Smearing and Unsmearing KKLT AdS Vacua Part II

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Based on arXiv:2212.05074 with Mariana Graña and Nicolas Kovensky

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Motivation

- KKLT is one of the leading ways to construct metastable de Sitter vacua in String Theory

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- The first step is done at the ten-dimensional level, but for the second step we have to resort to an effective four-dimensional theory.

  - Can we understand it from a ten-dimensional viewpoint?

Supersymmetry conditions for type II SUSY 4d vacua

- The Generalized Complex Geometry conditions for type IIB flux compactifications to four-dimensional flat and AdS vacua preserving N=1 SUSY read:

[Graña, Minasian, Petrini, Tomasiello '05]

$$d_H\left(e^{3A-\phi}\Psi_-\right) = 2i\mu e^{2A-\phi}\mathrm{Im}\,\Psi_+$$

$$d_H\left(e^{2A-\phi}\mathrm{Im}\,\Psi_+\right) = 0$$

$$d_H\left(e^{4A-\phi}\mathrm{Re}\,\Psi_+\right) = 3e^{3A-\phi}\mathrm{Re}\,[\bar{\mu}\,\Psi_-] + e^{4A} *_6 \alpha(F)$$

where $d_H = d + H\wedge$, $\tilde{F} = *_6\,\alpha(F)$, $*_6\alpha(\Psi_\pm) = i\Psi_\pm$, $\alpha(\omega_q) = (-1)^{\frac{q(q-1)}{2}}\omega_q$

- and the internal geometry is characterized by the polyforms $\Psi_\pm$:

$$\Psi_\pm \equiv -\frac{8i}{||\eta||^2}\sum_p \frac{1}{p!}\eta_\pm^{2\dagger}\gamma_{m_1\ldots m_p}\eta_+^1\, dy^{m_1}\wedge\cdots\wedge dy^{m_p}$$

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- These can be also understood as D- and F-flatness conditions for [Koerber, Martucci ’07, ’08]

$$W_{\mathrm{GCG}} = \pi\int_{M_6}\langle Z, dT\rangle\,, \qquad \langle A, B\rangle \equiv [A\wedge\alpha(B)]_6 \qquad \longrightarrow \qquad \langle W_{\mathrm{GCG}}\rangle = \mu\mathcal{N}$$

where the holomorphic fields are

$$Z = e^B e^{3A-\phi}\Psi_-\,, \qquad T = e^B(C + ie^{-\phi}\,\mathrm{Re}\,\Psi_+)$$

and the Kähler potential is given by

$$\mathcal{K} = -3\log\mathcal{N}\,, \qquad \mathcal{N} = 4\pi\int_{M_6} e^{2A-2\phi}\mathrm{vol}_6$$

Supersymmetry conditions with localized terms

- We include a stack of space-time filling D7-branes wrapping an internal four-cycle Σ and undergoing gaugino condensation. The supersymmetry conditions are modified as follows

$$d_H \left(e^{3A-\phi} \Psi_- \right) = 2i\mu e^{2A-\phi} \mathrm{Im}\, \Psi_+ - 2i \langle S \rangle \delta^{(2)} \left[\Sigma_4 \right] ,$$

$$d_H \left(e^{2A-\phi} \mathrm{Im}\, \Psi_+ \right) = 0 ,$$

$$d_H \left(e^{4A-\phi} \mathrm{Re}\, \Psi_+ \right) = 3 e^{3A-\phi} \mathrm{Re}\, \left[\bar{\mu}\, \Psi_- \right] + e^{4A} *_6 \alpha(F) - e^{A} \delta^{(0)} \left[\Sigma_4 \right] \mathrm{Re}\, \left[\langle \bar{S} \rangle \Psi_- \right]$$

- Here $\langle S \rangle$ is the VEV of the condensate superfield related to the gaugino bilinear by

$$\langle S \rangle = \frac{1}{16\pi^2} \langle \lambda \lambda \rangle$$

- $\delta^{(2)}[\Sigma_4]$ is the localized 2-form Poincaré dual to the four-cycle wrapped by the branes and

$$\delta^{(0)} [\Sigma_4]\, \mathrm{vol}_6 = \langle \mathrm{Re}\, \Psi_+, \delta^{(2)} [\Sigma_4] \rangle$$

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- The on-shell value of the GCG superpotential is still

$$\langle W_{\mathrm{GCG}}\rangle = \mu\mathcal{N}$$

➢ It encodes all ingredients relevant to the effective action, including non-perturbative terms.

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- For example: for SU(3) structure $\Psi_- = \Omega$, $\Psi_+ = \exp(iJ)$, the first equation becomes

$$d_H\left(e^{3A-\phi}\Omega\right) = 2i\mu e^{2A-\phi}\left(J - \frac{1}{6}J\wedge J\wedge J\right) - 2i\langle S\rangle\delta^{(2)}\left[\Sigma_4\right]$$

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- Hard to do in practice and so we can either:
  - ▪ Study small perturbations in the deviation from SU(3) structure on top of the flat solution
  - ➢ Or simplify the problem by smearing the source

Smeared D7-branes

- We solve the modified supersymmetry equations for type IIB AdS4 compactifications with gaugino condensates on smeared D7-branes. The smearing procedure amounts to replacing

$$\delta^{(2)}[\Sigma_4] \to \gamma e^{2A-\phi} J\,, \qquad \delta^{(0)}[\Sigma_4] \to 3\gamma e^{2A-\phi}$$

where γ is fixed by requiring

$$\sigma_4 = \int_{M_6} \langle e^{-\phi} \mathrm{Re}\,\Psi_+, -\delta^{(2)}[\Sigma_4] \rangle = -\int_{M_6} \langle e^{-\phi} \mathrm{Re}\,\Psi_+, \gamma e^{2A-\phi} J \rangle \quad \Rightarrow \quad \gamma = -\frac{4\pi\sigma_4}{3\mathcal{N}}$$

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➢ We can have an SU(3) structure solution with $\mu = \gamma\langle S\rangle$.

Smeared D7-branes

- The rest of the supersymmetry conditions give

  - First SUSY equation

$$d_H\left(e^{3A-\phi}\Omega\right) = 2\mu e^{2A-\phi}\left(J - \frac{1}{6}J\wedge J\wedge J\right) - 2i\gamma\langle S\rangle e^{2A-\phi}J \longrightarrow \begin{cases} d\left(e^{3A-\phi}\Omega\right) &= 0 \\ H\wedge\Omega &= -\frac{\mu}{4}e^{-A}\bar{\Omega}\wedge\Omega \end{cases}$$

  - Second SUSY equation

$$d_H\left(e^{2A-\phi}(J - \frac{1}{6}J\wedge J\wedge J)\right) = 0 \longrightarrow \begin{cases} d\left(e^{2A-\phi}J\right) &= 0 \\ H\wedge J &= 0 \end{cases}$$

  - Third SUSY equation

$$d_H\left(e^{4A-\phi}\mathrm{Re}\,\Psi_+\right) = 3e^{3A-\phi}\mathrm{Re}\,[\bar{\mu}\,\Psi_-] + e^{4A} *_6\alpha(F) - e^{A}\delta^{(0)}[\Sigma_4]\,\mathrm{Re}\,[\langle\bar{S}\rangle\Psi_-] \longrightarrow e^{-\phi}H = F_3$$

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- The KKLT relation between the total and the non-perturbative superpotentials is precisely reproduced

$$\langle W \rangle = \mu \mathcal{N} = \gamma \mathcal{N} \langle S \rangle = -\frac{4\pi}{3} \sigma_4 \langle S \rangle = -\frac{2}{3} \left(\frac{2\pi}{N} \right) \sigma_4 \langle W_{NP} \rangle$$

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- There is no obstruction to scale separation if small (0,3) flux is possible.

Localized solution

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[Dymarsky, Martucci '11]

  - E.g. :

$$G_3^{(0,3)} \sim \langle S \rangle \delta^{(0)}[\Sigma_4]\bar{\Omega} \longrightarrow \int_{M_6} G_3 \wedge \star_6 \bar{G}_3 \sim \int_{M_6} |S|^2 \left(\delta^{(0)}[\Sigma_4]\right)^2 \text{vol}_6 + \cdots$$

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- The issue has been studied without reaching a common conclusion.

[Kachru, Kim, McAllister, Zimet '19]

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- One can evaluate the action without knowing the solution:

[Lüst, Marchesano, Martucci, Tsimpis '08]

$$S_{\text{eff}} = \int_{X_4} d^4x\sqrt{-g}\left(\frac{1}{2}\mathcal{N}R_4 - 2\pi V_{\text{eff}}\right)$$

On-shell action

- The effective potential is given by

$$V_{\rm eff} = -\frac{1}{2}\int_{M_6} {\rm vol}_6 e^{4A}[\tilde{F} - e^{-4A} d_H(e^{4A-\Phi}{\rm Re}\,\Psi_+)]^2 + \frac{1}{2}\int_{M_6} {\rm vol}_6 [d_H(e^{2A-\Phi}{\rm Im}\,\Psi_+)]^2 + \frac{1}{2}\int_{M_6} {\rm vol}_6 e^{-2A}|d_H(e^{3A-\Phi}\Psi_-)|^2 - \frac{1}{4}\int_{M_6} e^{-2A}\left(\frac{|\langle\Psi_+, d_H(e^{3A-\Phi}\Psi_-)\rangle|^2}{{\rm vol}_6} + \frac{|\langle\bar{\Psi}_+, d_H(e^{3A-\Phi}\Psi_-)\rangle|^2}{{\rm vol}_6}\right)$$

- The different contributions to the on-shell action in the absence of the gaugino condensate

	$\lvert\mu\rvert^2 e^{2A-2\phi}{\rm vol}_6$
1st	18
2nd	8
3rd	-32
Total	**-6**

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$$V_{\text{eff}} = -6|\mu|^2 \int_{M_6} e^{2A-2\phi}\text{vol}_6 \quad \Rightarrow \quad 2\pi V_{\text{eff}} = \Lambda\mathcal{N}$$

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$$-\frac{1}{4}\int_{M_6} e^{-2A}\left(\frac{|\langle\Psi_+, d_H(e^{3A-\Phi}\Psi_-)\rangle|^2}{\text{vol}_6} + \frac{|\langle\bar{\Psi}_+, d_H(e^{3A-\Phi}\Psi_-)\rangle|^2}{\text{vol}_6}\right)$$

- The different contributions to the on-shell action including the effect of the localized gaugino condensate

	$\lvert\mu\rvert^2 e^{2A-2\phi}\text{vol}_6$	$\delta^{(0)}\,[\Sigma_4]\,\text{Re}\,[\bar{\mu}S]\,e^{-\phi}\text{vol}_6$	$\left(\delta^{(0)}\,[\Sigma_4]\right)^2 \lvert S\rvert^2 e^{-2A}\text{vol}_6$
1st	18	-12	2
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- The terms proportional to $\delta^{(0)}[\Sigma_4]$ cancel out, while the terms proportional to $\left(\delta^{(0)}[\Sigma_4]\right)^2$ don't.

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- We should also consider the brane action, which however does not contain terms with a square of delta functions.

[Graña, Kovensky, Retolaza '20]

$$S_{\lambda\lambda} = \int d^4x \left(\frac{i}{2} f\, \bar{\lambda}_+ \gamma^\mu \nabla_\mu \lambda_+ + \frac{1}{2} m_\lambda\, \bar{\lambda}_- \lambda_+ + c.c. \right) , \quad m_\lambda = -\frac{i}{8\pi} \int_{M_6} \delta^{(0)}[\Sigma_4]\, e^A \langle \Psi_-, F + i\, d_H \left(e^{-\phi} \mathrm{Re}\, \Psi_+ \right) \rangle$$

- The different contributions to the on-shell action including the effect of the localized gaugino condensate

	$\lvert\mu\rvert^2 e^{2A-2\phi} \mathrm{vol}_6$	$\delta^{(0)}\left[\Sigma_4\right] \mathrm{Re}\left[\bar{\mu} S\right] e^{-\phi} \mathrm{vol}_6$	$\left(\delta^{(0)}\left[\Sigma_4\right]\right)^2 \lvert S\rvert^2 e^{-2A} \mathrm{vol}_6$
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- Putting everything together, the effective potential should be:

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- Putting everything together, the effective potential should be:

[Hamada, Hebecker, Shiu, Soler '21]

$$V_{\text{eff}}^{\text{bulk}} + V_{\text{eff}}^{\lambda\lambda} + \frac{\gamma}{64\pi^4}\int_{M_6} \delta^{(0)}\left[\Sigma_4\right] e^{-\phi}|\lambda\lambda|^2\text{vol}_6 - \frac{2}{3}\int_{M_6} e^{-2A}|\langle S\rangle|^2\left(\delta^{(0)}\left[\Sigma_4\right]\right)^2\text{vol}_6$$

➢ No perfect square structure arises.

Thank you!