(s+(t/2))^3} + \frac{2}{M_{\text{pl}}^2 t} \quad \longrightarrow \quad c_2(0) > F_0 > -\mathcal{O}(M_{\text{pl}}^{-2} M^{-2})$$

Forward-limit bounds for gravitational S-matrix

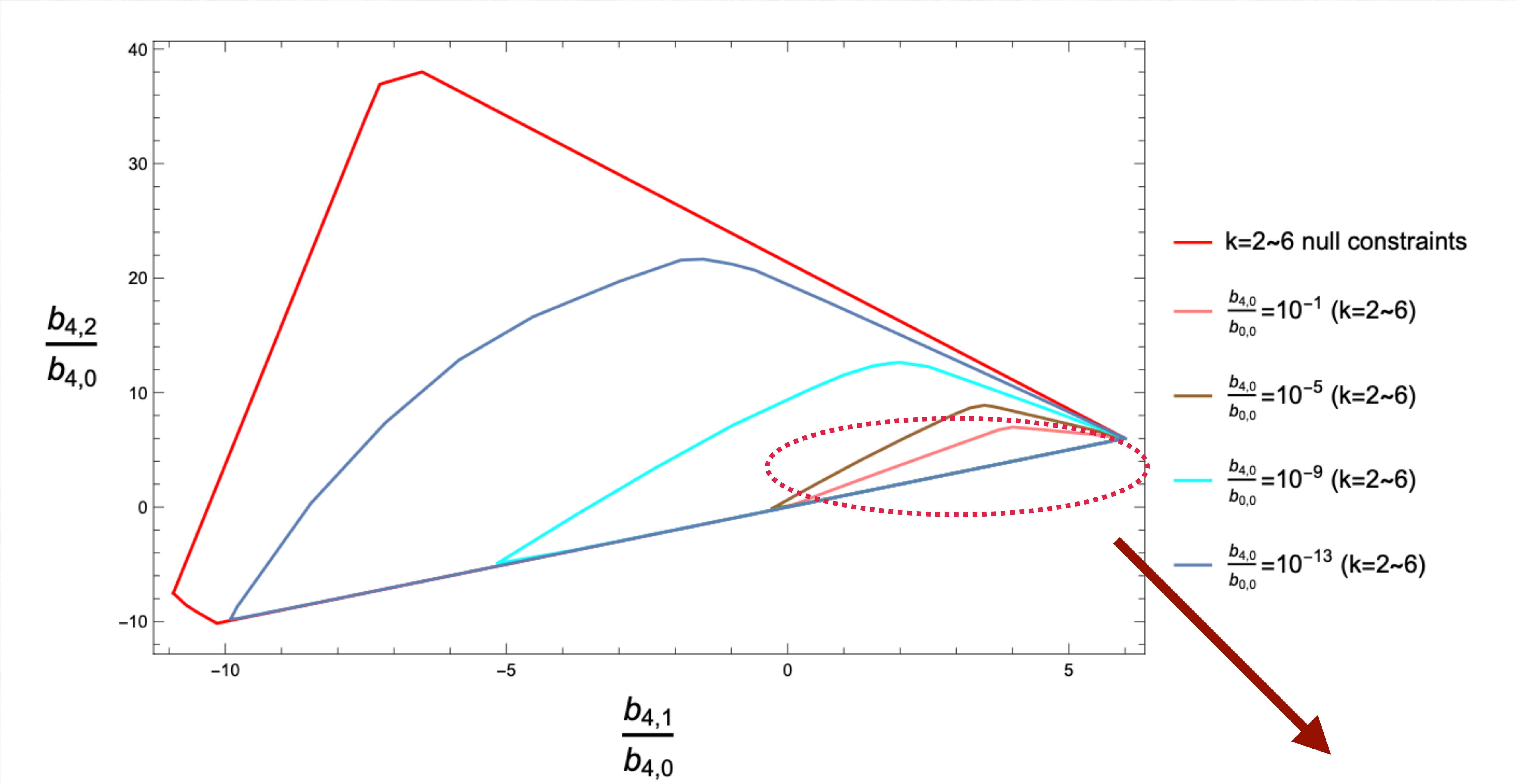
For fixed derivative couplings, with SDPB Li-Yuan Chiang, Y-t H, Wei Li, Laurentiu Rodina, He-Chen Weng 2105.02862

$D^8 R^4$

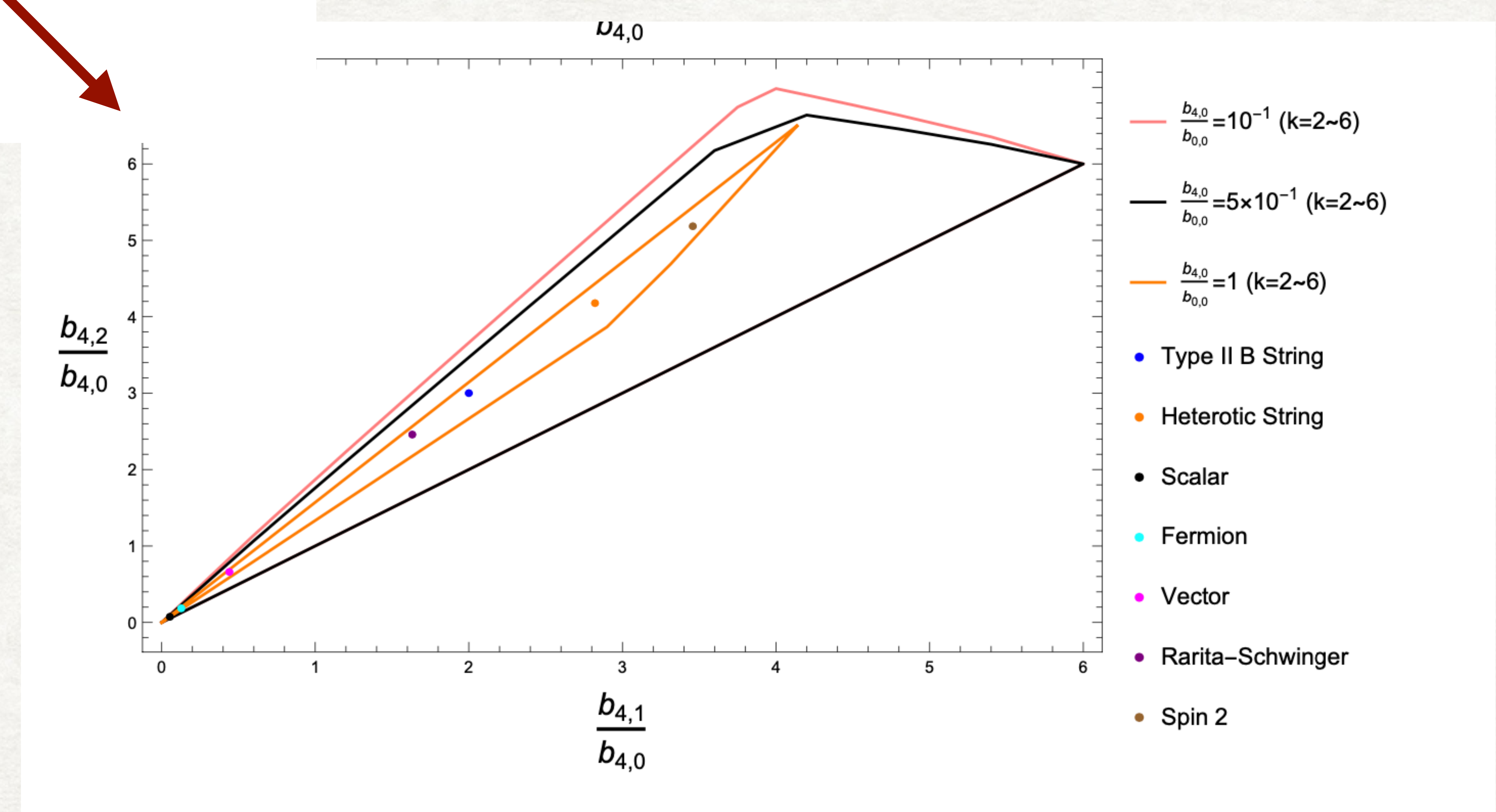
Z. Bern D. Kosmopoulos, A Zhiboedov 2103.12729



The space ``appears” to be still too large, but when viewed in the higher dimensions

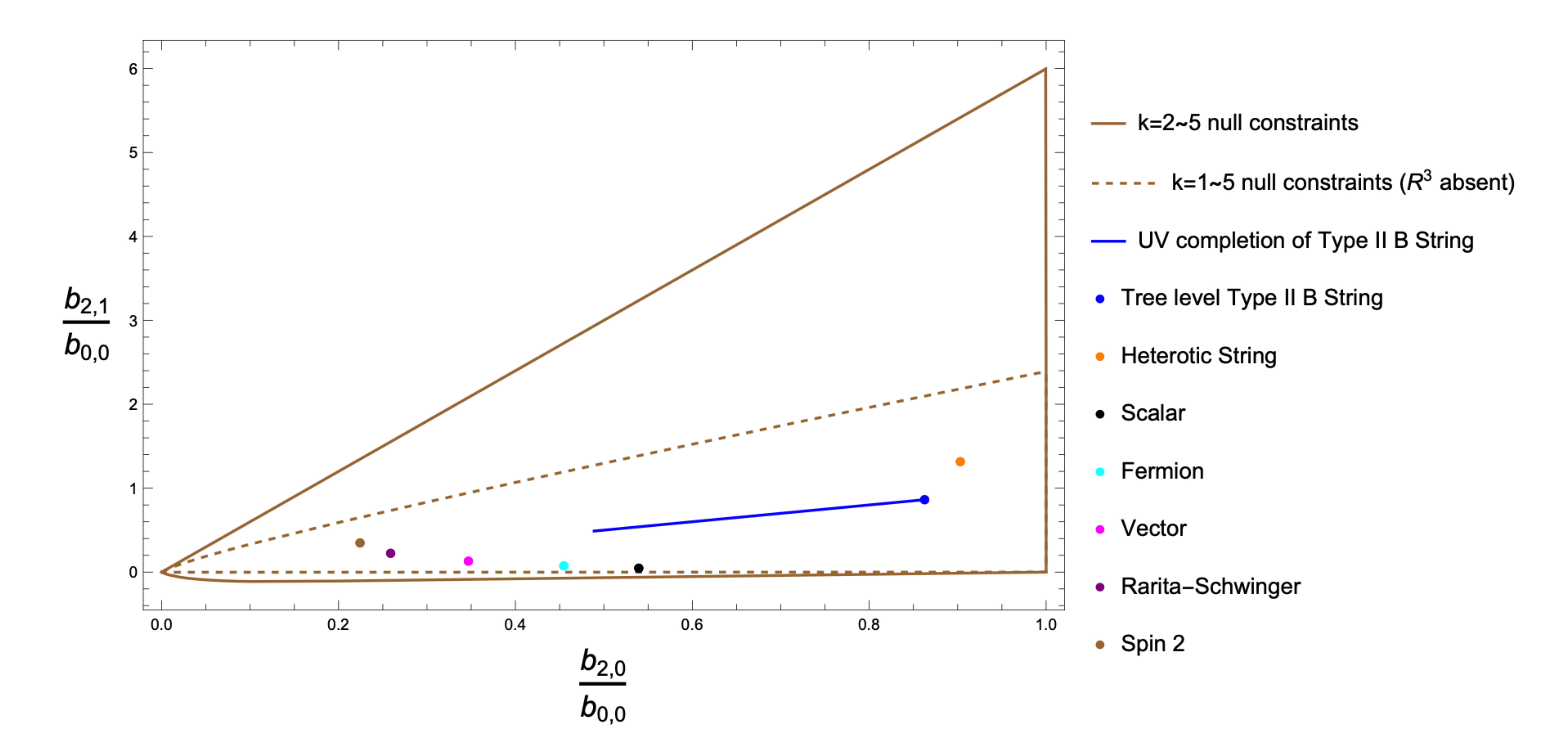


contour for extremal $\left(\frac{b_{4,1}}{b_{4,0}}, \frac{b_{4,2}}{b_{4,0}}\right)$ with $\frac{b_{4,0}}{b_{0,0}}$ held fixed.



The space is extremely flat, with a majority fine tuned

Indeed, when we consider ratios of couplings of different dimensions, the known theories start to span larger regions



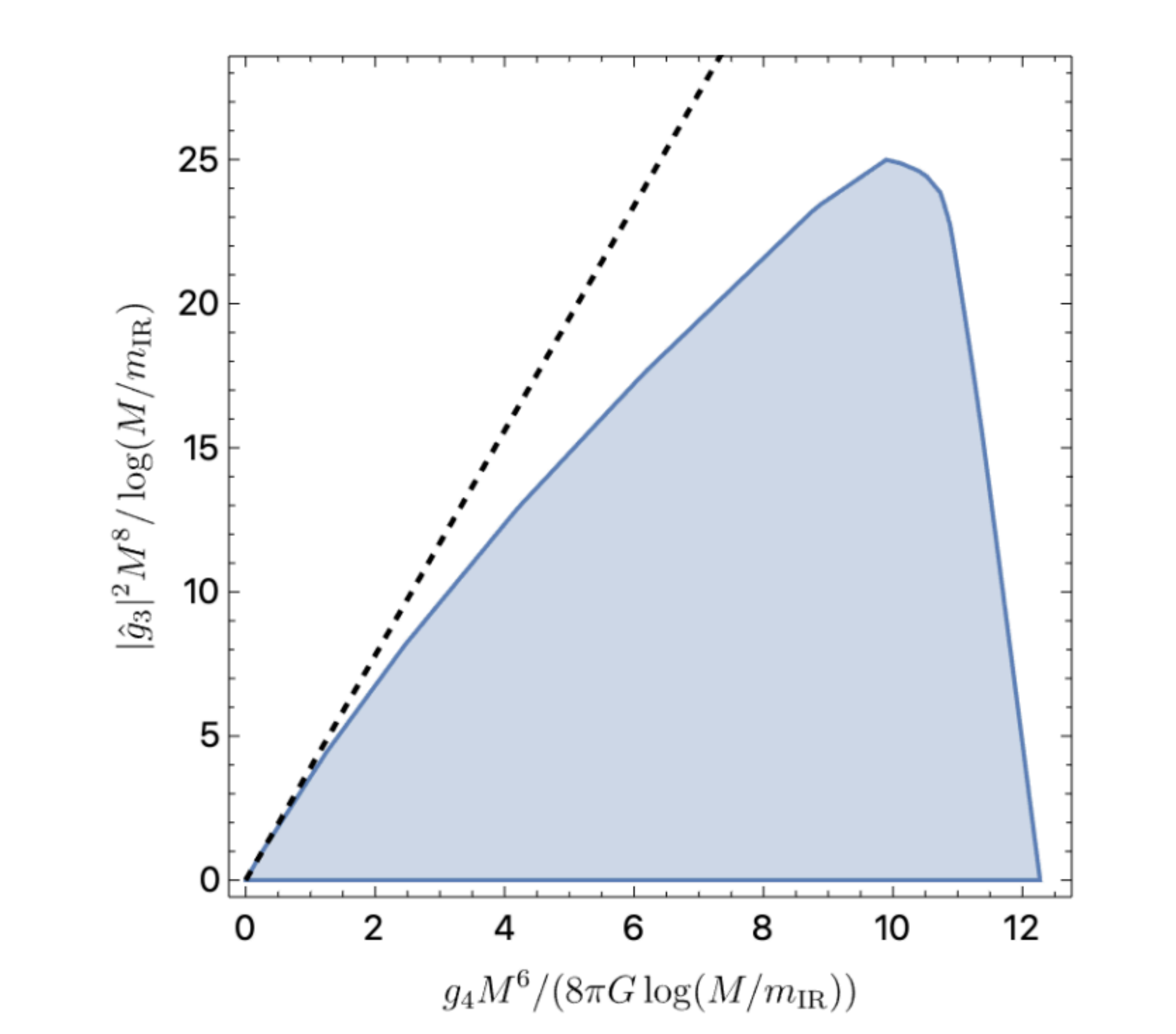
D^4R^4 operators normalized by R^4 coefficients.

Finite impact parameter bounds

Bounds with respect to G_N

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R - \frac{1}{3!}\left(\alpha_3 R^{(3)} + \tilde{\alpha}_3 \tilde{R}^{(3)}\right) + \frac{1}{4}\left(\alpha_4 (R^{(2)})^2 + \alpha_4'(\tilde{R}^{(2)})^2 + 2\tilde{\alpha}_4 R^{(2)}\tilde{R}^{(2)}\right) + \dots\Big]$$

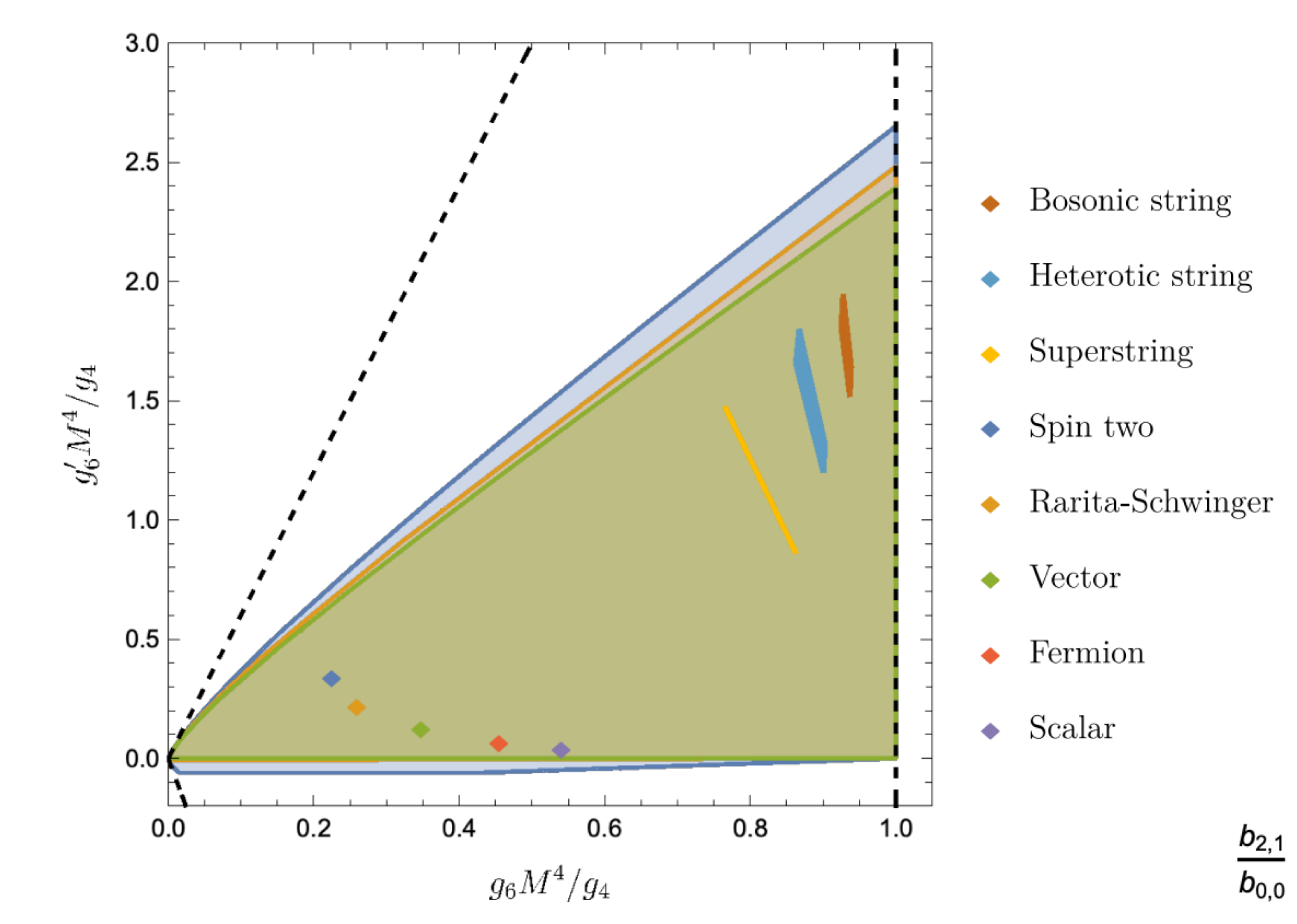
$$\widehat{g}_3 = \alpha_3 + i\tilde{\alpha}_3, \qquad g_4 = 8\pi G(\alpha_4 + \alpha_4'), \qquad \widehat{g}_4 = 8\pi G(\alpha_4 - \alpha_4' + i\tilde{\alpha}_4)$$

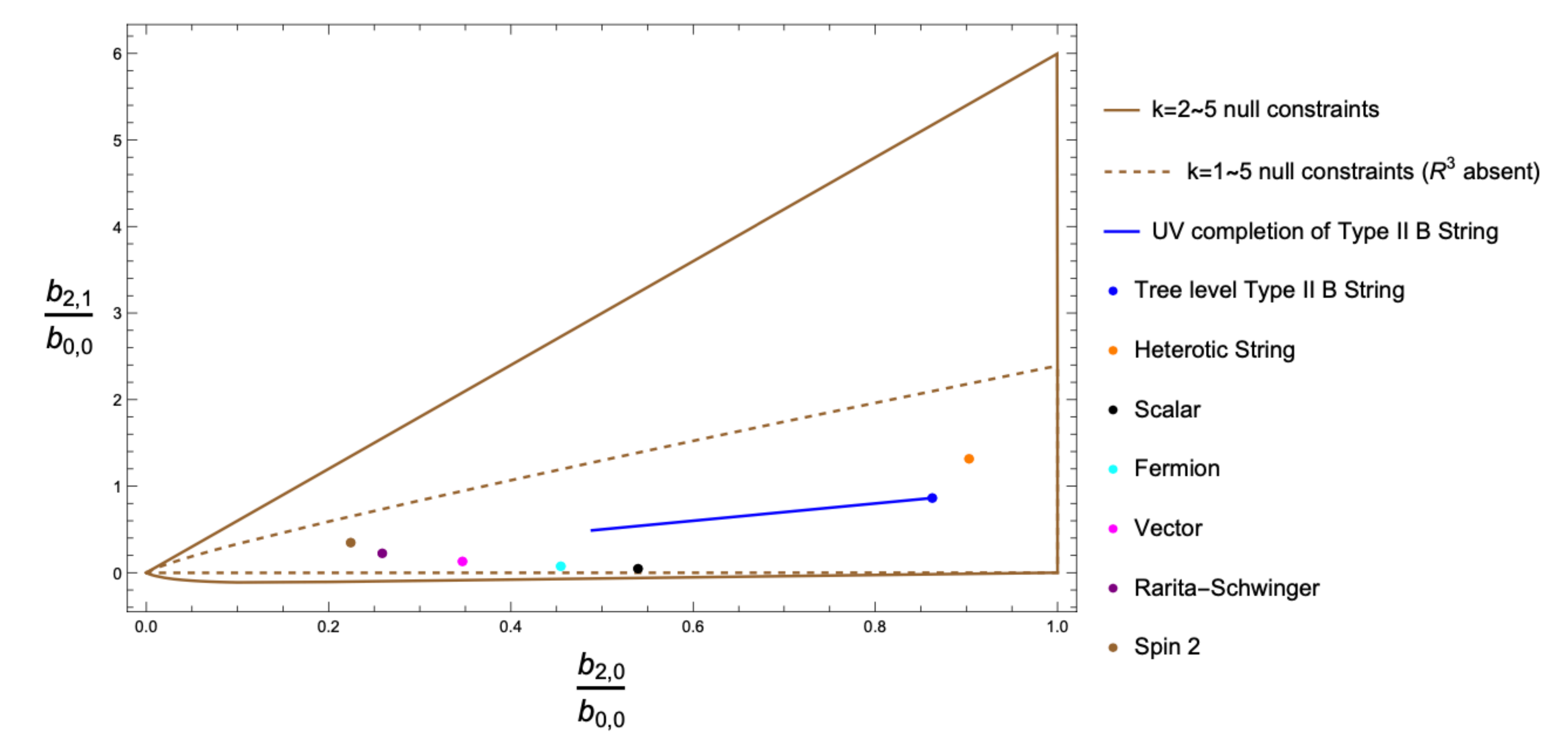


Caron-Huot, Li, Parra-Martinez, Simmons-Duffin 2201.06602

Matches the forward limit bounds

Caron-Huot, Li, Parra-Martinez, Simmons-Duffin 2201.06602





Impose further constraint?
Top-down ??

Perturbative sting amplitude

Being a theory of quantum gravity, string amplitudes are known to exhibit exponential suppression at high energies fixed angle scattering

Gross, Mende, PLB 303 issue 3,4 1988

$$A^{(4-\text{tachyon})}_{\text{closed}}(s,t,u) \simeq \frac{\sin(\pi t/2)\sin(\pi u/2)}{\sin(\pi s/2)}(stu)^{-3}\exp\left(-\frac{s\ln s + t\ln t + u\ln u}{4}\right)$$

This originated from the saddle point approximation to the world sheet integral

$$\int d^2z\,(1-z)^s(1-\bar{z})^s z^t \bar{z}^t \;\; f(z,\bar{z}) = \int d^2z\; e^{s\ln|1-z|^2 + t\ln|z|^2} \;\; f(z,\bar{z})$$

$$s\to\infty, \quad \frac{t}{s} = \text{fixed} \quad \longrightarrow \quad z = \frac{t}{u}$$

how do we incorporate this in our bootstrap?

Perturbative sting amplitude

For perturbative string, the amplitude is given by a world sheet integral of a 2D CFT

$$\int d^2z\ (1-z)^s(1-\bar{z})^s z^t \bar{z}^t\ \ f(z,\bar{z})$$

With the left-right factorizing, it can be written as a product of open string amplitudes (KLT)

$$\int_0^1 dz\ (1-z)^s z^t\ f(z,\bar{z})$$

We would like to explore the Koba-Nielson refactors on our EFT

Perturbative sting amplitude

Let us start with the pre-SL(2,C) fixed form

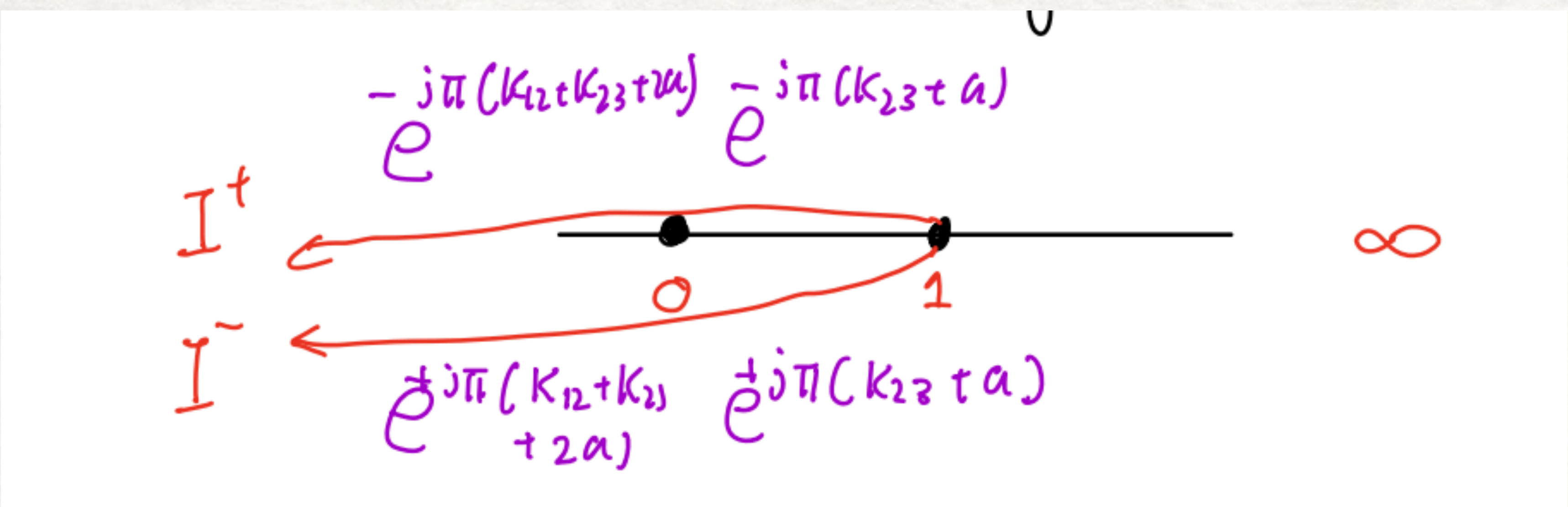
$$\int_{-\infty < z_1 < z_2 < z_3 < z_4 < \infty} \prod_{i=1}^{4} dz_i \qquad |z_i - z_j|^{k_{ij}} \quad f(z_i)$$

The only constraint on $f(z_i)$ to be permutation invariant, reflecting the use of identical vertex operator. After SL(2,C) fixing, distinct ordered amplitude takes the form

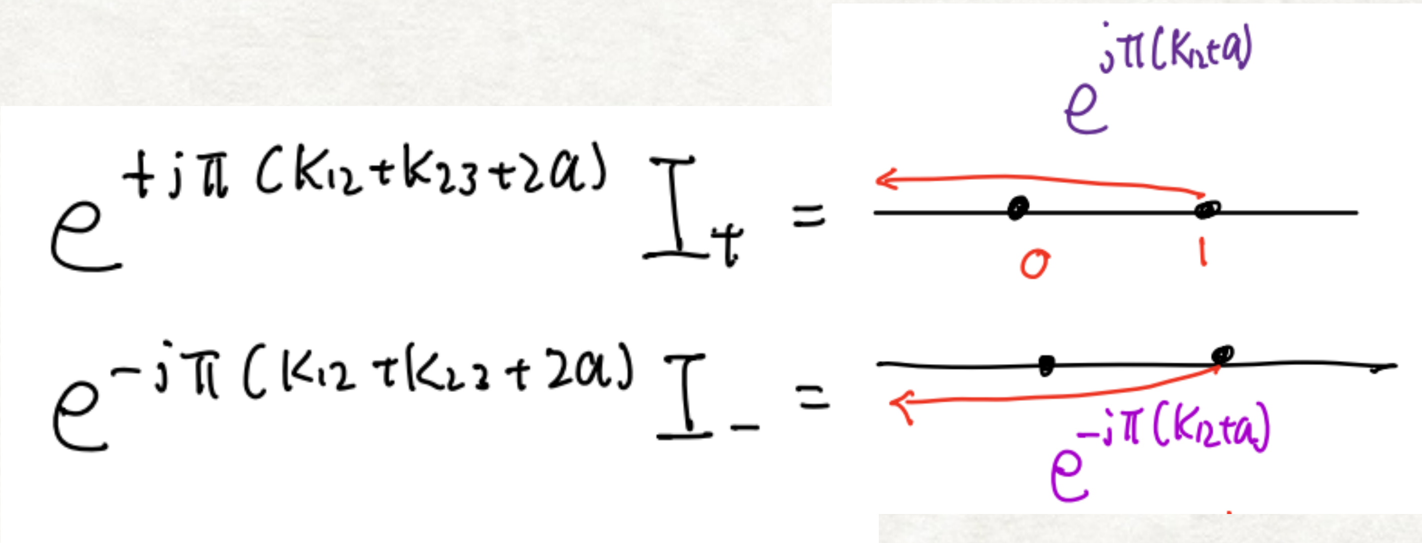
$$A(1234) = \int_0^1 dx \; x^{k_{12}+a} \, (1-x)^{k_{23}+a} \, f(x)$$

$$A(1324) = \int_1^\infty dx \; x^{k_{12}+a} \, (x-1)^{k_{23}+a} \, f(x)$$

The two amplitudes are obviously related by contour deformation $\quad I^+ = I^- = A(1324)$



→



→

$$\sin\pi(-k_{13}+a)\, A(1324) = -\sin\pi(k_{12}+a)\, A(1234)$$

Monodromy relations!

Bjerrum-Bohr, Damgaard, Vanhove, *Phys.Rev.Lett.* 103 (2009) 161602

Perturbative sting amplitude

For amplitudes with the massless pole we begin with trivial monodromy

$$A\left(s,u\right)+e^{i\pi s}A\left(s,t\right)+e^{-i\pi u}A\left(t,u\right)=0$$

Let's first consider maximal supersymmetry, $A(s,t)=\delta^{8}(Q)\;f(s,t)$

$$f(s,t)=-\frac{1}{st}+\left(\frac{b}{s}+\frac{b}{t}\right)+\left(c\frac{t}{s}+c\frac{s}{t}\right)+g_{00}+(g_{1,0}s+g_{1,1}t)+\sum_{k\geq q\geq 0}g_{k,q}s^{k-q}t^{q}$$

Crossing symmetry + monodromy relations imply b=0 and

$$\begin{pmatrix} g_{00} & & & & \\ g_{1,0} & g_{1,1} & & & \\ g_{2,0} & g_{2,1} & g_{2,2} & & \\ g_{3,0} & g_{3,1} & g_{3,2} & g_{3,3} & \\ g_{4,0} & g_{4,1} & g_{4,2} & g_{4.3} & g_{4,4} \end{pmatrix}=\begin{pmatrix} \frac{\pi^2}{6} & & & & \\ g_{1,0} & g_{1,0} & & & \\ \frac{\pi^4}{90} & \frac{\pi^4}{360} & \frac{\pi^4}{90} & & \\ g_{3,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & 2g_{3,0}-\frac{\pi^2}{6}g_{1,0} & g_{3,0} & \\ \frac{\pi^6}{945} & g_{4,1} & -\frac{\pi^6}{15120}+2g_{4,1} & g_{4,1} & \frac{\pi^6}{945} \end{pmatrix}$$

Now impose unitarity

Geometry from unitarity

At fixed-t the open string only s-channel thresholds are present, unitarity implies

$$g^s_{k,q} = \sum_a p_a \frac{v_{\ell_a,q}}{m_a^{2(k+1)}} \qquad G_\ell(1+2\delta) = v_{\ell,0}+v_{\ell,1}\delta+\cdots v_{\ell,\ell}\delta^\ell = \sum_{q=0}^{\ell} v_{\ell,q}\delta^q$$

We have a product geometry

$$\begin{pmatrix} g_{0,0} & & & \\ g_{1,0} & g_{1,1} & & \\ g_{2,0} & g_{2,1} & g_{2,2} & \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} = \sum_a p_a \begin{pmatrix} \frac{1}{m_a^2} \\ \frac{1}{m_a^4} \\ \frac{1}{m_a^6} \\ \frac{1}{m_a^8} \\ \vdots \end{pmatrix} \otimes (v_{\ell_a,0}, v_{\ell_a,1}, v_{\ell_a,2}, \cdots)$$

Note that v is a simple polynomial in J

$$v_{\ell,q} = \frac{2^q}{q!(2-q)!}\frac{(\alpha)_{\ell+q}}{\prod_{a=1}^{q}(\alpha+2a-1)} = \frac{\prod_{a=0}^{q}(J-a(a-1))}{(q!)^2} \qquad J = \ell(\ell+1).$$

After a linear transformation $\vec{a}^T = \vec{g}^T\mathbf{G}$ we have

at the core, the couplings are governed by the hull of product moments →

$$\begin{pmatrix} a_{0,0} & & & \\ a_{1,0} & a_{1,1} & & \\ a_{2,0} & a_{2,1} & a_{2,2} & \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} = \sum_a p_a \begin{pmatrix} \frac{1}{m_a^2} \\ \frac{1}{m_a^4} \\ \frac{1}{m_a^6} \\ \frac{1}{m_a^8} \\ \vdots \end{pmatrix} \otimes (1, J, J^2, J^3, \cdots)$$

≥ 0

Li-Yuan Chiang, Wei Li, He-Chen Wen, Laurentiu Rodina, Y-T H 2105.02862

EFT for perturbative string theory

Huang, Liu, Rodina, Wang, 2008.02293
Chiang, Huang, Weng in progress

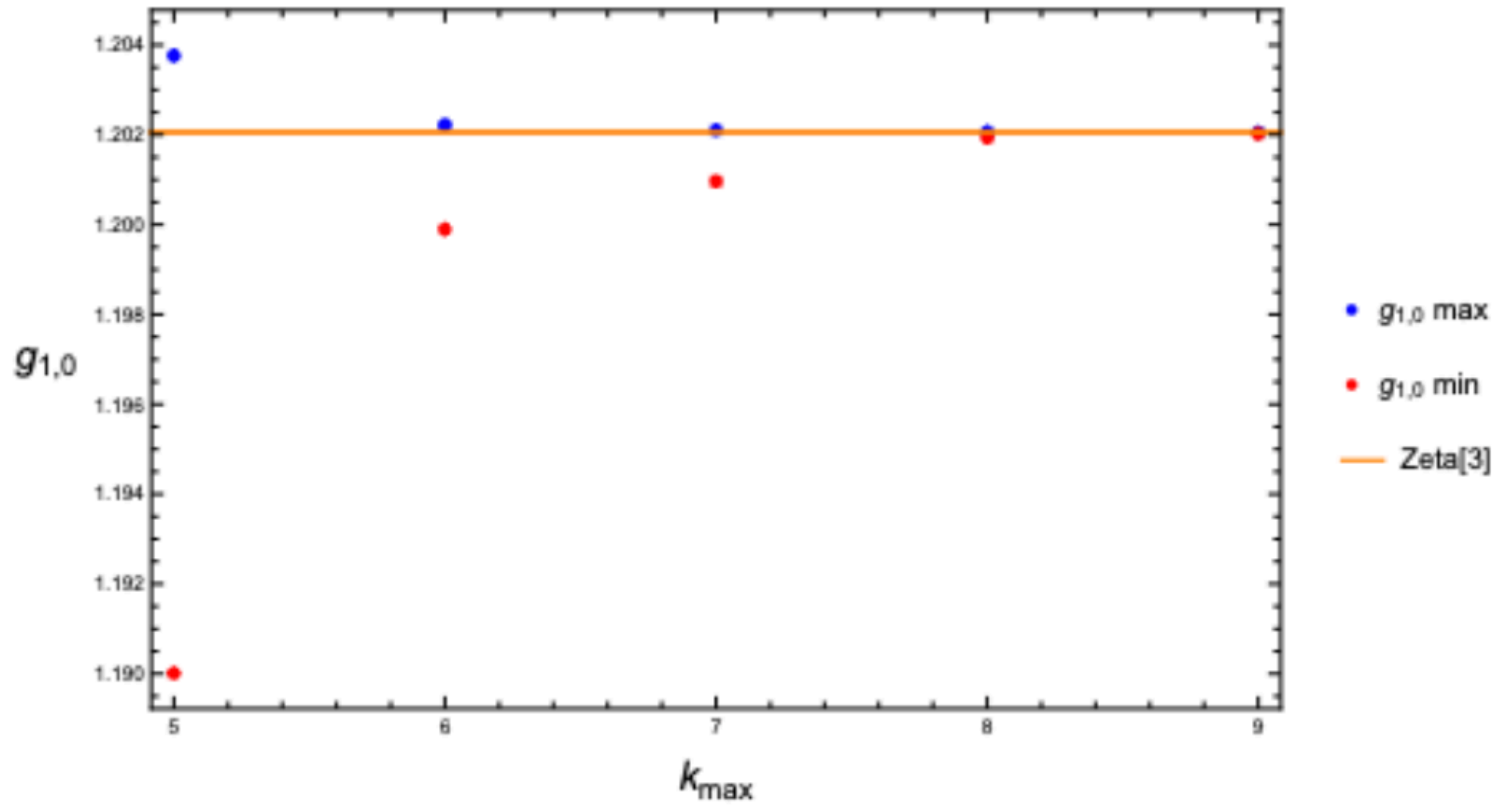


Figure 1. Bounds on $g_{1,0}$ plotted against the max derivative order.

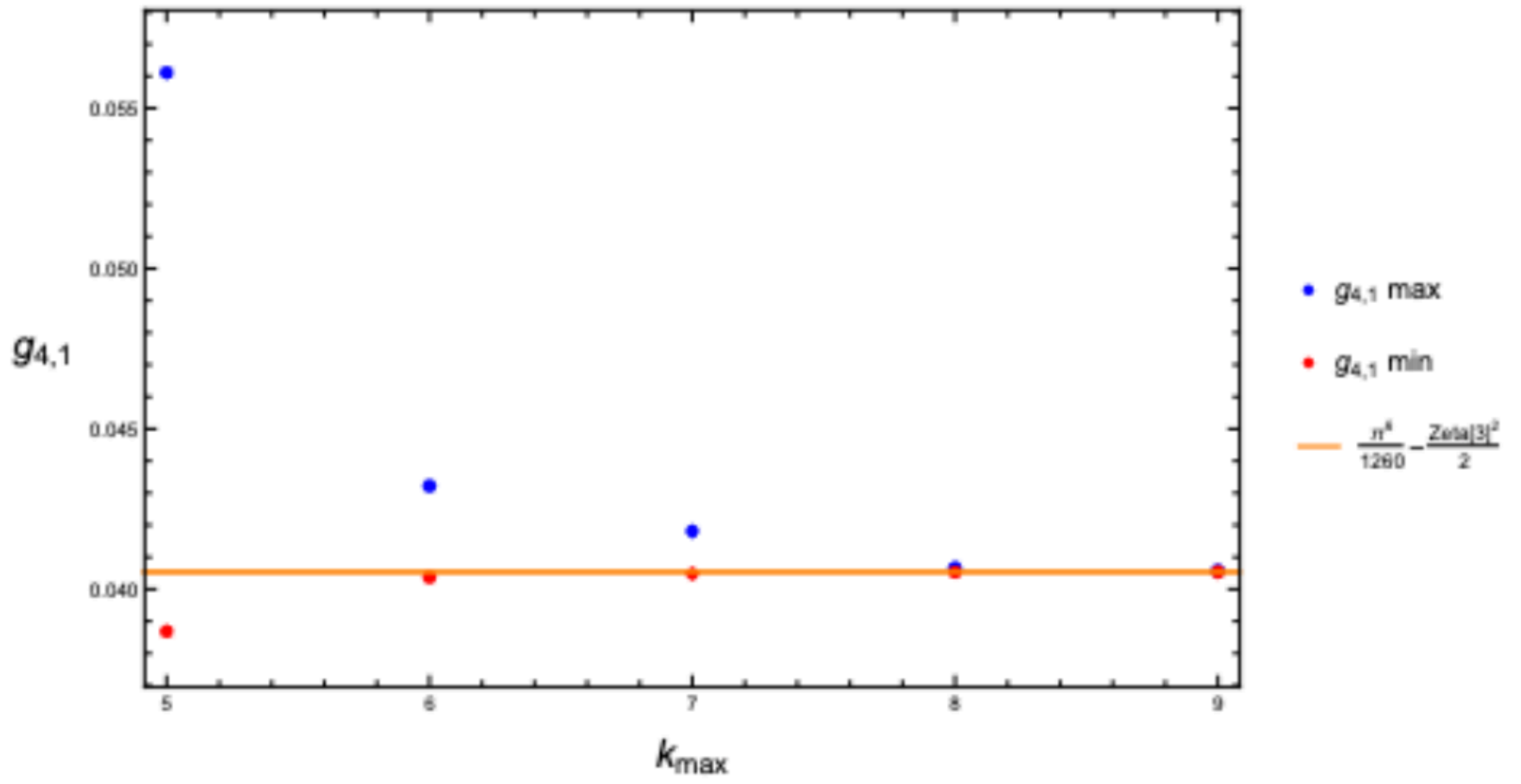


Figure 2. Bounds on $g_{4,1}$ plotted against the max derivative order.

$$g_{1,0} = \zeta(3) = 1.20205, \qquad \text{SDPB+monodromy}: \quad 1.20205 \geq g_{1,0} \geq 1.20201$$

The geometry of intersection between the monodromy plane and the EFThedron yields the four-point massless amplitude of Type-I superstring.

EFT for perturbative string theory

Huang, Liu, Rodina, Wang, 2008.02293
Chiang, Huang, Weng in progress

With trivial monodromy, we can easily KLT to obtain closed string EFT

$$M_{\text{closed string}}(s,t) = A(s,t)\sin(\pi s)\,A(s,-s-t) = \frac{-\pi}{st(s+t)} + \sum_{i,j=0}^{\infty} G_{i,j} s^{i-j} t^{i}$$

$$G_{0,0} = 2\pi g_{1,0}, \quad G_{2,0} = G_{2,1} = G_{2,2} = 2\pi g_{3,0},$$
$$G_{3,1} = G_{3,2} = -\pi\left(\frac{\pi^6}{630} + g_{1,0}^2 - 2g_{4,1}\right), \quad G_{3,0} = G_{3,3} = 0$$
$$G_{4,0} = G_{4,4} = 2\pi g_{5,0}, \quad G_{4,1} = G_{4,4} = 4\pi g_{5,0}, \quad G_{4,2} = 6\pi g_{5,0}$$

EFT for perturbative string theory

Chiang, Huang, Weng in progress

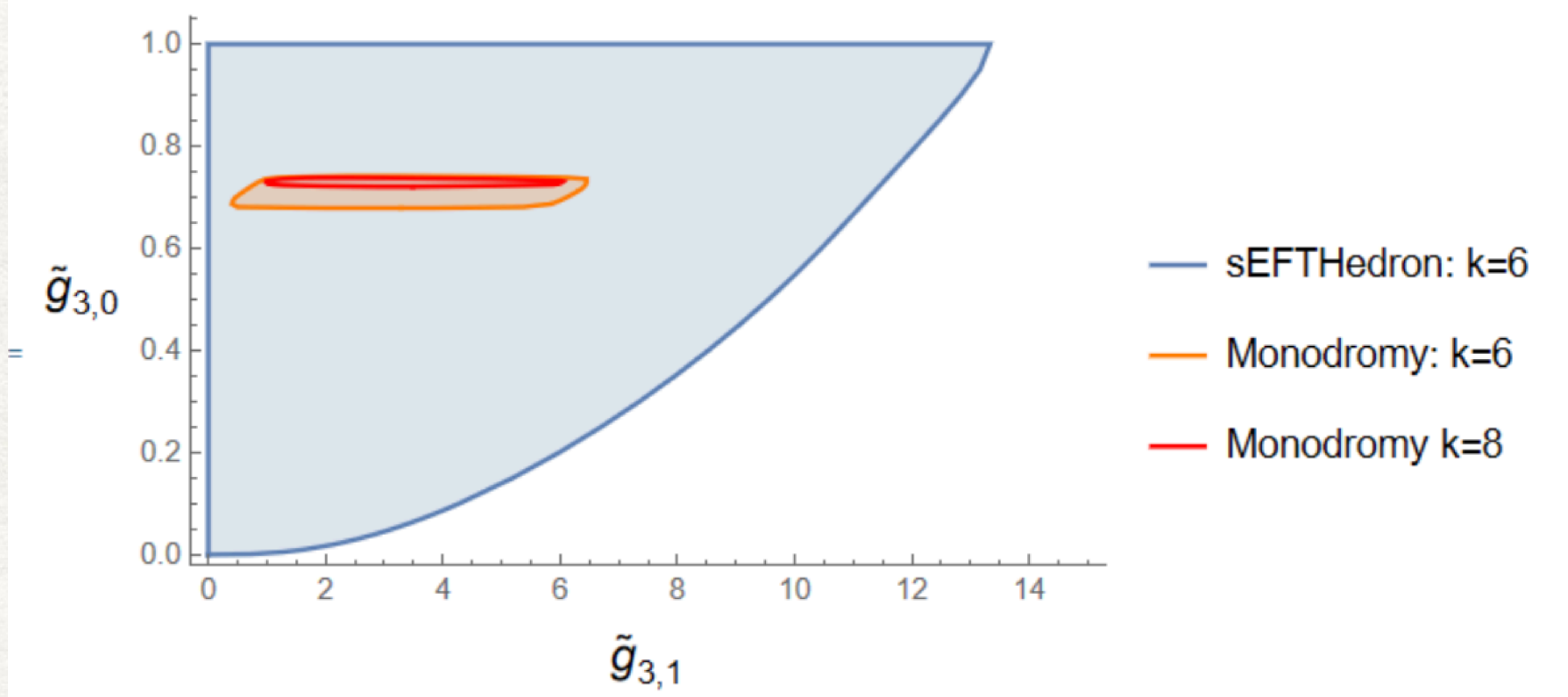
Now remove SUSY

$$A(s,t) = -\left(\frac{s}{t}+\frac{t}{s}\right) + b\left(\frac{1}{s}+\frac{1}{t}\right) + \sum_{k,q\geq 0} g_{k,q} s^{k-q} t^{q}.$$

Monodromy relations imply

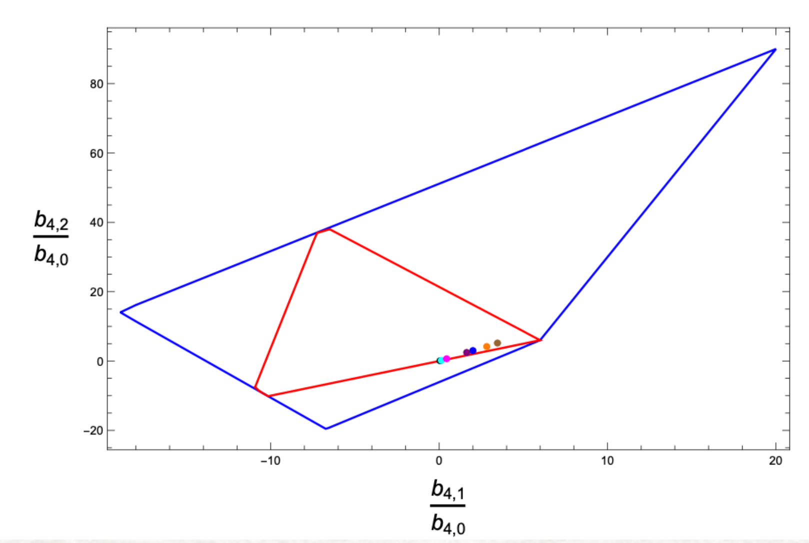
$$\begin{pmatrix} g_{2,0} & & & & \\ g_{3,0} & g_{3,1} & & & \\ g_{4,0} & g_{4,1} & g_{4,2} & & \\ g_{5,0} & g_{5,1} & g_{5,2} & g_{5,3} & \\ g_{6,0} & g_{6,1} & g_{6,2} & g_{6,3} & g_{6,4} \end{pmatrix} = \begin{pmatrix} \frac{\pi^2}{6} & & & & \\ g_{3,0} & g_{3,1} & & & \\ \frac{\pi^4}{90} & g_{4,1} & -\frac{\pi^4}{360}+2g_{4,1} & & \\ g_{5,0} & -\frac{3\pi^2}{10}g_{3,0}+\frac{\pi^2}{15}g_{3,1}+g_{5,0} & -\frac{\pi^2}{2}g_{3,0}+\frac{\pi^2}{12}g_{3,1}+5g_{5,0} & -\frac{\pi^2}{2}g_{3,0}+\frac{\pi^2}{12}g_{3,1}+5g_{5,0} & \\ \frac{\pi^6}{945} & g_{6,1} & \frac{\pi^6}{7560}-\frac{\pi^2}{6}g_{4,1}+3g_{6,1} & \frac{\pi^6}{3024}-\frac{\pi^2}{3}g_{4,1}+4g_{6,1} & \frac{\pi^6}{7560}-\frac{\pi^2}{6}g_{4,1}+3g_{6,1} \end{pmatrix}$$

Combined with unitarity

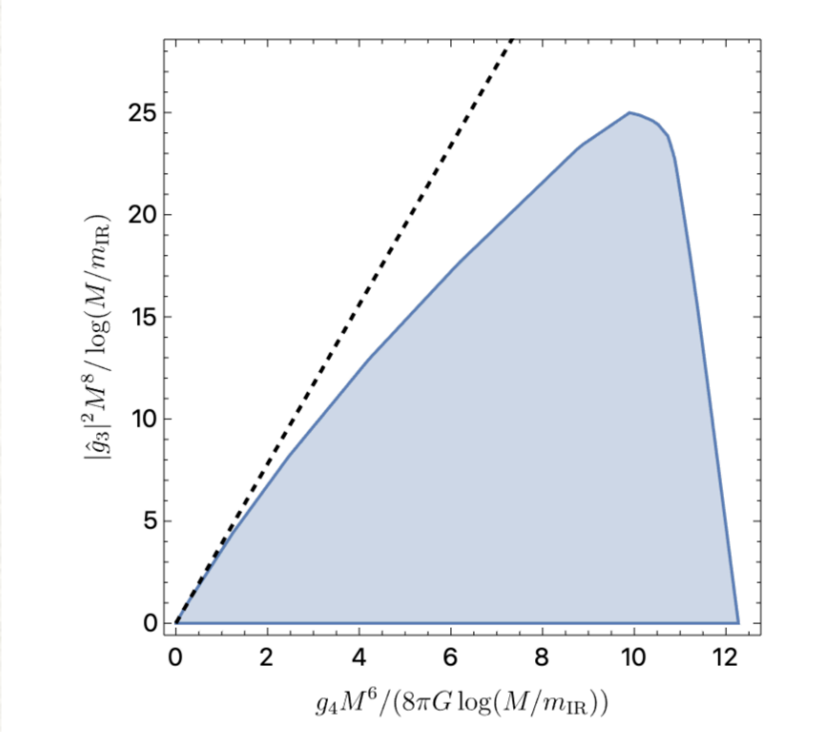


Summary

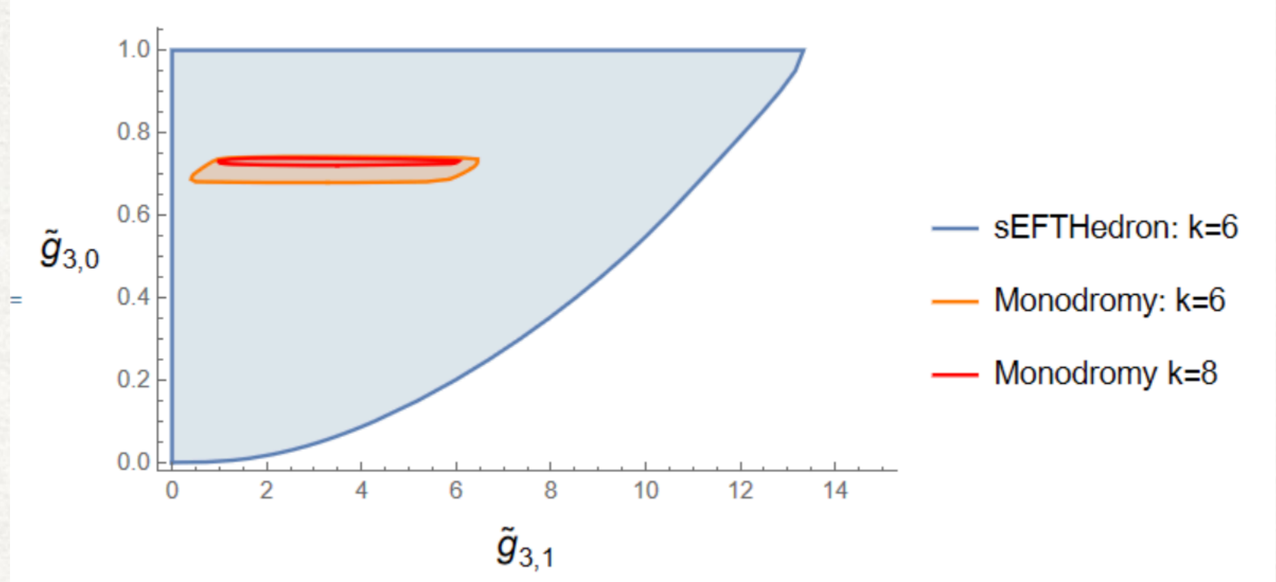
- Imposing unitarity and crossing constrains the space of consistent gravitational EFTs
- Forward limit bounds ratios of EFT coupling



- Finite impact parameters bounds couplings with respect to G_N



- Perturbative string EFTs can be carved out by monodromy relations



- Are there more constraints one can impose on EFT?

low spin dominance, consistent higher spin scattering.

- Is there a systematic way to explore consistent monodromy relations, enlarging the string EFT ?

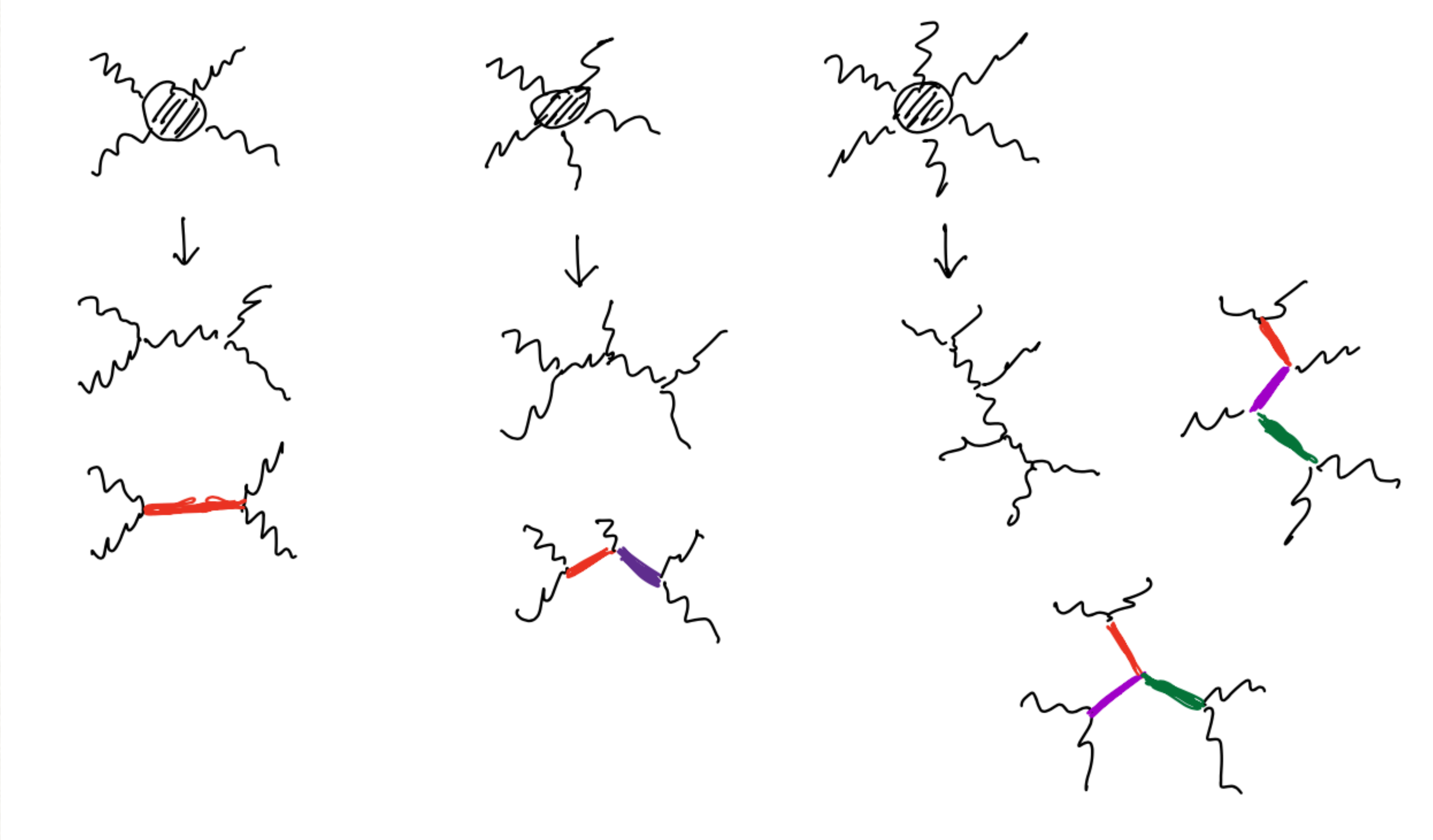
Summary

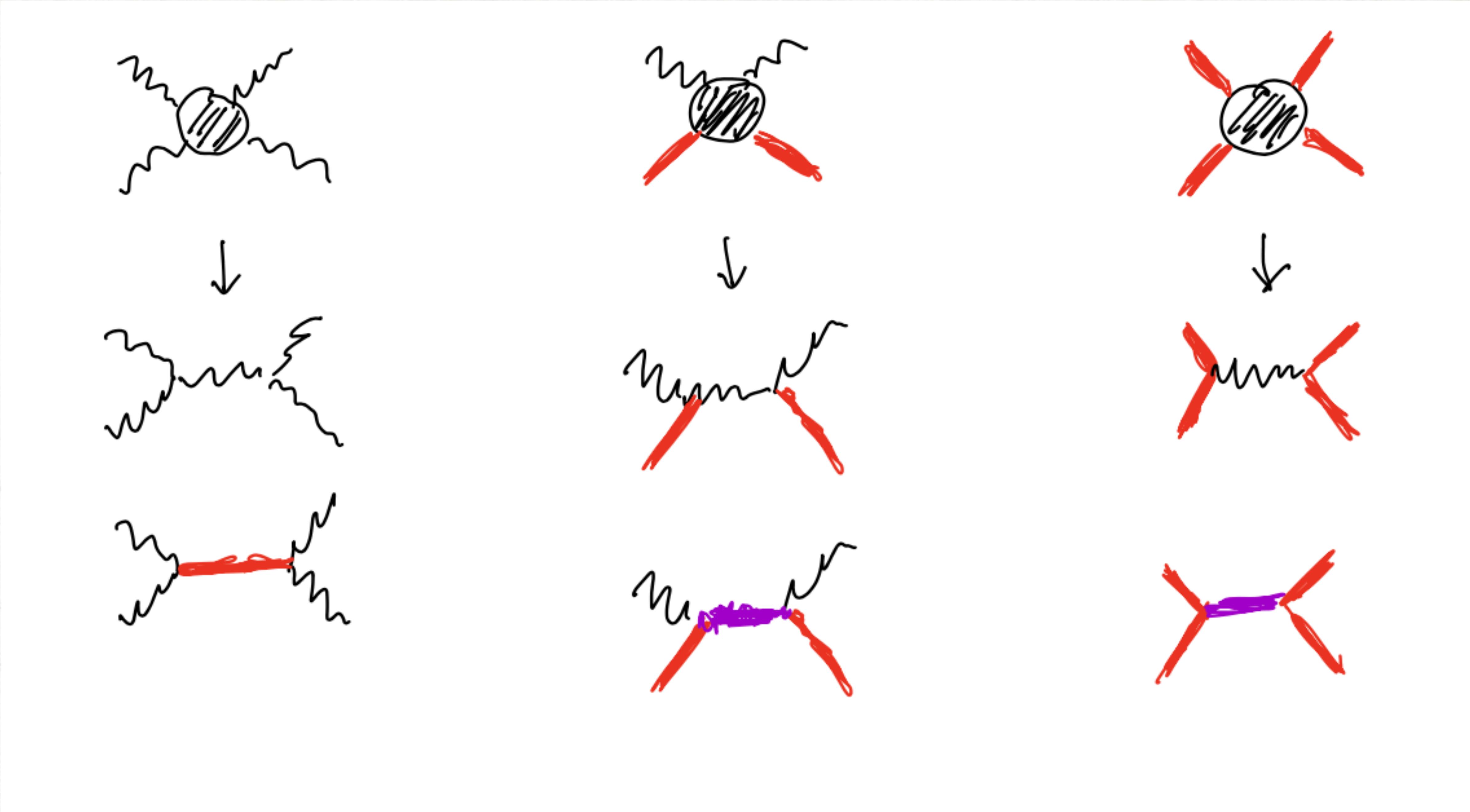
- Finite impact parameters bounds couplings with respect to G_N

- Perturbative string EFTs can be carved out by monodromy relations

Future Directions: mixed amplitudes

Consistent higher point generalizations → Consistent mixed amplitudes





ℓ	2	3	4	5	6	7	...
$h = 1, \tilde{\omega}_1^u(\ell)$	-	+	+	+	+	+	...
$h = 2, \tilde{\omega}_1^u(\ell)$			-	+	+	+	...
$h = 1, \tilde{\omega}_2^u(\ell)$	-	-	+	+	+	+	...
$h = 2, \tilde{\omega}_1^u(\ell)$			-	-	+	+	...

Consistent mixed (spin-4) amplitudes

For maximal SUSY in 10D

Andrea Guerrieri, João Penedones, and Pedro Vieira
Phys. Rev. Lett. 127, 081601, 2021

R^4 operator are non-renormalized, their coefficients are well defined

$$\frac{T(s,t,u)}{8\pi G_N = 64\pi^7 \ell_P^8} = s^4\left(\frac{1}{stu} + \alpha \ell_P^6 + O(s)\right)$$

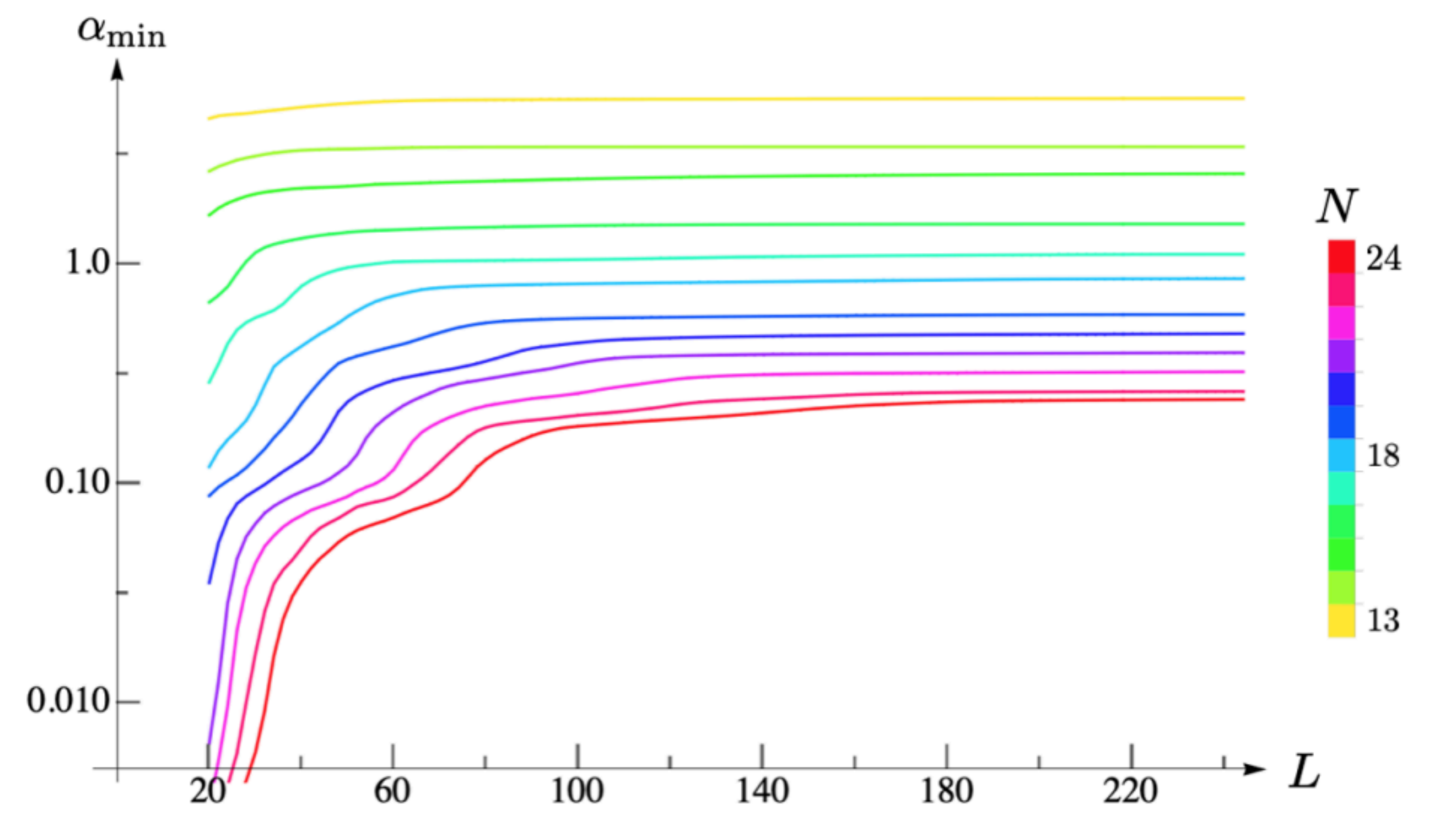
$$\frac{T}{8\pi G_N} = s^4\left(\underbrace{\frac{1}{stu}}_{\text{SUGRA}} + \underbrace{\prod_{A=s,t,u}(\rho_A+1)^2 \sum_{a+b+c\leq N}{}' \alpha_{(abc)}\rho_s^a\rho_t^b\rho_u^c}_{\text{UV completion}}\right)$$

$$\rho_s \equiv \frac{\sqrt{s_0}-\sqrt{-s}}{\sqrt{s_0}+\sqrt{-s}}$$

$$\alpha_{\min}^{\text{Boot}} \equiv \lim_{N\to\frac{\infty}{L}\to\infty} \alpha_{\min}(N,L) \approx 0.13 \pm 0.02.$$

$$\alpha^{\text{IIA}} = \frac{\zeta(3)}{32 g_s^{3/2}} + g_s^{1/2}\frac{\pi^2}{96} \geq \frac{\pi^{3/2}(\zeta(3))^{1/4}}{24\sqrt{3}} \approx 0.1403,$$

$$\alpha^{\text{IIB}} = \frac{1}{2^6}E_{\frac{3}{2}}(\tau,\bar\tau) \geq \frac{1}{2^6}E_{\frac{3}{2}}(e^{i\pi/3}, e^{-i\pi/3}) \approx 0.1389,$$



$$A_{\text{bosonic}}(1234) = \frac{\Gamma(-\frac{1}{2}s-1)\Gamma(-\frac{1}{2}t-1)}{\Gamma(\frac{1}{2}u+2)} K(1234),$$

A relation between tree amplitudes of closed and open strings ☆

H. Kawai, D.C. Lewellen, S.-H.H. Tye

$$\begin{aligned}
K(\zeta_1 k_1;\, \zeta_2 k_2;\, \zeta_3 k_3;\, \zeta_4 k_4) = (\tfrac{1}{2}s+1)(\tfrac{1}{2}t+1)(\tfrac{1}{2}u+1) \\
\times\Big[-K^{(\text{ss})} &+ \tfrac{1}{2}s\{\zeta_1\cdot k_3\,\zeta_2\cdot k_3(\zeta_3\cdot k_1\,\zeta_4\cdot k_1 + \zeta_3\cdot k_2\,\zeta_4\cdot k_2) \\
&+ \tfrac{1}{3}(\zeta_1\cdot k_2\,\zeta_2\cdot k_3\,\zeta_3\cdot k_1 - \zeta_1\cdot k_3\,\zeta_2\cdot k_1\,\zeta_3\cdot k_2)(\zeta_4\cdot k_1 - \zeta_4\cdot k_2)\} \\
&+ \tfrac{1}{2}t\{\zeta_2\cdot k_1\,\zeta_3\cdot k_1(\zeta_1\cdot k_3\,\zeta_4\cdot k_3 + \zeta_1\cdot k_2\,\zeta_4\cdot k_2) \\
&+ \tfrac{1}{3}(\zeta_1\cdot k_3\,\zeta_2\cdot k_1\,\zeta_3\cdot k_2 - \zeta_1\cdot k_2\,\zeta_2\cdot k_3\,\zeta_3\cdot k_1)(\zeta_4\cdot k_3 - \zeta_4\cdot k_2)\} \\
&+ \tfrac{1}{2}u\{\zeta_1\cdot k_2\,\zeta_3\cdot k_2(\zeta_2\cdot k_1\,\zeta_4\cdot k_1 + \zeta_2\cdot k_3\,\zeta_4\cdot k_3) \\
&+ \tfrac{1}{3}(\zeta_1\cdot k_2\,\zeta_2\cdot k_3\,\zeta_3\cdot k_1 - \zeta_1\cdot k_3\,\zeta_2\cdot k_1\,\zeta_3\cdot k_2)(\zeta_4\cdot k_3 - \zeta_4\cdot k_1)\} \\
&+ \frac{st}{4}\frac{1}{\frac{1}{2}u+1}(\zeta_1\cdot\zeta_3 - \zeta_1\cdot k_3\,\zeta_3\cdot k_1)(\zeta_2\cdot\zeta_4 - \zeta_2\cdot k_4\,\zeta_4\cdot k_2) \\
&+ \frac{tu}{4}\frac{1}{\frac{1}{2}s+1}(\zeta_1\cdot\zeta_2 - \zeta_1\cdot k_2\,\zeta_2\cdot k_1)(\zeta_3\cdot\zeta_4 - \zeta_3\cdot k_4\,\zeta_4\cdot k_3) \\
&+ \frac{su}{4}\frac{1}{\frac{1}{2}t+1}(\zeta_1\cdot\zeta_4 - \zeta_1\cdot k_4\,\zeta_4\cdot k_1)(\zeta_2\cdot\zeta_3 - \zeta_2\cdot k_3\,\zeta_3\cdot k_2) \\
&- \tfrac{1}{4}st(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_4) - \tfrac{1}{4}tu(\zeta_1\cdot\zeta_2)(\zeta_3\cdot\zeta_4) \\
&- \tfrac{1}{4}su(\zeta_1\cdot\zeta_4)(\zeta_2\cdot\zeta_3)\Big], \qquad (4.8)
\end{aligned}$$

$$\begin{aligned}
K^{(\text{ss})} = -\tfrac{1}{4}(&st\zeta_1\cdot\zeta_3\,\zeta_2\cdot\zeta_4 + su\zeta_2\cdot\zeta_3\,\zeta_1\cdot\zeta_4 + tu\zeta_1\cdot\zeta_2\,\zeta_3\cdot\zeta_4) \\
&+ \tfrac{1}{2}s(\zeta_1\cdot k_4\,\zeta_3\cdot k_2\,\zeta_2\cdot\zeta_4 + \zeta_2\cdot k_3\,\zeta_4\cdot k_1\,\zeta_1\cdot\zeta_3 + \zeta_1\cdot k_3\,\zeta_4\cdot k_2\,\zeta_2\cdot\zeta_3 \\
&+ \zeta_2\cdot k_4\,\zeta_3\cdot k_1\,\zeta_1\cdot\zeta_4) \\
&+ \tfrac{1}{2}t(\zeta_2\cdot k_1\,\zeta_4\cdot k_3\,\zeta_3\cdot\zeta_1 + \zeta_3\cdot k_4\,\zeta_1\cdot k_2\,\zeta_2\cdot\zeta_4 + \zeta_2\cdot k_4\,\zeta_1\cdot k_3\,\zeta_3\cdot\zeta_4 \\
&+ \zeta_3\cdot k_1\,\zeta_4\cdot k_2\,\zeta_2\cdot\zeta_1) \\
&+ \tfrac{1}{2}u(\zeta_1\cdot k_2\,\zeta_4\cdot k_3\,\zeta_3\cdot\zeta_2 + \zeta_3\cdot k_4\,\zeta_2\cdot k_1\,\zeta_1\cdot\zeta_4 + \zeta_1\cdot k_4\,\zeta_2\cdot k_3\,\zeta_3\cdot\zeta_4 \\
&+ \zeta_3\cdot k_2\,\zeta_4\cdot k_1\,\zeta_1\cdot\zeta_2) .
\end{aligned}$$