

FTheoryTools: A computational tool for analysis of singular elliptic fibrations

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String Phenomenology '23

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In collaboration with: Martin Bies
Matthias Zach, Anne Fröhbis-Krüger
OSCAR collaboration

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 - ▶ Intersection theory and fiber analysis

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Example: U(1)-restricted SU(5) Tate [Krause, Mayrhofer, Weigand '11]

$$y^2 + a_{1,0}xyz + a_{3,2}w^2yz^2 = x^3 + a_{2,1}wx^2z^2 + a_{4,3}w^3xz^4$$

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(Multivariate polynomial ring in 5 variables over QQ, QQMPolyRingElem[a10, a21,  
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julia> t_res, exceptionals, crepant = blowup_sequence(t, [[6, 7, 5], [2, 3, 1],  
[3, 4], [2, 4]])  
(ideal(-b_4_1*e_3*b_2_1*a1*z + ...), [ideal(x, y, w, e_1), ...], true)
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Literature models

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julia> resolve(t, 1)  
Scheme of a toric variety with fan spanned by  $[[1, 0, 0, 0, 0, -2, -3], \dots]$ 
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