Geometric Bounds on the 1-Form Gauge Sector

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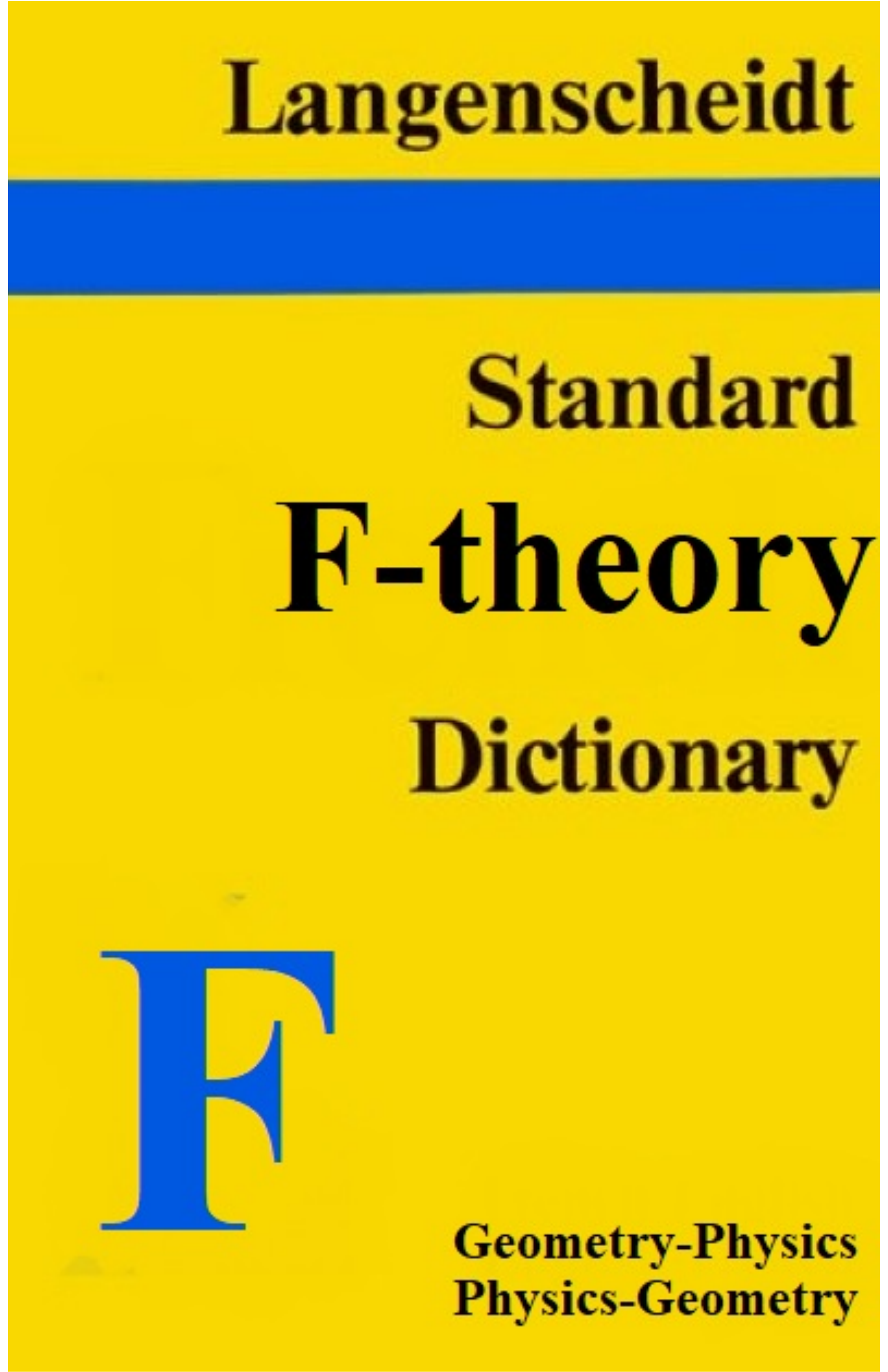
6D SUGRA Vacua and the Swampland

- Almost perfect geometric control over Landscape of $\mathcal{N} = (1{,}0)$ SUGRAs:

Elliptic Threefold

- Co dim 1 Singlrts
- Co dim 2 Singlrts
- Divisor classes
- ....
- ..



6D SUGRA

- Gauge Algebras
- Massless Spectra
- Anomalies
- ....
- ..

Some Open Matches in 6D

[See Hamada's talk!]

	Geometry	VS Physics	
• Finiteness of 6D SUGRA vacua	✓	✗	[Gross '93]
• Bounds on Massless Matter Representations	✓	✗	[Klevers, Morrison, Raghuram, Taylor'17]]
• Bounds on rk(G) + T & uncharged Hypers	✓	✗	[Grassi'23]
• Bounds on SU(N) groups	✓	✓	[Kim, Shiu, Vafa'19]
• Bounds on Abelian Group Rank $U(1)^{r \leq 18}$	✗	✓	[Lee, Weigand'19]

Bounds on $\pi_1(G) = Z$ (1-form gauge sector) $G = \frac{\tilde{G}}{Z} \times U(1)^k$

The 1-form gauge sector

[See Heckmann's talk]

- Centre **1-form gauge symmetries** ⟺ gauge group topology !
- 1-form gauge symmetries **constrain admissible field content**

Example

Reps	N	Adj	$Sym^k\Lambda^l$	$Sym^k\Lambda^{aN-k}$
$SU(N)$	✓	✓	✓	✓
$\frac{SU(N)}{\mathbb{Z}_N}$	✗	✓	✗	✓

Reps must be $\mathbb{Z}_N$ **neutral** !

Centre 1-form symmetries and Mordell-Weil

- Centre **1-form gauge symmetries** ⬌ gauge group topology !

[Aspwinall, Morrison'96; Mayrhover, Morrison, Till,Weigand'14]

- F-theory: **1-form gauge symmetry** ~ **finite Mordell Weil** Group of Geometry

$$\pi_1(G) = MW_{\text{tor}}(X_D) = \mathbb{Z}_K$$

- **Math Results**:

Bounds for elliptic D-folds:

D	K	
1	≤ 12	[Mazur'~60]
2	≤ 8	[Shimada; Miranda Persson'~60]
3	≤ 6	[Aspwinall, Morrison'96; Hajouji, **PKO**'19]
4	≤ 6	[Hajouji, **PKO**'19]

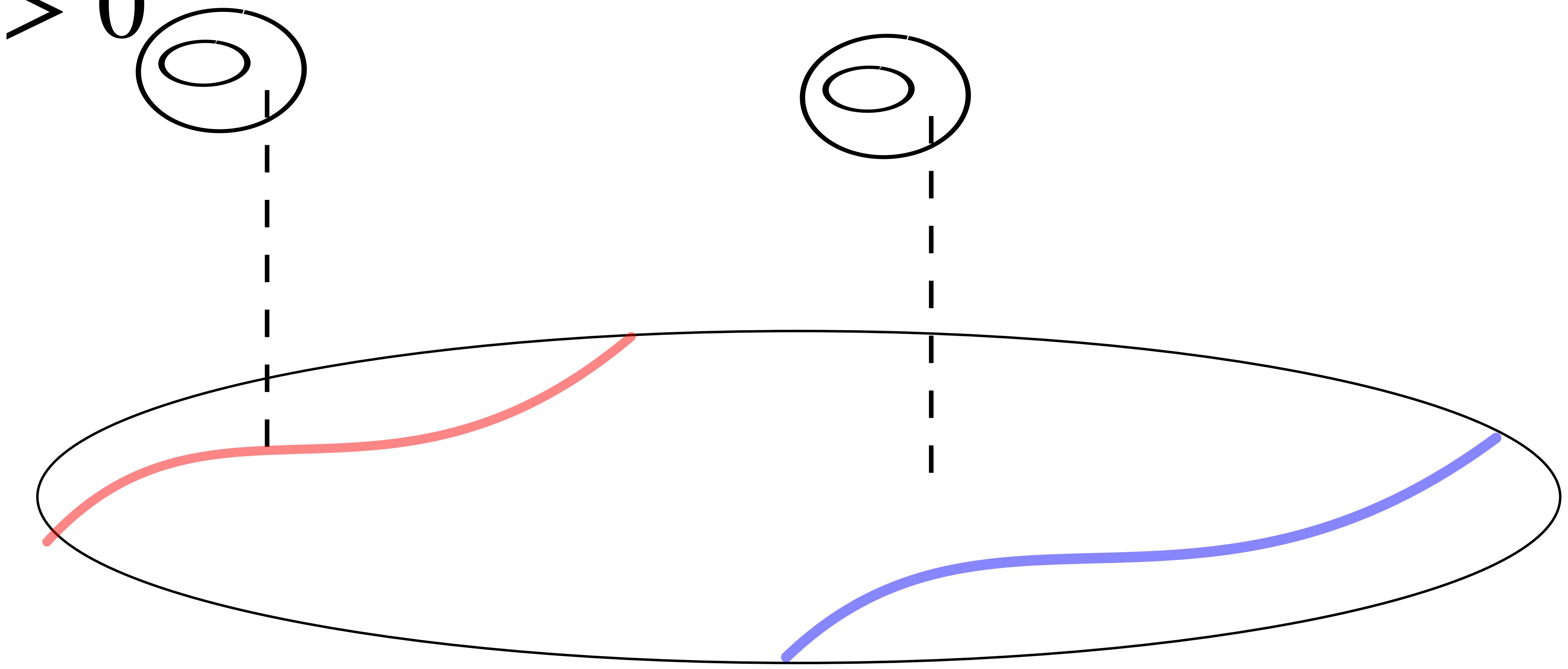
How to understand **physically** ?

F-theory and Heterotic Strings in 6D

- F-theory Geometry: Elliptic threefold X_3
- Facts: If #Tensors = $h^{1,1}(B_2) - 1 > 0$

➡ $\exists\ w \subset B_2$

with $w^2 = 0$ and $g_w = 0$



➡ (Almost) all geometries have **curve** w

➡ D3 brane wrap w yields 2D **Heterotic String** in 6D

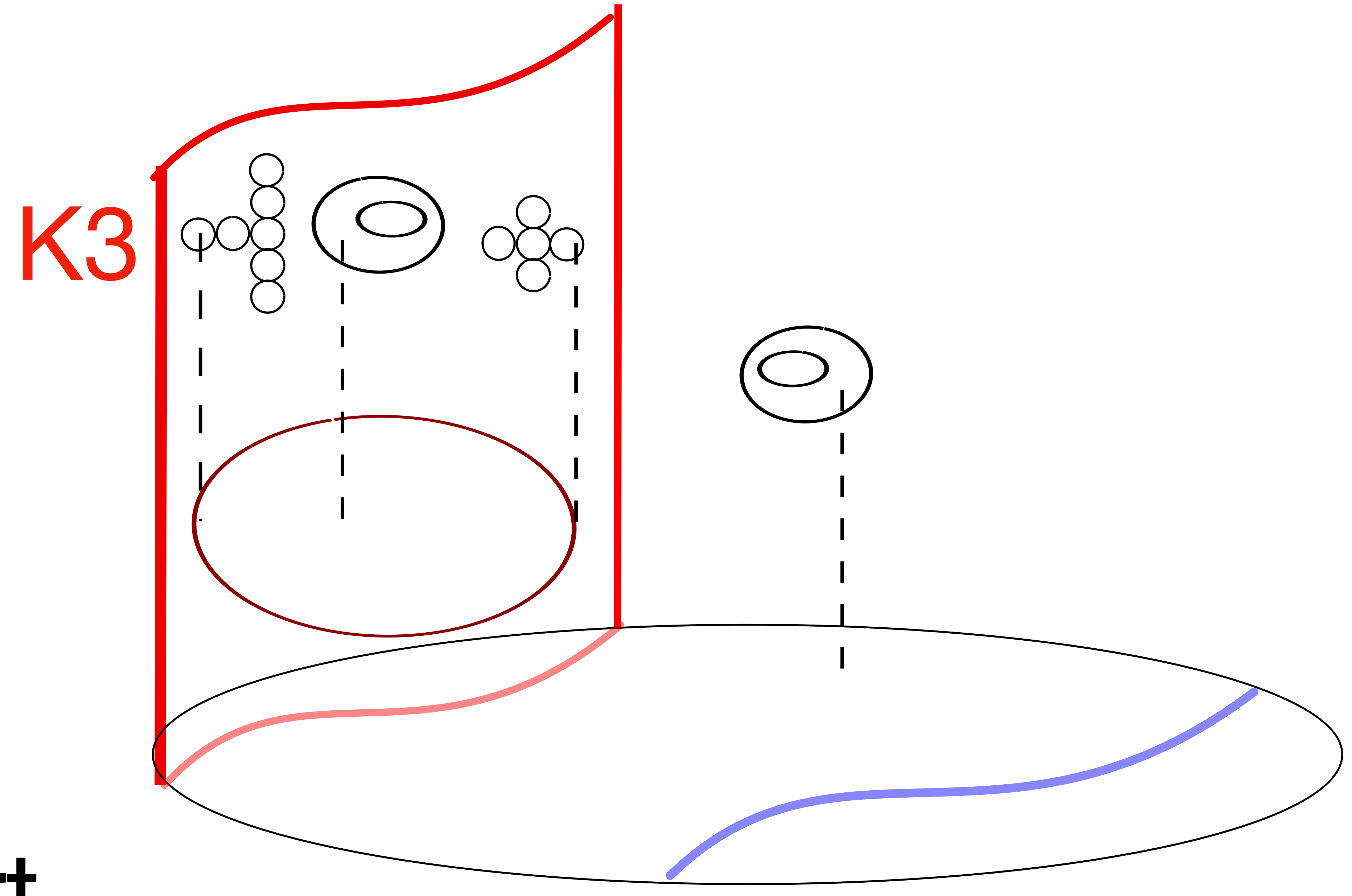
F-theory and Heterotic Strings in 6D

- The 3-fold X_3 has nested **K3 fibration**
- **Sections must be compatible**

$$MW(X_3) \subset MW(K3)$$

- **Global Structure** contained in **pert. Heterotic gauge group**

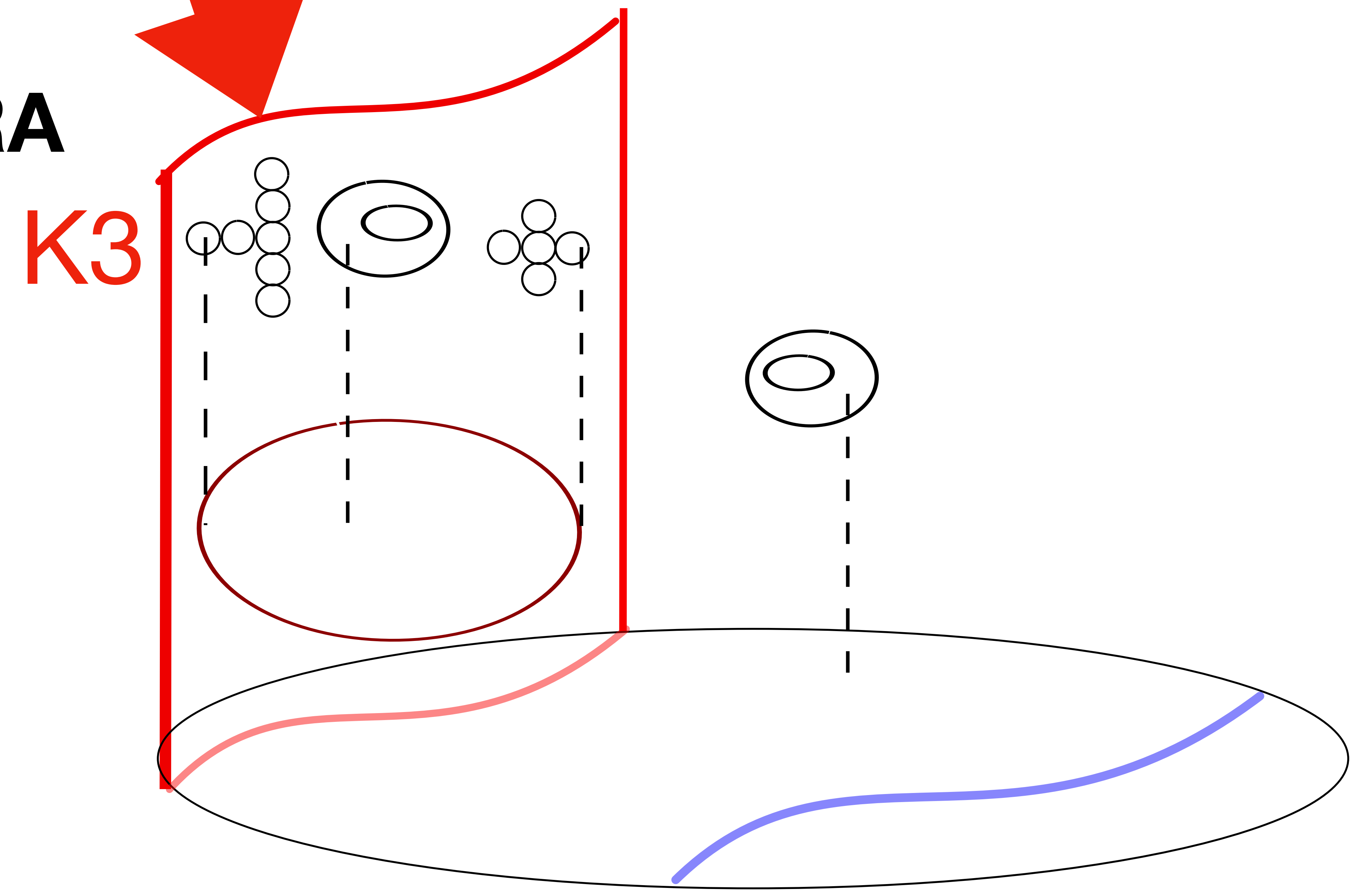
$$G = \frac{G_{\text{pert}} \times G_{\text{non-pert}}}{Z}$$



F-theory and Heterotic Strings in 6D

Like **8D SUGRA sub-sector**

- **Global Structure** on G_{pert} fixed by **8D SUGRA**



Fixed in 8D!

$$G = \frac{G_{\text{pert}} \times G_{\text{non-pert}}}{T}$$

- **8D anomaly for 1-form symmetries strongly constrains T!**

[Apruzzi, Dierigl, Lin'20; Cvetic, Dierigl, Lin, Zhang'20]

F-theory and Heterotic Strings in 6D

- **6D 1-form gauge** symmetry **T** can **not exceed** symmetry in 8D
- $T = \mathbb{Z}_{n>6}$ 8D: 1-form **anomalies strong, fixes embedding** k_i **uniquely**!

[Apruzzi, Dierigl, Lin'20; Cvetic, Dierigl, Lin, Zhang'20]

$$G = \frac{G_{\text{pert}} \times G_{\text{non-pert}}}{T}$$

8D 1-form Anomaly $\sum_i \alpha_{G_{i,pert}} k_i^2 = 0 \bmod 1$

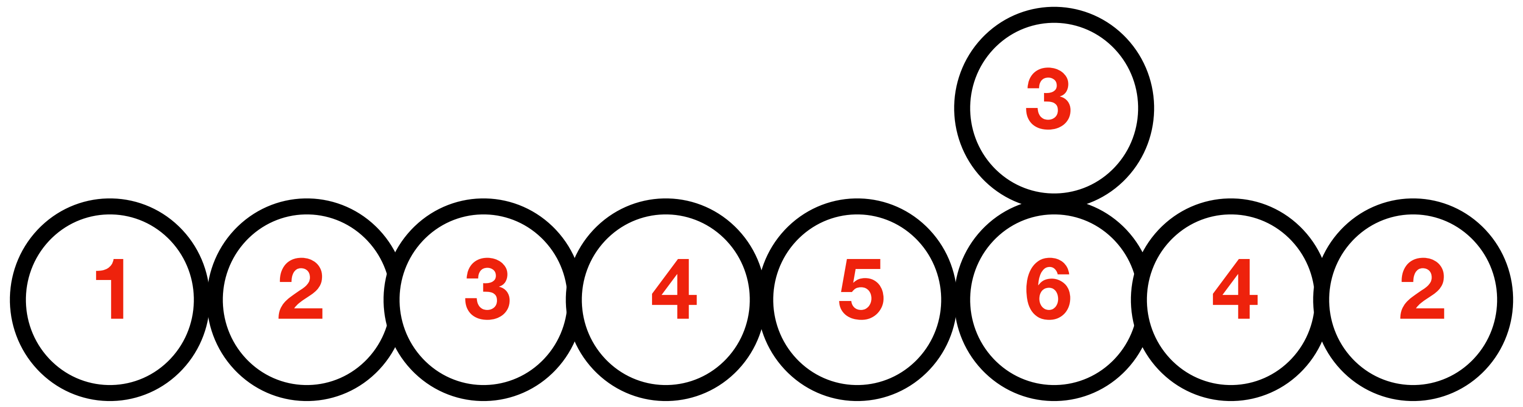
$$\alpha_{su(n)} = \frac{n-1}{2n}$$

- **But,** this also **fixes T neutral massless matter Reps**

➡ **Cant solve 6D Gauge Anomalies for** G_{pert}**!**

Summary and Outlook

- **K3 geometry**: Gauge group topology **constrained by perturbative heterotic 8D sector!**
- **Admissible** 1-form symmetries coincide **with highest Kac label** $\mathfrak{a}$ of E_8



$$E_8 \to \frac{G_1 \times G_2 \times G_3}{\mathfrak{a}}$$

- Max finite MW group of **rational elliptic surface** (E-string)

- **Constrain** discrete **0-form symmetries** in **similar way**? [Dierigl, **PKO**, Schimannek'22]
- **Exploit K3 constraints in P1 bundle base in four-folds?**