Interpolation and cosmological constant in non-SUSY heterotic strings

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- Y.K., NPB 990 (2023) 116160
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Introduction

- Where is SUSY breaking scale?
- \succ There is no evidence for SUSY in multi TeV scale according to the LHC.
- \succ It is interesting to consider SUSY is already broken at very high energies.

We have more non-SUSY vacua than SUSY ones in 10D:

- Type IIA
- Type IIB
- Type I
- Heterotic $E_8 \times E_8$

- Type 0A
- Type OBHeterotic SO(32)
- Heterotic SO(32) Heterotic $SO(16) \times E_8$
 - Heterotic $SO(16) \times SO(16)$

• Heterotic $E_7^2 \times SU(2)^2$

•

• Heterotic $SO(24) \times SO(8)$

difficulty: very large cosmological constant (vacuum energy density)

 $\Lambda^{(D)} \sim \mathcal{O}(M_s^D)$ M_s : string scale

We want to obtain small or vanishing cosmological const. without SUSY.

Our approach: "interpolating models"

- Low-dimensional models constructed by \mathbb{Z}_2 freely acting orbifolds
- The models are Non-SUSY where a radius *R* is finite.
- But SUSY can be restored in $R \rightarrow \infty$ (and/or 0) limits.



■ In SUSY restored region ($R \approx \infty$),

$$\Lambda^{(9)} = \frac{\xi}{R^9} (n_F - n_B) + \mathcal{O}(e^{-R}) \quad \begin{cases} \xi : \text{positive constant} \\ n_F, n_B : \# \text{ (massless fermion, boson)} \end{cases}$$

Itoyama, Taylor (1986)

 \succ *n_F* = *n_B* ⇒ exponentially suppressed cosmological constant

• The general heterotic models constructed by \mathbb{Z}_2 -twists:

- *d*-dim. compactified with $#(\mathbb{Z}_2$ -twisted directions) arbitrary
- With full set of moduli: $C_{ij} = G_{ij} + B_{ij} \& A_i^I$ turned on ($i = 10 - d, \dots, 9, I = 1, \dots, 16$)

(↔ with all marginal deformations considered)

Show various interpolation patterns in d = 1, 2

Find solutions of $n_F = n_B$ where cosmol. const. is exp. supp.

Analyze Wilson-line stability of the effective potential

Outline

- 1. Introduction
- 2. Non-SUSY heterotic strings with general Z₂ twists
- 3. Interpolation in 9D and 8D
- 4. Cosmological constant
- 5. Summary

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Narain lattice of SUSY heterotic strings on T^d

■ Modular inv. \Rightarrow internal momenta $P = Z\mathcal{E} \in \Gamma^{16+d,d}$

- Narain lattice: even self-dual lattice w/ Lorentz. sign. (16 + d, d)
- Labeled by an integer vector $Z = (q, \underline{m}, \underline{n}) \in Z^{16} \times Z^d \times Z^d$

winding numbers KK momenta

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- Turn on full moduli: $d(d + 16) = d^2 + 16d \Rightarrow (G_{ij} + B_{ij}) \& A_i^I$
- Consider a rectangular *d*-torus: $G_{ij} = R_i^2 \delta_{ij}$

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 $\geq P = (\ell_L, p_L; p_R)$

Narain, Sarmadi, Witten, (1986)

$$\begin{cases} \ell_L^I = \pi^I - m^i A_i^I, \\ p_{Li} = \frac{1}{\sqrt{2}R_i} \left(\pi \cdot A_i + n_i + m^j \left(G_{ij} + B_{ij} - \frac{1}{2}A_i \cdot A_j \right) \right) \\ p_{Ri} = \frac{1}{\sqrt{2}R_i} \left(\pi \cdot A_i + n_i - m^j \left(G_{ij} - B_{ij} + \frac{1}{2}A_i \cdot A_j \right) \right) \end{cases} \qquad 16 \begin{cases} \ell_L^I \\ p_{Li} \end{pmatrix} \\ d \end{cases}$$

 $\pi^{I} \equiv q^{I} \alpha_{16} \in \underline{\Gamma}^{16} \leftarrow Spin(32)/\mathbb{Z}_{2} \text{ or } E_{8} \times E_{8} \text{ lattice}$

Construction of non-SUSY heterotic strings

- Z₂ freely acting orbifold (stringy Scherk-Schwarz comp.)
 - Project out SUSY hetero on T^d by $\frac{1+(-)^F \alpha}{2}$ (+ twisted sec. added)

 $Z_2 \text{ generator}: (-)^F \alpha \quad \begin{cases} F: \text{ spacetime fermion } \#\\ \alpha: \text{ shift of order 2 such as } \alpha |P\rangle = e^{2\pi i P \cdot \delta} |P\rangle \end{cases}$

• δ is called a shift vector : $2\delta \in \Gamma^{16+d,d}$

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 - labeled by a vector $\hat{Z} = (\hat{q}^{I}, \hat{m}^{i}, \hat{n}_{i})$ whose components are <u>0 or 1</u> Non-SUSY strings depend on \hat{Z}
 - Splitting the Narain lattice $\Gamma^{16+d,d}$ into $\Gamma^{16+d,d}_+$ and $\Gamma^{16+d,d}_-$:

$$\Gamma^{16+d,d}_{+} = \left\{ P \in \Gamma^{16+d,d} \mid \delta \cdot P \in \mathbb{Z} \right\}$$

$$\Gamma^{16+d,d}_{-} = \left\{ P \in \Gamma^{16+d,d} \mid \delta \cdot P \in \mathbb{Z} + 1/2 \right\}$$

$$\Rightarrow \alpha \mid P \rangle = \begin{cases} + \mid P \rangle & \text{for } P \in \Gamma^{16+d,d}_{+} \\ - \mid P \rangle & \text{for } P \in \Gamma^{16+d,d}_{-} \end{cases}$$

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Bosons/Fermions live in $\Gamma_{+}^{16+d,d} / \Gamma_{-}^{16+d,d}$ respectively \longrightarrow SUSY breaking

1-loop partition function

• Heterotic strings on T^d (with maximal SUSY)



$$\begin{pmatrix} Z_B^{(8-d)} = \tau_2^{-\frac{8-d}{2}} (\eta \bar{\eta})^{-(8-d)} \\ Z_{\Gamma^{16+d,d}} = \eta^{-(16+d)} \bar{\eta}^{-d} \sum_{p \in \Gamma^{16+d,d}} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2} \\ q = e^{2\pi i \tau}, V_8, S_8, O_8, C_8: SO(8) \text{ characters} \end{pmatrix}$$

1-loop partition function

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Non-SUSY Heterotic strings

$$Z_{(\hat{Z})}^{SUSY} = Z_B^{(8-d)} \left\{ \underbrace{\bar{V}_8 Z_{\Gamma_{+}^{16+d,d}}}_{\text{vector}} - \underbrace{\bar{S}_8 Z_{\Gamma_{-}^{16+d,d}}}_{\text{spinor}} + \underbrace{\bar{O}_8 Z_{\Gamma_{\pm}^{16+d,d}+\delta}}_{\text{scalar}} - \underbrace{\bar{C}_8 Z_{\Gamma_{\mp}^{16+d,d}+\delta}}_{\text{co-spinor}} \right\}$$

$$\begin{pmatrix} Z_B^{(8-d)} = \tau_2^{-\frac{8-d}{2}} (\eta \bar{\eta})^{-(8-d)} \\ Z_{\Gamma^{16+d,d}} = \eta^{-(16+d)} \bar{\eta}^{-d} \sum_{p \in \Gamma^{16+d,d}} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2} \\ q = e^{2\pi i \tau}, V_8, S_8, O_8, C_8: SO(8) \text{ characters} \end{pmatrix}$$

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■ 9D Non-SUSY heterotic models (d = 1) Itoyama, Koga, Nkajima (2021) > $2^2 = 4$ classes exist $|\hat{\pi}|^2 + 2\hat{m}\hat{n}^t = 0 \pmod{4}$

• class (1):
$$|\hat{\pi}|^2 = 0 \pmod{4}$$
, $(\hat{m}; \hat{n}) = (0; 0)$



• class (2): $|\hat{\pi}|^2 = 0 \pmod{4}$, $(\hat{m}, \hat{n}) = (1; 0)$

$$10D \text{ non-SUSY} \xrightarrow{R} 10D \text{ SUSY}$$

• class (3):
$$|\hat{\pi}|^2 = 0 \pmod{4}$$
, $(\hat{m}, \hat{n}) = (0; 1)$

$$\frac{R}{0} \longrightarrow 10D \text{ non-SUSY}$$

• class (4):
$$|\hat{\pi}|^2 = 2 \pmod{4}$$
, $(\hat{m}, \hat{n}) = (1; 1)$



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• class (1):
$$|\hat{\pi}|^2 = 0 \pmod{4}$$
, $(\hat{m}; \hat{\pi}) = (0; 0)$
10D non-SUSY $\stackrel{R}{\longrightarrow}$ 10D non-SUSY
non-SUSY heterotic strings on a circle
• class (2): $|\hat{\pi}|^2 = 0 \pmod{4}$, $(\hat{m}, \hat{\pi}) = (1; 0)$
10D non-SUSY $\stackrel{R}{\longrightarrow}$ 10D SUSY
• class (3): $|\hat{\pi}|^2 = 0 \pmod{4}$, $(\hat{m}, \hat{\pi}) = (0; 1)$
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• class (4): $|\hat{\pi}|^2 = 2 \pmod{4}$, $(\hat{m}, \hat{\pi}) = (1; 1)$
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■ 9D Non-SUSY heterotic models (d = 1) Itoyama, Koga, Nkajima (2021)

$> 2^2 = 4$ classes exist



interpolation between SUSY and non-SUSY vacua

• class (4): $|\hat{\pi}|^2 = 2 \pmod{4}$, $(\hat{m}, \hat{n}) = (1; 1)$



■ 9D Non-SUSY heterotic models (d = 1) Itoyama, Koga, Nkajima (2021)

$> 2^2 = 4$ classes exist



SUSY restored at both of the endpoints

- 8D Non-SUSY heterotic models (d = 2) > $2^4 = 16$ classes exist
- class (2) & (1) : $|\hat{\pi}|^2 = 0 \pmod{4}, \ (\hat{m}; \hat{n}) = (1,0;0,0)$

• class (4) & (1) : $|\hat{\pi}|^2 = 2 \pmod{4}, (\hat{m}; \hat{n}) = (1,0; 1,0)$





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■ 8D Non-SUSY heterotic models (d = 2) > $2^4 = 16$ classes exist



10D (Non-)SUSY condition:

Limits of R_1, R_2	10D SUSY model	10D Non-SUSY model
$(R_1, R_2) \to (\infty, \infty)$	$\hat{m}^1 + \hat{m}^2 > 0$	$\hat{m}^1 + \hat{m}^2 = 0$
$(R_1, R_2) \to (\infty, 0)$	$\hat{m}^1 + \hat{n}_2 > 0$	$\hat{m}^1 + \hat{n}_2 = 0$
$(R_1, R_2) \to (0, \infty)$	$\hat{n}_1 + \hat{m}^2 > 0$	$\hat{n}_1 + \hat{m}^2 = 0$
$(R_1, R_2) \to (0, 0)$	$\hat{n}_1 + \hat{n}_2 > 0$	$\hat{n}_1 + \hat{n}_2 = 0$

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Cosmological Constant

• 1-loop cosmological constant (effective potential) :

$$\Lambda^{(10-d)} = -\frac{1}{2} (2\pi\sqrt{\alpha'})^{-(10-d)} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_{(\hat{Z})}^{SUSY}$$

Fundamental Region :

$$\mathcal{F} = \left\{ \tau = \tau_1 + i\tau_2 \in \mathbb{C} \ \middle| \ -\frac{1}{2} \le \tau_1 \le \frac{1}{2}, \ |\tau| \ge 1 \right\}$$

• Assignment of (\hat{m}, \hat{n}) :

$$\begin{aligned} & (\hat{m}^{a_{(2)}}, \hat{n}_{a_{(2)}}) = \begin{pmatrix} 1, 0 \end{pmatrix} & \text{for } a_{(2)} = 1, \dots, D_2 \\ & (\hat{m}^{a_{(4)}}, \hat{n}_{a_{(4)}}) = \begin{pmatrix} 1, 1 \end{pmatrix} & \text{for } a_{(4)} = D_2 + 1, \dots, D \\ & (\hat{m}^{b_{(3)}}, \hat{n}_{b_{(3)}}) = \begin{pmatrix} 0, 1 \end{pmatrix} & \text{for } b_{(3)} = D + 1, \dots, D + D_3 \\ & (\hat{m}^{b_{(1)}}, \hat{n}_{b_{(1)}}) = \begin{pmatrix} 0, 0 \end{pmatrix} & \text{for } b_{(1)} = D + D_3 + 1, \dots, d \end{aligned} \right\} \quad \text{Non-SUSY at } R_b \to \infty$$

Formula for cosmological constant

■ Consider $D \ge 1$, all $R_i \approx \infty$: SUSY is restored

· Up to exponentially suppressed terms,

$$\Lambda^{(10-d)} \sim -\frac{4! \cdot 2^{d-1}}{\pi^{15-d} (\sqrt{\alpha'})^{10-d}} \left(\prod_{i=1}^{d} R_i \right) \sum_{n} \left\{ \sum_{a=1}^{D} (2n_a - 1)^2 R_a^2 + \sum_{b=D+1}^{d} (2n_b)^2 R_b^2 \right\}^{-5} \times 8 \left(24 + \sum_{\pi \in \Delta_g} \exp\left[2\pi i \left\{ \sum_{a=1}^{D} (2n_a - 1)(\pi \cdot A_a) + \sum_{b=D+1}^{d} n_b(\pi \cdot A_b) \right\} \right] \right)$$

 Δ_g : nonzero roots of SO(32) or $E_8 \times E_8$, not Δ_{\pm} \Rightarrow CC does not depend on all the other endpoint models

massless condition $\Lambda^{(10-d)} \sim \frac{4! \cdot 2^{d-1}}{\pi^{15-d}} \left(\prod_{i=1}^{d} R_i \right) \sum_{\overrightarrow{n}} \left\{ \sum_a (2n_a - 1)^2 R_a^2 + \sum_b (2n_b)^2 R_b^2 \right\}^{-5} (n_F - n_B)$ $2\pi \cdot A_a \in \mathbb{Z}$ $\pi \cdot A_b \in \mathbb{Z}$

■SUSY *SO*(32) endpoint model:

 $\Delta_{SO(32)} = \left\{ \left(\underline{\pm}, \pm, 0^{14} \right) \right\}$

Simplest configurations:

- A_a^I (a = 1, ..., D) are the same configuration
- $A_b^I(b = D + 1, ..., d)$ are taken to be 0

$$A_a = \left(0^p, \left(\frac{1}{2}\right)^q\right) \ (p+q=16), \ A_b = \left(0^{16}\right)$$

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- $D \in 2\mathbf{Z}$: $n_F n_B = -504 \neq 0$
- $D \in 2\mathbb{Z} + 1$: $n_F n_B = 4pq \{2p(p-1) + 2q(q-1)\} 24$ $n_F = n_B \Rightarrow (p,q) = (9,7), (7,9)$

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Cosmological constant is exponentially suppressed when the gauge group is $SO(18) \times SO(14)$

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Summary

- (10 d)D Non-SUSY models are constructed by orbifolding by $(-)^{F} \alpha$ (α : shift of order 2 in Narain lattice)
- Various interpolations are shown in d = 2 case
- Cosmological constant of (10 d)D Non-SUSY models in $R_i \approx \infty$ is

$$\Lambda^{(10-d)} \sim \frac{4! \cdot 2^{d-1}}{\pi^{15-d}} \left(\prod_{i=1}^{d} R_i \right) \sum_{\overrightarrow{n}} \left\{ \sum_a (2n_a - 1)^2 R_a^2 + \sum_b (2n_b)^2 R_b^2 \right\}^{-5} (n_F - n_B) + \mathcal{O}(e^{-R/\sqrt{\alpha'}})$$

• Find the configurations of WLs which give exp. supp. cosmol. const.

<u>Out look</u>

Higher-loop correction, (meta)stable vacua, cosmology, ...