arXiv : 2305.16709 [hep-th]

PHENOMENOLOGY IN MAGNETIZED ORBIFOLD MODELS

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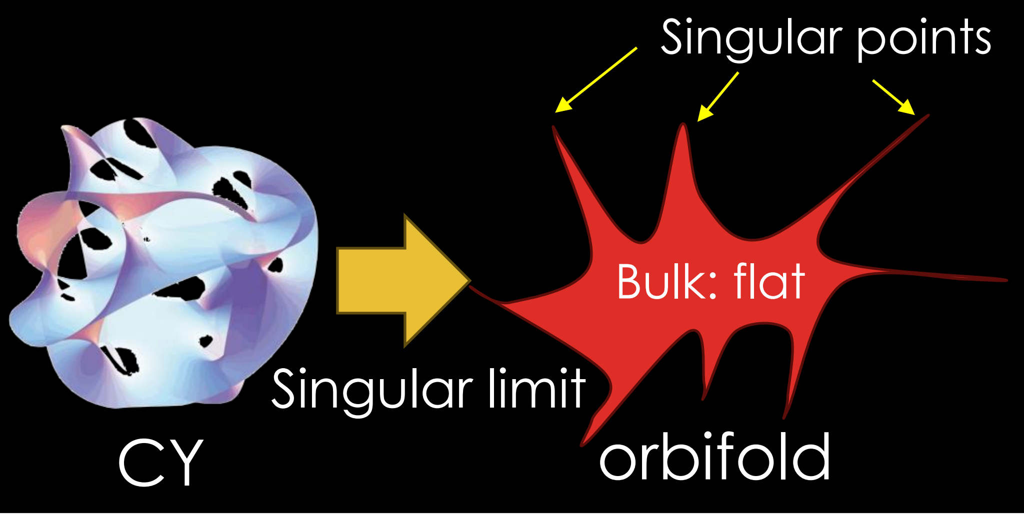
INTRODUCTION

- The most general solution = Calabi-Yau (CY)

⇒ so complicated to analyze zero-modes

- singular limit of CY ⇒ bulk : flat

⇒ orbifold ⇒ concrete zero-modes



- Previous ⇒ magnetized factorizable orbifold ⇒ models on magnetized 2D or 4D

H. Abe, K. S. Choi, T. Kobayashi, H. Ohki(2008)
Kikuchi, Kobayashi, Nasu, Takada, Uchida(2022)

- Our research ⇒ NON-factorizable T^6/Z_N $(N = 7\ or\ 12)$ with flux

⇒ discuss counting zero-modes numbers only on magnetized 6D

INTRODUCTION

the Dirac eq. on 6D ⇒ zero-modes ψ on 6D

Introduce flux N in magnetized torus/orbifold models

- tachyonic-free 4D chiral theory
- Generation structure
- Yukawa couplings (ψ's overlap integral)

……

ψ on magnetized torus are found in

D. Cremades, L. E. Ibanez, F. Marchesano (2004)

I. Antoniadis, A. Kumar, B. Panda (2009)

Flux may be Useful for realizing 4D EFT with the SM !

⇒ By counting zero-modes numbers,

first check if magnetized T^6/Z_N realize three-generation models

COMPACTIFICATION

How to construct orbifold

T^2 :
modulus, flux, $SL(2,Z) = Sp(2,Z)$ modular symmetry

Ex. T^2/Z_3 ⇔ divide T^2 into Z_3

$SL(2,Z)$: generator S, T ⇒ $(ST)^3 = 1$

$$S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

ST can introduce Z_3 element ⇒ construct Z_3 orbifold !

<u>Next present how to count zero-modes in Z_N sector</u>

COMPACTIFICATION

Zero-modes ψ on T^2 are degenerated by flux N

ψ in Z_N sectors ⇔ ψ with Z_N charge

$\rho' s$ eigenvalues ⇔ Z_N charge.

$$\begin{bmatrix} \psi^0 \\ \psi^1 \\ \cdots \\ \psi^{N-1} \end{bmatrix} \Rightarrow \rho(ST) \begin{bmatrix} \psi^0 \\ \psi^1 \\ \cdots \\ \psi^{N-1} \end{bmatrix}$$

Representation of ST

the numbers of $\rho' s$ eigenvalues

⇒degenerated zero-modes numbers in Z_N sector

extend these ideas from T^2/Z_N to T^6/Z_N !

COMPACTIFICATION

- T^6

moduli (3×3 symmetric matrix)

flux N (3×3 symmetric matrix)

$Sp(6, Z)$ modular symmetry generators : S , T_i $(i = 1, \cdots, 6)$

- The lattice of Z_7, Z_{12} orbifolds

$\Leftrightarrow$ $SU(7)$, E_6 root lattice respectively.

D. G. Markushevich, M. A. Olshanetsky ,A. M. Perelomov (1987)

Katsuki, Kawamura,Kobayashi et al. (1989)

L. E. Ibanez, J. Mas, H. P. Nilles, F. Quevedo (1988)

Constructing T^6/Z_7 and T^6/Z_{12} by S , T_i + introducing flux N

$\Rightarrow$ count zero-modes numbers in magnetized Z_7 and Z_{12} orbifolds !!

NON-FACTORIZABLE ORBIFOLD (T^6/Z_{12})

$$\left(ST_1T_2T_3^{-1}T_5T_6\right)^{12} = 1$$

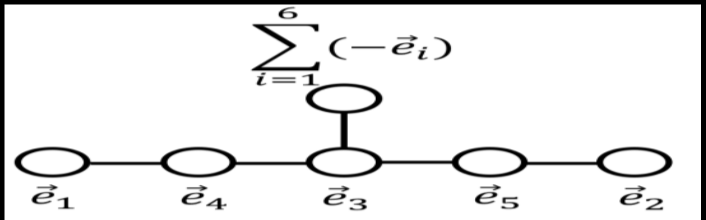
Complex structure moduli

$$\Omega_{12} = \begin{bmatrix} -\frac{1}{2} - \frac{\sqrt{3}}{6}i & \frac{\sqrt{3}}{3}i & -\frac{1}{2} + \frac{\sqrt{3}}{6}i \\ \frac{\sqrt{3}}{3}i & -\frac{1}{2} - \frac{\sqrt{3}}{6}i & -\frac{1}{2} + \frac{\sqrt{3}}{6}i \\ -\frac{1}{2} + \frac{\sqrt{3}}{6}i & -\frac{1}{2} + \frac{\sqrt{3}}{6}i & \frac{1}{2} - \frac{\sqrt{3}}{6}i \end{bmatrix}$$

Flux n_{11}, n_{12}, n_{13}: all even or odd

$$N = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{12} & n_{11} & n_{13} \\ n_{13} & n_{13} & n_{11} + n_{12} - 2n_{13} \end{bmatrix}$$

The lattice corresponds to E_6



fluxes realizing three-generation models with no tachyonic-modes？

⇒ **Only following <u>two fluxes</u>!**

$$N_1 = \begin{bmatrix} -3 & 3 & 1 \\ 3 & -3 & 1 \\ 1 & 1 & -2 \end{bmatrix} \qquad N_2 = \begin{bmatrix} -5 & 13 & 3 \\ 13 & -5 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

We find three-generation models on magnetized T^6/Z_{12} !

NON-FACTORIZABLE ORBIFOLD(T^6/Z_7)

$$(ST_3T_4T_5)^7 = 1$$

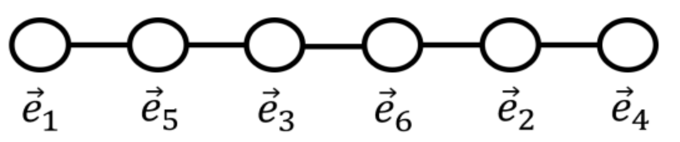
Complex structure moduli

$$\Omega_7 = \begin{bmatrix} -\frac{2}{\sqrt{7}}i & -\frac{1}{2}+\frac{\sqrt{7}}{14}i & \frac{i}{\sqrt{7}} \\ -\frac{1}{2}+\frac{\sqrt{7}}{14}i & -\frac{i}{\sqrt{7}} & -\frac{1}{2}+\frac{3\sqrt{7}}{14}i \\ \frac{i}{\sqrt{7}} & -\frac{1}{2}+\frac{3\sqrt{7}}{14}i & -\frac{1}{2}-\frac{\sqrt{7}}{14}i \end{bmatrix}$$

Flux n_{11}, n_{22}, n_{33}: even

$$N = \begin{bmatrix} n_{11} & n_{33}-n_{22} & n_{22}-n_{11} \\ n_{33}-n_{22} & n_{22} & n_{33}-n_{11} \\ n_{22}-n_{11} & n_{33}-n_{11} & n_{33} \end{bmatrix}$$

The lattice corresponds to $SU(7)$



No three-generation model
And
No tachyonic-free model

magnetized T^6/Z_{12} can realize three-generation models

T^6/Z_7 with flux may not be suitable for realizing realistic models

CONCLUSION

- We count zero-modes numbers on non-factorizable T^6/Z_N with flux.
- We construct models by $Sp(6, Z)$ generators and flux N
- stable three-generation models are found on T^6/Z_{12}, not on T^6/Z_7
- In the future, we will analyze Yukawa couplings on T^6/Z_N with flux.

THANK YOU FOR YOUR ATTENTION !

APPENDIX

- Fermion masses :

Yukawa couplings + Higgs VEVs

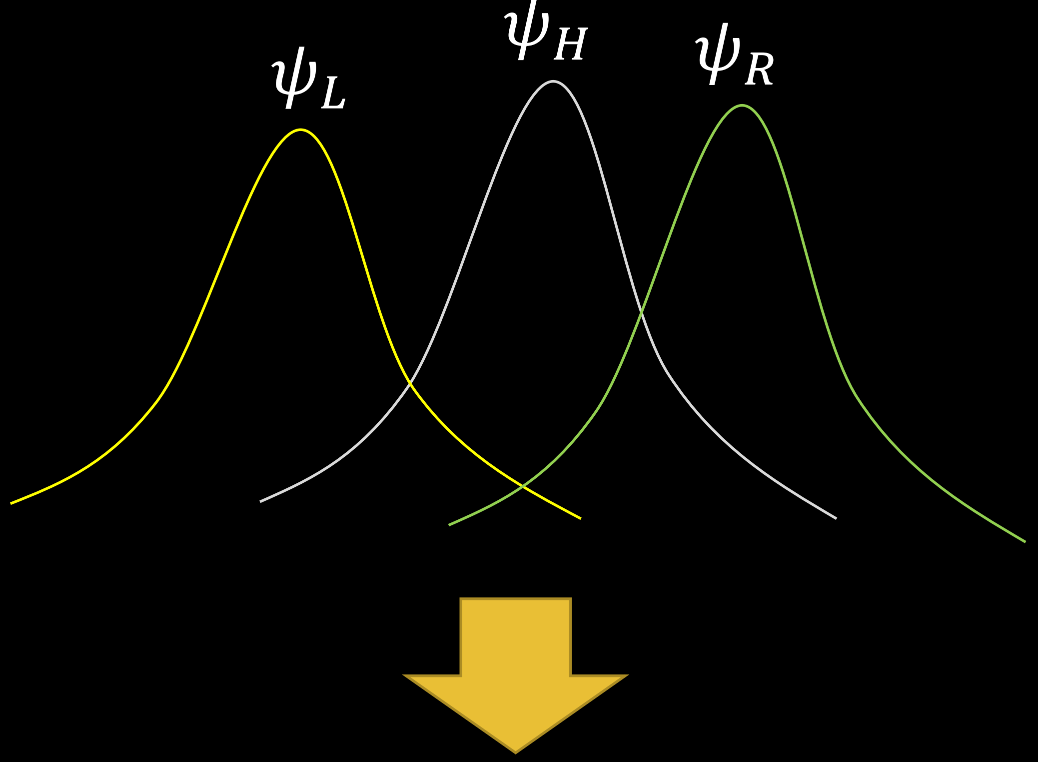
- Yukawa couplings :

overlap integral of Left, Right, and Higgs zero-modes on T^6/Z_{12} (ψ_L, ψ_R, ψ_H)

- Considering zero-modes chirality,

need zero-modes with the other

▪we want to realize mass hierarchy.



Introduce $Y_{ijk} \sim \int d^3z\, d^3\bar{z}\, \psi_L\, \psi_R\, \psi_H^*$

APPENDIX

$tr\,\rho(ST_3T_4T_5) = -1, +1, -\sqrt{7}i$ $|n_{ii}| \leq 400$, $|\det N| \leq 1.0 \times 10^{10} \Rightarrow$ NO SOLUTION

$tr\,\rho\left(ST_1T_2T_3^{-1}T_5T_6\right) = -1, +1, -\sqrt{3}i$ $|n_{1i}| \leq 400 \Rightarrow$ two fluxes

$$G = ST_1T_2T_3^{-1}T_5T_6$$

$$N_1 = \begin{bmatrix} -3 & 3 & 1 \\ 3 & -3 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$\det N = 12$, $\mathrm{tr}\rho(G) = \mathrm{tr}\rho(G^2) = -\sqrt{3}i$, $\mathrm{tr}\rho(G^3) = 6$, $\mathrm{tr}\rho(G^4) = \sqrt{3}i$, $\mathrm{tr}\rho(G^6) = 12$

$[3, 0, 0, 0, 3, 0, 1, 0, 3, 0, 2, 0]$

$$N_2 = \begin{bmatrix} -5 & 13 & 3 \\ 13 & -5 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

$\det N = 36$, $\mathrm{tr}\rho(G) = \mathrm{tr}\rho(G^2) = -\sqrt{3}i$, $\mathrm{tr}\rho(G^3) = 18$, $\mathrm{tr}\rho(G^4) = \sqrt{3}i$, $\mathrm{tr}\rho(G^6) = 36$

$[9, 0, 2, 0, 9, 0, 3, 0, 9, 0, 4, 0]$.