

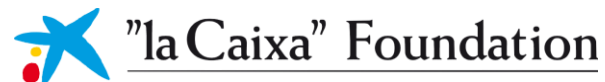
# Stringy compactifications with non-trivial torsional homology

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String Phenomenology 2023 – Daejeon

04/07/2023



# In a Nutshell

- Torsion Cohomology classes represented with delta forms
- EFTs may have access to the smeared delta forms

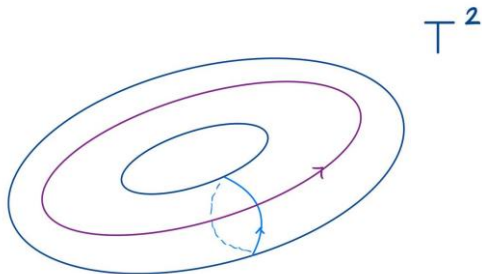
The Linking Numbers of calibrated torsional cycles can be computed via the smeared delta forms

[Gonzalo F.C., Fernando M., M.Z. 23]

# What is torsion (co)homology?

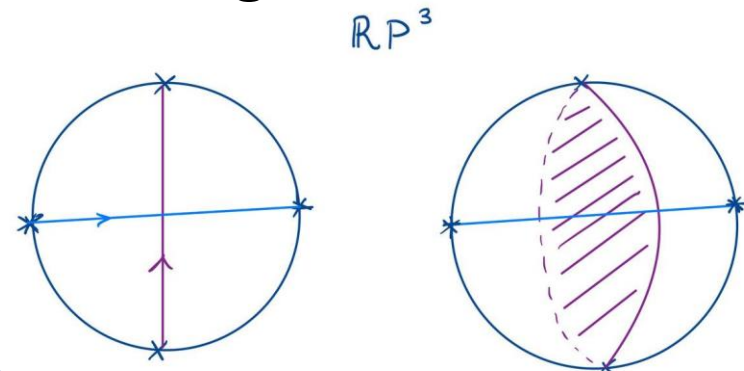
$$H_{\text{free}}(X_n, \mathbb{Z})$$

- Counts # Cycles
- Not trivial
- Intersection numbers



$$H_{\text{tor}}(X_n, \mathbb{Z})$$

- Counts # Torsion Cycles
- Trivial with N windings
- Linking numbers



# Smearred delta forms

- Bump Delta forms = **Poincare Duals** of a cycle

$$\int_{\Pi_p} \omega_p = \int_{X_n} \omega_p \wedge \delta(\Pi_p)$$

- Expansion in a basis of smooth forms

$$\delta(\Pi_p) = \sum_i c_i b_{n-p}^i \quad c_i = \int_{\Pi_p} \star b_{n-p}^i$$

- **Smearing**

$$\delta^{\text{sm}}(\Pi_p) = \sum_{\lambda_i \ll \ell_s m_{kk}} c_i b_{n-p}^i$$

# Smeared Linking Number

- Working definition [Horowitz, Srednicki 90]

$$L(\Pi_{n-p-1}, \Pi_p) = \int_{X_n} d^{-1} \delta(\Pi_{n-p-1}) \wedge \delta(\Pi_p)$$

- Expansion

$$\delta(\Pi_p) = \sum_i c_i b_{n-p}^i \quad \delta(\Pi_{n-p-1}) = \sum_i e_i \frac{1}{\lambda_i} d \star b_{n-p}^i \quad \Rightarrow \quad L = \sum_i \frac{c_i e_i}{\lambda_i}$$

- Smearing

$$L(\Pi_{n-p-1}, \Pi_p) = \sum_i \frac{c_i e_i}{\lambda_i}$$

# The Proposal

The **Smearred Linking Numbers** of calibrated torsional cycles is equal to the **Exact Linking Number**

$$L = L^{\text{sm}}$$

Based on:

- Explicit examples (twisted torus)
- Supersymmetric EFTs structures [**Fernando's talk**]

# Motivations

1. The  $\delta^{\text{sm}}$  enter in the EFTs as **massive modes**
  - Form a **basis** for dimensional reduction
2. The  $\mathbb{Z}_n$  factors of  $H(X_n, \mathbb{Z})$  produce  $\mathbb{Z}_n$  gauge symmetries via Stuckelberg like Lagrangians
  - It is possible to extract  $L^{\text{sm}}$  from the Lagrangian
  - If  $L = L^{\text{sm}}$  we can extract **topological information!**

## A Simple Example: $\tilde{T}^6 = \tilde{T}^3 \times T^3$

- Realized in type 2A as half-flat manifold mirror dual to  $T^6$  in type 2B with electric NS flux [Gurrieri, Louis, Micu, Waldram 03]

$$ds_{\tilde{T}^3}^2 = \delta_{ab} e^a \otimes e^b$$

$$e^1 = r_1 dx^1 \quad e^2 = r_2 dx^2 \quad e^3 = r_3 (dx^3 + N x^2 dx^1)$$

$$\Pi_2^{\text{tor}}, \Pi_3^{\text{tor}} \Rightarrow \Pi_1^{\text{tor}} \times \Pi_1, \Pi_1^{\text{tor}} \times \Pi_2$$

$$L(\Pi_1^{\text{tor}}, \Pi_1^{\text{tor}}) = L^{\text{sm}}(\Pi_1^{\text{tor}}, \Pi_1^{\text{tor}})$$



# Proof:

Step 0: we consider a 3-dim manifold such that

$$\xi \text{ killing, } \star d\xi = f\xi, \quad \xi^2 = 1, \quad f \in \mathbb{R}$$

Step 1: we consider a basis of scalars such that

$$\{\phi_i\} \text{ scalars, } \Delta\phi_i = \sigma_i^2\phi_i, \quad \mathcal{L}_\xi\phi_i = i\mu_i\phi_i, \quad \sigma_i^2, \mu_i \in \mathbb{R}$$

$$\Rightarrow \phi_i = e^{i\mu_i\theta} \tilde{\phi}_i, \quad \mu_i \in \mathbb{Z}, \quad \theta \sim \theta + 2\pi$$

## Step 2: we build a basis of 1-forms

- We define

$$A_i = d\phi_i, \quad B_i = \star d(\phi_i \xi), \quad C_i = \star dB_i$$

- Closed under  $\star d$

$$\star dB_i = C_i, \quad \star dC_i = \sigma_i^2 B_i + fC_i$$

- Diagonalize  $\Delta D_i^\pm = (\lambda_i^\pm)^2 D_i^\pm$

$$D_i^\pm = \left( \frac{1}{2} \pm \frac{f}{2\sqrt{f^2 + 4\sigma_i^2}} \right) C_i \pm \frac{\sigma_i^2}{\sqrt{f^2 + 4\sigma_i^2}} B_i, \quad (\lambda_i^\pm)^2 = \sigma_i^2 + \frac{f^2}{2} \pm \frac{f}{2} \sqrt{f^2 + 4\sigma_i^2}$$

### Step 3: Replacing in the Linking number

$$L(\Pi_1, \tilde{\Pi}_1) = \frac{1}{\lambda_0} K_0 \tilde{K}_0 + \sum_{i, \mu=0} \left[ \frac{K_i^+ \tilde{K}_i^+}{\lambda_i^+} + \frac{K_i^- \tilde{K}_i^-}{\lambda_i^-} \right] + \sum_{i, \mu \neq 0} \left[ \frac{K_i^+ \tilde{K}_i^+}{\lambda_i^+} + \frac{K_i^- \tilde{K}_i^-}{\lambda_i^-} \right]$$

With

$$K_i^\pm = \int_{\Pi_1} D_i^\pm, \quad K_0 = \int_{\Pi_1} D^0$$

blue and red term cancels

$$L^{\text{sm}} \equiv L = \frac{1}{N}$$

# Conclusions

- **Non-Renormalization theorem**, verified in examples, conjectured to be valid in calibrated SUSY EFT. Evidences based on how torsion leaks into the EFTs.
- Prescription to build a **basis** for dimensional reduction in presence of **massive modes**

Our findings suggest that **torsion** in Cohomology may result in **measurable** physics in the EFT