Towards a complete classification of 6D supergravities

Gregory J. Loges

[230x.xxxxx] w/ Yuta Hamada

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KEK

The plan

- Motivation
- 6D SUGRA & consistency conditions
- Multigraphs, cliques & classification
- Thoughts & To-Do list

Motivation

- Anomaly cancellation very constraining in dimension $D = 2, 6, 10$
- With < 9 tensor multiplets the number of anomaly-free 6D, $\mathcal{N} = (1, 0)$ supergravities is known to be finite [Kumar, Taylor – 09] [Kumar, Morrison, Taylor – 10]
- Classification for $T = 0, 1$ and/or simple gauge groups is well understood [Avramis, Kehagias – 05] [Kumar, Park, Taylor – 10] ...
- String universality: much recent progress in $D > 6$ [Kim, Tarazi, Vafa – 19] [Kim, Shiu, Vafa – 19] [Montero, Vafa – 21] [Bedroya, Hamada, Montero, Vafa – 22] ...

Goal: Bottom-up classification ("collecting data")

Low-energy data for 6D, $\mathcal{N} = (1,0)$ supergravity

- Supermultiplets

$$\text{gravity} : (g, \Psi, B^-) \qquad \text{tensor} : (B^+, \chi, \phi) \qquad \text{vector} : (A, \lambda) \qquad \text{hyper} : (\psi, \varphi)$$

- Choice of gauge group G, number of tensors T and hypermultiplet irreps $\mathcal{H}$
- Gravity/gauge/mixed anomalies can be cancelled via Green-Schwarz mechanism:

$$\hat{I}_8 = \hat{I}_{\text{grav}} - T\,\hat{I}_{\text{tensor}} + \hat{I}_{1/2}^{\text{Adj}} - \sum_R \hat{I}_{1/2}^R \qquad = Y_4 \cdot Y_4$$

$$Y_4 = -\frac{1}{2} b_0 \,\text{tr}(\mathcal{R}^2) + \sum_i \frac{2b_i}{\lambda_i} \text{tr}(\mathcal{F}_i^2)\,, \qquad Y_4, b_0, b_i \in \mathbb{R}^{1,T}$$

- Gauge kinetic terms $-j \cdot b_i \,\text{tr}(\mathcal{F}_i^2)$ for $j \in \text{SO}(1,T)/\,\text{SO}(T)$
- $\Lambda = \bigoplus_I \mathbb{Z}\, b_I \subset \mathbb{R}^{1,T}$ is an integral lattice [Kumar, Morrison, Taylor – 10]

Anomaly cancellation

$\lambda_i \operatorname{tr}_R \mathcal{F}_i^2 = A_R^i \operatorname{tr} \mathcal{F}_i^2$

$\lambda_i^2 \operatorname{tr}_R \mathcal{F}_i^4 = B_R^i \operatorname{tr} \mathcal{F}_i^4 + C_R^i (\operatorname{tr} \mathcal{F}_i^2)^2$

From matching terms in $\hat{I}_8 = Y_4 \cdot Y_4$:

- $H_{\text{ch}} - V \leq H - V = 273 - 29T$
- $\sum n_R^i B_R^i - B_{\text{Adj}}^i = 0$ for each G_i in $G = G_1 \times G_2 \times \cdots \times G_k$
- The Gram matrix $\mathbb{G}_{IJ} = b_I \cdot b_J$ is determined,

$$\text{e.g.} \qquad b_0 \cdot b_0 = 9 - T\,, \qquad b_0 \cdot b_i = \tfrac{1}{6}\big(\textstyle\sum n_R^i A_R^i - A_{\text{Adj}}^i\big)\,, \quad \ldots$$

but there need to actually exist vectors b_I which realize $\mathbb{G}$:

$$\Lambda \subset \mathbb{R}^{1,T} \qquad \Longrightarrow \qquad n_+(\mathbb{G}) \leq 1\,, \quad n_-(\mathbb{G}) \leq T$$

Additional consistency conditions

- $\Lambda \subseteq \Gamma \subset \mathbb{R}^{1,T}$ where Γ is unimodular (integral and $|\det \Gamma| = 1$) [Seiberg, Taylor – 11]
  Unimodular lattices of indefinite signature $(1,T)$ have a simple classification:

$$\mathrm{I}_{1,T} \cong \mathbb{Z}^{T+1}, \qquad \mathrm{II}_{1,T} \cong \left\{ x \in \mathbb{Z}^{T+1} \cup (\mathbb{Z} + \tfrac{1}{2})^{T+1} \mid \sum x^\alpha \in 2\mathbb{Z} \right\} \quad (T \equiv 1 \mod 8)$$

- $\exists j \in \mathbb{R}^{1,T}$ with $j \cdot j = 1$ such that $j \cdot b_i > 0$
  $\longrightarrow$ convex geometries and quadratic programming

- Witten and global anomalies – easily avoided

Main idea

All of these consistency conditions except for the gravitational anomaly condition behave nicely upon "decomposition"

$$G = G_1 \times G_2 \times \cdots \times G_k,$$
$$\Lambda = \langle b_0, b_1, \ldots, b_k \rangle \subseteq \Gamma \subset \mathbb{R}^{1,T},$$
$$\exists j : j \cdot b_i > 0, \text{ etc}$$

$$G' = G_1 \times G_2 \times \cdots \times G_m,$$
$$\Lambda' = \langle b_0, b_1, \ldots, b_m \rangle \subseteq \Gamma \subset \mathbb{R}^{1,T},$$
$$\exists j : j \cdot b_{i \leq m} > 0, \text{ etc}$$

$$G'' = G_{m+1} \times G_{m+2} \times \cdots \times G_k,$$
$$\Lambda'' = \langle b_0, b_{m+1}, \ldots, b_k \rangle \subseteq \Gamma \subset \mathbb{R}^{1,T},$$
$$\exists j : j \cdot b_{i > m} > 0, \text{ etc}$$

Motivating example

$$G = \mathrm{SU}(8) \times \mathrm{SU}(9) \times \mathrm{SU}(10)$$

$$T = 3$$

$$\mathcal{H} = 5 \times (\square, \bullet, \bullet) + \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}, \bullet, \bullet\right) + \left(\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}, \bullet, \bullet\right)$$
$$+ (\bullet, \square, \bullet) + (\bullet, \square\square, \bullet) + 2 \times \left(\bullet, \bullet, \begin{smallmatrix}\square\\\square\end{smallmatrix}\right) + 2 \times (\square, \bullet, \square)$$

$$\mathbb{G} = \begin{pmatrix} 6 & 5 & -1 & 2 \\ 5 & 3 & 0 & 2 \\ -1 & 0 & -1 & 0 \\ 2 & 2 & 0 & 0 \end{pmatrix}$$

$$H_{\mathrm{ch}} - V = 186 = 273 - 29T$$

Motivating example

$$G = \mathrm{SU}(8)\times\mathrm{SU}(9)\times\mathrm{SU}(10)$$

$$T = 3$$

$$\mathcal{H} = 5\times(\square,\bullet,\bullet) + \left(\begin{smallmatrix}\square\\\square\end{smallmatrix},\bullet,\bullet\right) + \left(\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix},\bullet,\bullet\right) + (\bullet,\square,\bullet) + (\bullet,\square\square,\bullet) + 2\times\left(\bullet,\bullet,\begin{smallmatrix}\square\\\square\end{smallmatrix}\right) + 2\times(\square,\bullet,\square)$$

$$\mathbb{G} = \begin{pmatrix} 6 & 5 & -1 & 2 \\ 5 & 3 & 0 & 2 \\ -1 & 0 & -1 & 0 \\ 2 & 2 & 0 & 0 \end{pmatrix}$$

$$H_{\mathrm{ch}} - V = 186 = 273 - 29T$$

$$G_1 = \mathrm{SU}(8) \quad T = 3 \quad \mathbb{G}_1 = \left(\begin{smallmatrix} 6 & 5 \\ 5 & 3 \end{smallmatrix}\right)$$

$$\mathcal{H}_1 = 25\times\square + \begin{smallmatrix}\square\\\square\end{smallmatrix} + \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$$

$$(H_{\mathrm{ch}} - V)_1 = 221$$

$$G_2 = \mathrm{SU}(9) \quad T = 3 \quad \mathbb{G}_2 = \left(\begin{smallmatrix} 6 & -1 \\ -1 & -1 \end{smallmatrix}\right)$$

$$\mathcal{H}_2 = \square + \square\square$$

$$(H_{\mathrm{ch}} - V)_2 = -26$$

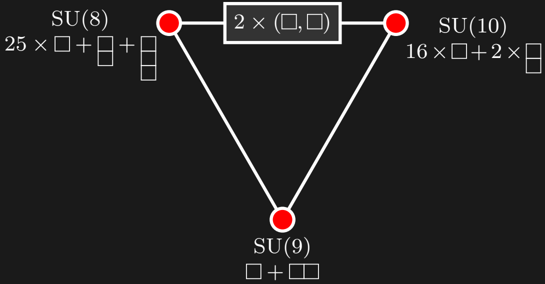
$$G_3 = \mathrm{SU}(10) \quad T = 3 \quad \mathbb{G}_3 = \left(\begin{smallmatrix} 6 & 2 \\ 2 & 0 \end{smallmatrix}\right)$$

$$\mathcal{H}_3 = 16\times\square + 2\times\begin{smallmatrix}\square\\\square\end{smallmatrix}$$

$$(H_{\mathrm{ch}} - V)_3 = 151$$

$$20\times(\square,\bullet,\bullet) + 16\times(\bullet,\bullet,\square) \longrightarrow 2\times(\square,\bullet,\square)$$

Motivating example



$$G = \mathrm{SU}(8) \times \mathrm{SU}(9) \times \mathrm{SU}(10)$$

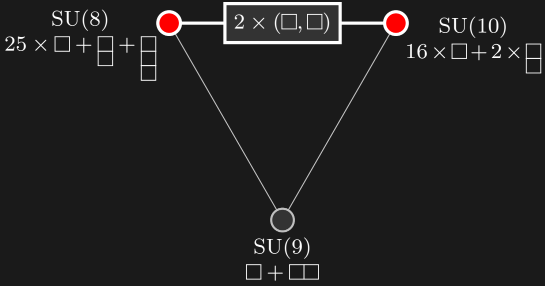
$$T = 3$$

$$\mathcal{H} = (\cdots) + 2 \times (\square, \bullet, \square)$$

$$H_{\mathrm{ch}} - V = 186 \leq 273 - 29T$$

$$\mathbb{G} = \left(\begin{array}{c|ccc} 6 & 5 & -1 & 2 \\ \hline 5 & 3 & 0 & 2 \\ -1 & 0 & -1 & 0 \\ 2 & 2 & 0 & 0 \end{array}\right)$$

Motivating example



$$G = \mathrm{SU}(8) \times \mathrm{SU}(10)$$

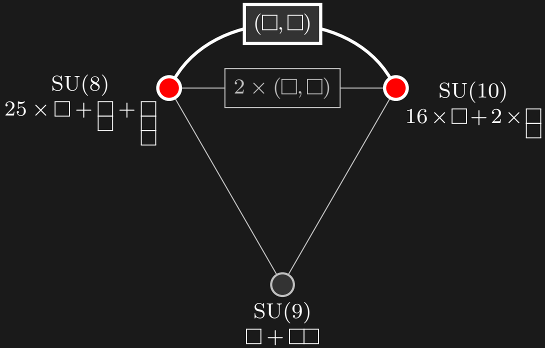
$$T = 3$$

$$\mathcal{H} = (\cdots) + 2 \times (\square, \square)$$

$$H_{\mathrm{ch}} - V = 212 > 273 - 29T$$

$$\mathbb{G} = \left(\begin{array}{c|cc} 6 & 5 & 2 \\ \hline 5 & 3 & 2 \\ 2 & 2 & 0 \end{array}\right)$$

Motivating example



$$G = \mathrm{SU}(8) \times \mathrm{SU}(10)$$

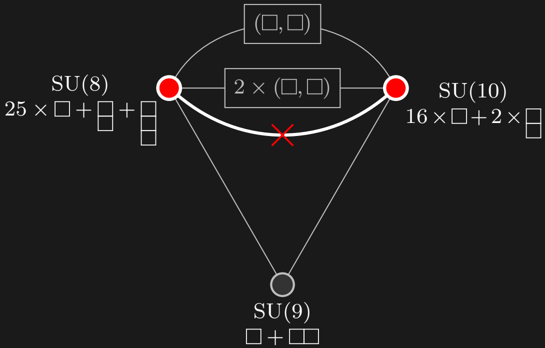
$$T = 3$$

$$\mathcal{H} = (\cdots) + (\square, \square)$$

$$H_{\mathrm{ch}} - V = 292 > 273 - 29T$$

$$\mathbb{G} = \left(\begin{array}{c|cc} 6 & 5 & 2 \\ \hline 5 & 3 & 1 \\ 2 & 1 & 0 \end{array}\right)$$

Motivating example



$$G = \mathrm{SU}(8) \times \mathrm{SU}(10)$$

$$T = 3$$

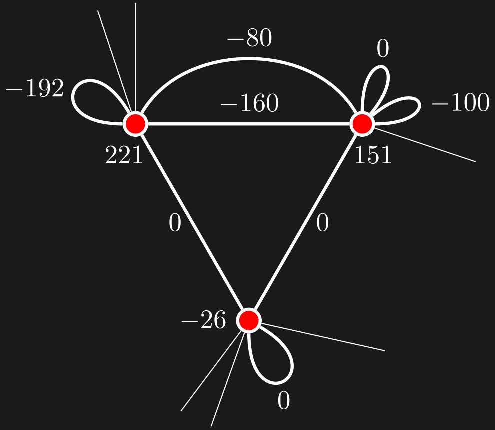
$$\mathcal{H} = (\cdots)$$

$$H_{\mathrm{ch}} - V = 372 > 273 - 29T$$

$$\mathbb{G} = \left(\begin{array}{c|cc} 6 & 5 & 2 \\ \hline 5 & 3 & 0 \\ 2 & 0 & 0 \end{array}\right)$$

$$\Lambda \not\subset \mathbb{R}^{1,4}$$

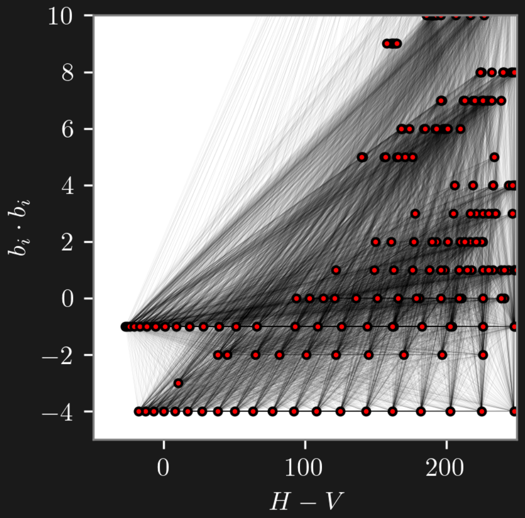
Motivating example



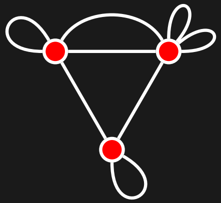
Classification outline

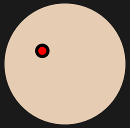
- Construct vertices/edges of multigraph for each T
  ($\Leftrightarrow$ theories with 1 and 2 simple factors)
- Recursively construct k-cliques
  ($\Leftrightarrow$ theories with k simple factors)

  Along the way, check that...
  - $\Lambda \subseteq \Gamma \subset \mathbb{R}^{1,T}$
  - j exists
  - $n_R \geq 0$
- $H_{\text{ch}} - V$ can increase/decrease as vertices are added to a clique: need a way to determine if a k-clique can *never* be extended to satisfy $H_{\text{ch}} - V \leq 273 - 29T$

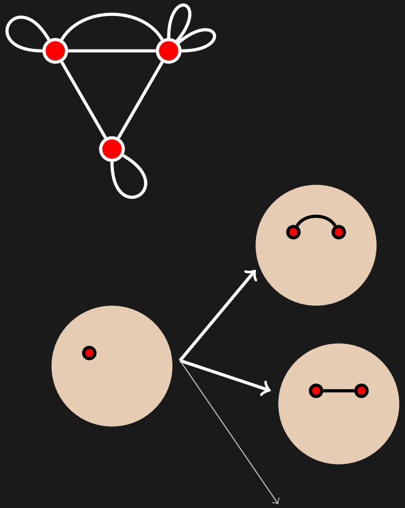


Clique construction: augment & prune

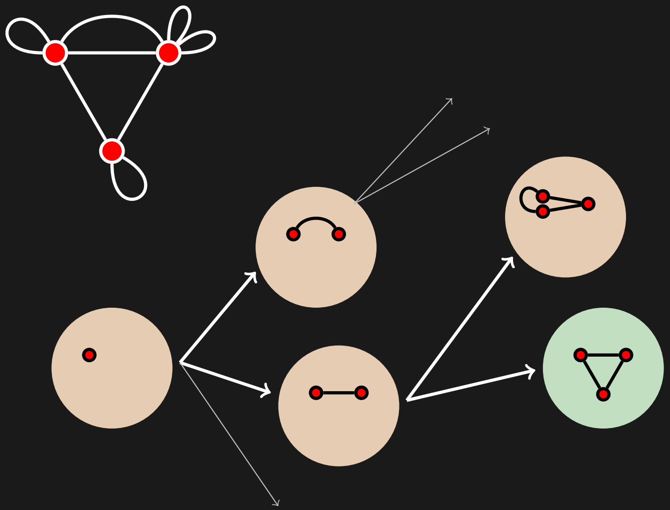




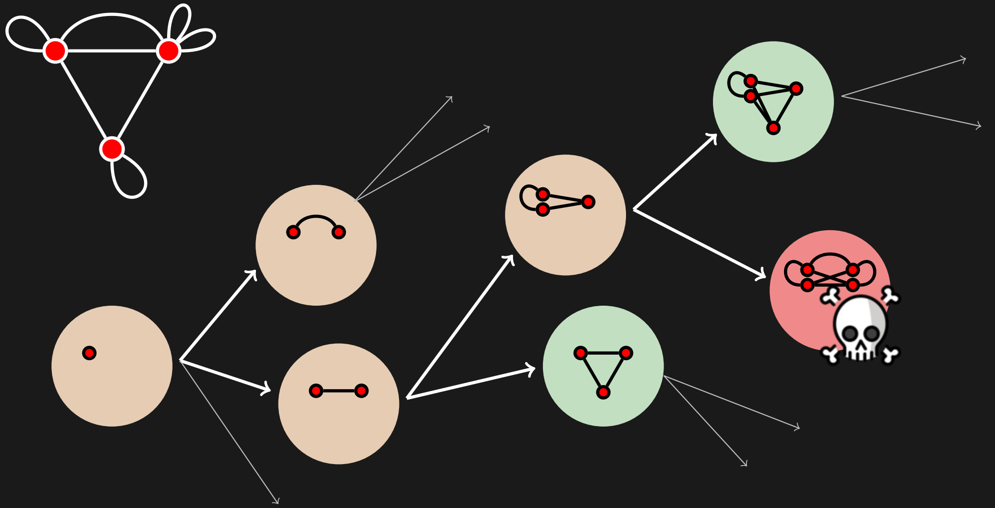
Clique construction: augment & prune



Clique construction: augment & prune



Clique construction: augment & prune



Some preliminary results ($T = 2$)

- $T = 2$ and *any* gauge group built from simple factors of rank $7 \leq r \leq 20$:

$$\mathrm{SU}(8 \to 21)\,, \quad \mathrm{SO}(14 \to 41)\,, \quad \mathrm{Sp}(7 \to 20)\,, \quad E_7\,,\ E_8$$

- $\approx$19,000 consistent theories
- 6 theories have $k = 6 \gg T$ simple factors, e.g.

$$G = \mathrm{SU}(8)^4 \times \mathrm{SU}(16) \times E_7$$

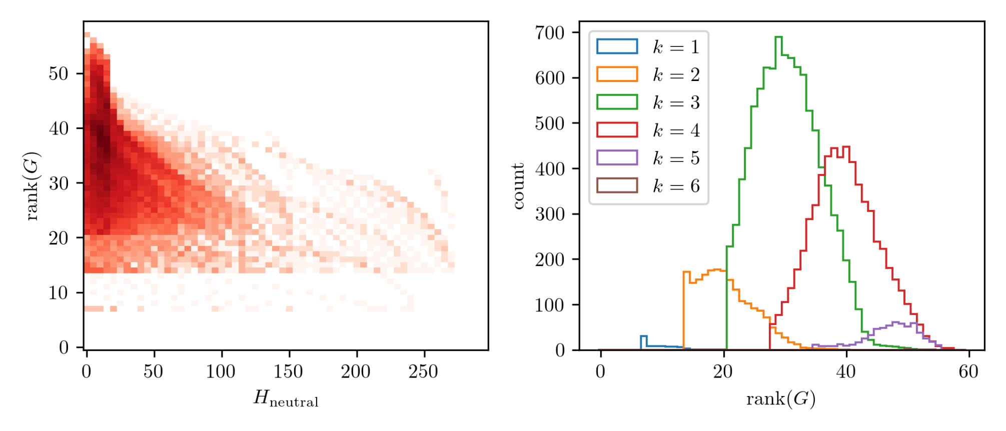
$$\mathcal{H} = \Big(2 \times (\underline{\mathbf{28}}, \underline{\mathbf{1}}, \underline{\mathbf{1}}, \underline{\mathbf{1}}; \underline{\mathbf{1}}; \underline{\mathbf{1}}) + (\underline{\mathbf{8}}, \underline{\mathbf{1}}, \underline{\mathbf{1}}, \underline{\mathbf{1}}; \underline{\mathbf{16}}; \underline{\mathbf{1}}) + (3 \text{ others})\Big) + 2 \times (\underline{\mathbf{1}}^4; \underline{\mathbf{1}}; \underline{\mathbf{56}})$$

- The maximum rank for these hyperparameters is 57:

$$G = \mathrm{SU}(13) \times \mathrm{SU}(13) \times \mathrm{SU}(16) \times \mathrm{SU}(19)$$

$$\mathcal{H} = 5 \times (\underline{\mathbf{13}}, \underline{\mathbf{1}}^3) + (\underline{\mathbf{91}}, \underline{\mathbf{1}}^3) + (\underline{\mathbf{1}}, \underline{\mathbf{13}}, \underline{\mathbf{16}}, \underline{\mathbf{1}}) + (\underline{\mathbf{1}}^2, \underline{\mathbf{16}}, \underline{\mathbf{19}}) + 2 \times (\underline{\mathbf{1}}, \underline{\mathbf{78}}, \underline{\mathbf{1}}^2) + 2 \times (\underline{\mathbf{1}}^3, \underline{\mathbf{171}})$$

Some preliminary results ($T = 2$)



Summary & To-Do

- Continue to larger T (parallelization)
- Additional conditions from BPS strings (e.g. see [Kim, Shiu, Vafa – 19]): cannot be imposed at intermediate steps
- Gauss-Bonnet term $\propto j \cdot b_0$: how often is the sign fixed?
- Kodaira condition: which/how many theories have an F-theory realization?

Summary & To-Do

- Consistency conditions (anomalies, unimodularity, positivity, ...)
- Anomaly-free theories $\Leftrightarrow$ k-cliques in multigraph (with certain properties)
- Complete, deterministic classification (modulo technical limitations)
- (∗) We've constructed an extremely large infinite family of anomaly-free theories with essentially any gauge group and any hypers for $T \gg 1$ [Hamada, GL – to appear]

<u>What's new</u>:

- All representations
- Any number of simple gauge factors
- Any T (in principle)

<u>Technical limitations</u>:

- No SU(2) or SU(3) factors
- No (3+)-charged hypers (e.g. $(\square, \square, \square)$)
- None of...

$$(E_6 + k \times \underline{\mathbf{27}},\ k \leq 1)\,, \qquad (E_8 + \emptyset)\,,$$
$$(E_7 + \tfrac{k}{2} \times \underline{\mathbf{58}},\ k \leq 3)\,, \qquad (F_4 + \emptyset)$$

Thanks!

$$\lambda_i \, \mathrm{tr}_R(\mathcal{F}^2) = A_R \, \mathrm{tr}(\mathcal{F}^2) \qquad \lambda_i \, \mathrm{tr}_R(\mathcal{F}^4) = B_R \, \mathrm{tr}(\mathcal{F}^4) + C_R \left[\mathrm{tr}(\mathcal{F}^2) \right]^2$$

Gravitational
$$\begin{cases} H_{\mathrm{ch}} - V \leq 273 - 29T \\ b_0 \cdot b_0 = 9 - T \end{cases}$$

Gauge
$$\begin{cases} 0 = \sum_R n_R^i B_R^i - B_{\mathrm{Adj}}^i \\ b_i \cdot b_i = \frac{1}{3} \left(\sum_R n_R^i C_R^i - C_{\mathrm{Adj}}^i \right) \\ b_i \cdot b_j = \sum_{R,S} n_{(R,S)}^{i,j} A_R^i A_S^j \end{cases}$$

Mixed
$$b_0 \cdot b_i = \frac{1}{6} \left(\sum_R n_R^i A_R^i - A_{\mathrm{Adj}}^i \right)$$

- There are three non-trivial homotopy groups:

$$\pi_6\big(\mathrm{SU}(2)\big) \cong \mathbb{Z}_{12}\,, \qquad \pi_6\big(\mathrm{SU}(3)\big) \cong \mathbb{Z}_6\,, \qquad \pi_6\big(G_2\big) \cong \mathbb{Z}_3$$

These groups are subject to an additional constraint:

$$\sum\nolimits_R n_R^i C_R^i - C_{\mathrm{Adj}}^i \equiv 0 \mod 12\,,\ 6\,,\ 3 \qquad \text{i.e.} \qquad b_i \cdot b_i \in 4\mathbb{Z}\,,\ 2\mathbb{Z}\,,\ \mathbb{Z}$$

- There can be half-hypermultiplets for quaternionic representations, but only if A_R is even. An odd number of half-hypermultiplets is anomalous if A_R is odd (e.g. fundamentals of $\mathrm{Sp}(N)$ must occur as full-hypermultiplets)

$$G = \mathrm{SU}(10) \qquad \left(\sum n_R B_R = B_{\mathrm{Adj}}\right)$$

$$\mathcal{H}_{\mathrm{SU}(10)} = 7 \times \underline{\mathbf{10}} + 5 \times \underline{\mathbf{45}} + 8 \times \underline{\mathbf{120}} + 3 \times \underline{\mathbf{210}} + \tfrac{1}{2} \times \underline{\mathbf{252}} + 2 \times \underline{\mathbf{825}} + \underline{\mathbf{990}} + 2 \times \underline{\mathbf{1848}}$$

$$H - V = 8248$$

$$\mathbb{G} = \begin{bmatrix} 9 - T & 473 \\ 473 & 895 \end{bmatrix}$$

$$G' = \mathrm{SU}(10) \times E_8^m \qquad \left(\sum n_R B_R = B_{\mathrm{Adj}}\right)$$

$$\mathcal{H}_{\mathrm{SU}(10)} = 7 \times \underline{\mathbf{10}} + 5 \times \underline{\mathbf{45}} + 8 \times \underline{\mathbf{120}} + 3 \times \underline{\mathbf{210}} + \tfrac{1}{2} \times \underline{\mathbf{252}} + 2 \times \underline{\mathbf{825}} + \underline{\mathbf{990}} + 2 \times \underline{\mathbf{1848}}$$

$$(H-V)' = 8248 - 248m$$

$$\mathbb{G}' = \begin{bmatrix} 9-T' & 473 & -10_m \\ 473 & 895 & 0_m \\ -10_m & 0_m & -12\,\mathbb{I}_{m\times m} \end{bmatrix} \qquad \frac{10^2}{12}\, m + T < T' \leq \frac{273 - 8248 + 248m}{29}$$

and take $j = j^1 b_1 - (b_2 + b_3 + \cdots + b_{m+1})$ with $j^1 > \sqrt{12m/895}$.