Fano 3-Folds, Ypk Manifolds and Brane Brick Models

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UNIST | DEPARTMENT OF MATHEMATICAL SCIENCES

Collaborations

Sebastian
Franco

CUNY, New York

Dongwook
Ghim

YITP, Kyoto

Franco-Seong JHEP 2208:008, 2022 [2203.15816]

Franco-Ghim-Seong JHEP 2303:050, 2023 [2212.02523]

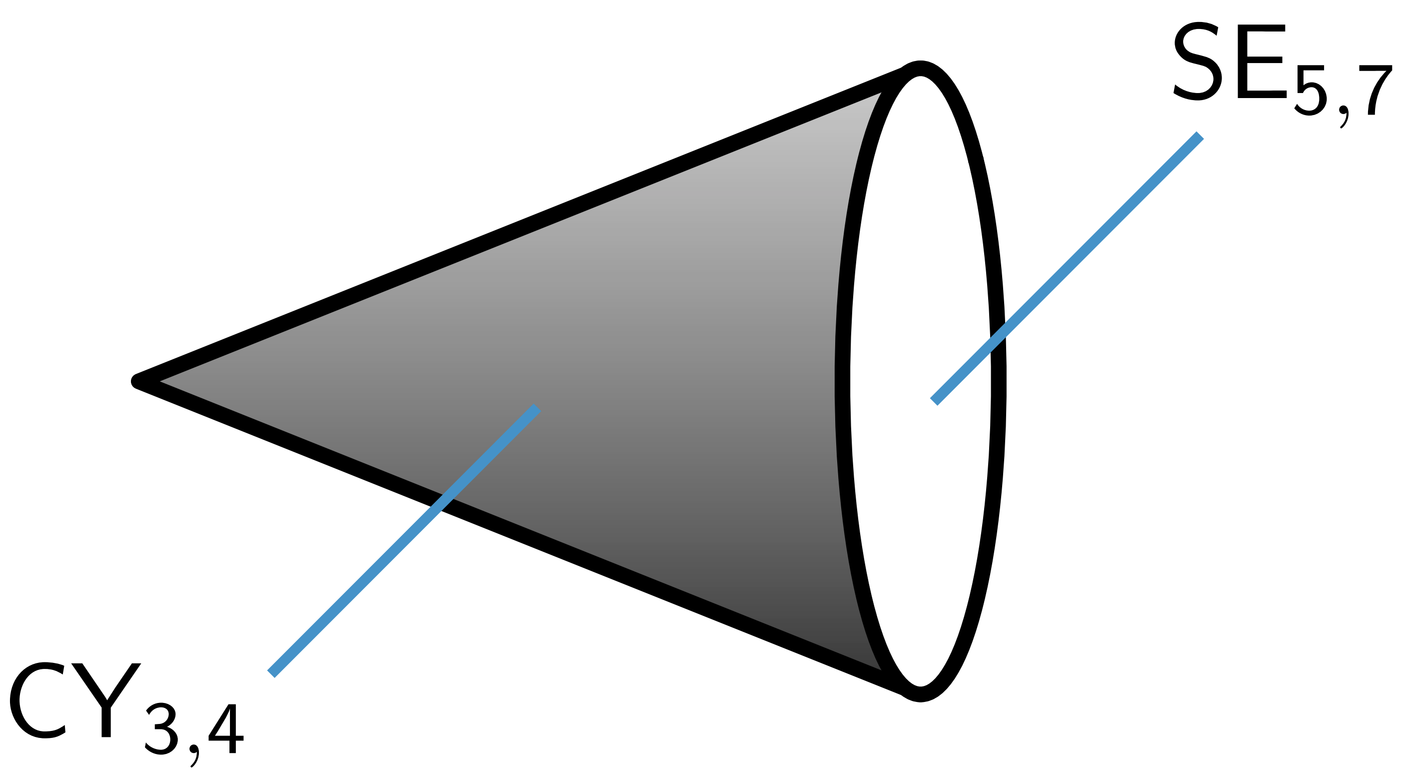
Toric Calabi-Yau Geometries

Non-Compact Calabi-Yaus

- Cone over Sasaki-Einstein Manifolds

Cone $\mathrm{CY}_3 = \mathcal{C}(\mathrm{SE}_5)$

$\mathrm{CY}_4 = \mathcal{C}(\mathrm{SE}_7)$



Metric $ds^2_{\mathrm{CY}} = dr^2 + r^2 ds^2_{\mathrm{SE}}$

Example Conifold with Sasaki-Einstein base $T^{1,1}$

Toric Calabi-Yaus

- Toric Ideals

Variety $\mathbb{C}[x_i] / \left\langle \prod_a x_a - \prod_b x_b \right\rangle$

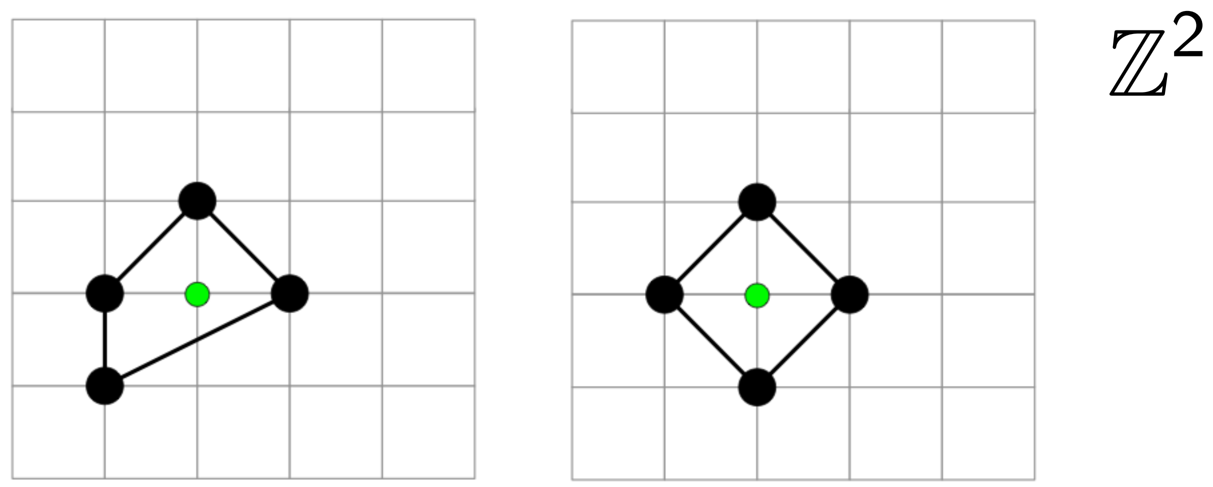
binomial relations

Example Conifold $\mathbb{C}[M_{ij}] / \langle \det M \rangle$ $i, j = 1, 2$

- Toric Diagrams

A toric variety is characterized by a convex lattice polytope

Example



Brane Probes

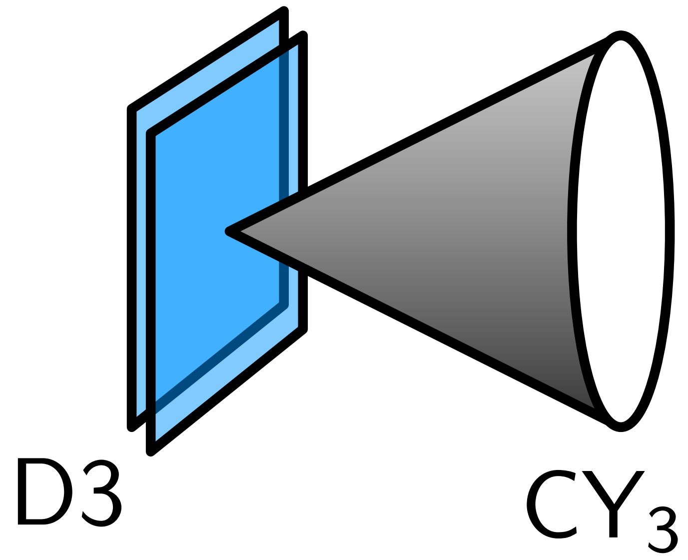
[Franco-Hanany-Kennaway-Vegh-Wecht 2005]

[Franco-Lee-Seong 2015] [Franco-Lee-Seong-Vafa 2016]

Calabi-Yau 3-folds

4d N=1 worldvolume gauge theory

Brane Tilings (Dimers)

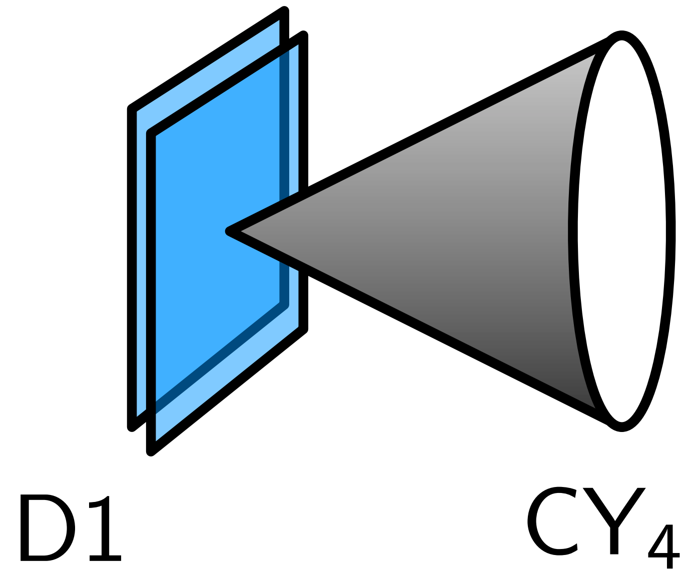


	0	1	2	3	4	5	6	7	8	9
D3	×	×	×	×	·	·	·	·	·	·
CY_3	·	·	·	·	×	×	×	×	×	×

Calabi-Yau 4-folds

2d (0,2) worldvolume gauge theory

Brane Brick Models

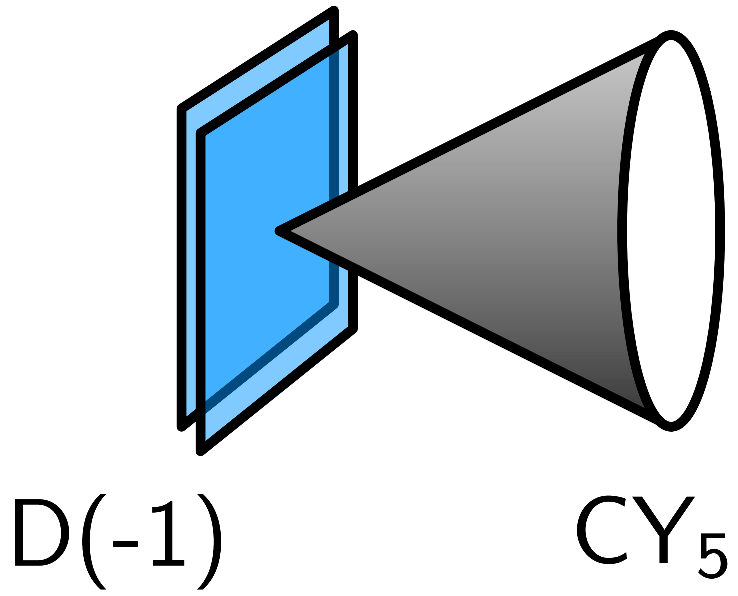


	0	1	2	3	4	5	6	7	8	9
D1	×	×	·	·	·	·	·	·	·	·
CY_4	·	·	×	×	×	×	×	×	×	×

Calabi-Yau 5-folds

0d N=1 worldvolume gauge theory

Brane Hyper-Brick Models



	0	1	2	3	4	5	6	7	8	9
D(-1)	·	·	·	·	·	·	·	·	·	·
CY_5	×	×	×	×	×	×	×	×	×	×

Brane Probes

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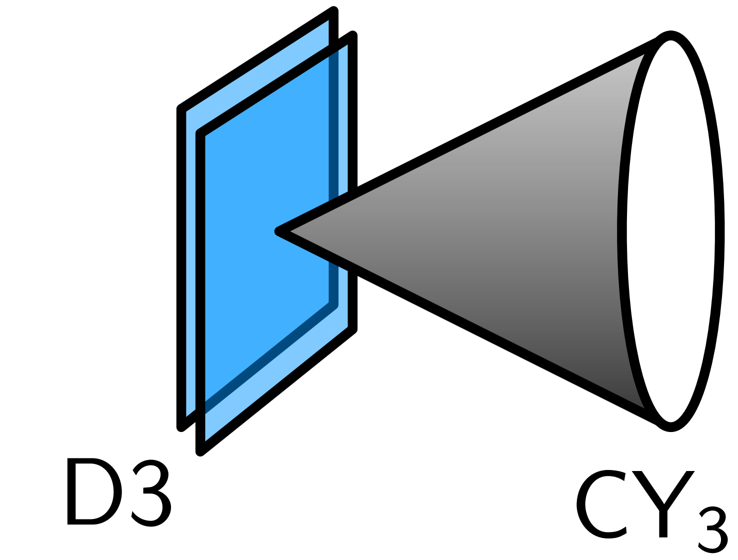
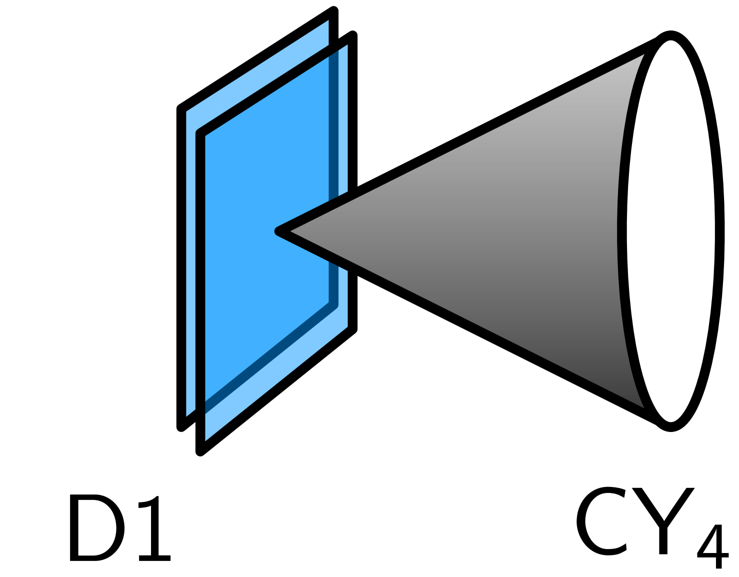


	0	1	2	3	4	5	6	7	8	9
D3	×	×	×	×	·	·	·	·	·	·
CY_3	·	·	·	·	×	×	×	×	×	×

Calabi-Yau 4-folds

2d (0,2) worldvolume gauge theory
Brane Brick Models



	0	1	2	3	4	5	6	7	8	9
D1	×	×	·	·	·	·	·	·	·	·
CY_4	·	·	×	×	×	×	×	×	×	×

Calabi-Yau 5-folds

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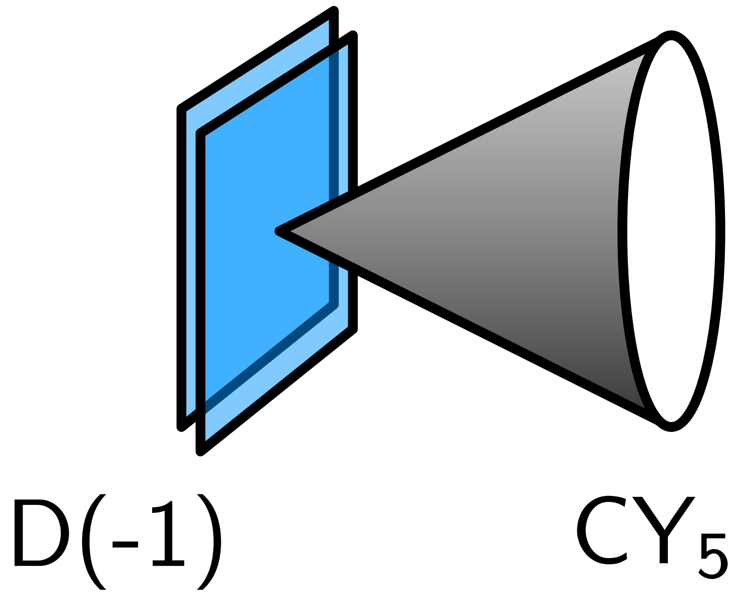
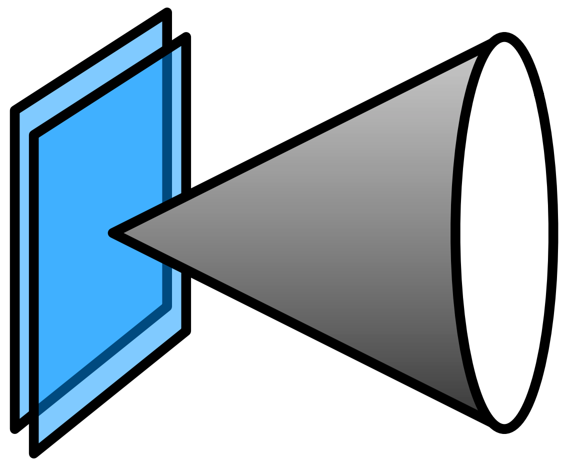


	0	1	2	3	4	5	6	7	8	9
D(-1)	·	·	·	·	·	·	·	·	·	·
CY_5	×	×	×	×	×	×	×	×	×	×

Brane Construction

[Franco-Lee-Seong 2015]

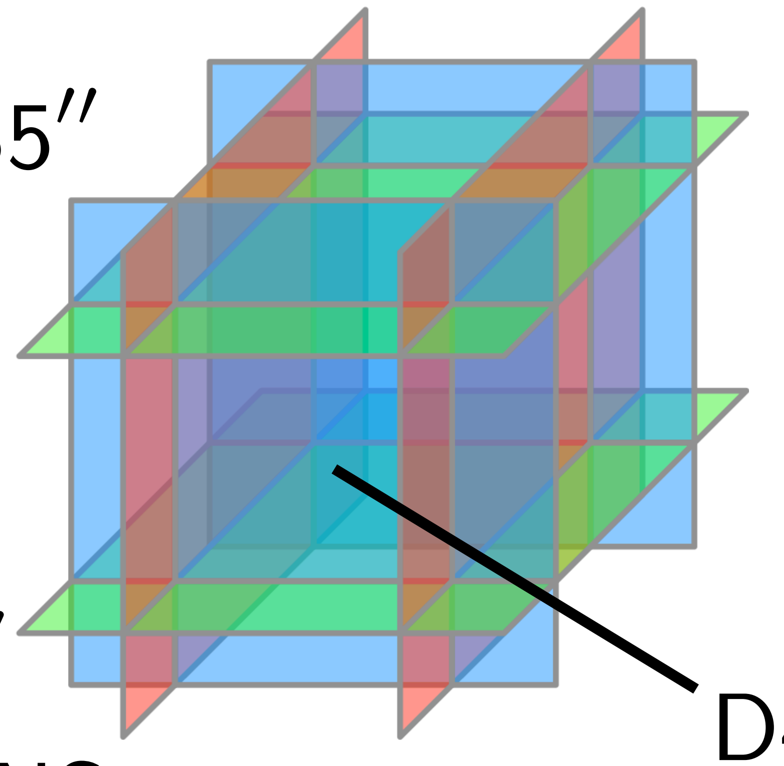
Calabi-Yau 4-folds



	0	1	2	3	4	5	6	7	8	9
D1	×	×	·	·	·	·	·	·	·	·
CY_4	·	·	×	×	×	×	×	×	×	×

T-Duality

	0	1	2	3	4	5	6	7	8	9
D4	×	×	—	—	$\Sigma(x,y,z)$	—	—	—	·	·
NS5	×	×	—	—	$\Sigma(x,y,z)$	—	—	—	·	·



Holomorphic Surface $\Sigma \ : \ P(x,y,z) = 0$

Example $P(x,y,z) = (x + 1/x) + (y + 1/y) + (z + 1/z) + c$

Brane Construction

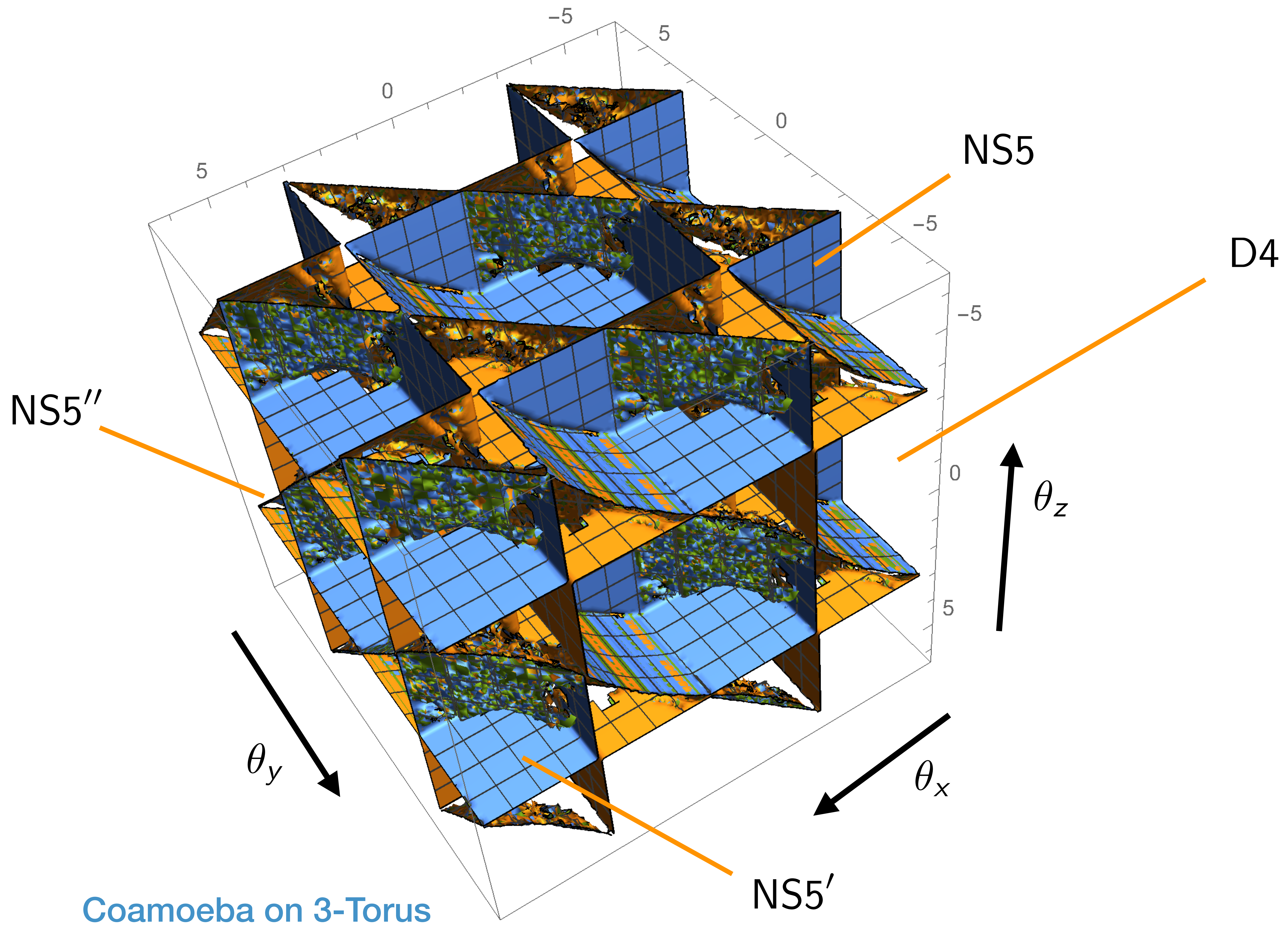
[Franco-Lee-Seong-Vafa 2016]

Calabi-Yau 4-folds

- Tropical Geometry

Coamoeba Map

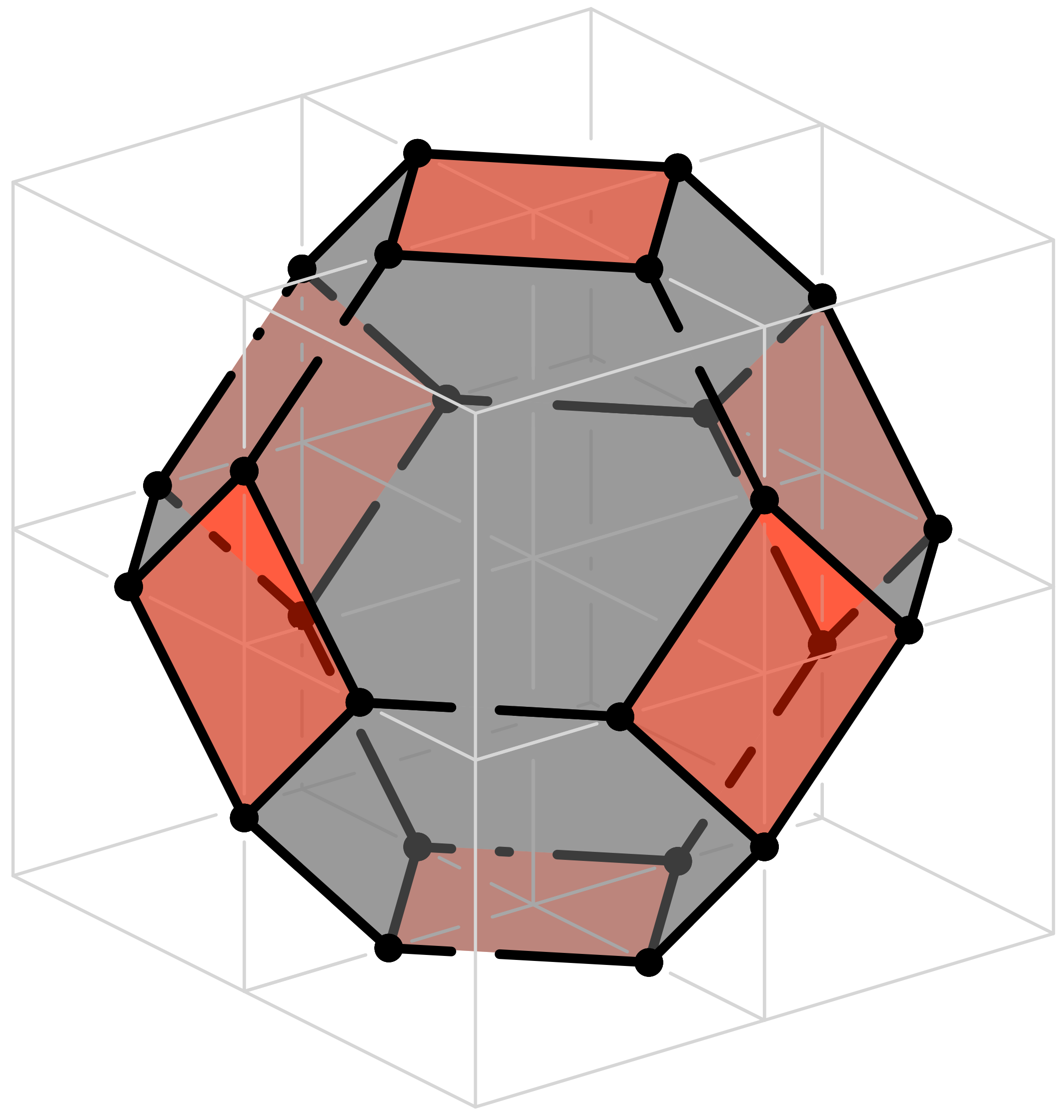
$$\Sigma \to T^3$$



Coamoeba on 3-Torus

Brane Brick Models

[Franco-Lee-Seong 2015]



Brane Brick Models

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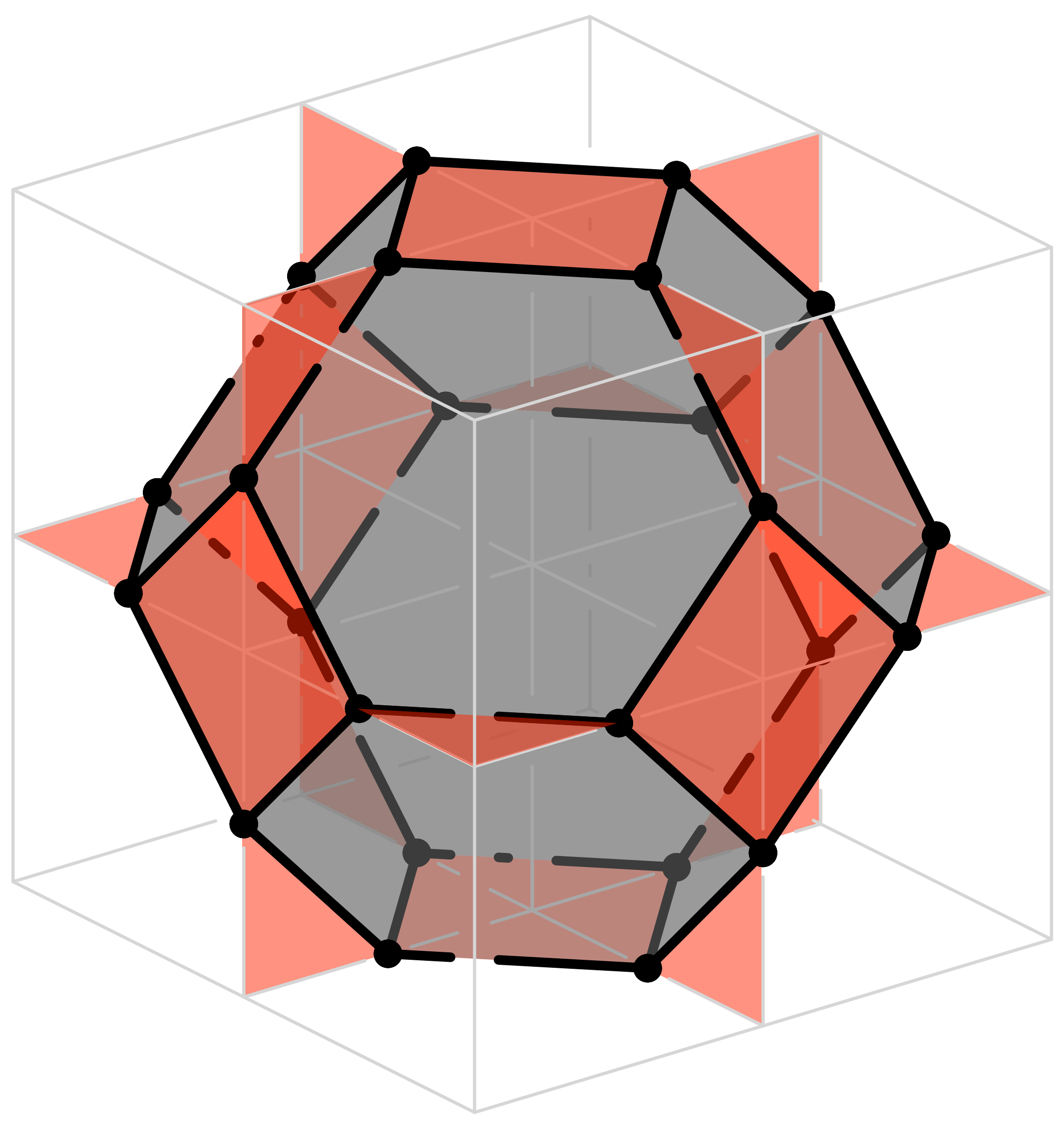

Brane Brick Models

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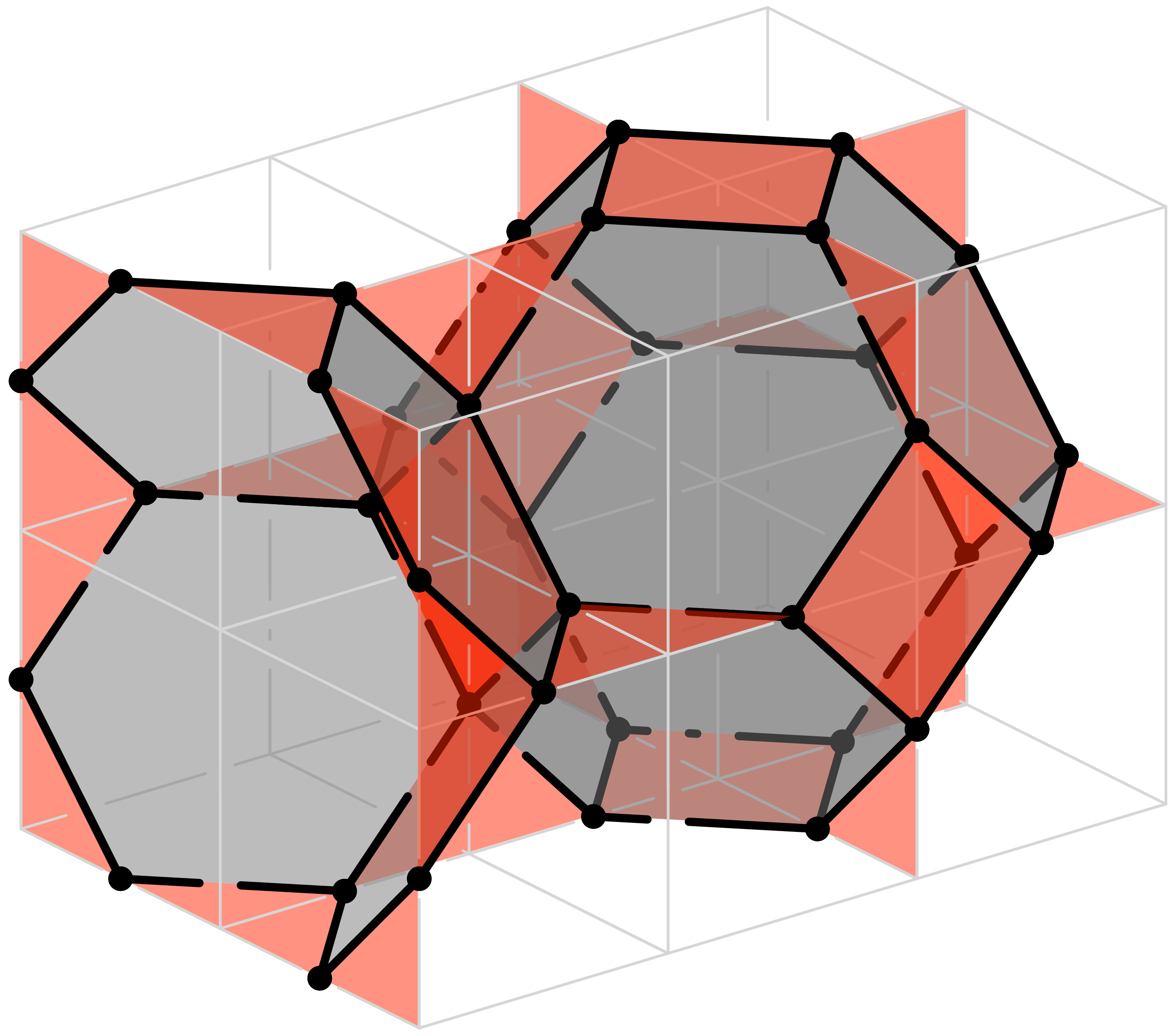

Brane Brick Models

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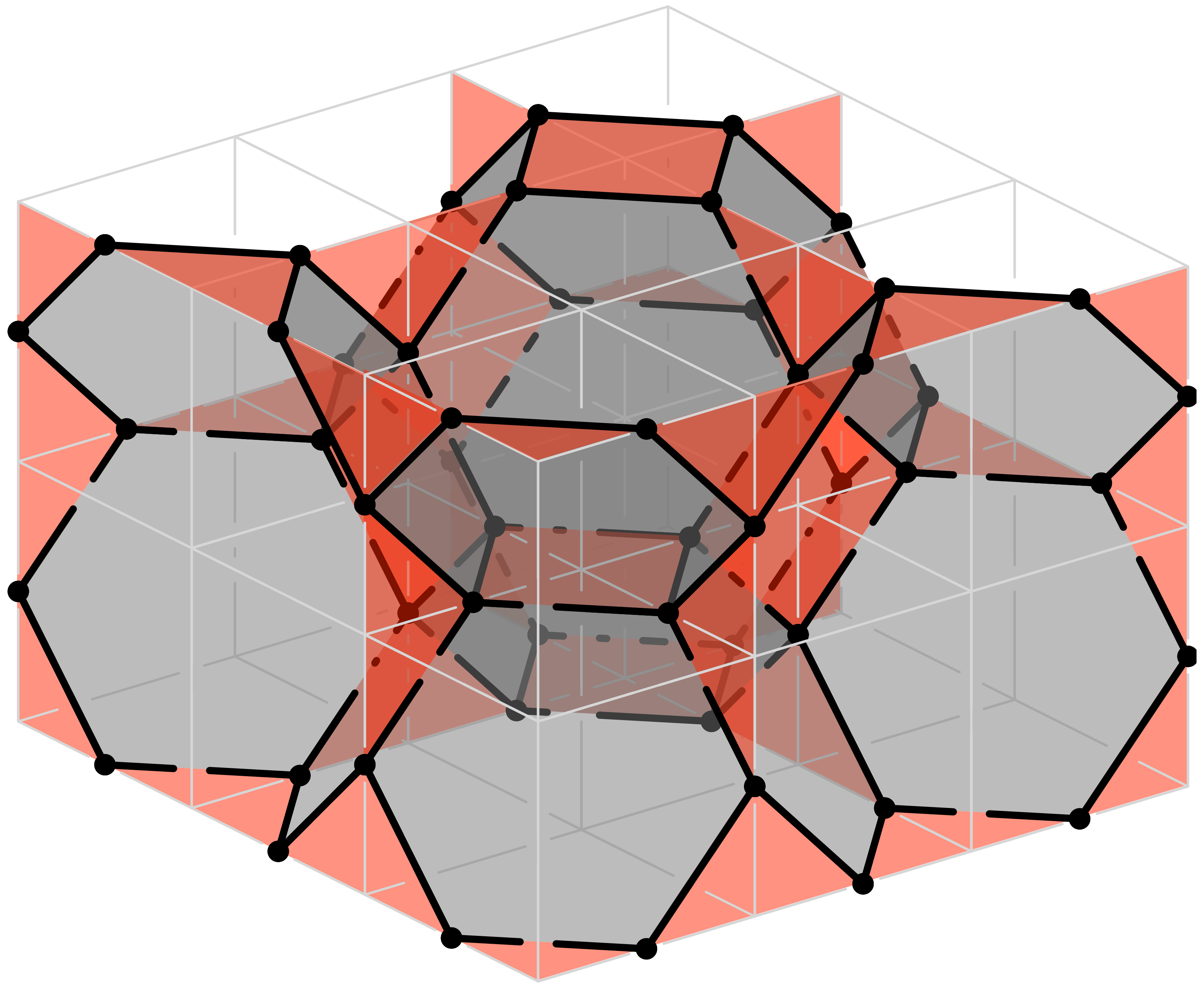


Brane Brick Models

[Franco-Lee-Seong 2015]

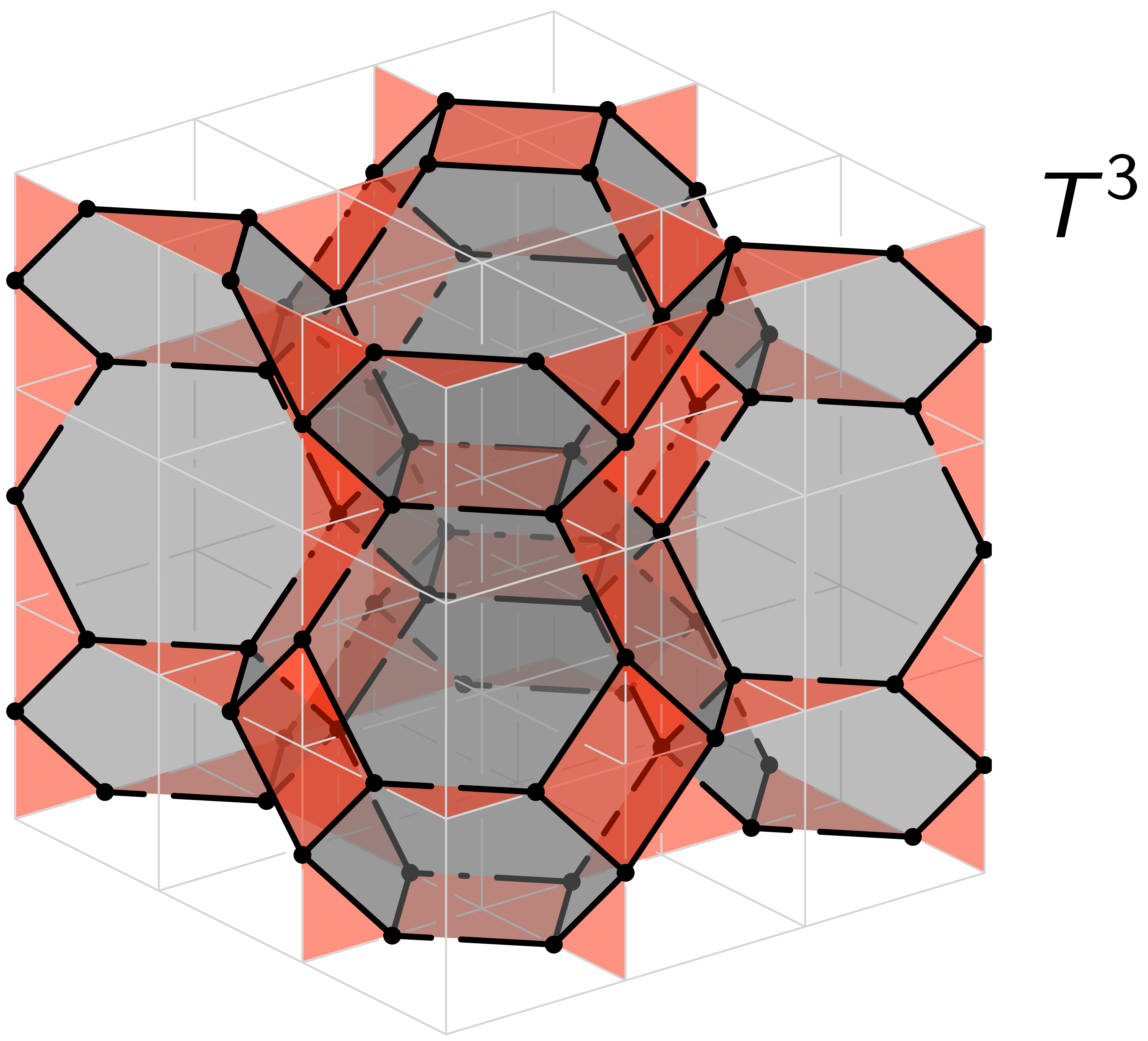
- Geometry

$\mathbb{C}^4/\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2} \times \mathbb{Z}_{n_3}$

order of the orbifold

$n_1 * n_2 * n_3 = \#(\text{truncated octahedra})$

- Brane Brick Model



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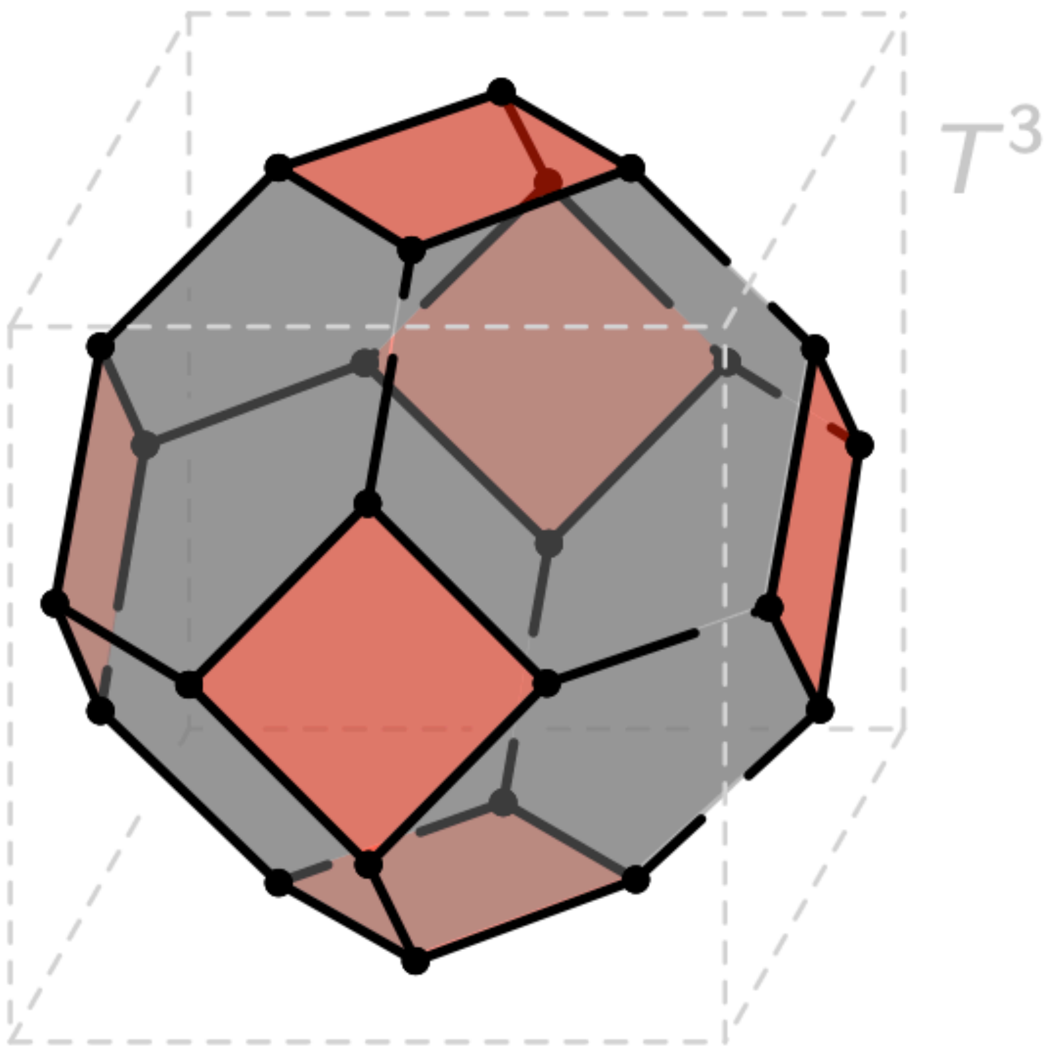
Brane Brick Models

[Franco-Lee-Seong 2015]

Polytopes

- **U(N) Gauge Group**

Convex Polytopes with even-sided Faces

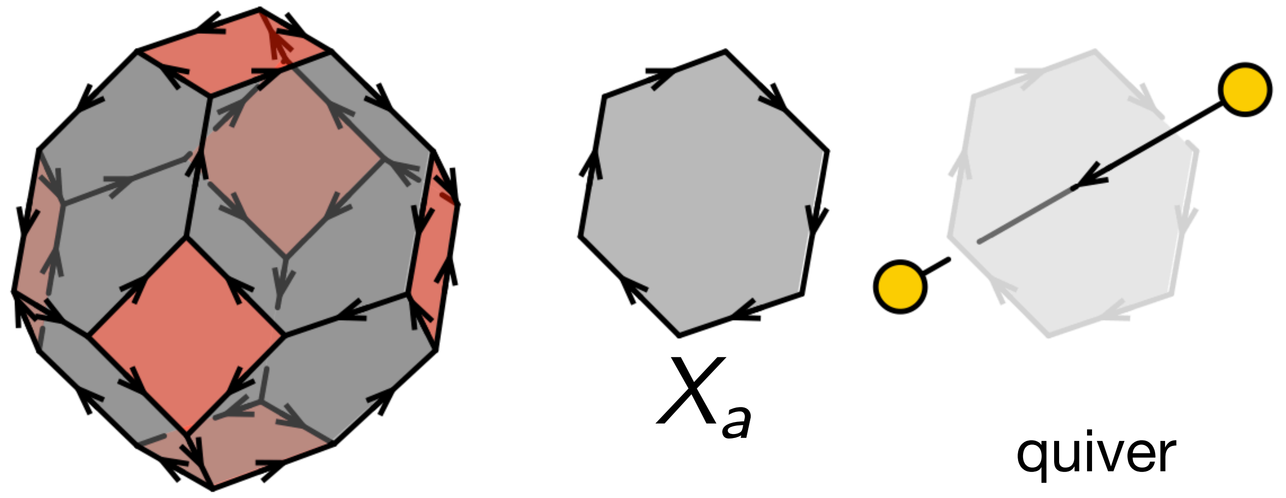


truncated octahedron

Faces

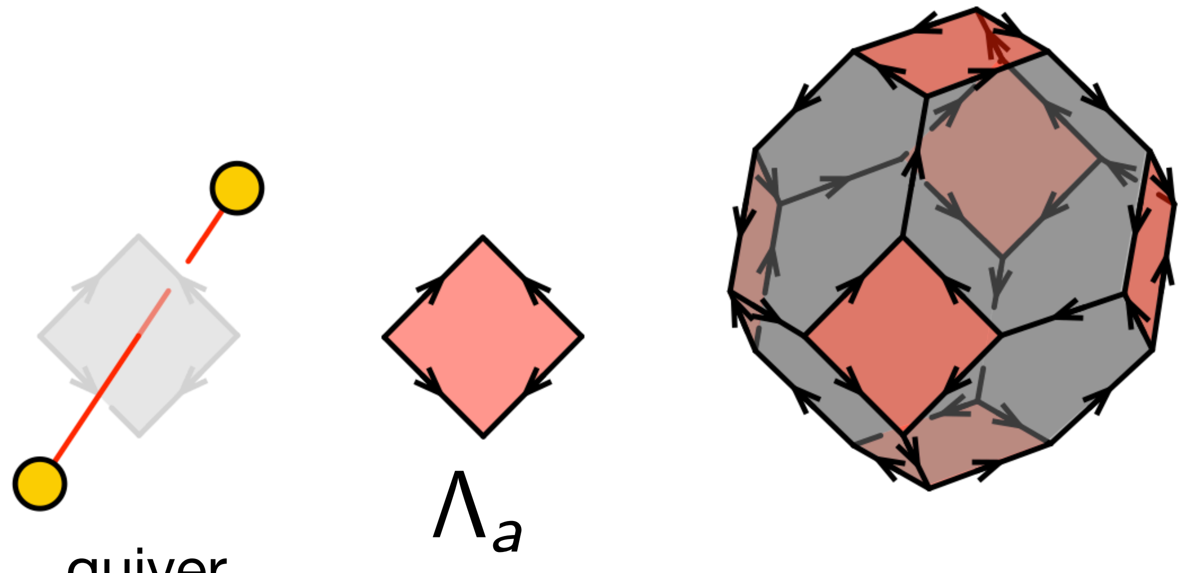
- **2d (0,2) Chiral**

Even-sided Faces



- **2d (0,2) Fermi**

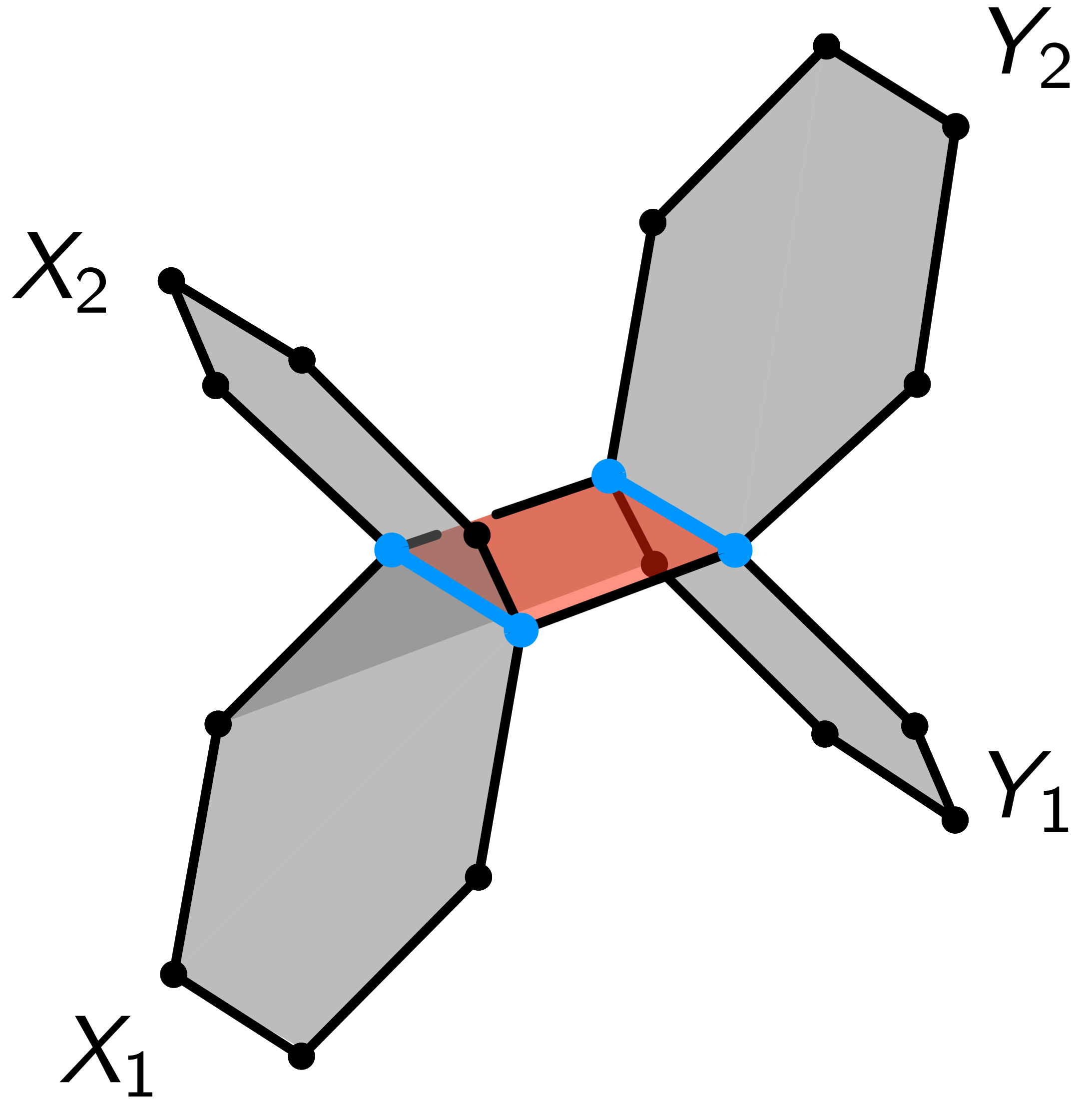
Square Faces



Edges

- **J, E - Terms**

Faces attached to Edges form monomials in J,E-Term relations



$$J \; : \; X_1 X_2 - Y_1 Y_2$$

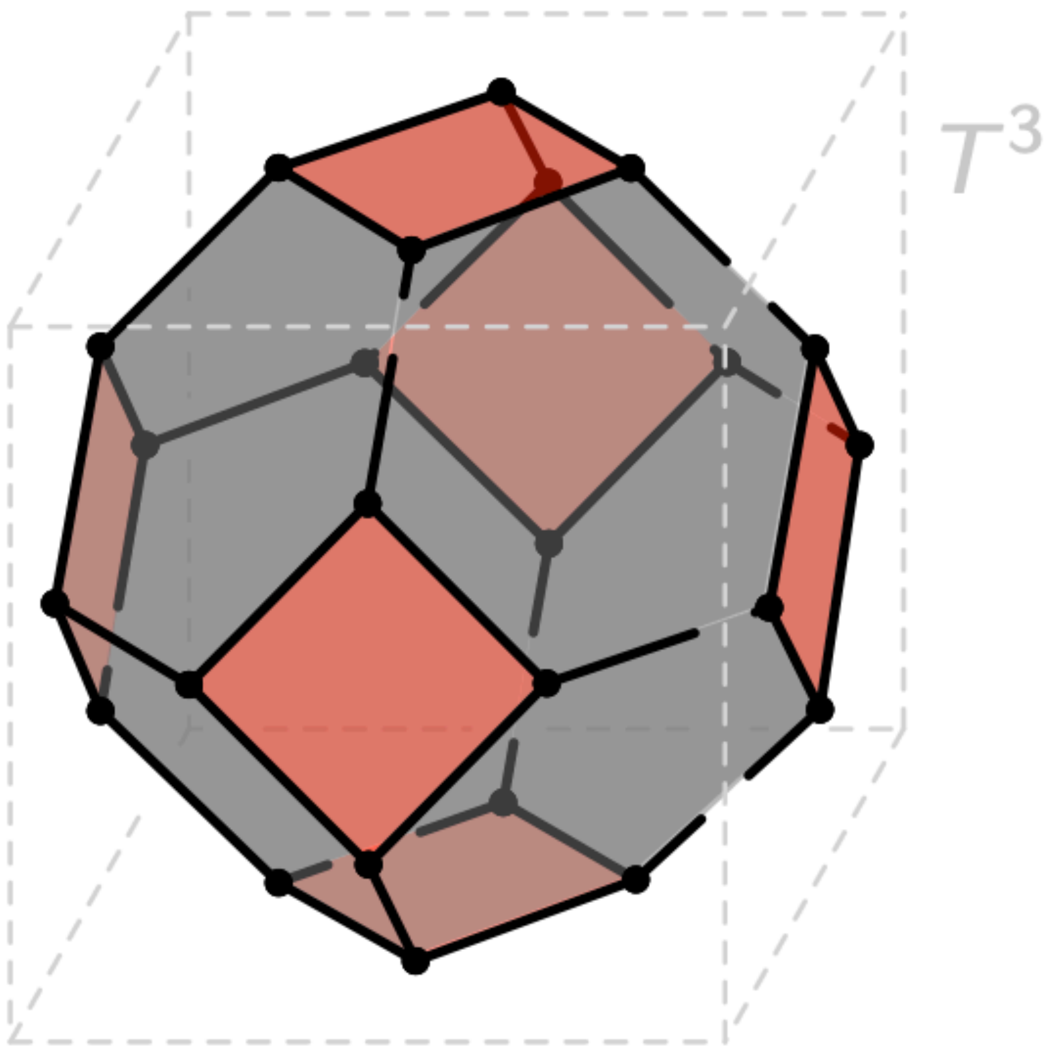
Brane Brick Models

[Franco-Lee-Seong 2015]

Polytopes

- **U(N) Gauge Group**

Convex Polytopes with even-sided Faces



truncated octahedron

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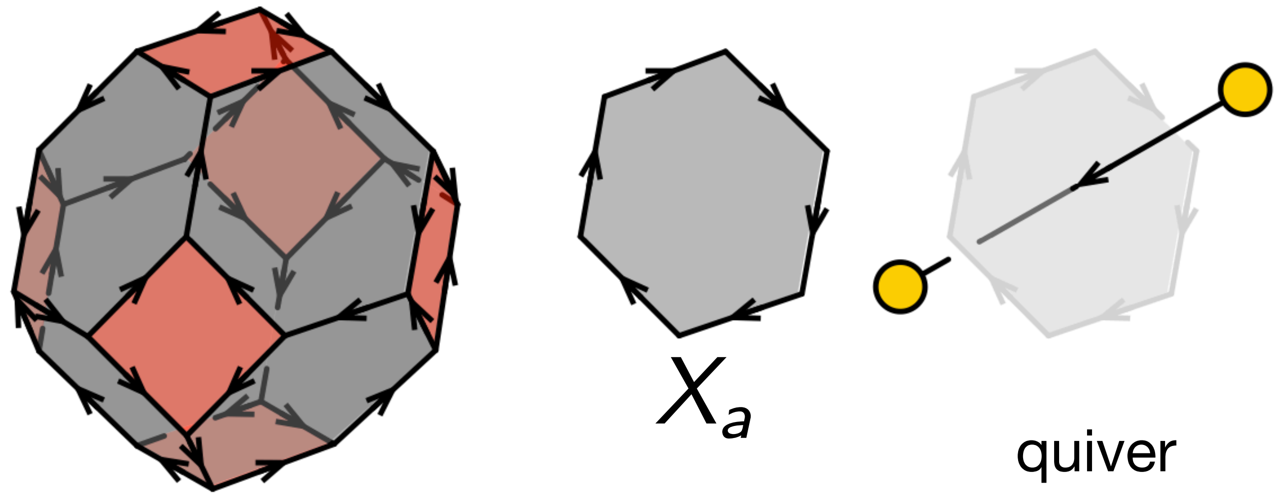
Even-sided Faces



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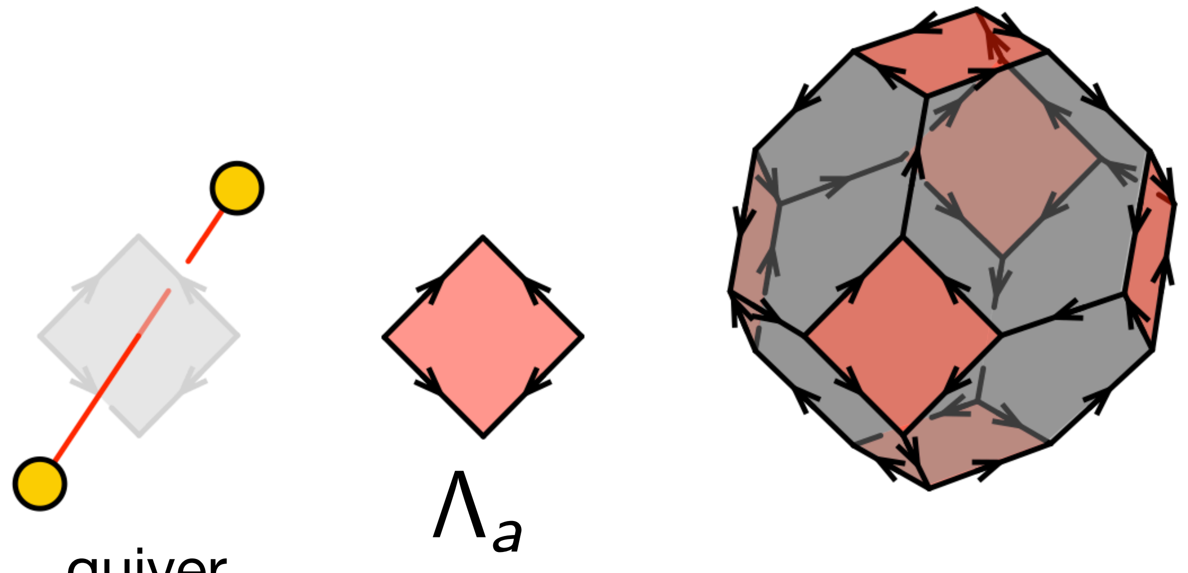

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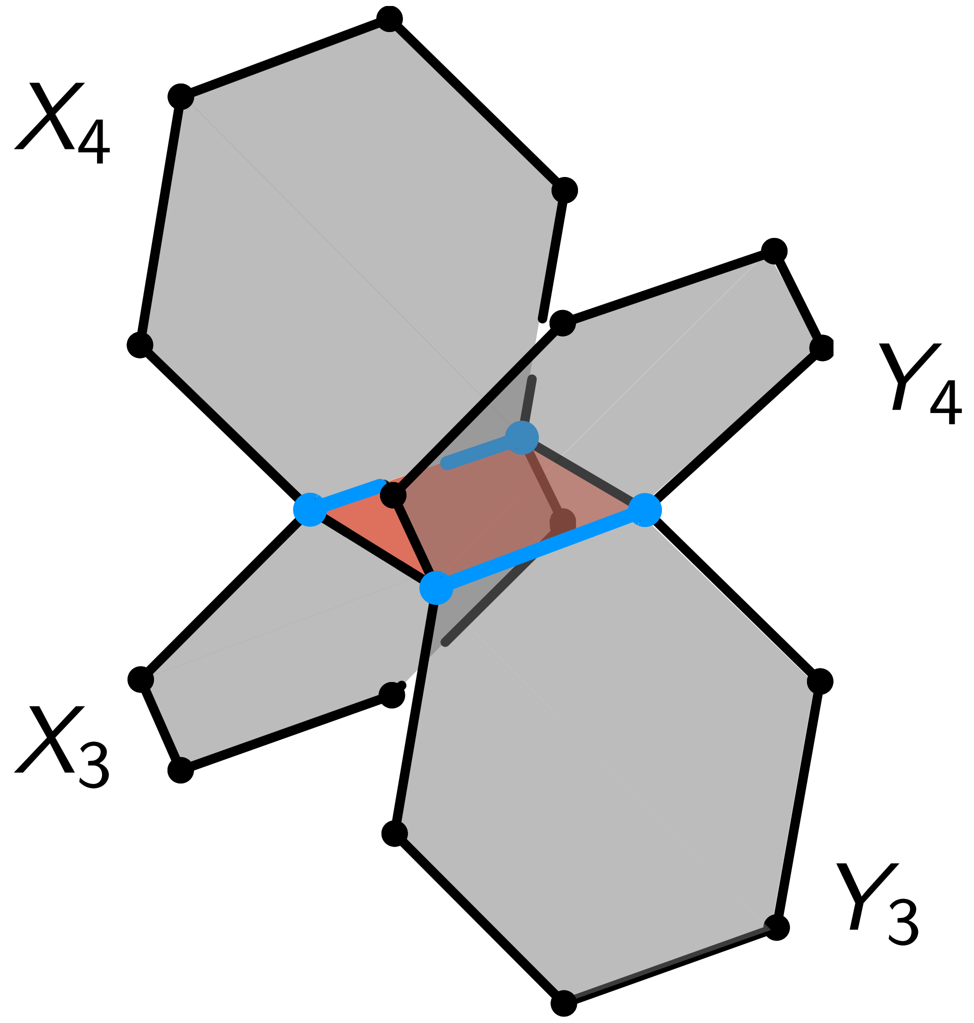


$$E \ : \ X_3 X_4 - Y_3 Y_4$$

Fano 3-Folds

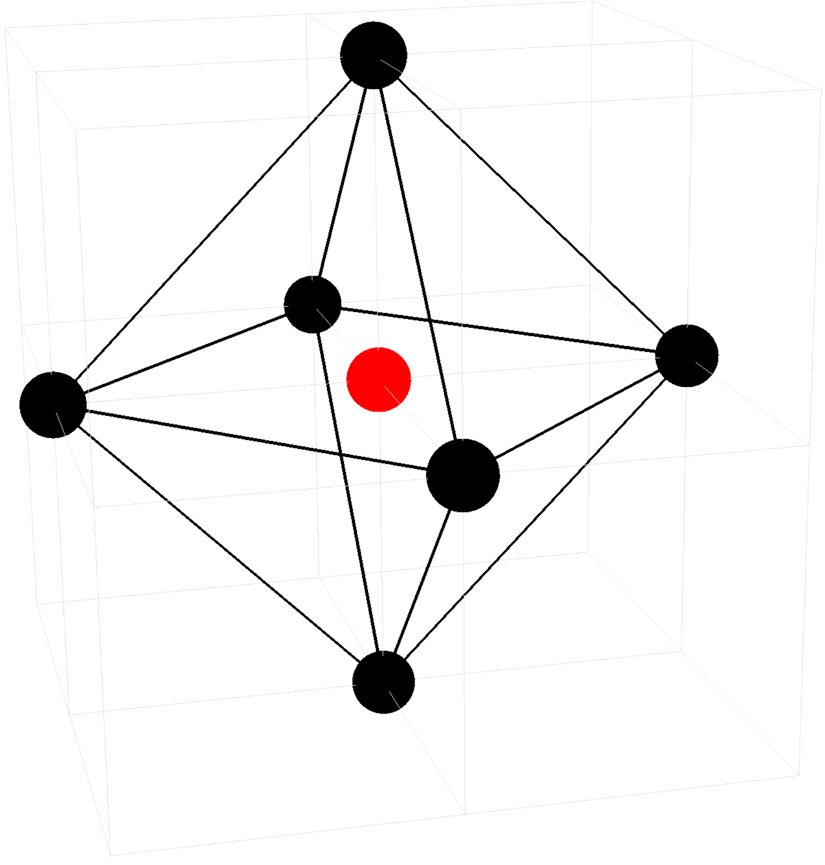
[Franco-Seong 2022]

Examples

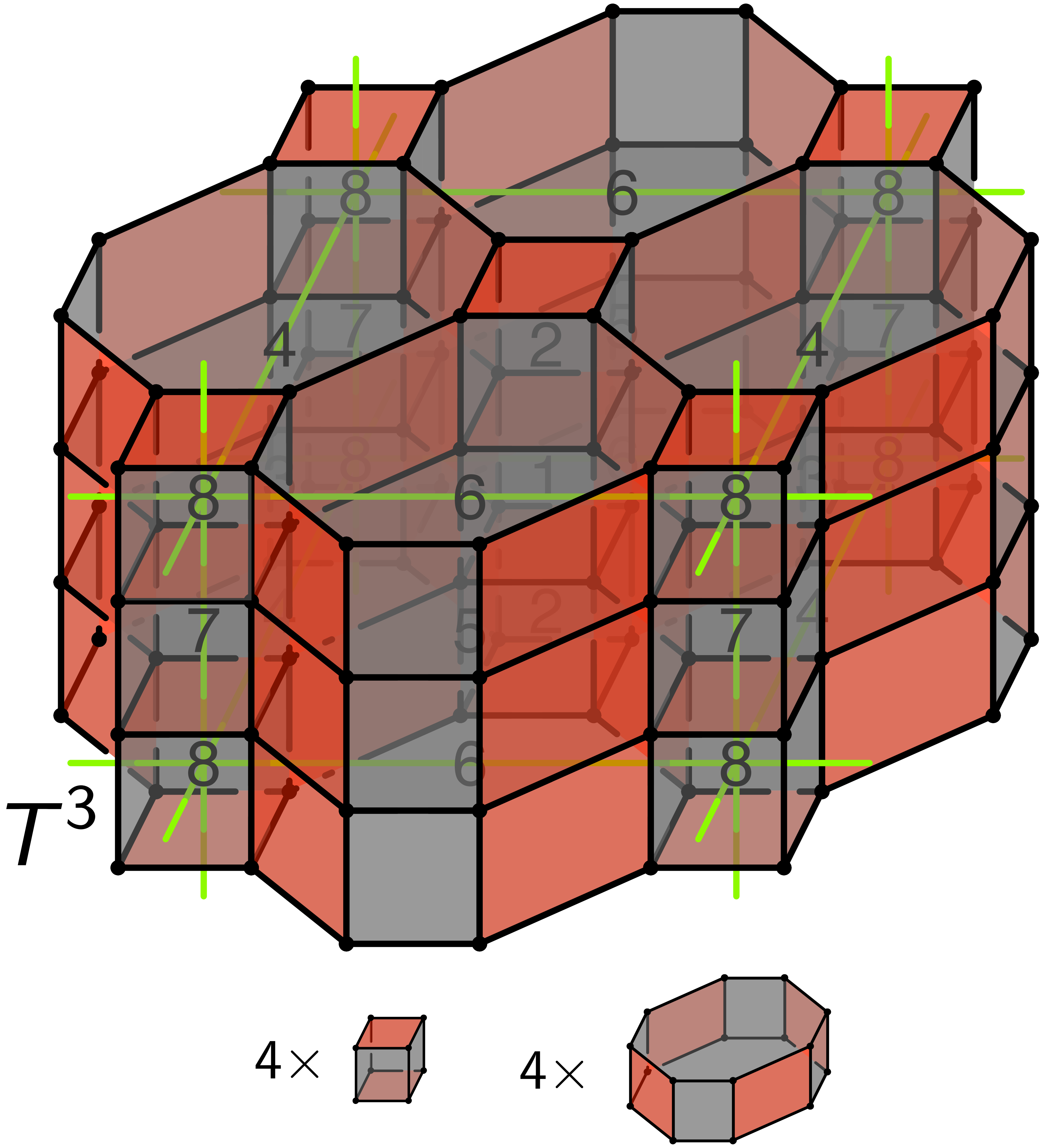
- Geometry

$Q^{1,1,1}/\mathbb{Z}_2$

- Toric Diagram



- Brane Brick Model



- J,E-Terms

8 U(N) Gauge Groups, with 24 Chiral Fields

	J	E
Λ^1_{12} :	$Y_{23}Y_{34}X_{41} - X_{23}Y_{34}Y_{41}$	$P_{15}X_{52} - Q_{15}R_{52}$
Λ^2_{12} :	$X_{23}X_{34}Y_{41} - Y_{23}X_{34}X_{41}$	$P_{15}Y_{52} - Q_{15}S_{52}$
Λ^1_{27} :	$X_{78}Y_{85}Y_{52} - Y_{78}Y_{85}X_{52}$	$P_{26}X_{67} - X_{23}P_{37}$
Λ^2_{27} :	$Y_{78}X_{85}X_{52} - X_{78}X_{85}Y_{52}$	$P_{26}Y_{67} - Y_{23}P_{37}$
Λ^3_{27} :	$X_{78}Y_{85}S_{52} - Y_{78}Y_{85}R_{52}$	$X_{23}Q_{37} - Q_{26}X_{67}$
Λ^4_{27} :	$Y_{78}X_{85}R_{52} - X_{78}X_{85}S_{52}$	$Y_{23}Q_{37} - Q_{26}Y_{67}$
Λ^1_{38} :	$Y_{85}Y_{52}X_{23} - X_{85}Y_{52}Y_{23}$	$P_{37}X_{78} - X_{34}P_{48}$
Λ^2_{38} :	$X_{85}X_{52}Y_{23} - Y_{85}X_{52}X_{23}$	$P_{37}Y_{78} - Y_{34}P_{48}$
Λ^3_{38} :	$Y_{85}S_{52}X_{23} - X_{85}S_{52}Y_{23}$	$X_{34}Q_{48} - Q_{37}X_{78}$
Λ^4_{38} :	$X_{85}R_{52}Y_{23} - Y_{85}R_{52}X_{23}$	$Y_{34}Q_{48} - Q_{37}Y_{78}$
Λ^1_{45} :	$X_{52}Y_{23}Y_{34} - Y_{52}Y_{23}X_{34}$	$P_{48}X_{85} - X_{41}P_{15}$
Λ^2_{45} :	$Y_{52}X_{23}X_{34} - X_{52}X_{23}Y_{34}$	$P_{48}Y_{85} - Y_{41}P_{15}$
Λ^3_{45} :	$R_{52}Y_{23}Y_{34} - S_{52}Y_{23}X_{34}$	$X_{41}Q_{15} - Q_{48}X_{85}$
Λ^4_{45} :	$S_{52}X_{23}X_{34} - R_{52}X_{23}Y_{34}$	$Y_{41}Q_{15} - Q_{48}Y_{85}$
Λ^1_{56} :	$Y_{67}Y_{78}X_{85} - X_{67}Y_{78}Y_{85}$	$R_{52}Q_{26} - X_{52}P_{26}$
Λ^2_{56} :	$X_{67}X_{78}Y_{85} - Y_{67}X_{78}X_{85}$	$S_{52}Q_{26} - Y_{52}P_{26}$

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Fano 3-Folds

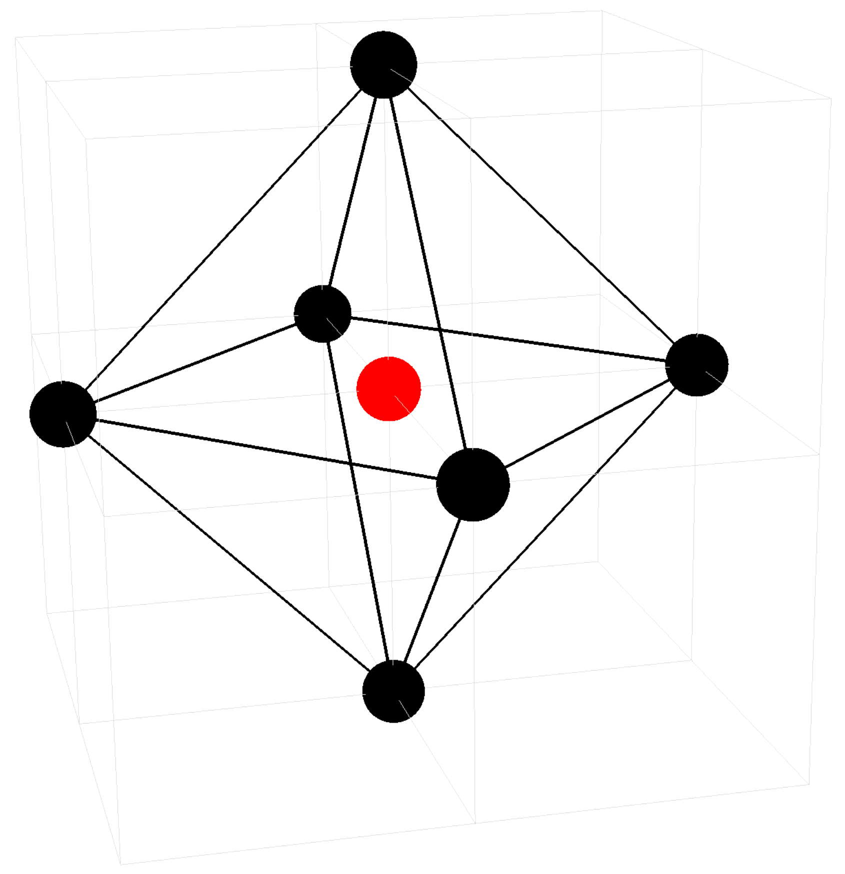
[Franco-Seong 2022]

Examples

Geometry

$Q^{1,1,1}/\mathbb{Z}_2$

Toric Diagram



J,E-Terms

8 U(N) Gauge Groups, with 24 Chiral Fields

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Λ^1_{12} :	$Y_{23}Y_{34}X_{41}$	$-$	$X_{23}Y_{34}Y_{41}$	$P_{15}X_{52}$	$-$	$Q_{15}R_{52}$
Λ^2_{12} :	$X_{23}X_{34}Y_{41}$	$-$	$Y_{23}X_{34}X_{41}$	$P_{15}Y_{52}$	$-$	$Q_{15}S_{52}$
Λ^1_{27} :	$X_{78}Y_{85}Y_{52}$	$-$	$Y_{78}Y_{85}X_{52}$	$P_{26}X_{67}$	$-$	$X_{23}P_{37}$
Λ^2_{27} :	$Y_{78}X_{85}X_{52}$	$-$	$X_{78}X_{85}Y_{52}$	$P_{26}Y_{67}$	$-$	$Y_{23}P_{37}$
Λ^3_{27} :	$X_{78}Y_{85}S_{52}$	$-$	$Y_{78}Y_{85}R_{52}$	$X_{23}Q_{37}$	$-$	$Q_{26}X_{67}$
Λ^4_{27} :	$Y_{78}X_{85}R_{52}$	$-$	$X_{78}X_{85}S_{52}$	$Y_{23}Q_{37}$	$-$	$Q_{26}Y_{67}$
Λ^1_{38} :	$Y_{85}Y_{52}X_{23}$	$-$	$X_{85}Y_{52}Y_{23}$	$P_{37}X_{78}$	$-$	$X_{34}P_{48}$
Λ^2_{38} :	$X_{85}X_{52}Y_{23}$	$-$	$Y_{85}X_{52}X_{23}$	$P_{37}Y_{78}$	$-$	$Y_{34}P_{48}$
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Λ^1_{56} :	$Y_{67}Y_{78}X_{85}$	$-$	$X_{67}Y_{78}Y_{85}$	$R_{52}Q_{26}$	$-$	$X_{52}P_{26}$
Λ^2_{56} :	$X_{67}X_{78}Y_{85}$	$-$	$Y_{67}X_{78}X_{85}$	$S_{52}Q_{26}$	$-$	$Y_{52}P_{26}$

Hilbert Series

Generating function of gauge invariant operators

$$HS(Q^{1,1,1}/\mathbb{Z}_2) = \frac{1+19t^6-63t^{12}+43t^{18}+43t^{24}-63t^{30}+19t^{36}+t^{42}}{(1-t^6)^8}$$

$$= 1+27t^6+125t^{12}+343t^{18}+729t^{24}+\ldots$$

$3\times 3\times 3 = 27$

$5\times 5\times 5 = 125$

$7\times 7\times 7 = 343$

$$= \sum_{n=0}^{\infty}(2n+1)^3 t^{6n}$$

Fano 3-Folds

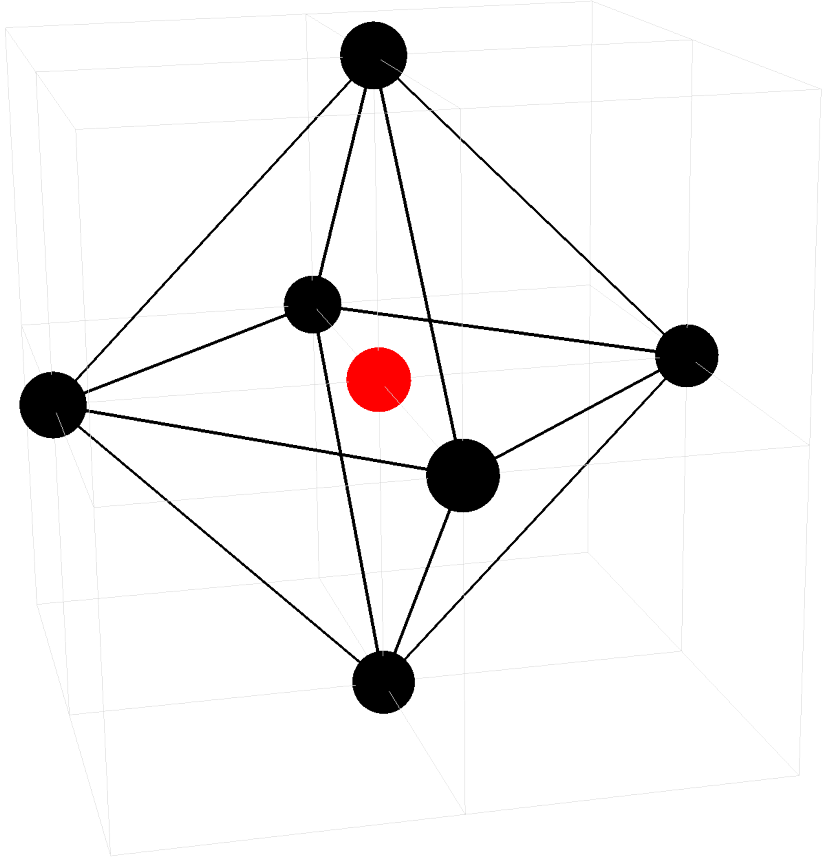
[Franco-Seong 2022]

Examples

- **Geometry**

$Q^{1,1,1}/\mathbb{Z}_2$

- **Toric Diagram**



- **Hilbert Series**

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$3\times3\times3=27$

$5\times5\times5=125$

$7\times7\times7=343$

$$=\sum_{n=0}^{\infty}(2n+1)^3 t^{6n}$$

- **Global Symmetry**

Characters of Irreducible Representations

$SU(2)$

$[2] = x+1+\frac{1}{x}$ adjoint

$[4] = x^4+x^2+1+\frac{1}{x^2}+\frac{1}{x^4}$

$[6] = x^6+x^4+x^2+1+\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}$

dimension $\dim[2n]=2n+1$

$$HS(Q^{1,1,1}/\mathbb{Z}_2) = \sum_{n=0}^{\infty}[2n][2n][2n]t^{6n}$$

Global Symmetry $SU(2)_x\times SU(2)_y\times SU(2)_z\times U(1)_t$

Fano 3-Folds

[Franco-Seong 2022]

Examples

- Hilbert Series and Global Symmetry

$$HS(Q^{1,1,1}/\mathbb{Z}_2) = \sum_{n=0}^{\infty} [2n]_x [2n]_y [2n]_z t^{6n}$$

$$= 1 + \Big(1 + x^2y^2z^2 + \frac{x^2z^2}{y^2} + \frac{x^2y^2}{z^2} + \frac{x^2}{y^2z^2} + \frac{y^2z^2}{x^2} + \frac{z^2}{x^2y^2} + \frac{y^2}{x^2z^2} + \frac{1}{x^2y^2z^2}$$
$$+x^2y^2 + \frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{x^2y^2} + x^2z^2 + \frac{x^2}{z^2} + \frac{z^2}{x^2} + \frac{1}{x^2z^2} + x^2 + \frac{1}{x^2} + y^2$$
$$+z^2 + \frac{z^2}{y^2} + \frac{y^2}{z^2} + \frac{1}{y^2z^2} + y^2 + \frac{1}{y^2} + z^2 + \frac{1}{z^2}\Big) t^6 + \dots$$

We can collect exponents for terms corresponding to generators

$$x^{m_1} y^{m_2} z^{m_3} \rightarrow (m_1, m_2, m_3)$$

character weights

scaling

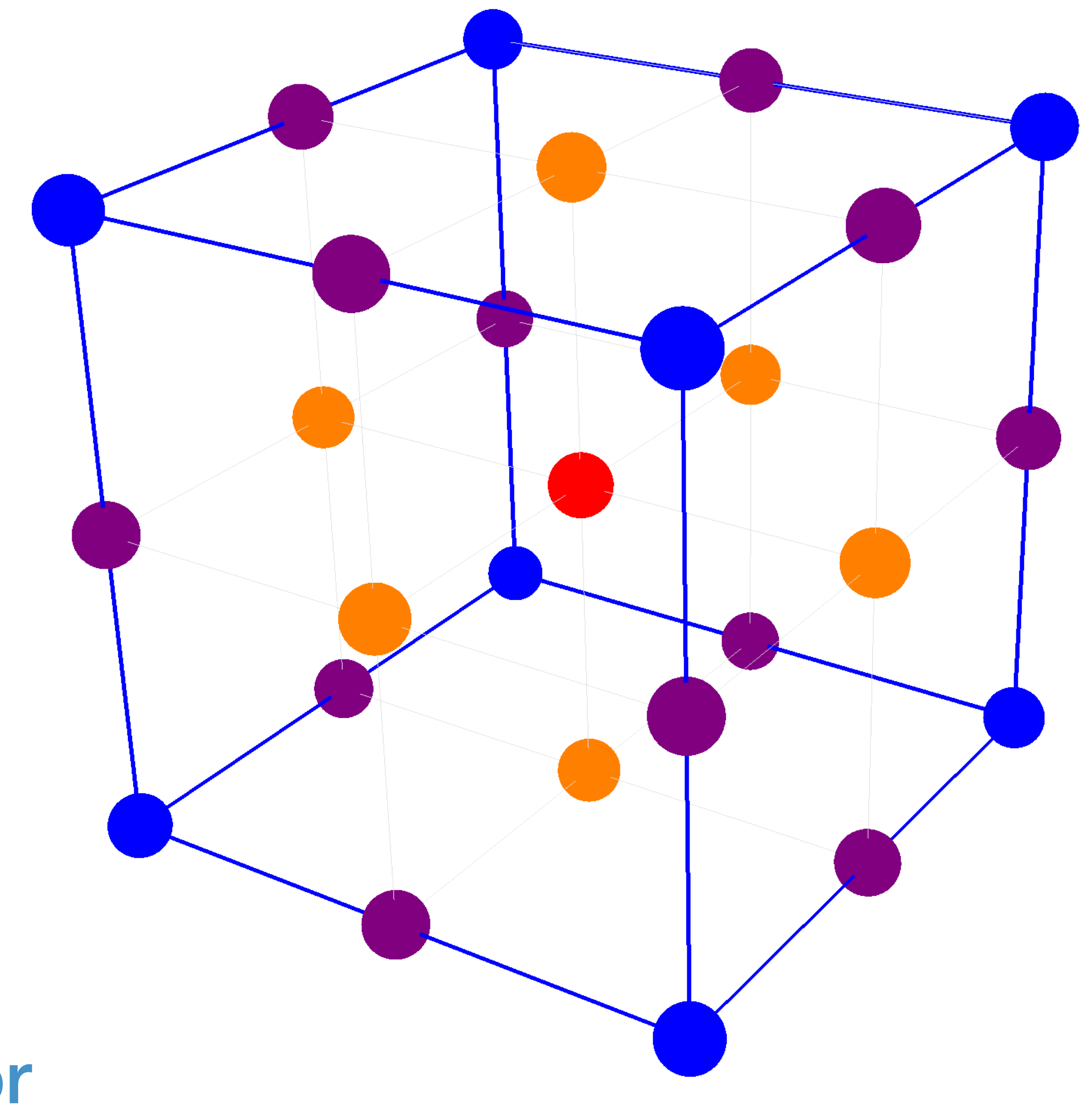
$(0,-2,0) \rightarrow (0,-1,0)$

$(0,2,0) \rightarrow (0,1,0)$

$(0,-2,-2) \rightarrow (0,-1,-1)$

generator lattice

The character weights of generators form a convex polygon



Fano 3-Folds

[Franco-Seong 2022]

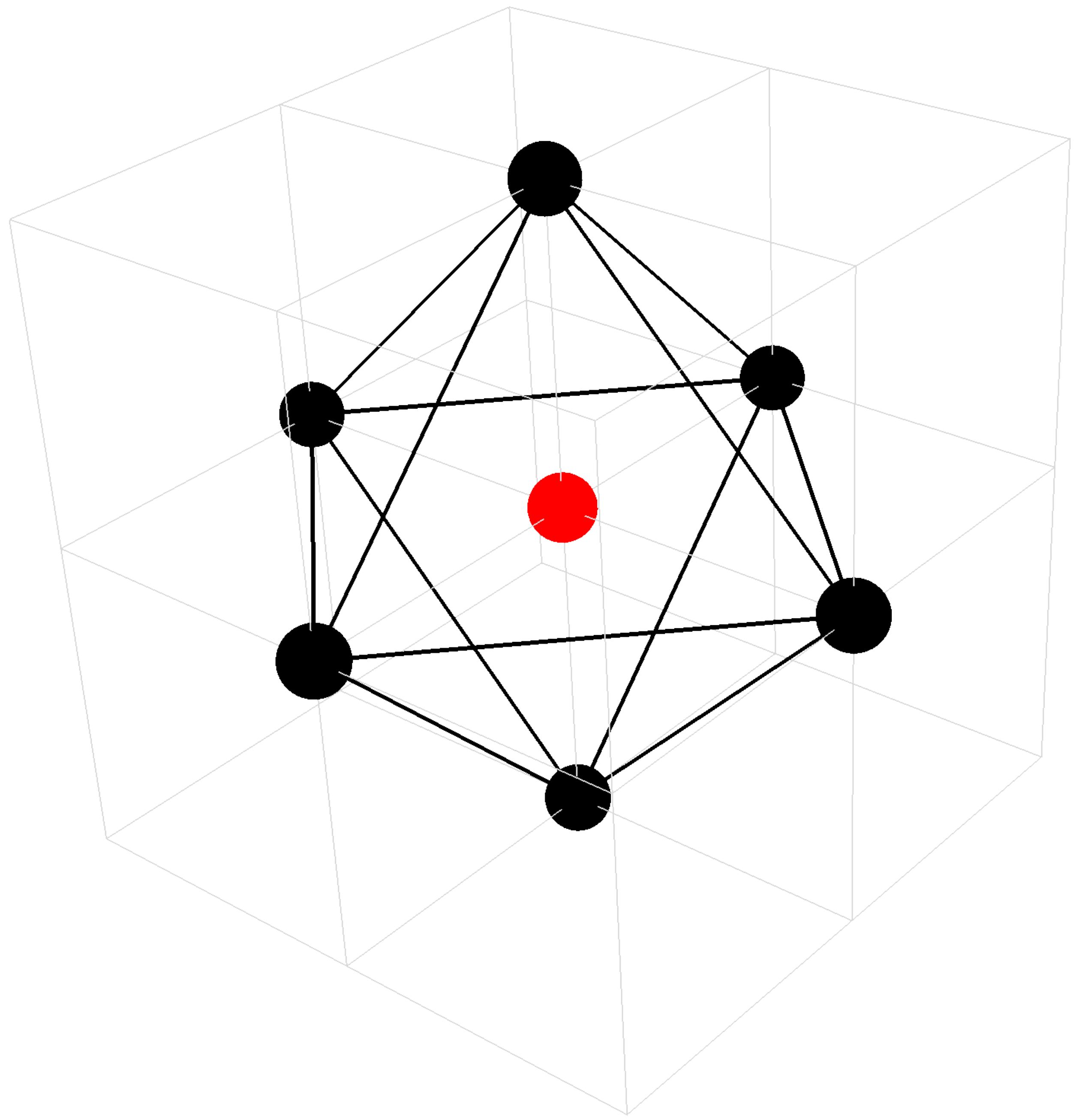
Examples

- Geometry

$Q^{1,1,1}/\mathbb{Z}_2$

- Toric Diagram

A Reflexive Polytope

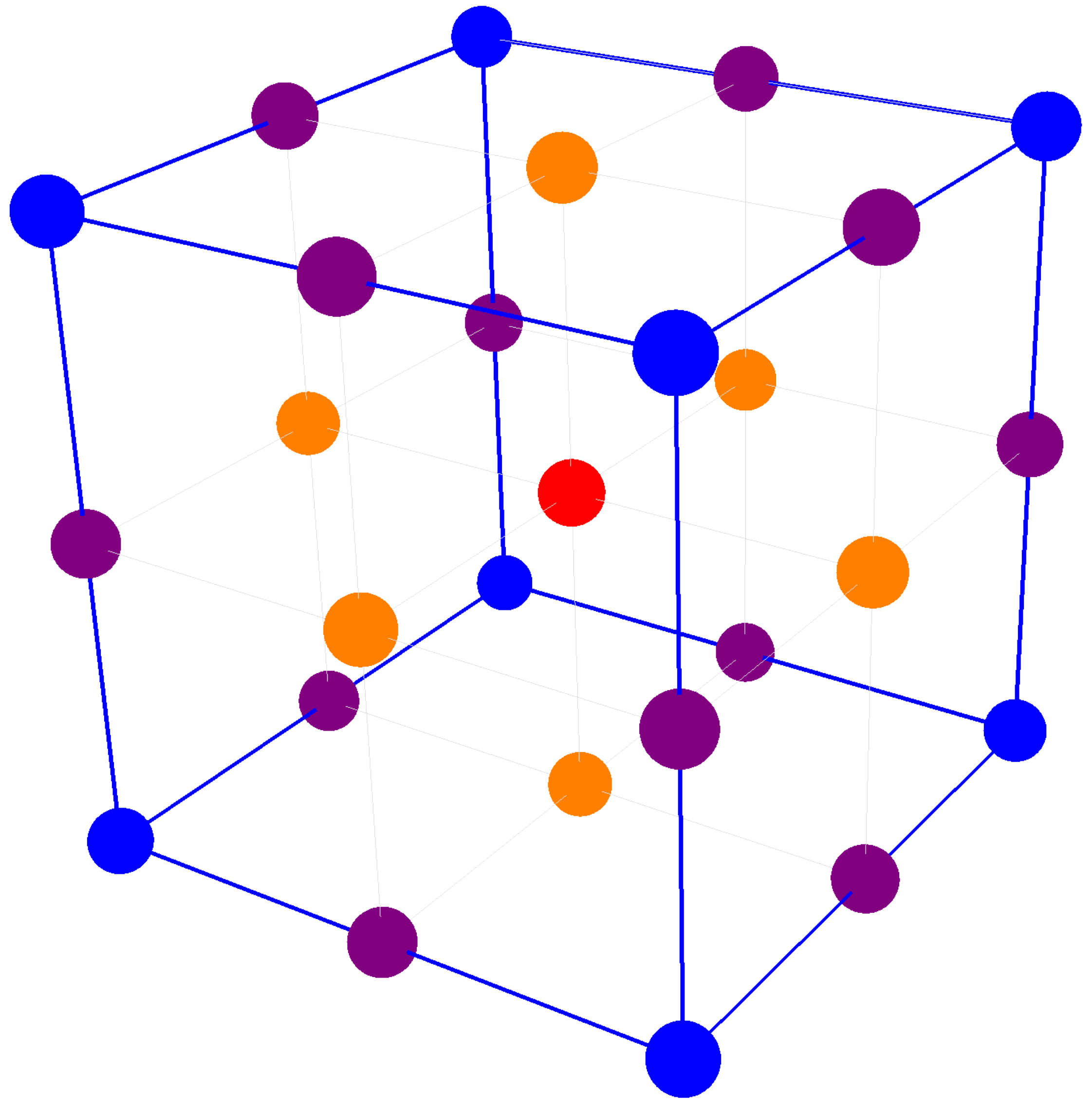


Dual Polytopes

property of reflexive polytopes

- Lattice of Generators

A Reflexive Polytope



Fano 3-Folds

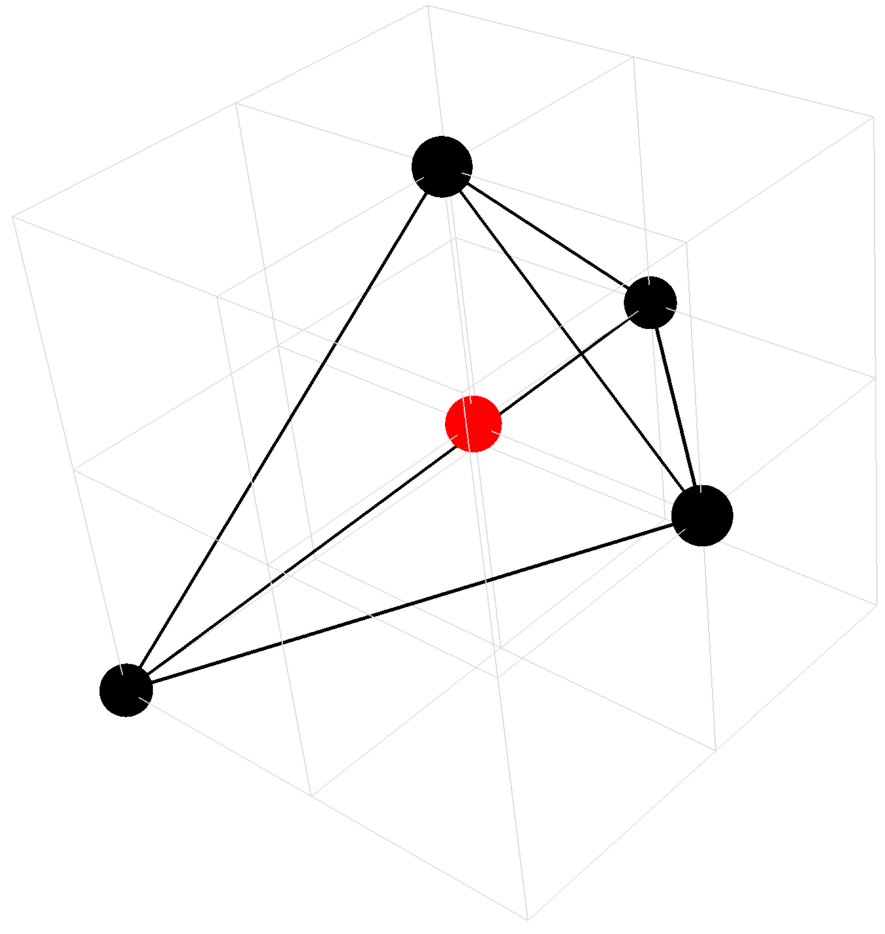
[Franco-Seong 2022]

Examples

- **Geometry**

$$\mathbb{C}^4/\mathbb{Z}_4\ (1,1,1,1)$$

- **Toric Diagram**



- **Hilbert Series**

series expansion

$$HS(\mathbb{C}^4/\mathbb{Z}_4\ (1,1,1,1)) = \frac{1+31t^4+31t^8+t^{12}}{(1-t^4)^4}$$

$$= 1 + 35t^4 + 165t^8 + 455t^{12} + 969t^{16} + \ldots$$

$[4,0,0]$ $[8,0,0]$ $[12,0,0]$

Characters of Irreducible Representations

$SU(4)$

$$= \sum_{n=0}^{\infty}[4n,0,0]t^{4n}$$

Global Symmetry $SU(4)_{x_i} \times U(1)_t$

- **Generator Lattice**

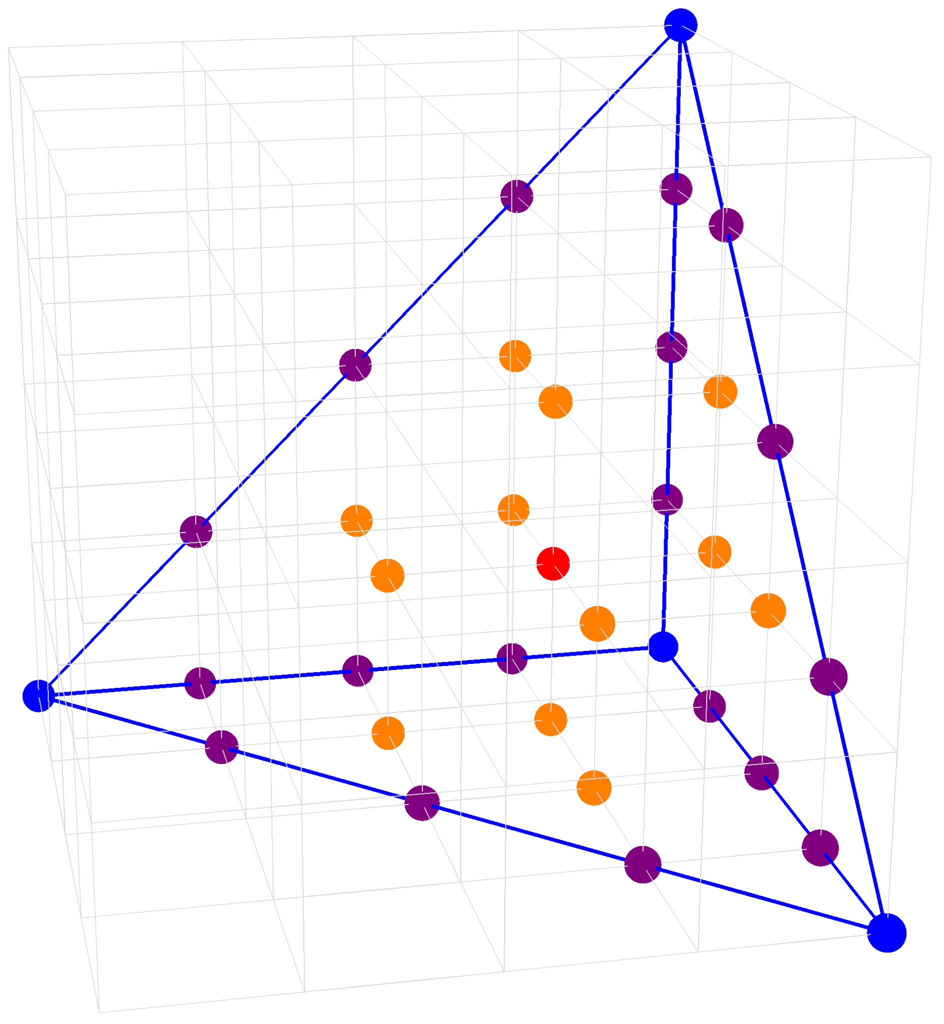
generators

$$\dim[4,0,0]_{SU(4)} = 35$$

$$\downarrow$$

$$(x_1, x_2, x_3)$$

generator lattice is a convex polytope



35 points

Fano 3-Folds

[Franco-Seong 2022]

Examples

- Geometry

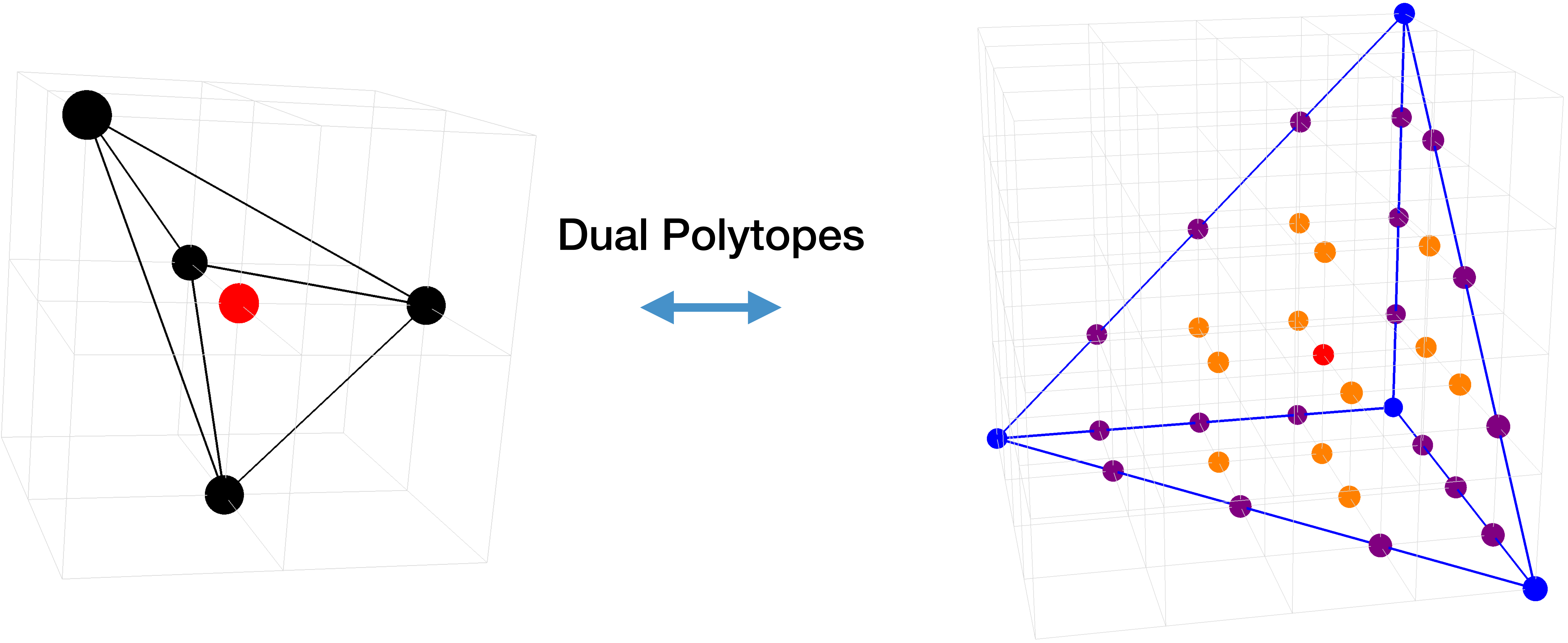
$\mathbb{C}^4/\mathbb{Z}_4 \ (1,1,1,1)$

- Toric Diagram

A Reflexive Polytope

- Lattice of Generators

A Reflexive Polytope



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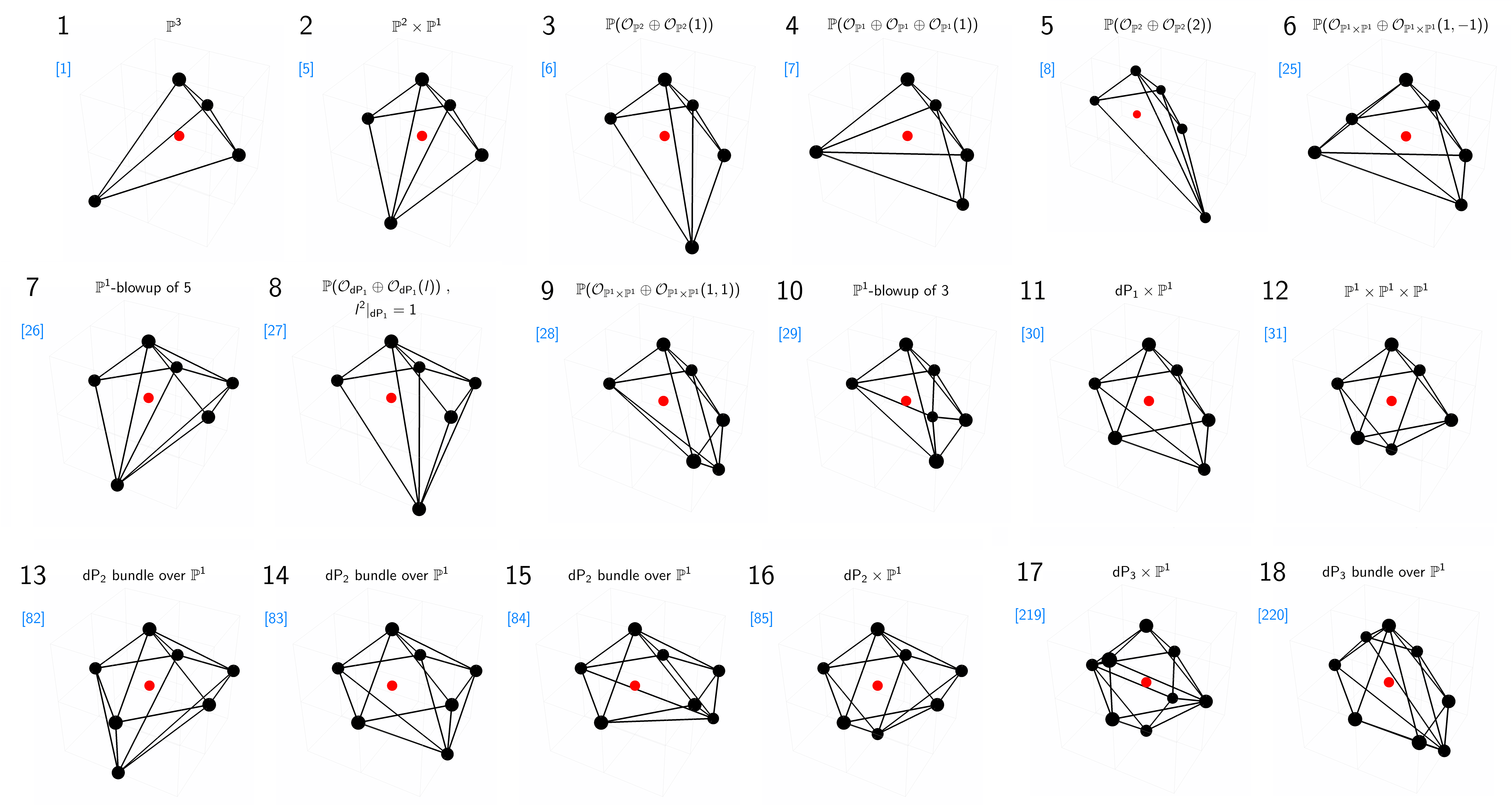
Fano 3-Folds

[Franco-Seong 2022]

Regular Reflexive Polytopes (CY4)

18 Polytopes and their Brane Brick Models



Infinite Families of Geometries

[Gauntlett-Martelli-Sparks-Waldram 2004]

"For every $2n$-dimensional Kähler-Einstein manifold B_{2n} there is an infinite family of Sasaki-Einstein manifolds Y_{2n+3}."

n=1 $\mathbb{CP}^1$: $Y^{p,q}$ [Gauntlett-Martelli-Sparks-Waldram 2004]

n=2 2 Kähler-Einstein manifolds: $\mathbb{CP}^2$ and $\mathbb{CP}^1 \times \mathbb{CP}^1$ [Gauntlett-Martelli-Sparks-Waldram 2004] [Martelli-Sparks 2008]

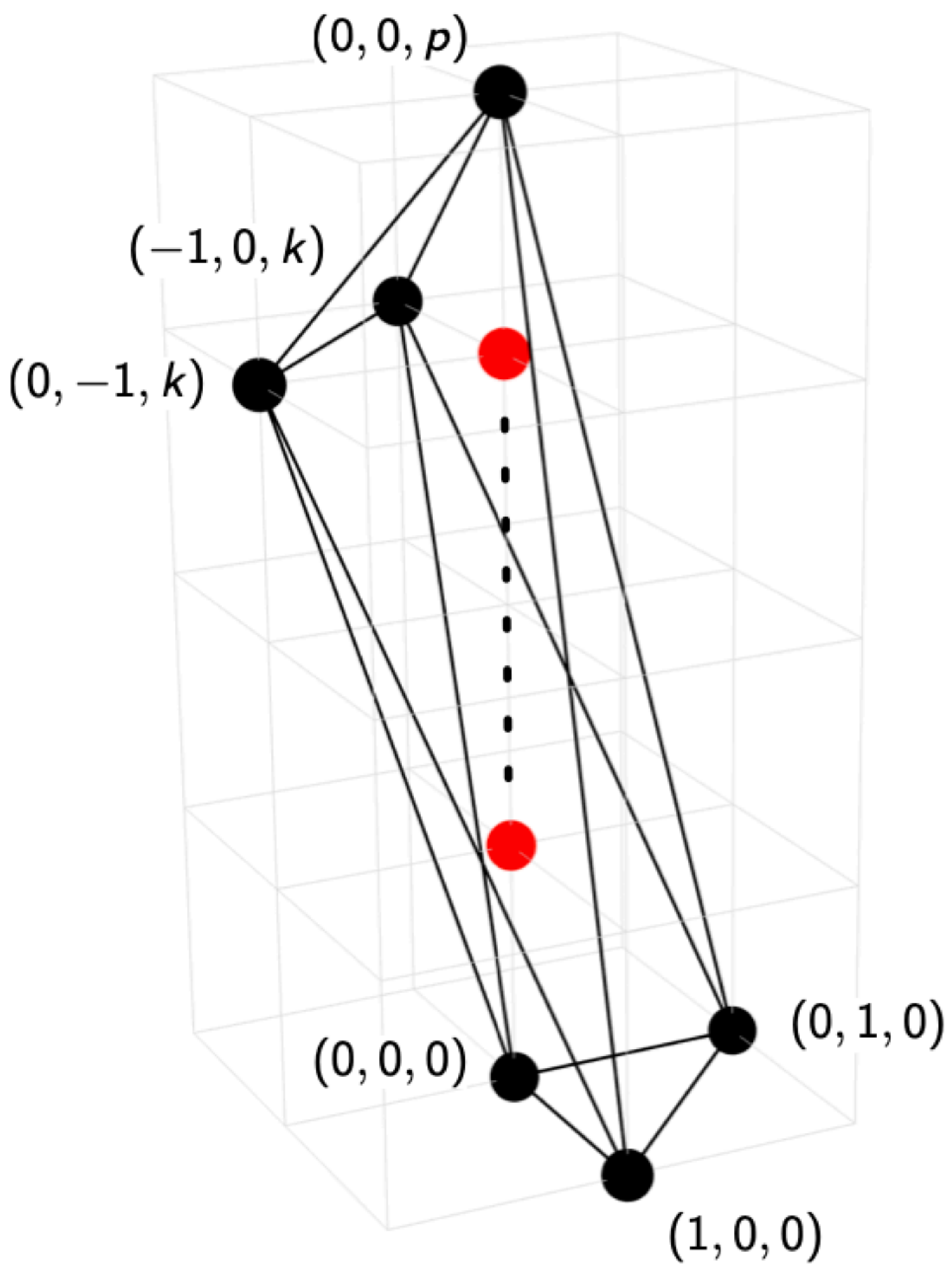
	$Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$	$Y^{p,k}(\mathbb{CP}^2)$
cone	toric CY_4	toric CY_4
isometry	$SU(2) \times SU(2) \times U(1)^2$	$SU(3) \times U(1)^2$
parameters	$0 \leq k \leq p$	$0 \leq k \leq \frac{3}{2}p$

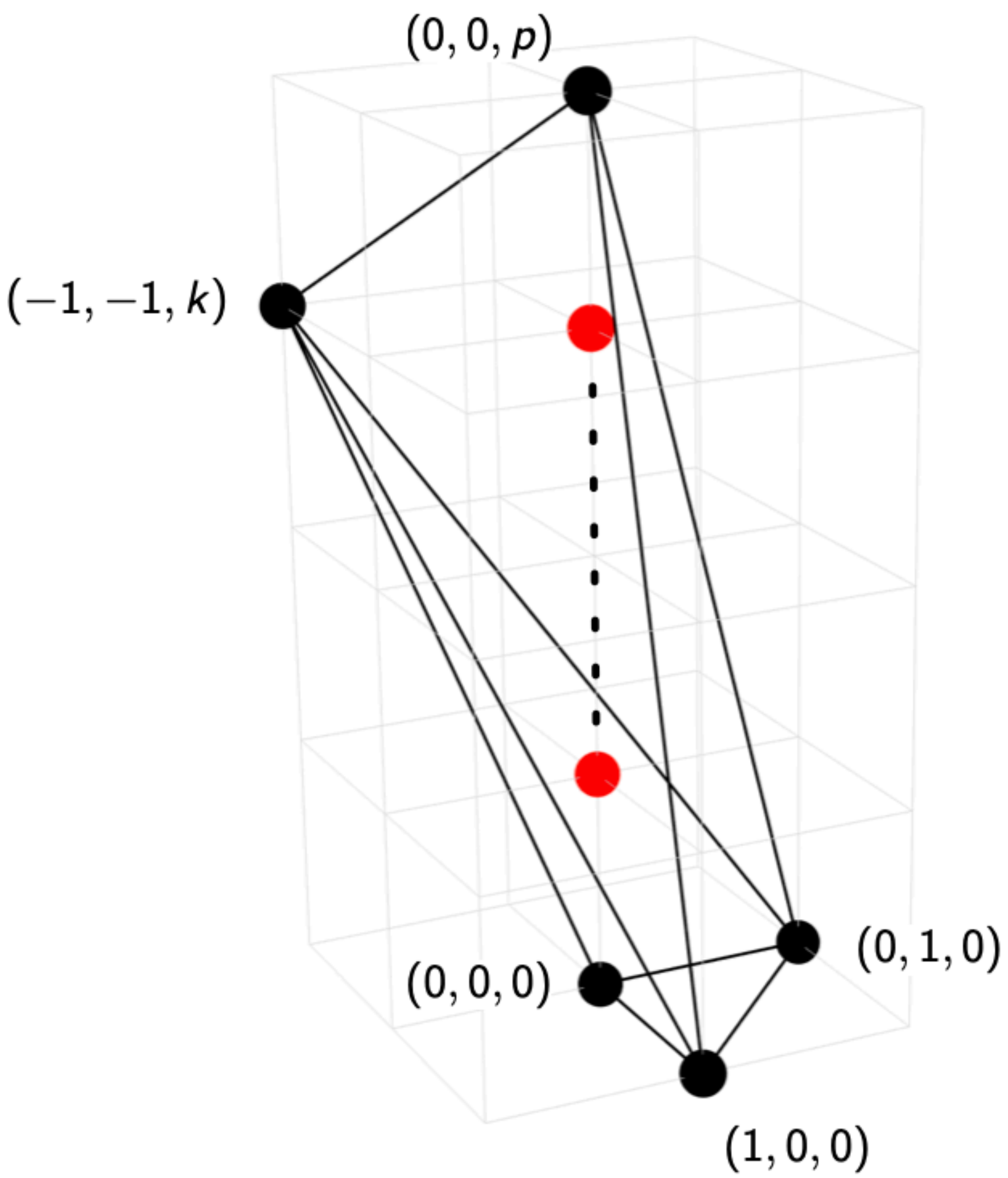
Ypk(CP1xCP1) and Ypk(CP2)

$Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$

$Y^{p,k}(\mathbb{CP}^2)$

- toric diagrams



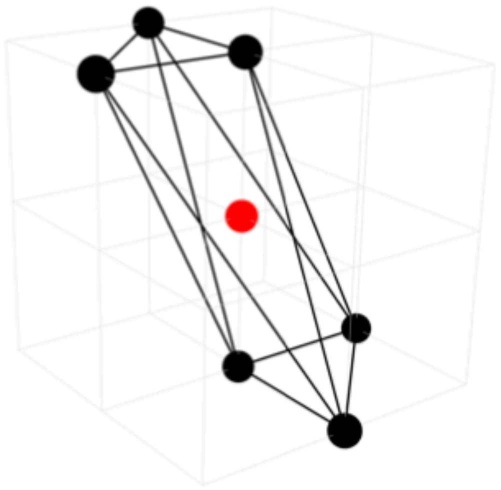


Ypk(CP1xCP1) and Ypk(CP2)

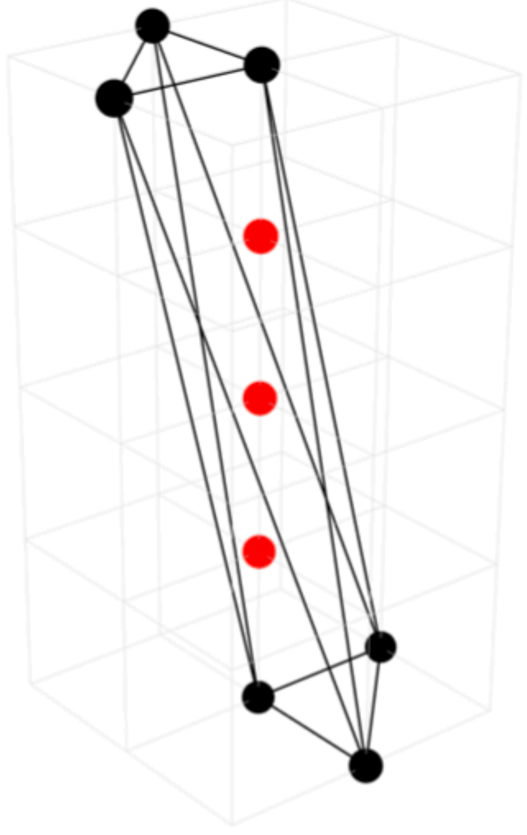
$Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$

- orbifolds

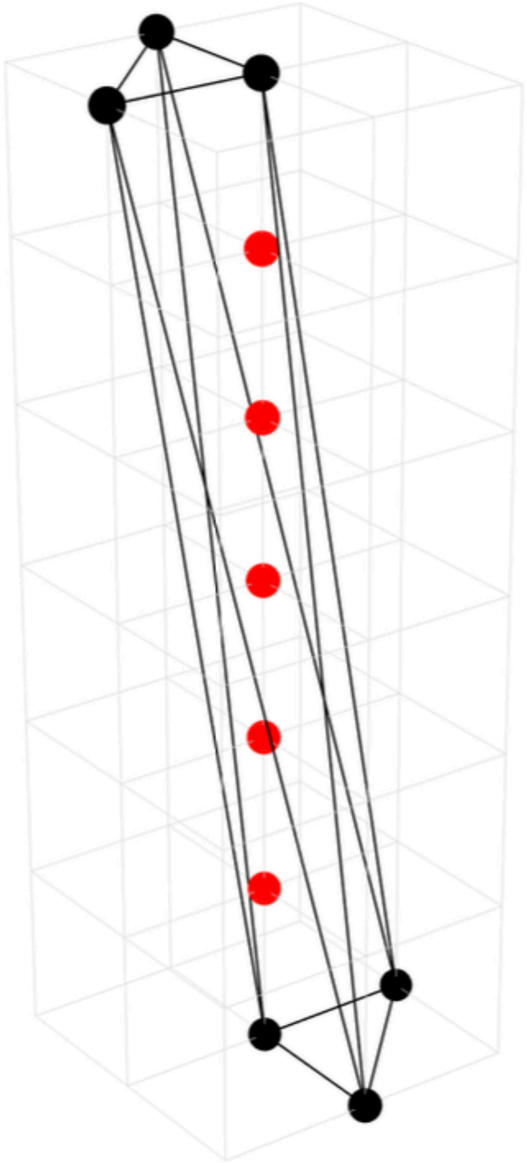
$$p = k = 2m, \ \ m \in \mathbb{Z}^+$$



$Y^{2,2}(\mathbb{CP}^1 \times \mathbb{CP}^1)$
$\simeq Q^{1,1,1}/\mathbb{Z}_2 \times \mathbb{Z}_1$



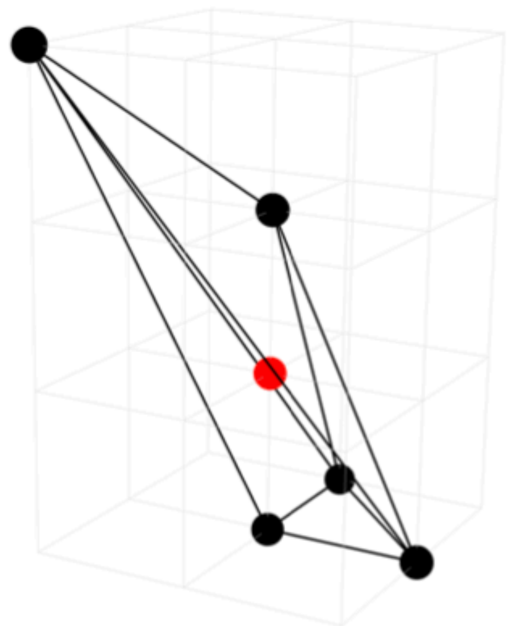
$Y^{4,4}(\mathbb{CP}^1 \times \mathbb{CP}^1)$
$\simeq Q^{1,1,1}/\mathbb{Z}_2 \times \mathbb{Z}_2$



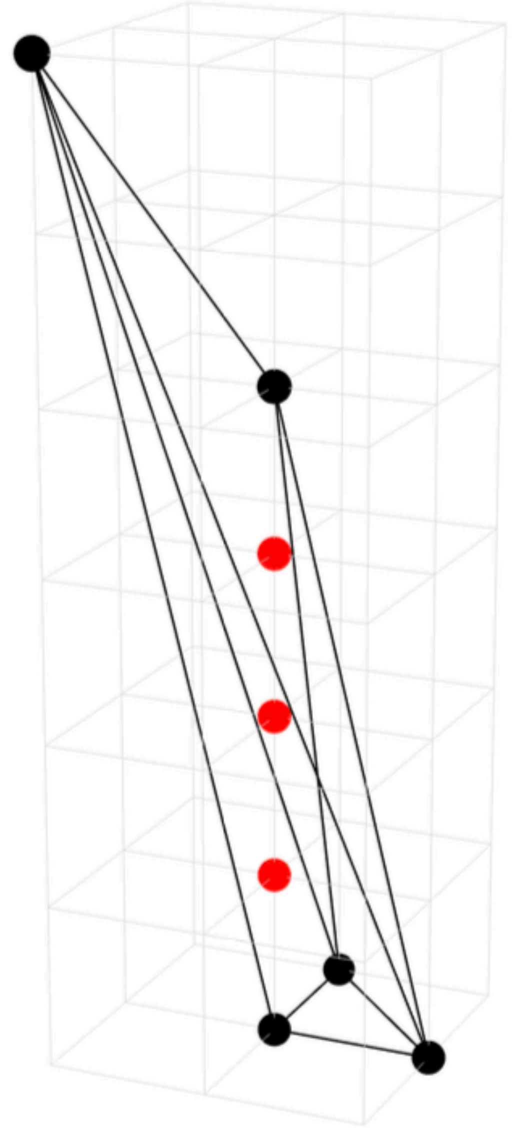
$Y^{6,6}(\mathbb{CP}^1 \times \mathbb{CP}^1)$
$\simeq Q^{1,1,1}/\mathbb{Z}_2 \times \mathbb{Z}_3$

$Y^{p,k}(\mathbb{CP}^2)$

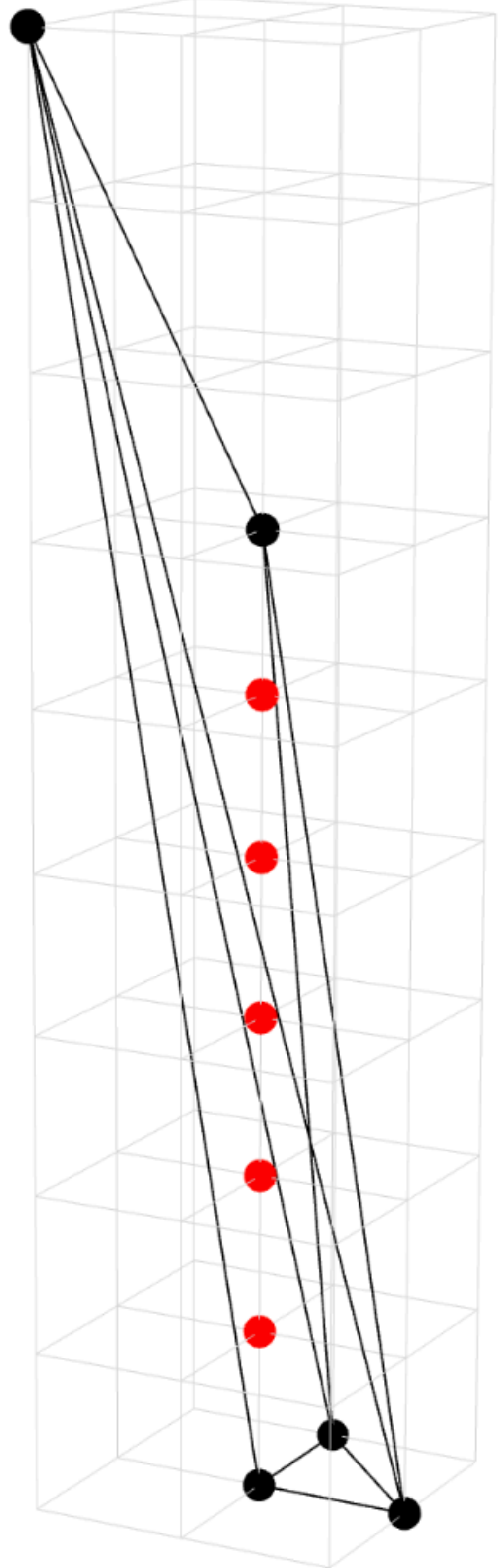
$$p = \frac{2}{3}k = 2m, \ \ m \in \mathbb{Z}^+$$



$Y^{2,3}(\mathbb{CP}^2)$
$\simeq M^{3,2}$



$Y^{4,6}(\mathbb{CP}^2)$
$\simeq M^{3,2}/\mathbb{Z}_2$

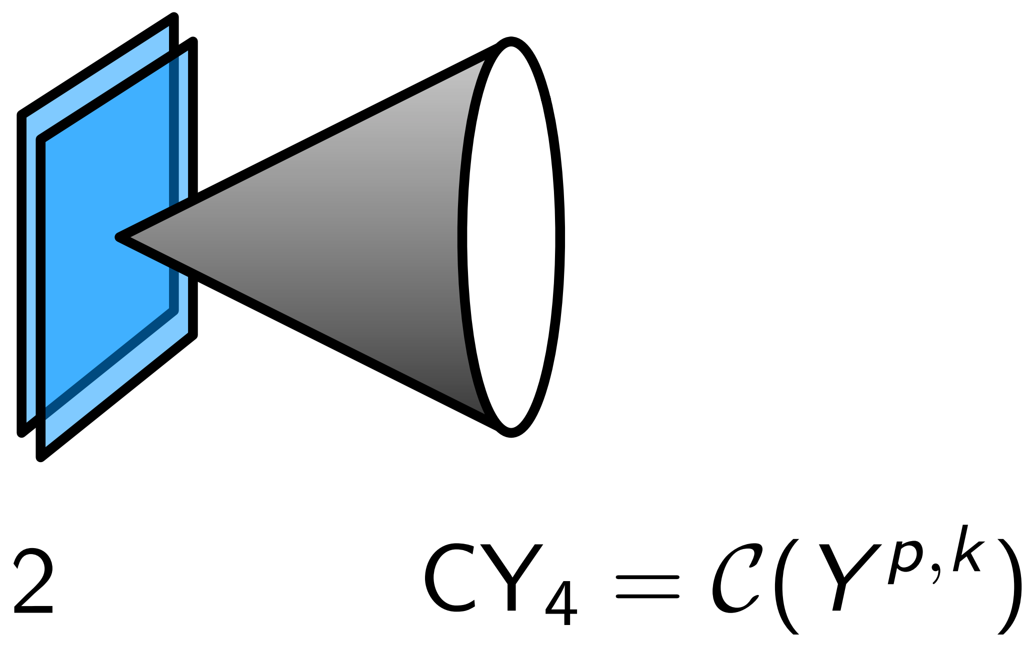


$Y^{6,9}(\mathbb{CP}^2)$
$\simeq M^{3,2}/\mathbb{Z}_3$

Ypk(CP1xCP1) and Ypk(CP2)

Theory on M2-Brane

worldvolume theory is a 3d Chern-Simons theory
(Chern-Simons level weighted Brane Tiling)



- fully identified for $Y^{p,k}(\mathbb{CP}^2)$

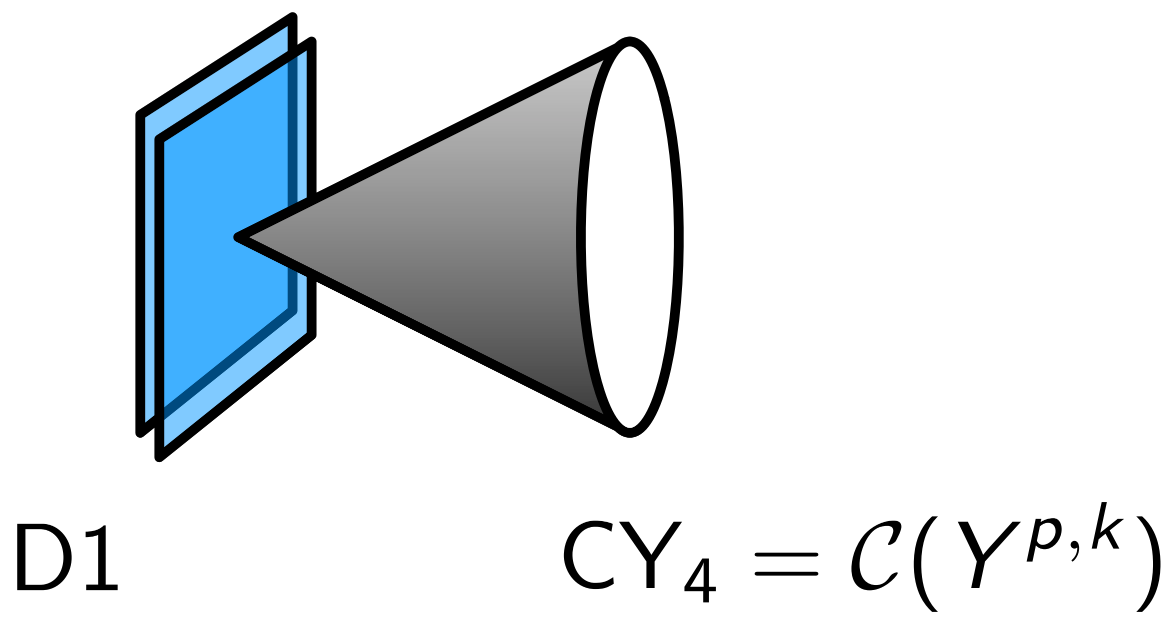
  [Benini-Closset-Cremonesi 2011]

- **partially** identified for $Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$

  [Benini-Closset-Cremonesi 2011]

Theory on D1-Brane

worldvolume theory is a 2d (0,2) quiver gauge theory
(Brane Brick Model)



- fully identified for **both** $Y^{p,k}(\mathbb{CP}^2)$ **and** $Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$

  [Franco-Ghim-Seong 2022]

Ypk(CP1xCP1) and Ypk(CP2)

[Franco-Ghim-Seong 2022]

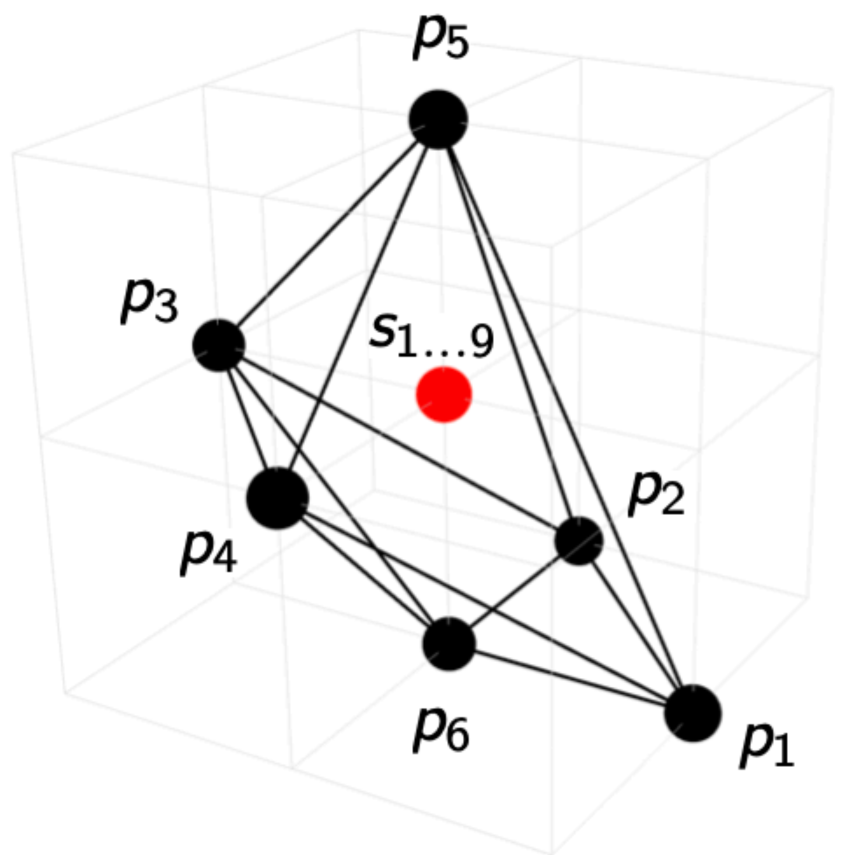
$$Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$$

Examples

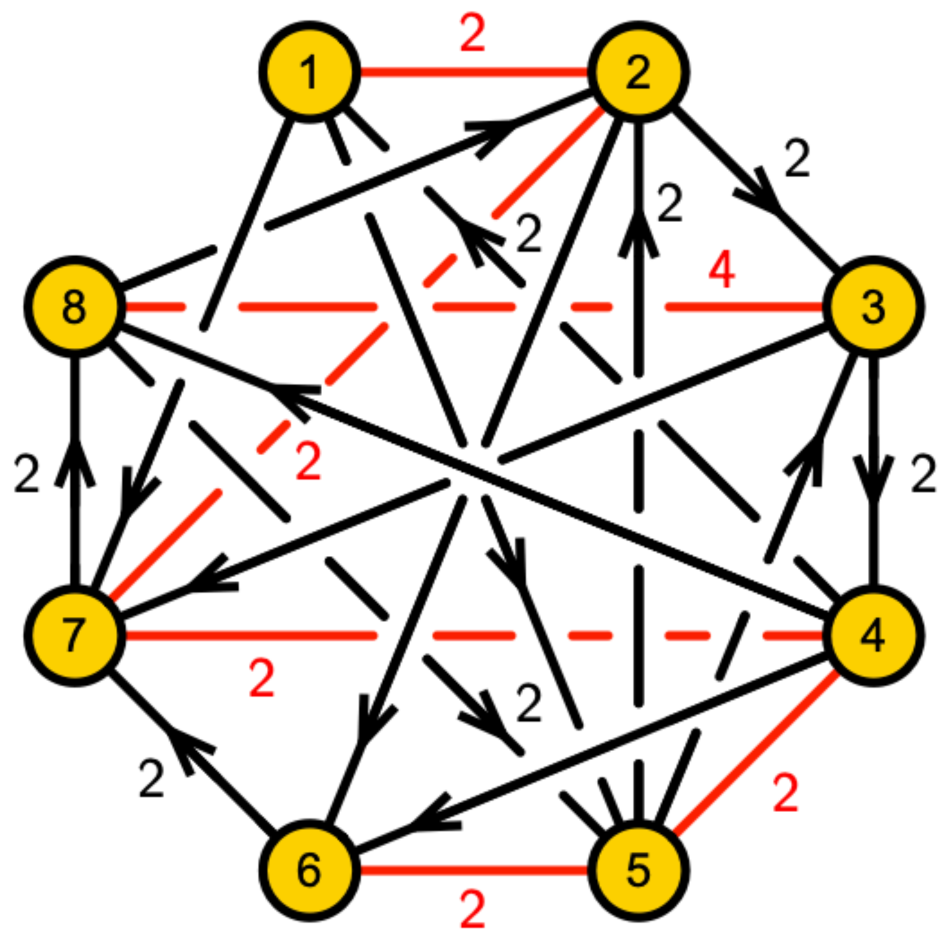
- **Geometry**

$Y^{2,1}(\mathbb{CP}^1 \times \mathbb{CP}^1)$

- **Toric Diagram**



- **Quiver**



- **J,E-Terms**

	J	E
Λ^{i}_{12} :	$\epsilon_{ij}\epsilon_{kl}X^k_{23}X^j_{34}X^l_{41}$	$P_{17}X^i_{78}Q_{82} - Q_{15}X^i_{52}$
Λ^{2i}_{27} :	$\epsilon_{ij}\epsilon_{kl}X^k_{78}X^j_{85}X^l_{52}$	$X^i_{23}Q_{37} - Q_{26}X^i_{67}$
Λ^{2i}_{38} :	$\epsilon_{ij}\epsilon_{kl}X^k_{85}X^j_{52}X^l_{23}$	$X^i_{34}Q_{48} - Q_{37}X^i_{78}$
Λ^{2i}_{45} :	$\epsilon_{ij}\epsilon_{kl}X^k_{52}X^j_{23}X^l_{34}$	$X^i_{41}Q_{15} - Q_{48}X^i_{85}$
Λ^{1i}_{47} :	$\epsilon_{ij}\epsilon_{kl}X^k_{78}X^j_{85}Q_{53}X^l_{34}$	$X^i_{41}P_{17} - P_{46}X^i_{67}$
Λ^{i}_{56} :	$\epsilon_{ij}\epsilon_{kl}X^k_{67}X^j_{78}X^l_{85}$	$X^i_{52}Q_{26} - Q_{53}X^i_{34}P_{46}$
Λ^{2i}_{83} :	$\epsilon_{ij}\epsilon_{kl}X^k_{34}X^j_{41}P_{17}X^l_{78}$	$X^i_{85}Q_{53} - Q_{82}X^i_{23}$

$i, j, k, l = 1, 2$

$$SU(2) \times SU(2) \times U(1)^2$$

Global Symmetry

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Ypk(CP1xCP1) and Ypk(CP2)

[Franco-Ghim-Seong 2022]

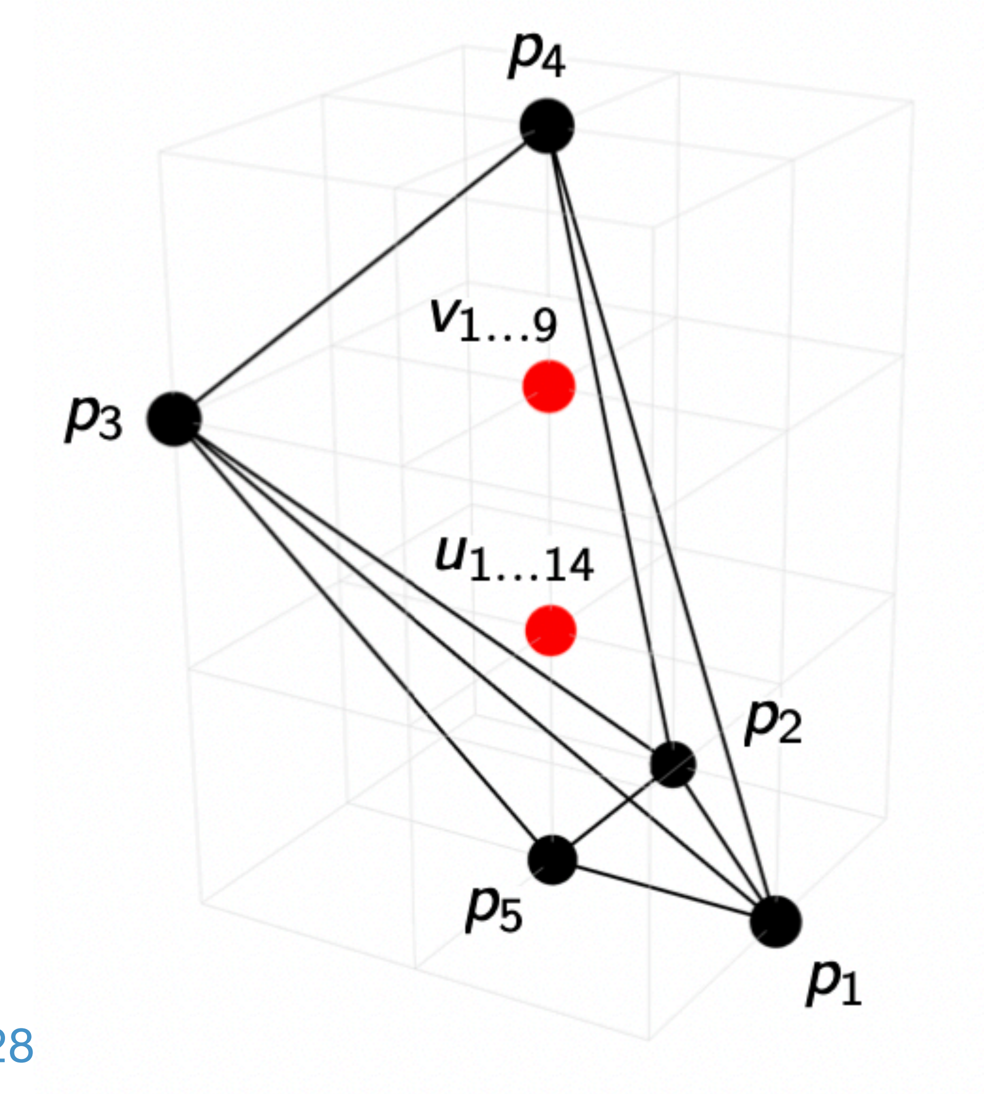
$Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$

Examples

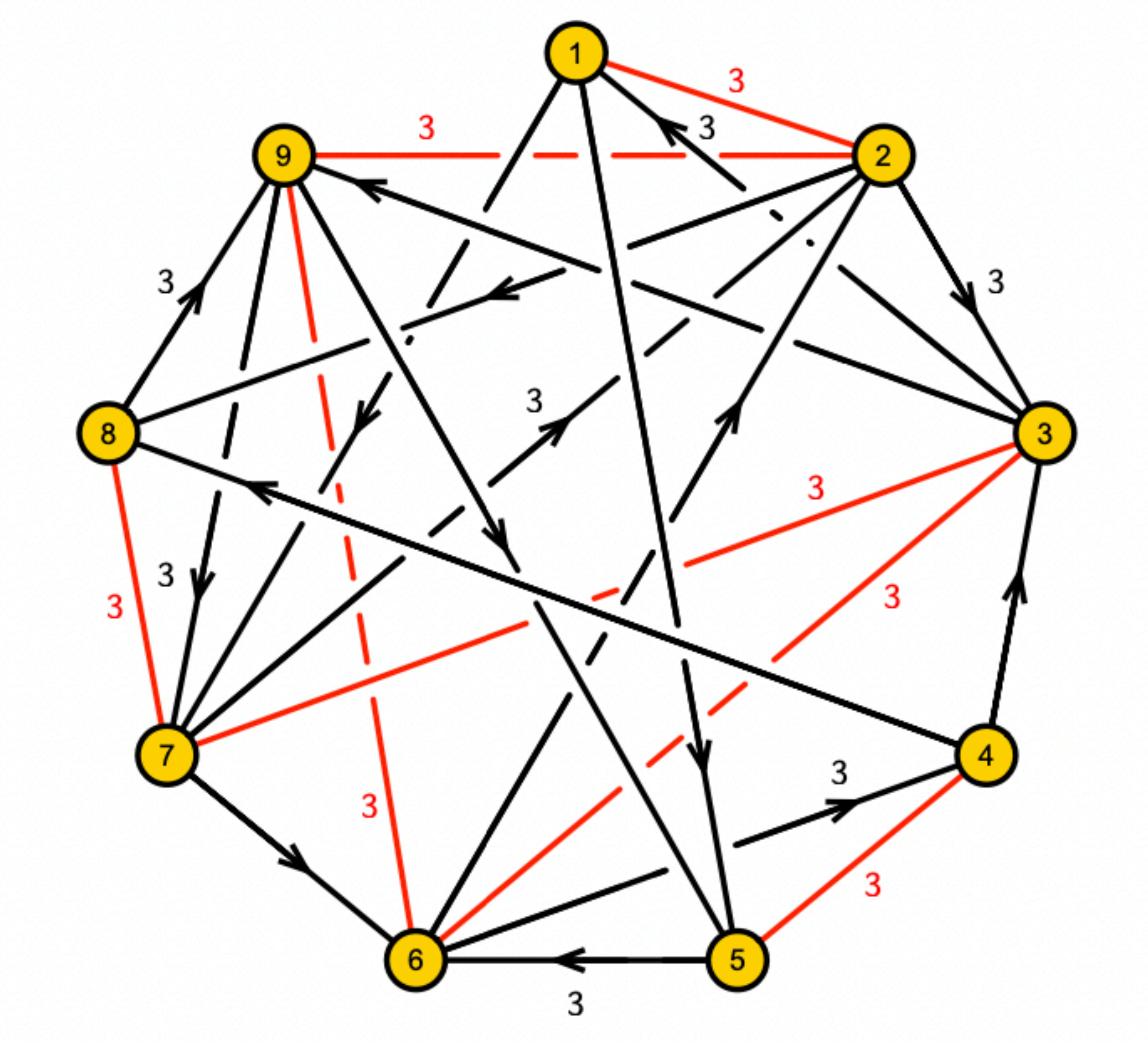
- Geometry

$Y^{3,2}(\mathbb{CP}^2)$

- Toric Diagram



- Quiver



- J,E-Terms

	J	E
Λ^{i}_{12} :	$\epsilon_{ijk} X^{j}_{23} X^{k}_{31}$	$P_{15} X^{i}_{56} Q_{62} - Q_{17} X^{i}_{72}$
Λ^{2i}_{29} :	$\epsilon_{ijk} X^{j}_{97} X^{k}_{72}$	$X^{i}_{23} Q_{39} - Q_{28} X^{i}_{89}$
Λ^{2i}_{37} :	$\epsilon_{ijk} X^{j}_{72} X^{k}_{23}$	$X^{i}_{31} Q_{17} - Q_{39} X^{i}_{97}$
Λ^{i}_{45} :	$\epsilon_{ijk} X^{j}_{56} X^{k}_{64}$	$P_{48} X^{i}_{89} Q_{95} - Q_{43} X^{i}_{31} P_{15}$
Λ^{2i}_{63} :	$\epsilon_{ijk} X^{j}_{31} P_{15} X^{k}_{56}$	$X^{i}_{64} Q_{43} - Q_{62} X^{i}_{23}$
Λ^{i}_{78} :	$\epsilon_{ijk} X^{j}_{89} X^{k}_{97}$	$X^{i}_{72} Q_{28} - Q_{76} X^{i}_{64} P_{48}$
Λ^{2i}_{96} :	$\epsilon_{ijk} X^{j}_{64} P_{48} X^{k}_{89}$	$X^{i}_{97} Q_{76} - Q_{95} X^{i}_{56}$

$i, j, k = 1, 2, 3$

$SU(3) \times U(1)^2$

Global Symmetry

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Ypk(CP1xCP1) and Ypk(CP2)

[Franco-Ghim-Seong 2022]

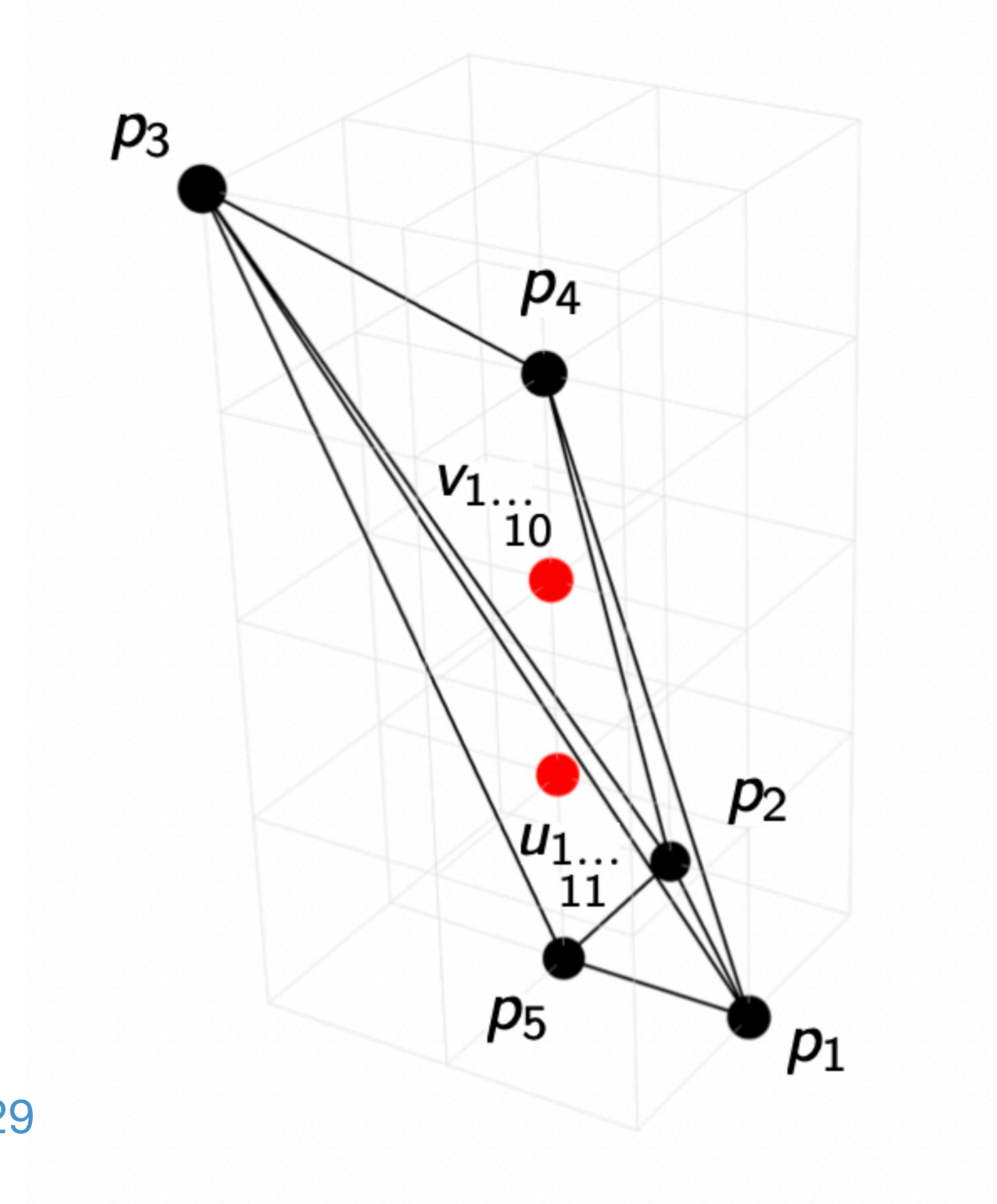
$$Y^{p,k}(\mathbb{CP}^1 \times \mathbb{CP}^1)$$

Examples

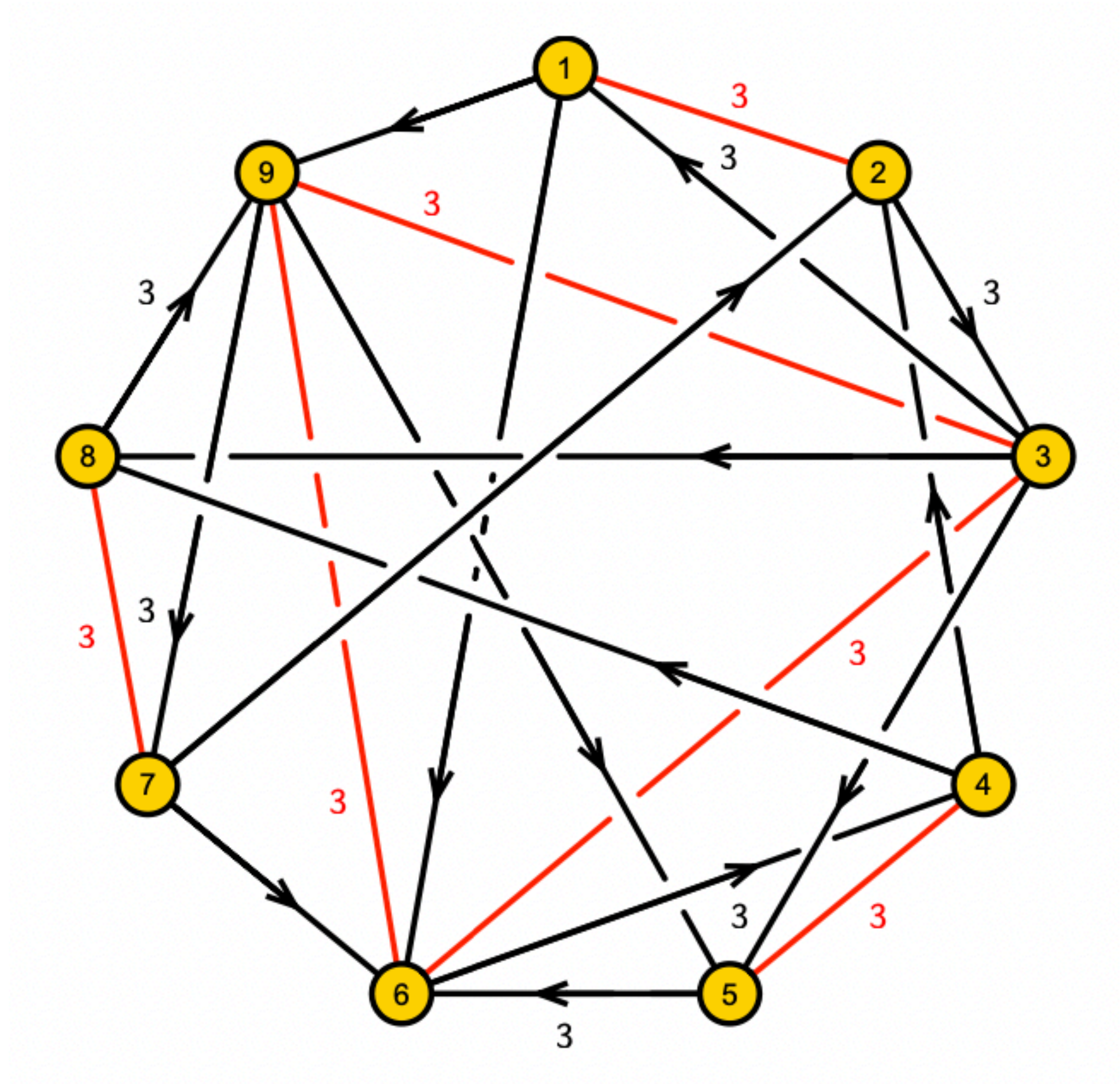
- Geometry

$Y^{3,4}(\mathbb{CP}^2)$

- Toric Diagram



- Quiver



- J,E-Terms

	J	E
Λ^i_{12} :	$\epsilon_{ijk} X^j_{23} X^k_{31}$	$P_{16} X^i_{64} Q_{42} - Q_{19} X^i_{97} P_{72}$
Λ^{1i}_{36} :	$\epsilon_{ijk} X^j_{64} Q_{42} X^k_{23}$	$X^i_{31} P_{16} - P_{35} X^i_{56}$
Λ^{2i}_{39} :	$\epsilon_{ijk} X^j_{97} P_{72} X^k_{23}$	$X^i_{31} Q_{19} - Q_{38} X^i_{89}$
Λ^i_{45} :	$\epsilon_{ijk} X^j_{56} X^k_{64}$	$P_{48} X^i_{89} Q_{95} - Q_{42} X^i_{23} P_{35}$
Λ^i_{78} :	$\epsilon_{ijk} X^j_{89} X^k_{97}$	$P_{72} X^i_{23} Q_{38} - Q_{76} X^i_{64} P_{48}$
Λ^{2i}_{96} :	$\epsilon_{ijk} X^j_{64} P_{48} X^k_{89}$	$X^i_{97} Q_{76} - Q_{95} X^i_{56}$

$i, j, k = 1, 2, 3$

$SU(3) \times U(1)^2$

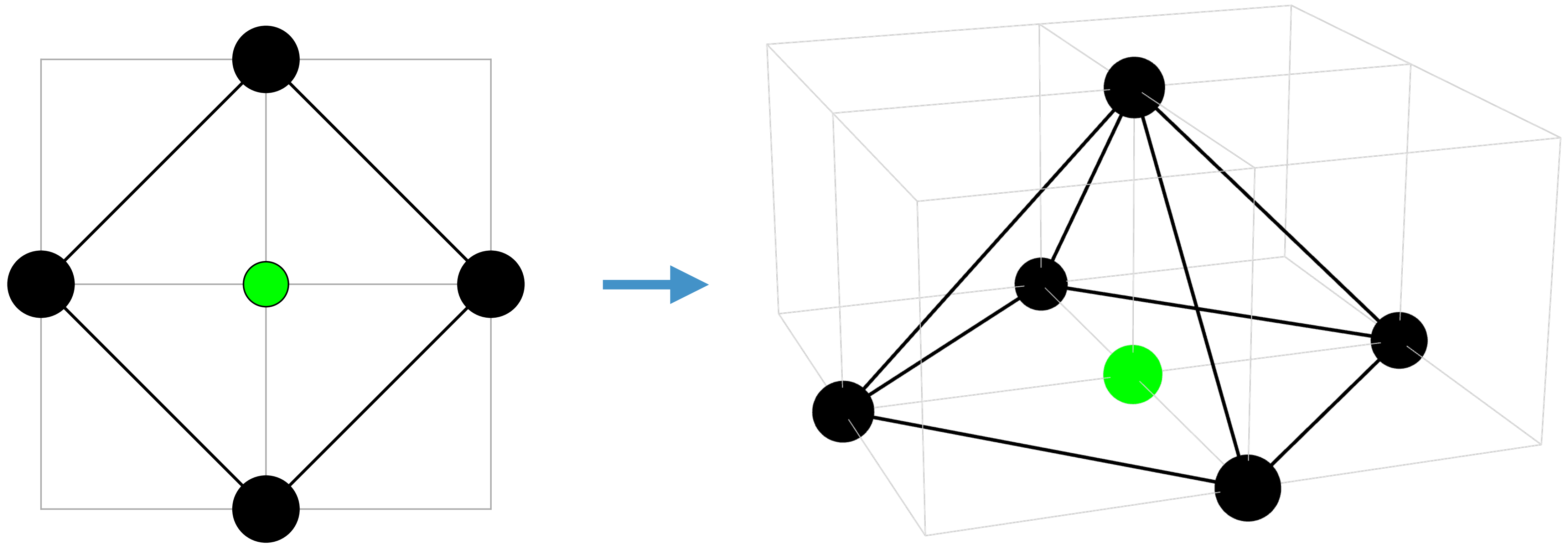
Global Symmetry

Evolution of Brane Brick Model Constructions

Dimensional Reduction

[Franco-Ghim-Lee-Seong-Yokoyama 2015]

CY_3 $\rightarrow$ $CY_3 \times \mathbb{C}$



4d N=1 quiver gauge theory
Brane Tiling

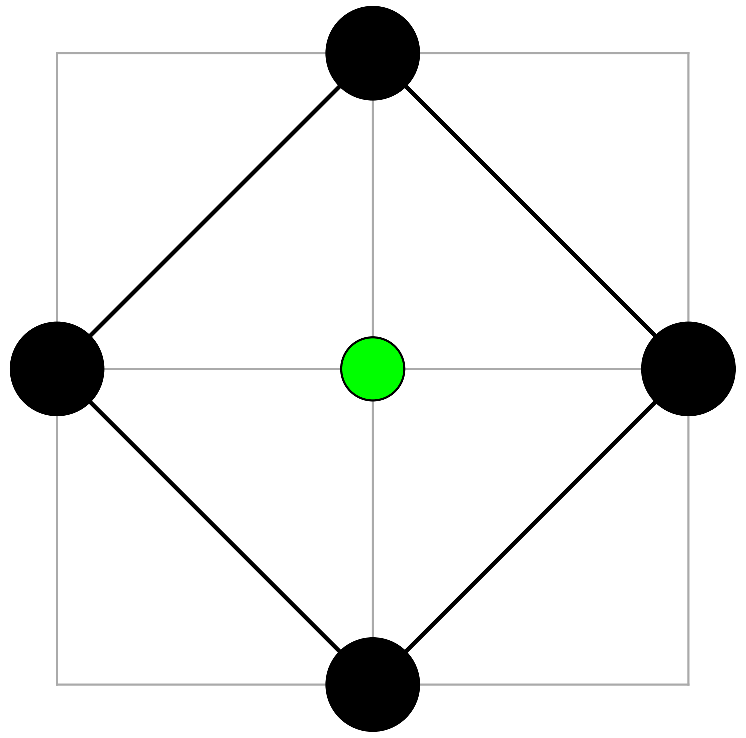
2d (2,2) quiver gauge theory
Brane Brick Model

Evolution of Brane Brick Model Constructions

Dimensional Reduction

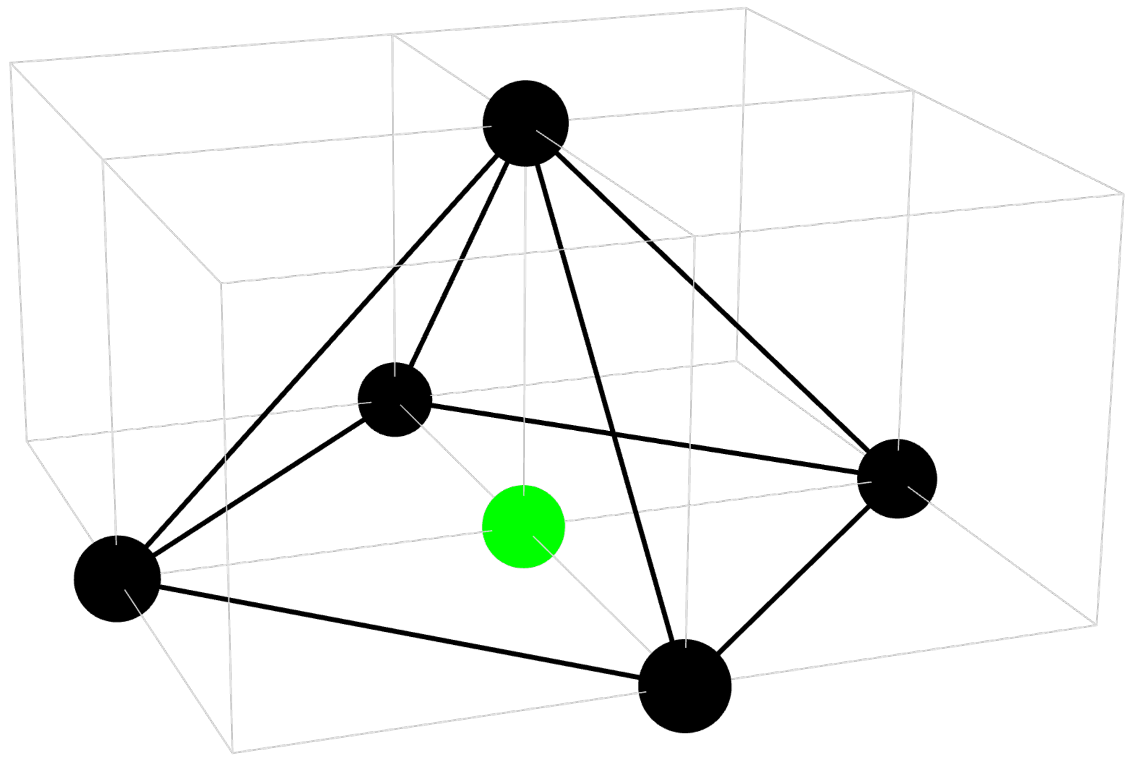
[Franco-Ghim-Lee-Seong-Yokoyama 2015]

CY_3



4d N=1 quiver
gauge theory
Brane Tiling

$\mathrm{CY}_3 \times \mathbb{C}$

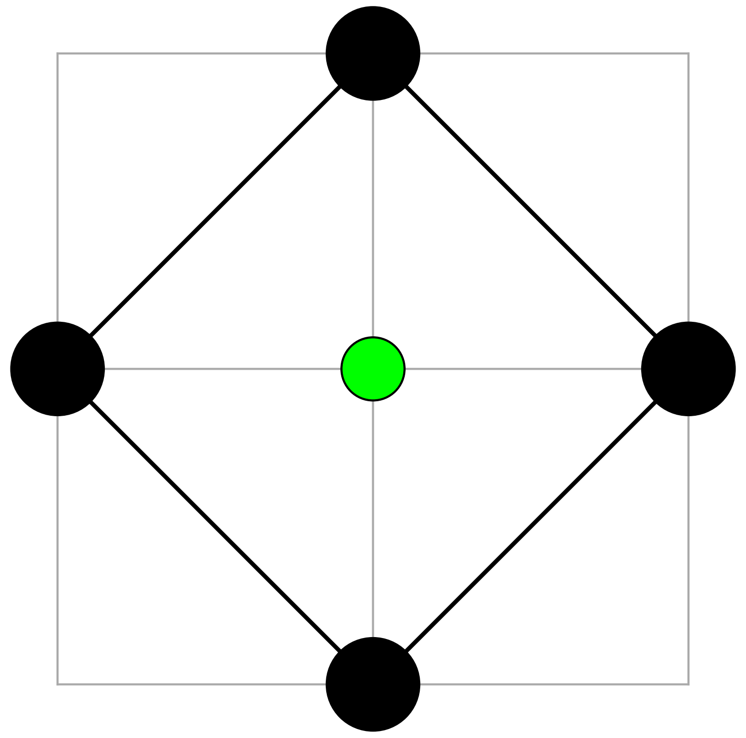


2d (2,2) quiver
gauge theory
Brane Brick Model

Orbifold Reduction

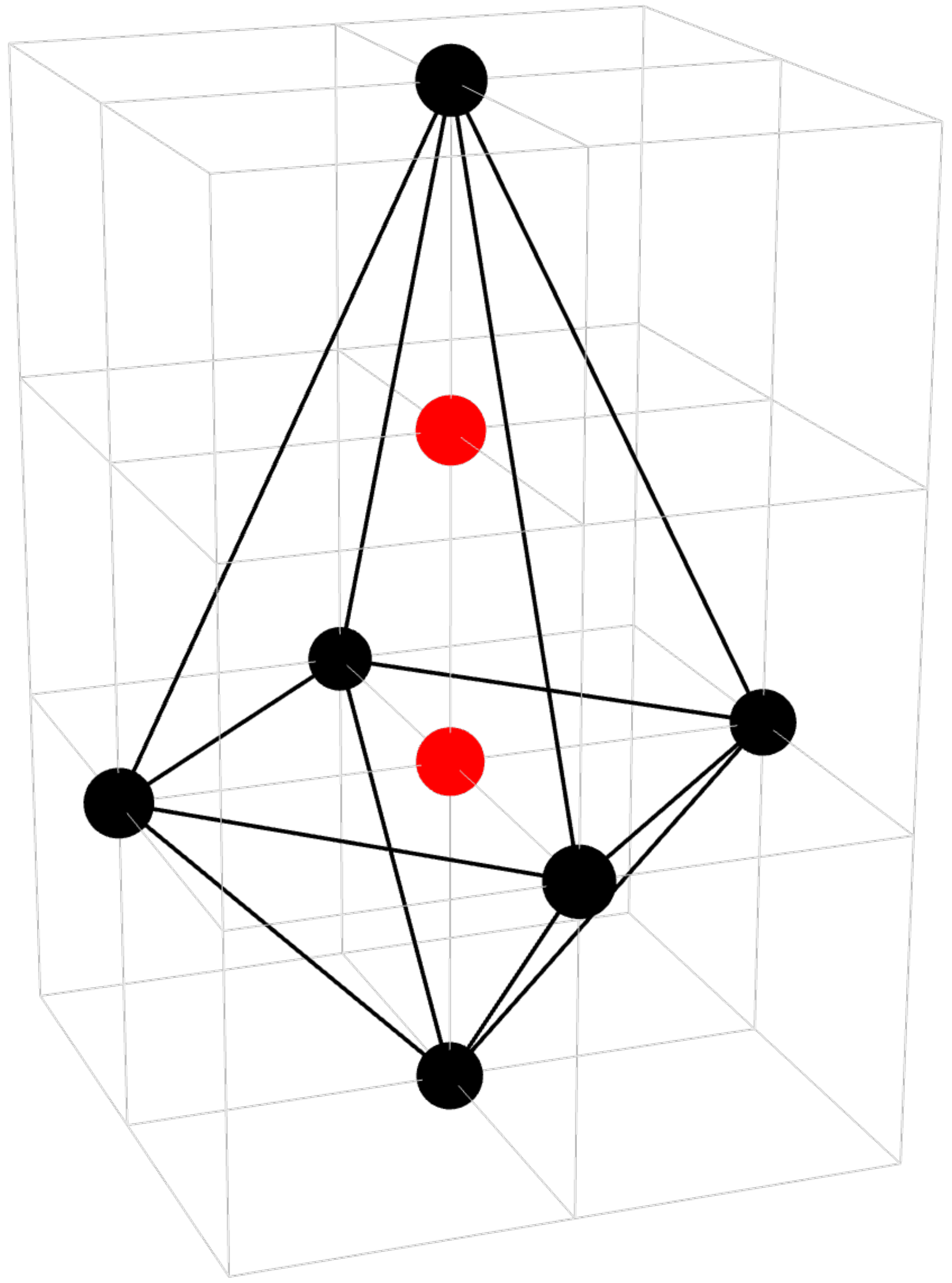
[Franco-Lee-Seong 2016]

CY_3



4d N=1 quiver
gauge theory
Brane Tiling

$\mathrm{CY}_4 = \mathcal{C}_{++-}(\mathrm{CY}_3)$



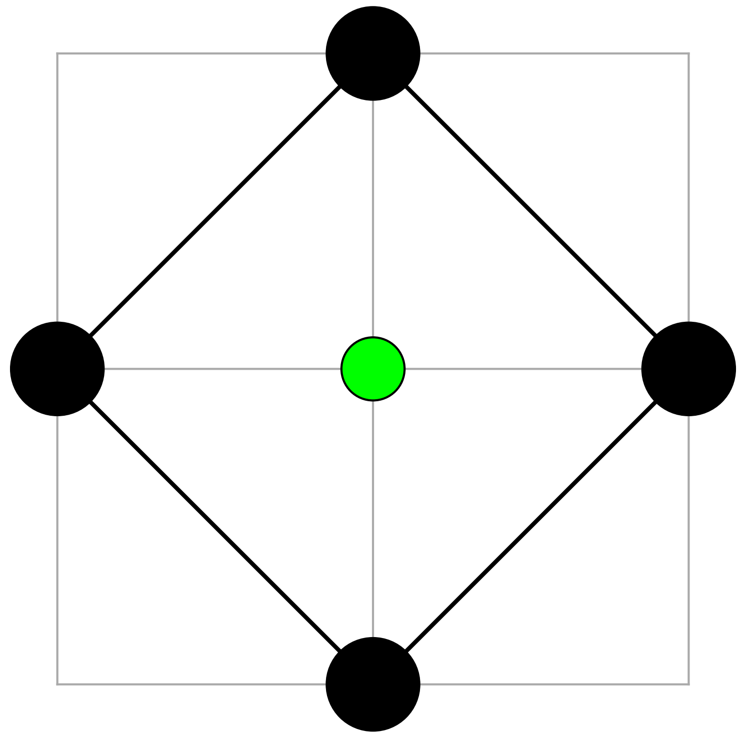
2d (0,2) quiver
gauge theory
Brane Brick Model

Evolution of Brane Brick Model Constructions

3D Printing

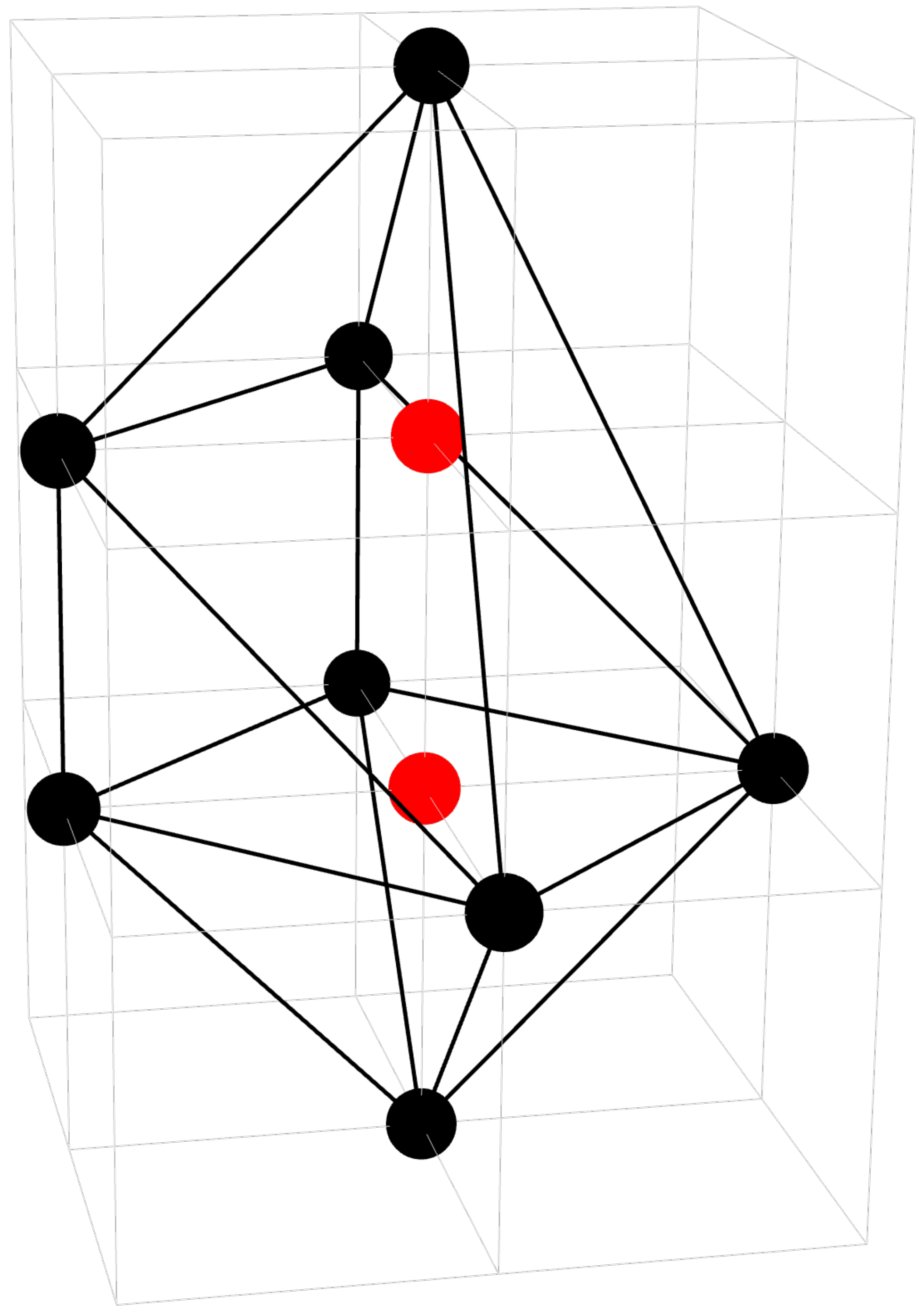
[Franco-Hasan 2018]

CY_3



4d N=1 quiver
gauge theory
Brane Tiling

CY_4

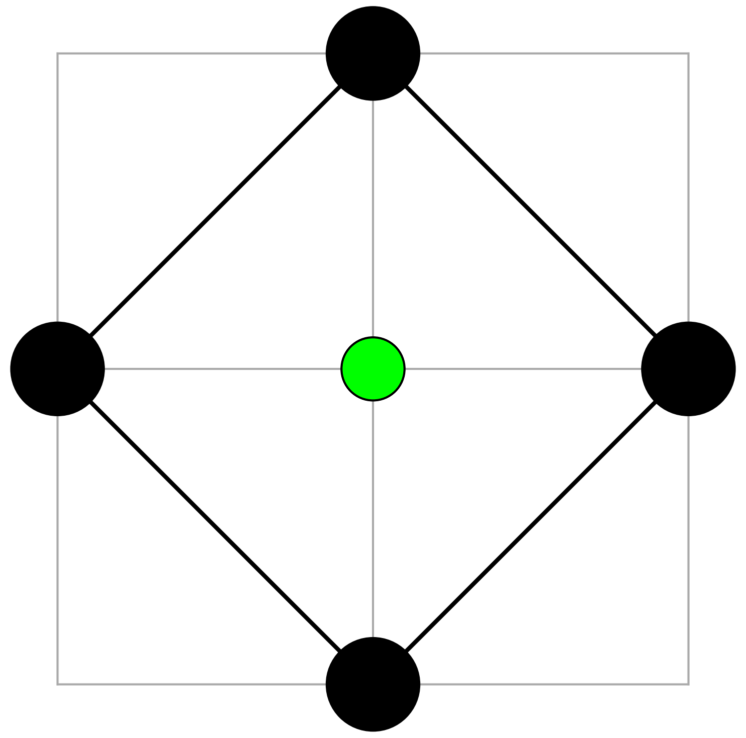


2d (0,2) quiver
gauge theory
Brane Brick Model

Ypk Manifolds

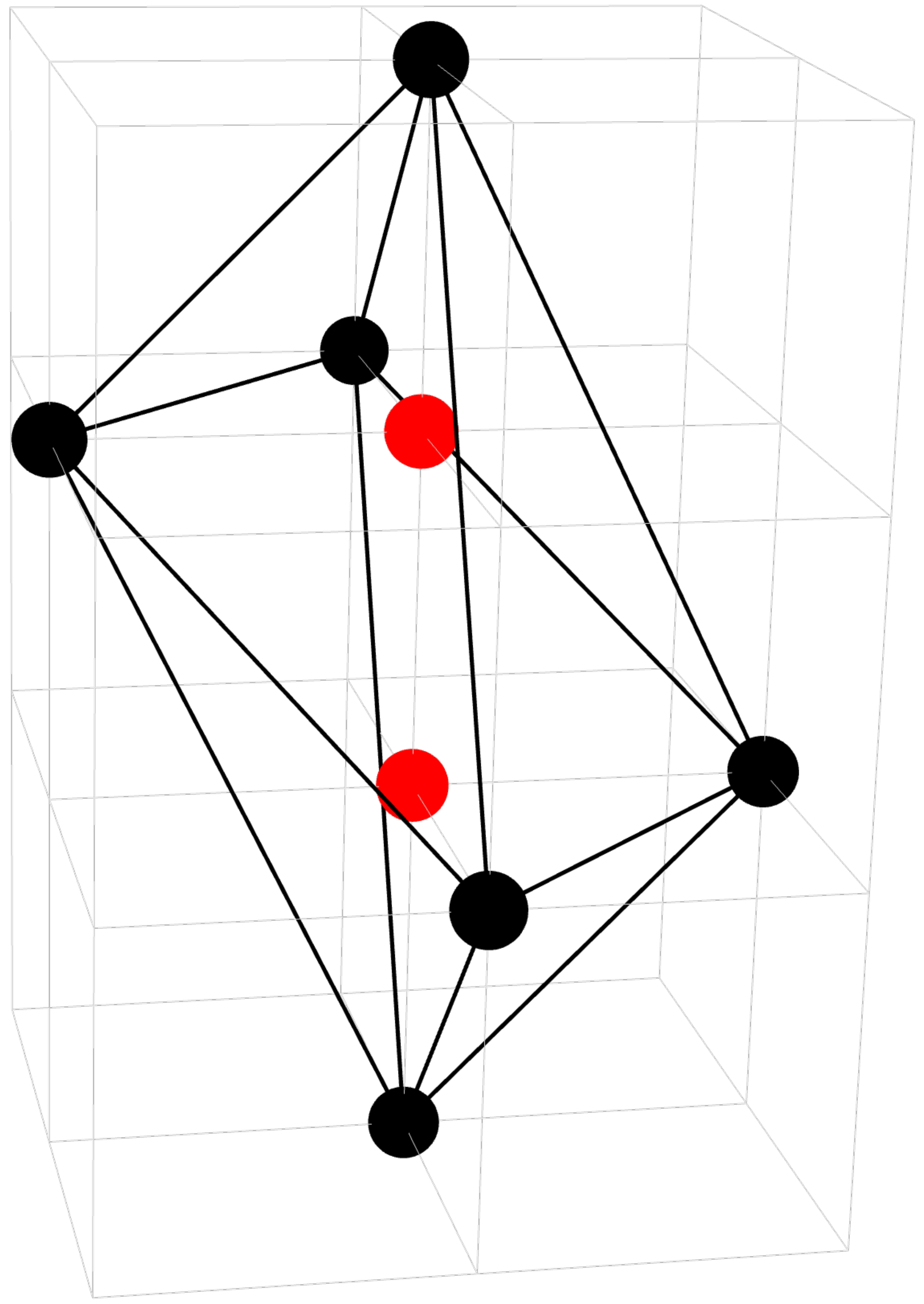
[Franco-Ghim-Seong 2022]

CY_3



4d N=1 quiver
gauge theory
Brane Tiling

CY_4



2d (0,2) quiver
gauge theory
Brane Brick Model

Thank You