Back to Heterotic Little String Theories on ALE Spaces

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Based on • arXiv:2209.10551, 2212.05311, 2307.xxxxx with Del Zotto & Oehlmann

Work in progress with Braun, Del Zotto & Oehlmann

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What is 6d LST?

- LSTs are **intermediate** between local and gravitational theories
  $\rightarrow$ they are related by decompactification:

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- **Goal - geometric engineering in F-theory**:
  1. Determine novel Heterotic ALE instantonic LSTs
  2. Study 2-group structure and T-dual network

T-duality and 2-groups

- LSTs have a continuous 2-group symmetry structure:

$$\to \left(\mathscr{P}^{(0)} \times SU(2)_R^{(0)} \times \prod_a F_a^{(0)} \right) \times_{\widehat{\kappa}_{\mathscr{P}}, \widehat{\kappa}_R, \widehat{\kappa}_{F_a}} U(1)_{LST}^{(1)}$$

- T-dual LSTs must have **matching** (very constraining):

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- T-dual LSTs must have **matching** (very constraining):

1. $\widehat{\kappa}_{\mathscr{P}}$, $\widehat{\kappa}_R$ and $\widehat{\kappa}_{F_a}$ (formula determined by GS mechanism)

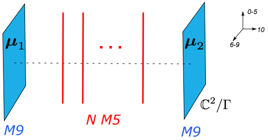
$$\widehat{\kappa}_F = -\sum_{I=1}^{n_T+1} N_I \eta^{IA} \qquad \widehat{\kappa}_R = \sum_{I=1}^{n_T+1} N_I h^\vee_{\mathfrak{g}_I} \qquad \widehat{\kappa}_{\mathscr{P}} = -\sum_{I=1}^{n_T+1} N_I (\eta^{II} - 2)$$

2. Coulomb branch dimension and amounts of Wilson line parameters

$$\textbf{Dim(CB)} = T + \text{rk}(G)\,, \qquad \textbf{Dim(WL)} = \text{rk}(G_F)$$

Generic $E_8 \times E_8$ heterotic instantonic LSTs in HW picture

- The exceptional LSTs in M-theory ← A stack of N M5 branes [Hořava, Witten'95]:



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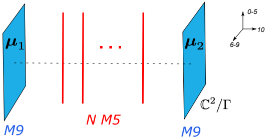

- The resulting theory depends on a choice of a flat connection encoded in:

$$\mu_a \colon \pi_1(S^3/\Gamma_{\mathfrak{g}}) \simeq \Gamma_{\mathfrak{g}} \to E_8\,, \quad \text{for } \mu_a \simeq id\,, \text{ see [Aspinwall, Morrison'97]}$$

  - The LST ↔ A generalized quiver:

$$\mathcal{K}_N(\mu_1, \mu_2; \mathfrak{g}) = \mathcal{T}(\mu_1, \mathfrak{g}) \overset{\mathfrak{g}}{\text{———}} \mathcal{T}_N(\mathfrak{g}, \mathfrak{g}) \overset{\mathfrak{g}}{\text{———}} \mathcal{T}(\mu_2, \mathfrak{g})$$

  - Determine fractional instantons by **F-theory** [Del Zotto, Heckman, Tomasiello, Vafa'14]
  - Matching criteria to **chart the T-dualities**:

6D Heteorotic LST from F-theory

- An elliptic fibered CY threefold X $\leftrightarrow$ 6d LSTs in F-theory:

$$\begin{array}{ccc} T^2 \rightarrow & X \\ & \downarrow \pi \\ & B_2 \end{array} , \quad \begin{array}{ccc} T^2 \rightarrow & X \\ & \downarrow \widetilde{\pi} \\ & \widetilde{B_2} \end{array}$$

  - Obtain the same 5d theory (with inequivalent 6d uplifts) after circle reduction
  - Geometrically realize T-duality between these two 6d Theories

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- Consider a **non-trivial global structure** (Different fibre type $K3/F_i$) [Aspinwall, Morrison'98...]

$$MW(X) = \mathbb{Z}^r \times MW(X)_{Tor} \Rightarrow G_T = \frac{G_F \times G}{MW_{Tor}}$$

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- Break the E_8 flavor factors via a **discrete holonomy** $\mu_i = \mathbb{Z}_n$
(focus on the rank preserving case in this work)

$\mathbb{Z}_2$ Discrete Holonomy LSTs

- Consider a breaking to $\mathfrak{e}_7 \times \mathfrak{su}_2$ and $\mathfrak{so}_{4N+8}^M$ gaugings:

$$[\mathfrak{e}_7] \; \underbrace{\underset{[\mathfrak{su}_2]}{\overset{\mathfrak{so}_{4N+8}}{1}} \; \overset{\mathfrak{so}_{4N+8}}{2} \; \dots \; \overset{\mathfrak{so}_{4N+8}}{2} \; \underset{[\mathfrak{su}_2]}{\overset{\mathfrak{so}_{4N+8}}{1}}}_{M\times} \; [\mathfrak{e}_7]$$

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- Has two more inequivalent toric fibrations, first:

$$\begin{array}{c} \\ [\mathfrak{so}_{24}] \end{array} \begin{array}{c} \\ \overset{\mathfrak{sp}_{2N+M+3}}{1} \end{array} \begin{array}{c} \overset{\mathfrak{sp}_{2N+M-3}}{1} \\ \overset{\mathfrak{so}_{8N+4M+4}}{4} \end{array} \underbrace{\ldots \; \overset{\mathfrak{sp}_{4(N-k)+2M-4}}{1} \; \overset{\mathfrak{so}_{8(N-k)+4M-4}}{4} \; \ldots \; \overset{\mathfrak{sp}_{2M}}{1} \; \overset{\mathfrak{so}_{4M+4}}{4^*}}_{2N\times} \begin{array}{c} \overset{\mathfrak{sp}_{M-3}}{1^*} \\ \overset{\mathfrak{sp}_{M-1}}{1} \end{array} \begin{array}{c} \\ [\mathfrak{so}_8] \end{array}$$

This chain has the full $\mathfrak{so}_{4N+8}^{(1)}$ topology! $\leftrightarrow$ **Fiber-Base-Duality**

$\mathbb{Z}_2$ Discrete Holonomy LSTs

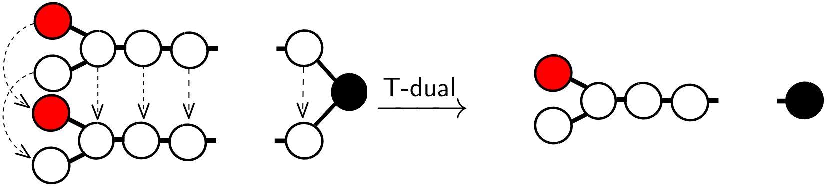
- The third fibration has the quiver:

$$[\mathfrak{su}_{16} \times \mathfrak{u}_1] \quad \overset{\mathfrak{su}_{4N+2M+6}}{2}$$

$$\overset{\mathfrak{su}_{8N+4M-4}}{2} \quad \underbrace{\overset{\mathfrak{su}_{8N+4M-12}}{2} \quad \overset{\mathfrak{su}_{8N+4M-20}}{2} \quad \ldots \quad \overset{\mathfrak{su}_{4M+4}}{2} \quad \overset{\mathfrak{sp}_{2M-2}}{1}}_{N\times}$$

$$\overset{\mathfrak{su}_{4N+2M-2}}{2}$$

- $\mathfrak{so}^{(1)}_{4N+8}$ base shape is folded to an $\mathfrak{su}^{(2)}_{N+3}$



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- The third fibration has the quiver:

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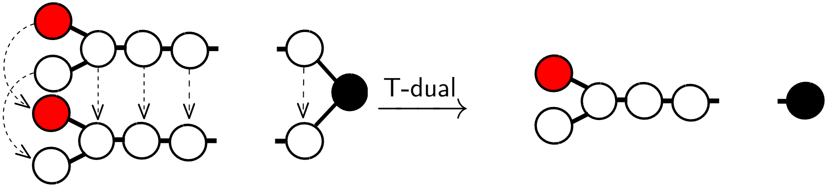

- The 2-groups and CB dimension are matched and given below as

$$\mathrm{Dim(CB)} = 4N^2 + 4NM + 8N + 6M + 2\,, \quad \widehat{\kappa}_{\mathscr{R}} = 8N^2 + 8NM + 8N + 8M + 2$$

Heterotic LSTs and K3 fibration

1. An elliptic fibered CY has a nested fibration:

$$\begin{array}{ccc} K3 \rightarrow & X \\ & \downarrow \\ & \mathbb{C} \end{array} \qquad \text{with} \qquad \begin{array}{ccc} T^2 \rightarrow & K3 \\ & \downarrow \\ & \mathbb{P}^1 \end{array}$$

2. Q: Can we obtain certain information of 6D LST from K3 only?

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   [Friedman, Morgan, Witten'97, Harder, Thompson'97]

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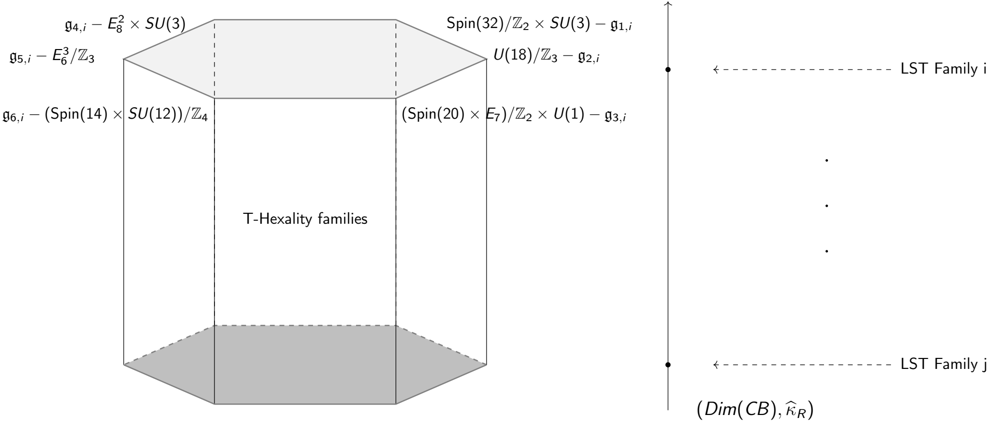
5. Add non-trivial compact curves to X $\to$ **K3 degeneration** $\to$ 6d LST quiver

Stratification of 6D LST families from the same K3



- $(Dim(CB), \widehat{\kappa}_R) \rightarrow$ Stratification of 6D LST families from the same generic K3

Geometric engineering of heterotic strings

- No full M5 branes but only the M9 fractions:

  1. Orbi-instanton quiver → Reduced theories → Fuse two reduced theories:

$$\mathcal{K}_0(\boldsymbol{\mu}_1, \boldsymbol{\mu}_2; \mathfrak{g}) = \mathcal{T}_{red}(\boldsymbol{\mu}_1; \mathfrak{g}) \overset{\mathfrak{g}}{\text{———}} \widehat{\mathcal{T}}_{red}(\boldsymbol{\mu}_2; \mathfrak{g})$$

$$= [\mathfrak{f}(\boldsymbol{\mu}_1)] \; \overset{\mathfrak{h}_1}{n_1} \; \overset{\mathfrak{h}_2}{n_2} \; \dots \; \overset{\mathfrak{h}_{r_1}}{n_{r_1}} \; \overset{\mathfrak{g}}{m_{r_1+1}} \; \overset{\widehat{\mathfrak{h}}_{r_2}}{\widehat{n}_{r_2}} \; \dots \; \overset{\widehat{\mathfrak{h}}_2}{\widehat{n}_2} \; \overset{\widehat{\mathfrak{h}}_1}{\widehat{n}_1} \; [\mathfrak{f}(\boldsymbol{\mu}_2)]$$

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2. For A-type singuarities:

$$\mathcal{K}_0(E_8, E_8; \mathfrak{su}_k) = [E_8]\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{su}_3}{2} \cdots \overset{\mathfrak{su}_{k-1}}{2}\ \underset{N_f=2}{\overset{\mathfrak{su}_k}{2}}\ \overset{\mathfrak{su}_{k-1}}{2} \cdots \overset{\mathfrak{su}_3}{2}\ \overset{\mathfrak{su}_2}{2}\ 2\ 1\ [E_8]$$

3. If k is even, the T-dual theory is

$$\widetilde{\mathcal{K}}(\mathfrak{so}_{32}; \mathfrak{su}_k) = [\mathfrak{so}_{32}]\ \overset{\mathfrak{sp}_{2k}}{1}\ \overset{\mathfrak{su}_{4k-8}}{2}\ \overset{\mathfrak{su}_{4k-16}}{2} \cdots \overset{\mathfrak{su}_8}{2}\ 1\ [\mathfrak{su}_2]$$

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Fronzen Singularity and Incomplete fusion

- In M-theory, can partially freeze the $\mathcal{X}_{\mathfrak{g}}$ singularities by torsional C_3 fluxes at infinity

$$\int_{S^3/\Gamma_{\mathfrak{g}}} C = \frac{\ell}{d}$$

- This results subalgbra

$\mathfrak{g}$	$\mathfrak{so}_{2k+8}$	$\mathfrak{e}_6$		$\mathfrak{e}_7$			$\mathfrak{e}_8$				
$\frac{l}{d}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}, \frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}, \frac{2}{3}$	$\frac{1}{4}, \frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}, \frac{2}{3}$	$\frac{1}{4}, \frac{3}{4}$	$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$	$\frac{1}{6}, \frac{5}{6}$
$\mathfrak{h}$	$\mathfrak{sp}_k$	$\mathfrak{su}_3$	—	$\mathfrak{so}_7$	$\mathfrak{su}_2$	—	$\mathfrak{f}_4$	$\mathfrak{g}_2$	$\mathfrak{su}_2$	—	—

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$\mathfrak{h}$	$\mathfrak{sp}_k$	$\mathfrak{su}_3$	—	$\mathfrak{so}_7$	$\mathfrak{su}_2$	—	$\mathfrak{f}_4$	$\mathfrak{g}_2$	$\mathfrak{su}_2$	—	—

- $\rightarrow$ An incomplete fusion (decouple the quiver at the $i-$th node)

$$\mathcal{T}_{red}(\boldsymbol{\mu};\mathfrak{g}) = [\mathfrak{f}(\boldsymbol{\mu})]\ \overset{\mathfrak{h}_1}{n_1}\ \overset{\mathfrak{h}_2}{n_2}\ \ldots \overset{\mathfrak{h}_i}{n_i} \ldots\ \overset{\mathfrak{h}_r}{n_r}\ [\mathfrak{g}]$$

Exceptional non-simply laced fusion

- Consider $\mathcal{T}_{red}(\mathfrak{e}_6 : \mathfrak{e}_8) \to$ Read off the unbroken gauge algebra: e.g $\mathfrak{f}_4$

$$[\mathfrak{e}_6]\, 1\, \overset{\mathfrak{su}_3}{2}\, 1\, \underbrace{\overset{\mathfrak{f}_4}{5}\, 1\, \overset{\mathfrak{g}_2}{3}\, \overset{\mathfrak{sp}_2}{2}\, 2\, 1\, [\mathfrak{e}_8] \quad \text{and} \quad [\mathfrak{e}_8]\, 1\, 2\, \overset{\mathfrak{sp}_1}{2}\, \overset{\mathfrak{g}_2}{3}\, 1\, \overset{\mathfrak{f}_4}{5}}_{\text{Fusion}}\, 1\, \overset{\mathfrak{su}_3}{2}\, 1\, [\mathfrak{e}_6]$$

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- Yield LST :

$$[\mathfrak{e}_6]\;1\;\overset{\mathfrak{su}_3}{3}\;1\;\underset{[N_F=1]}{\overset{\mathfrak{f}_4}{4}}\;1\;\overset{\mathfrak{su}_3}{3}\;1\;[\mathfrak{e}_6]$$

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- This LST has a T-dual partner

$$[\mathfrak{so}_{20}]\,\overset{\mathfrak{sp}_4}{1}\,\overset{\mathfrak{so}_{12}}{4}\,\underset{[\mathfrak{su}_2^2]}{1}\,2\,2\,[\mathfrak{su}_2]$$

Outlook

1. Turn towards non-heterotic LSTs given by systems without M9 branes
2. Incorporate the possibility of twisted compactifications
3. Relate heterotic LSTs to the classification of degenerate K3 surfaces and study unexplored reducible K3 fibrations occur in the geometry of LSTs

Thank you!

Categorical Aspects of Symmetries (AUGUST 14-25, 2023)
NORDITA Stockholm (Sweden)