A dataset of R-symmetric Wess-Zumino models

Zheng Sun

Sichuan University

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(based on Brister-Kou-Li-Sun 2204.05767, Sun 2207.13933 and related works)

Outline

SUSY breaking in Wess-Zumino models, R-symmetries, the Nelson-Seiberg theorem and its extensions

The dataset of R-symmetric Wess-Zumino models, the data structure, the brute-force algorithm and the search result

Summary and outlook

Supersymmetry (SUSY)

What is SUSY?

- Poincare $\to$ super-Poincare algebra, fermions $\leftrightarrow$ bosons.
- e.g., the chiral multiplet $\hat{\Phi} = (\phi, \psi, F)$, SUSY transformation $\delta_\xi \hat{\Phi} = (\sqrt{2}\xi\psi,\ -i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\phi - \sqrt{2}\xi F,\ -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\xi})$.
- Also expressed as superfields with fermionic coordinates $\theta, \bar{\theta}$, e.g. the chiral field $\Phi(x^\mu, \theta, \bar{\theta}) = \phi + \sqrt{2}\theta\psi - \theta\theta F + \cdots$.
- And the vector multiplet $V = (A_\mu, \lambda, \bar{\lambda}, D)$, etc.
- The SUSY invariant action (4-d $N = 1$) $\to$ physics.

Why and where is SUSY?

- Pheno reasons: Higgs mass hierarchy, dark matter, etc.
- Math reasons: Coleman-Mandula no-go, string tachyons, etc.
- SUSY breaking $\to$ messengers $\to$ the Standard Model (SM).
- Ingredients of strings and string pheno, flux vacua, etc.

Wess-Zunmino models (WZ) for F-term SUSY breaking

WZ and their supergravity (SUGRA) extensions

- A superpotential $W(\phi_i)$ for scalar ϕ's of chiral Φ's.
- A Kähler potential $K(\phi_i^*, \phi_j)$ and $K^{\bar{i}j} = (\partial_{\phi_i^*}\partial_{\phi_j}K)^{-1}$.
- The SUSY Lagrangian $\mathcal{L} \Rightarrow V = K^{\bar{i}j}(\partial_{\phi_i}W)^*\partial_{\phi_j}W \geq 0$.
- SUSY breaking $\Leftrightarrow \langle V \rangle > 0 \Leftrightarrow \langle -F_i^* \rangle = \langle \partial_{\phi_i}W \rangle \neq 0$.
- In SUGRA, $V = e^{K/M_{\mathrm{P}}^2}(K^{\bar{i}j}(D_{\phi_i}W)^*D_{\phi_j}W - 3W^*W/M_{\mathrm{P}}^2)$, SUSY breaking $\Leftrightarrow \langle D_{\phi_i}W \rangle = \langle \partial_{\phi_i}W + W\partial_{\phi_i}K/M_{\mathrm{P}}^2 \rangle \neq 0$, in particular $\langle \partial_{\phi_i}W \rangle = \langle W \rangle = 0 \Leftrightarrow$ SUSY and $\langle V \rangle = 0$.

The Nelson-Seiberg theorem (NS) for WZ

- An $U(1)$ R-symmetry $\theta \to e^{i\alpha}\theta, \mathrm{d}\theta \to e^{-i\alpha}\mathrm{d}\theta, \phi \to e^{iR(\phi)\alpha}\phi$.
- $\mathcal{L}_W = \int W\mathrm{d}^2\theta + \mathrm{h.c.}$ is R-invariant $\Rightarrow R(W) = 2$.
- Generically, F-term SUSY breaking at the true vacuum $\Rightarrow$ R-symmetries, $\Leftarrow$ spotaneous R-breaking at the vacuum.

  (Nelson-Seiberg hep-ph/9309299)

NS revised (NSr) and other extensions

Classification of R-symmetric renormalizable WZ

- $N_2 \leq N_0 \Rightarrow$ SUSY, R-preserving and $\langle W \rangle = 0$, same for discrete R-symmetries, generically $N_2 \leq N_0 \Leftrightarrow$ SUSY (NSr).
  (Sun 1109.6421, Kang-Li-Sun 1209.1059)
- $N_0 < N_2 \leq N_0 + N_\pm \Rightarrow$ SUSY, R-breaking and $\langle W \rangle = 0$.
  (Sun-Tan-Yang 1904.09589, 2106.08879, Amariti-Sauro 2005.02076)
- $N_2 > N_0 + N_\pm \Rightarrow$ SUSY breaking generically, but SUSY "counterexamples" with special R-charges exist.
  (Brister-Sun 2203.05464, Nakayama-Yoshida 2303.00951)
- No $N_2 \leq N_0 + N_\pm$ counterexample with special R-charges.
  (Li-Sun 2107.09943)

Other types of R-symmetric WZ

- Smooth W with an R-symmetry: same result as above.
- Sigular W with an R-symmetry $\Rightarrow$ SUSY breaking generically.
  (Li-Sun 2006.00538)

Generic R-symmetric WZ

Notation conventions

- The generic superpotential with an R-symmetry:

$$
\begin{aligned}
W &= X_i(a_i + b_{ij}Y_j + c_{ijk}Y_jY_k + d_{(r)ijk}P_{(r)j}Q_{(-r)k}) + W_1, \\
W_1 &= \xi_{ijk}X_iX_jA_k + \rho_{ijk}X_iA_jA_k \\
&\quad + (\mu_{ij} + \nu_{ijk}Y_k)A_iA_j + \lambda_{ijk}A_iA_jA_k \\
&\quad + \sigma_{(r)ijk}P_{(r)i}A_jA_k + \tau_{(r)ijk}Q_{(-r)i}A_jA_k.
\end{aligned}
$$

- $P_{(r)}$'s and $Q_{(-r)}$'s with $R(P_{(r)}) = -R(Q_{(-r)}) = r > 0, r \neq 2$, both P's and Q's only appear linearly in cubic terms of W_1.
- X's with $R(X) = 2$, Y's with $R(Y) = 0$, A's for other fields.
- $N_2 = N_X$, $N_0 = N_Y$, $N_\pm = \sum_r (N_{P(r)} + N_{Q(-r)} - 1)$.

Genericity conditions

- Non-generic parameters need tuning, uninteresting for pheno.
- Non-generic R-charges, or special R-charge assignments.

Applications of WZ

SUSY pheno

- WZ as an effective theory for SUSY breaking, mediating to the SUSY Standard Model for pheno.
- E.g. in gauge mediation, $W = FX + (m^{ij} + n^{ij}X)\Phi_i\Phi_j$, messengers Φ_i are charged in SM gauge symmetries.

String pheno

- Type IIB strings compactified on a Calabi-Yau 3-fold (CY), effectively described by 4-d $N = 1$ SUGRA.
- Fluxes on CY generate $W = \int G_3 \wedge \Omega$ for moduli stabilization.
- Discrete R-symmetries from CY with enhanced symmetries lead to $\langle \partial_{\phi_i} W \rangle = \langle W \rangle = 0$ and SUSY vacua with $\langle V \rangle = 0$.
  (Dine-O'Neil-Sun hep-th/0501214, Dine-Sun hep-th/0506246)
- Non-perturbative effects generate low-scale SUSY breaking and a small positive vacuum energy density $\langle V \rangle \gtrsim 0$.
  (Demirtas-Kim-McAllister-Moritz 1912.10047)

The dataset of R-symmetric Wess-Zumino models

Motivation

- NSr allows a fast scan of vacua by field counting without the explicit form of W or vacuum solutions, but counterexamples with "non-generic" R-charges may introduce error.
- A dataset of models with up to N fields may estimate such error and have other pheno and string pheno applications.

The data structure

- Model data: the R-charge assignment $\hat{r} = \{r_1, \dots r_N\}$.
- $W = \hat{c} \cdot \hat{\phi}^{\hat{a}} = c_1 \phi_1^{a_1^1} \dots \phi_N^{a_N^1} + \dots + c_m \phi_1^{a_1^m} \dots \phi_N^{a_N^m}$, with generic undetermined coefficients $\hat{c}$, identified by the exponent array $\hat{a} = \{\hat{a}^1, \dots, \hat{a}^m\} = \{\{a_1^1, \dots, a_N^1\}, \dots, \{a_1^m, \dots, a_N^m\}\}$.
- The R-symmetry constraint $\hat{a}^s \cdot \hat{r} = a_1^s r_1 + \dots + a_N^s r_N = 2$ and the renormalization constraint $a_1^s + \dots + a_N^s \leq 3$ can be used to construct $\hat{a}$ from $\hat{r}$ or determine $\hat{r}$ from $\hat{a}$.

A brute-force search

Outline of the search algorithm

- Step 1: generate all M possible monomials with N fields satisfying the renormalization constraint.
- Step 2: for each N-tuple of M monomials from Step 1, try to solve the R-symmetry constraint equations and record the R-charge assignment $\hat{r}$ if the solution is unique.
- Step 3: for each recorded R-charge assignment $\hat{r}$, construct W from $\hat{r}$ by including all monomials from Step 1 satisfying the R-symmetry constraint.

Redundant data

- Symmetries in fields, e.g. $W = a\phi_1 + b\phi_2^2$ or $a\phi_2 + b\phi_1^2$.
- Symmetries in terms, e.g. $W = a\phi_1 + b\phi_2^2$ or $a\phi_2^2 + b\phi_1$.
- Reducible models, e.g. $W = a\phi_1 + b\phi_2^2 = W_1(\phi_1) + W_2(\phi_2)$.
- Eliminating redundancies reduces the size of the database.

Generating data

Demonstration for small N's

- $N = 1 \Rightarrow 3$ models: $\hat{r} = \{2\}, \{1\}, \{\frac{2}{3}\} \Rightarrow W = a\phi_1, a\phi_1^2, a\phi_1^3$.
- $N = 2 \Rightarrow M = 9$: $\phi_1, \phi_1^2, \phi_1^3, \phi_2, \phi_1\phi_2, \phi_1^2\phi_2, \phi_2^2, \phi_1\phi_2^2, \phi_2^3$.
- Fixing all R-charges and adding all possible monomials, e.g., $\{\phi_1, \phi_1\phi_2\} \Rightarrow \hat{r} = \{2, 0\} \Rightarrow W = a\phi_1 + b\phi_1\phi_2 + c\phi_1\phi_2^2$.
- 7 models: $\hat{r} = \{2, 0\}, \{2, -2\}, \{0, 1\}, \{\frac{1}{2}, 1\}, \{\frac{2}{3}, \frac{2}{3}\}, \{\frac{2}{3}, \frac{4}{3}\}, \{1, 1\} \Rightarrow W = a\phi_1 + b\phi_1\phi_2 + c\phi_1\phi_2^2, a\phi_1 + b\phi_1^2\phi_2, a\phi_2^2 + b\phi_1\phi_2^2, a\phi_2^2 + b\phi_1^2\phi_2, a\phi_1^3 + b\phi_1^2\phi_2 + c\phi_1\phi_2^2 + d\phi_1^3, a\phi_1^3 + b\phi_1\phi_2, a\phi_1^2 + b\phi_1\phi_2 + c\phi_2^2$.
- In general $M \sim N^3$, $\binom{M}{N} \sim N^{3N}$ N-tuples $\Rightarrow$ EXP complexity, but valid and non-redundant data are few.

Data classification

- $\hat{r} \Rightarrow N_2 \leq N_0$ or $N_2 > N_0$, $\partial_i W = 0 \Rightarrow$ SUSY or no SUSY.
- Existence of SUSY breaking vacua is not checked, e.g., the model $\hat{r} = \{2, -2\}$ has only a SUSY runaway exists.

The search result

List of models

- 859 models with $N \leq 5$, about one day running time.

N	1	2	3	4	5	Total
$N_2 \leq N_0$, SUSY	2	6	19	81	448	556
$N_2 > N_0$, no SUSY	1	1	7	38	237	284
$N_2 > N_0$, SUSY	0	0	0	1	18	19
Total	3	7	26	120	703	859

(Brister-Kou-Li-Sun 2204.05767)

- Most models match the NSr criteria $N_2 \leq N_0 \Leftrightarrow$ SUSY except 19 "counterexamples" of type $N_2 > N_0$ and SUSY.
- Most "counterexamples" has $N_0 < N_2 \leq N_0 + N_\pm$ except two: $\hat{r} = \{2, -2, 4, -4, 8\}$ and $\hat{r} = \{2, \frac{2}{3}, \frac{4}{3}, -\frac{4}{3}, \frac{8}{3}\} \Rightarrow$
$W = a\phi_1 + b\phi_1^2\phi_2 + c\phi_2\phi_3 + d\phi_1\phi_3\phi_4 + e\phi_2\phi_4\phi_5$ and
$W = a\phi_1 + b\phi_2^3 + c\phi_2\phi_3 + d\phi_1\phi_3\phi_4 + e\phi_2\phi_4\phi_5$,
$N_\pm = 0$, SUSY at $\phi_1 = \phi_2 = 0, \phi_3\phi_4 = -\frac{a}{d}, \phi_5 = -\frac{c\phi_3}{e\phi_4}$.
(Nakayama-Yoshida 2303.00951)

Summary and outlook

NS and extensions

- SUSY breaking $\Rightarrow$ R, $\Leftarrow$ spontaneous broken R (NS).
- R and $N_2 \leq N_0$ $\Rightarrow$ SUSY and R-preserving (NSr: "$\Leftrightarrow$").
- R and $N_0 < N_2 \leq N_0 + N_\pm$ $\Rightarrow$ SUSY and R-breaking.
- R and $N_2 > N_0 + N_\pm$ $\Rightarrow$ SUSY breaking generically, but SUSY "counterexamples" with special R-charges exist.

The dataset of R-symmetric WZ

- 859 models with up to 5 fields, 19 counterexamples to NSr.
- All 19's have $N_2 > N_0$, SUSY, R-breaking and $\langle W \rangle = 0$, most have $N_2 \leq N_0 + N_\pm$, two exceptions covered in literature.
- Computational complexity $\sim N^{3N}$, need better algorithms.
- Metastable vacua and runaway models are not identified.
- Larger N, SUGRA for flux vacua and more applications . . . $\square$