String Pheno ’23

Life-time of Metastable Vacuum in String Theory and Trans-Planckian Censorship Conjecture

Based on arXiv:2305.00781 [hep-th]

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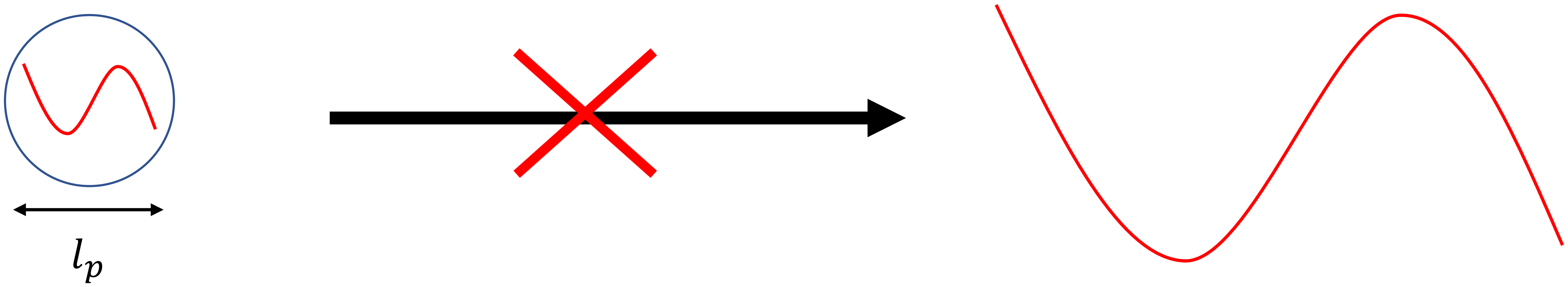
Trans-Planckian censorship conjecture and metastable vacua

- TCC forbids enlargement of sub-Planckian fluctuation to classical one during inflation. [Vafa et al., JHEP 09 123 (2020)]



- It gives strong constraints on lifetimes of metastable de-Sitter vacua.

[Bedroya et al.,arXiv:2008.07555[hep-th]]

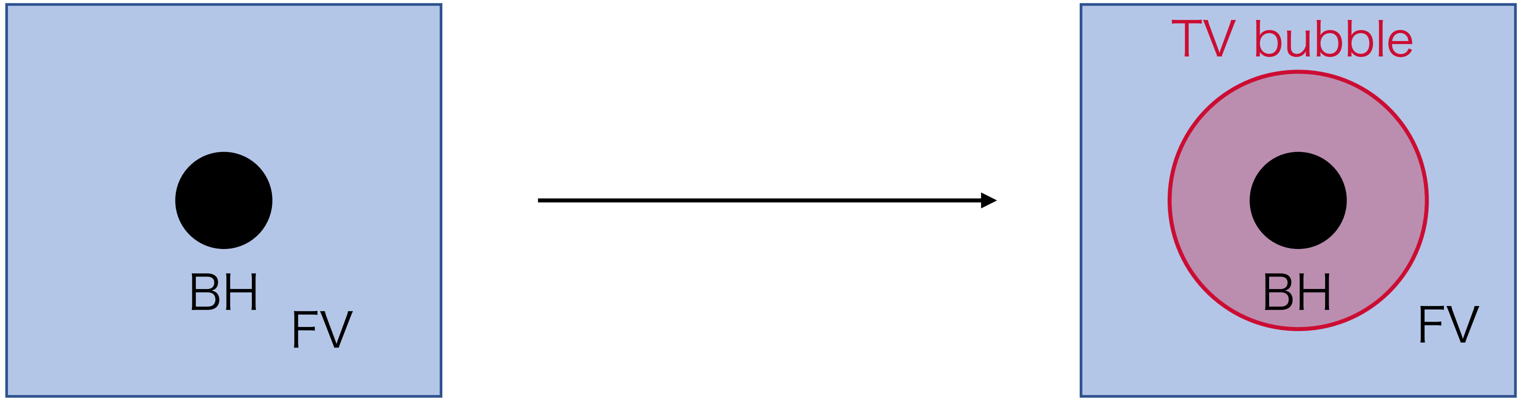
$$\tau < \frac{1}{H}\log\frac{M_{pl}}{H}$$ TCC condition

- In GUT and gravitational theory, monopoles and black holes are well known for acting as catalysts to enhance the instability of the vacuum.

[P. J. Steinhardt, Nucl. Phys. B 190, 583-616 (1981); Phys. Rev. D 24, 842 (1981)]

[Y. Hosotani, Phys. Rev. D 27, 789 (1983)] [U. A. Yajnik, Phys. Rev. D 34, 1237-1240 (1986)]

e.g. Black hole catalysis [R. Gregory, I. G. Moss and B. Withers, JHEP 03, 081 (2014)]



- Can metastable vacua with impurities circumvent swampland??

Brane setup

- D5 and antiD5-branes wrapped to conifolds form a metastable state.
  [F. Cachazo et al., Nucl. Phys. B 603, 3-41 (2001);C. Vafa, J. Math. Phys. 42, 2798-2817 (2001);M. Aganagic et al., Nucl. Phys. B 789, 382-412 (2008)]
- D3-branes which are wrapped to the internal space dissolve into D5-brane and become magnetic flux. ⟶ Catalyst!
  [A. Kasai et al., Phys. Rev. D 91, no.12, 126002 (2015)]

Table 1. brane configuration

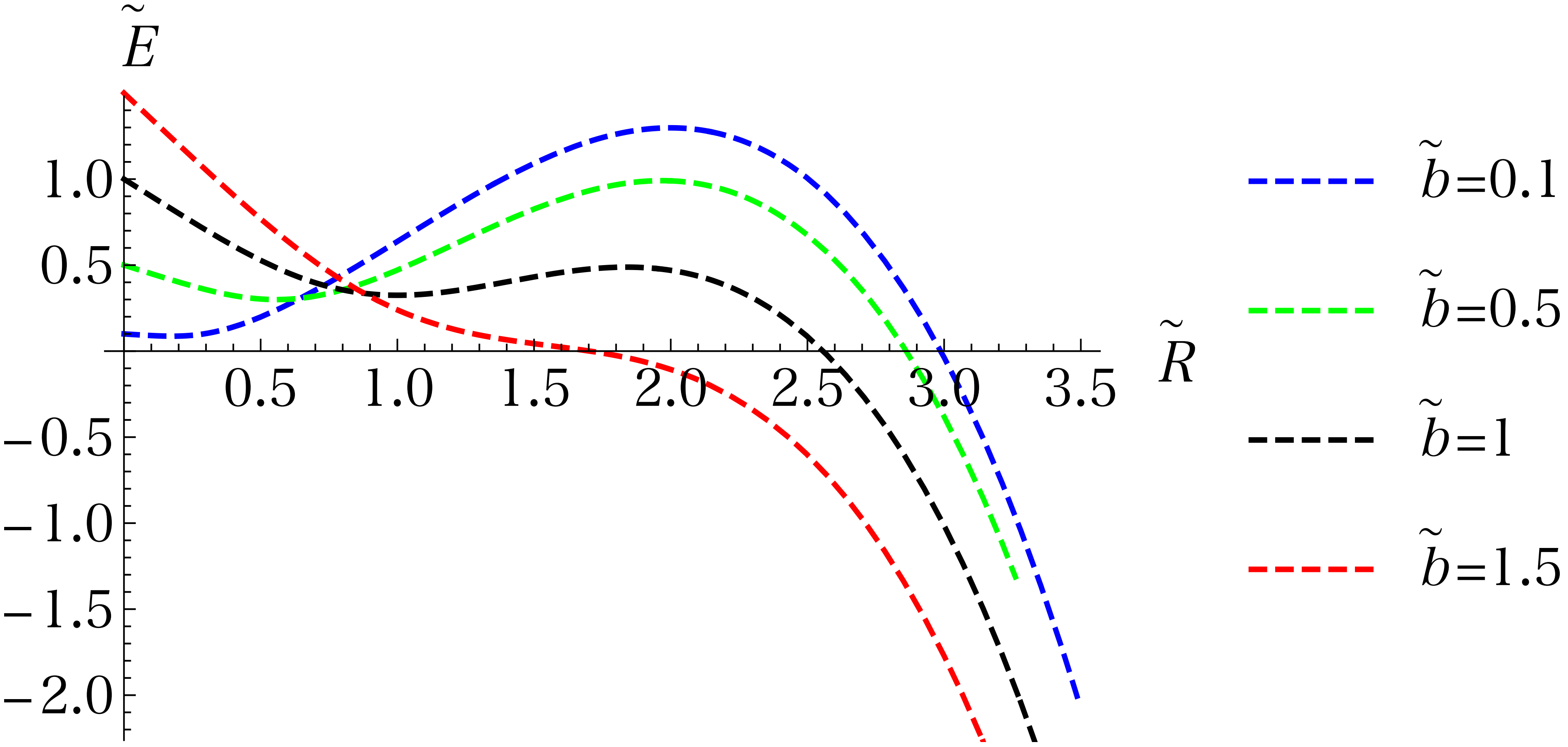
	0	1	2	3	4	5	6	7	8	9
D5/antiD5	○	○	○	○	△	△	△	×	×	×
DWD5	○	△	△	△	△	△	△	△	×	×
D3	○	×	×	×	△	△	△	△	×	×

S^2 (directions 1–3 of DWD5), S^3 (directions 4–7 of DWD5 and D3)

$$L_{\text{total}} = -T_{\text{DW}} 4\pi \sqrt{(R^4+b^2)\left(1-\dot{R}^2\right)} + 2T_{D5} r b \left[4\pi R \times {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{R^4}{b^2}\right)\right]$$

$$T_{\text{DW}} = T_{D5}\left[2\pi^2 L^2 \int_0^\pi d\psi_I \frac{2}{\pi}\sqrt{\sin^4\psi_I + \left(\frac{b_{\text{NS}}}{L^2}\right)^2}\right]$$

Brane setup



$\tilde{E}$
1.0
0.5
−0.5
−1.0
−1.5
−2.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
$\tilde{R}$
$\tilde{b}$=0.1
$\tilde{b}$=0.5
$\tilde{b}$=1
$\tilde{b}$=1.5

- Comparing the critical lifetime with the TCC condition, we obtained a specific restriction on the string scale.
- We derived a specific expression of the determinant factor for our noncanonical system. In this work, we determined the 1-loop pre-factor and found a behavior different from canonical theories.
- Unfortunately, 1-loop analysis breaks down when the potential barrier completely vanished. We used variation perturbation method instead and obtain a finite lifetime.

Contents

1. Introduction and Summary

2. 1-loop analysis
   - Comment on zeromode
   - Functional determinant

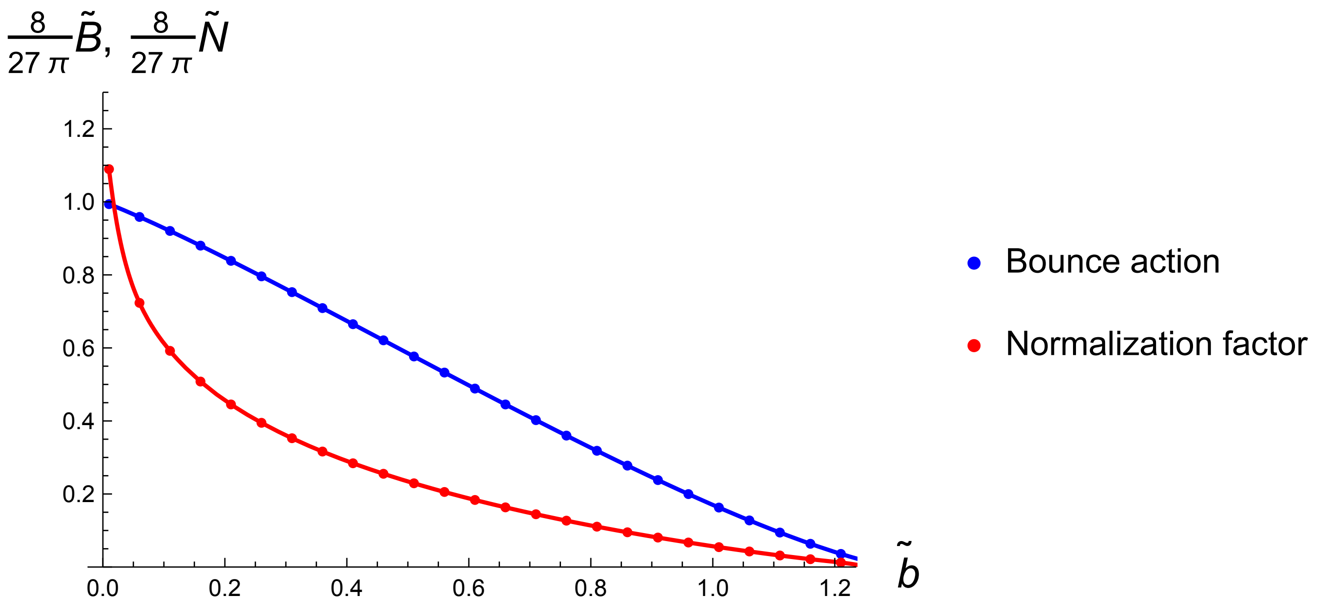
3. Reduction to cubic oscillator and comparison to TCC

- To get decay rate (lifetime), we must calculate bounce action, zero mode normalization constant and functional determinant.

[S. R. Coleman, Phys. Rev. D 15, 2929-2936 (1977) [erratum: Phys. Rev. D 16, 1248 (1977)]]

$$\Gamma = \sqrt{\frac{N_{\rm b}}{2\pi\hbar}} \left| \frac{\det' S''[\bar{x}_{\rm b}]}{\det S''[\bar{x}_{\rm triv}]} \right|^{-1/2} e^{-B/\hbar} = \sqrt{\frac{B}{2\pi\hbar}} \left| \frac{\det' S''[\bar{x}_{\rm b}]}{\det S''[\bar{x}_{\rm triv}]} \right|^{-1/2} e^{-B/\hbar}$$

- We must note that N_b doesn't correspond to B in noncanonical theories.



ζ function regulatization

$$S''[x] = -\frac{d}{dt}\left(\frac{\partial^2 L}{\partial \dot{x}^2}\frac{d}{dt}\right) + \frac{\partial^2 L}{\partial x^2} - \frac{d}{dt}\frac{\partial^2 L}{\partial x \partial \dot{x}} = -\frac{d}{dt}\left(P[x]\frac{d}{dt}\right) + Q[x]$$

canonical ⟶ m

- Functional determinant=infinite products of eigenvalues $\prod_n{}' \lambda_n$.
- One can treat the UV divergence via zeta function regularization.

$$\det{}' S''[x] = \exp\left(-\zeta'(s=0; S'')\right) \qquad \zeta(s; S'') = \sum_n \frac{1}{\lambda_n^s}$$

- The regularization procedure is well known for canonical case . How about non-canonical case?

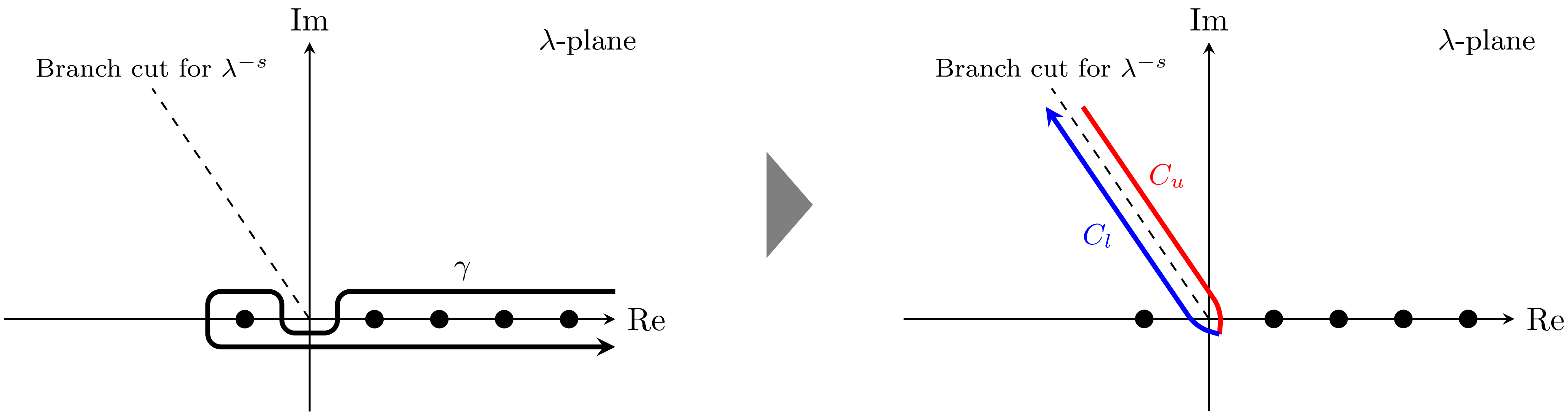
↑

need some developments !!

- Spectral zeta function can be written by a contour integration of the meromorphic function which has simple poles at eigenvalues.

Contour deformation method

[K. Kirsten et al., (2003);K. Kirsten et al., (2004)]



$$\zeta(s;L_{A,B}) = e^{is(\pi-\psi)}\frac{\sin(\pi s)}{\pi}\int_0^\infty dt t^{-s}\frac{d}{dt}\ln\left(F_{A,B}(te^{i\psi})t^{-m_0}e^{-im_0\psi}\right)$$

$$\frac{1}{2} < s < 1$$

- The asymptotic expansion of characteristic func. at large λ is order $\sqrt{\lambda}$.
- We can perform analytic continuation by subtracting N terms and add them back. [F. Gesztesy and K. Kirsten, (2019); G. Fucci et al., (2021)]

$$\zeta'\left(0; L_{A,B}\right) = i\pi n - \ln\left(2c\left|\frac{F_{m_0}}{\Gamma_{k_0}}\right|\right)$$

n : number of negative modes F_{m_0} : 1st order coefficient of small λ expansion

$$\det{}'S''[R_b] = -\left|F_{m_0}\right| = \frac{N_b}{P_{\min}\dot{R}_b(-\beta/2)\dot{\chi}(-\beta/2)} \cdot \frac{\chi(\beta/2) - \chi(-\beta/2)}{\dot{R}_b(-\beta/2)}$$

Removing scattering mode

- We need to extract a scattering mode which diverge at $\beta \to \infty$.

[M.Marino, "*Instanton and Large N*" (2015);ST, (2023)]

$$\frac{\chi(\beta/2)-\chi(-\beta/2)}{\dot{R}_b(-\beta/2)} \propto \exp\left[\sqrt{\frac{Q_{\min}}{P_{\min}}}\beta\right]$$

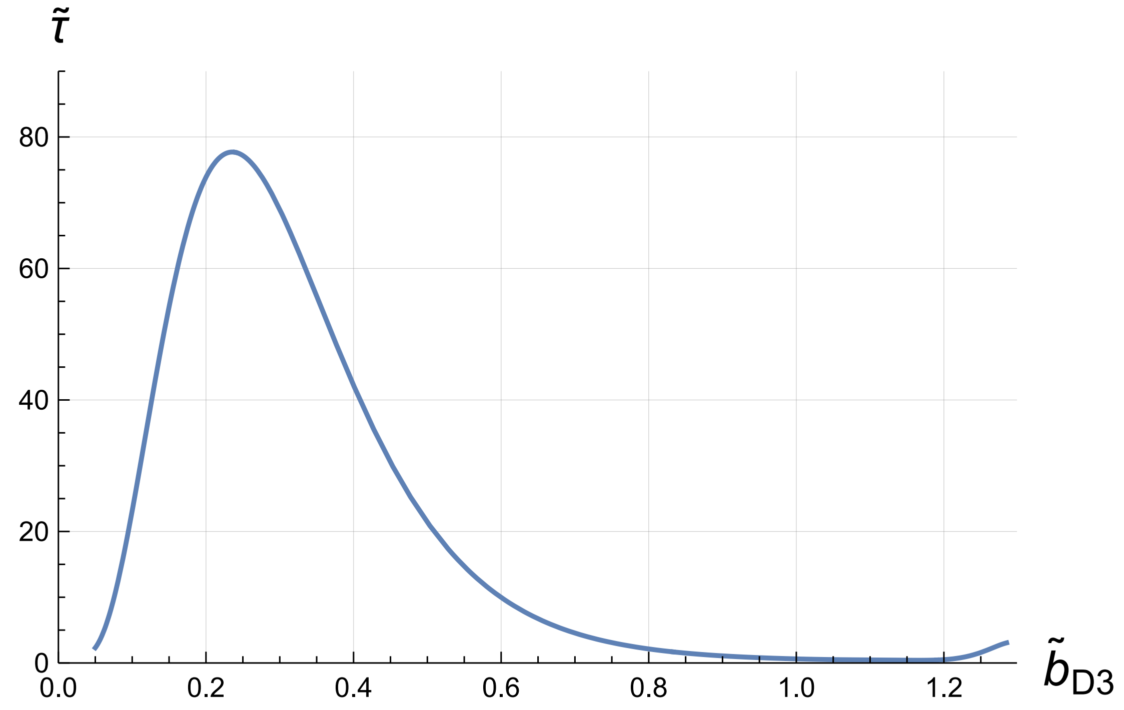
- The scattering mode can be regularized by reference determinant.

$$\frac{\det'\left[S''[\widetilde{R}_b]\right]}{\det\left[S''[\widetilde{R}_{\min}]\right]} = -\frac{N_b}{2\left(\widetilde{R}_{\max}-\widetilde{R}_{\min}\right)^2}\frac{P_{\min}^{1/2}}{Q_{\min}^{3/2}}\exp\left[-2\sqrt{\frac{Q_{\min}}{P_{\min}}}\int_{\widetilde{R}_{\min}}^{\widetilde{R}_{\max}}\left\{\frac{1}{\sqrt{(\dot{\widetilde{R}}_b)^2}}-\sqrt{\frac{P_{\min}}{Q_{\min}}}\frac{1}{\widetilde{R}-\widetilde{R}_{\min}}\right\}d\widetilde{R}_b\right]$$

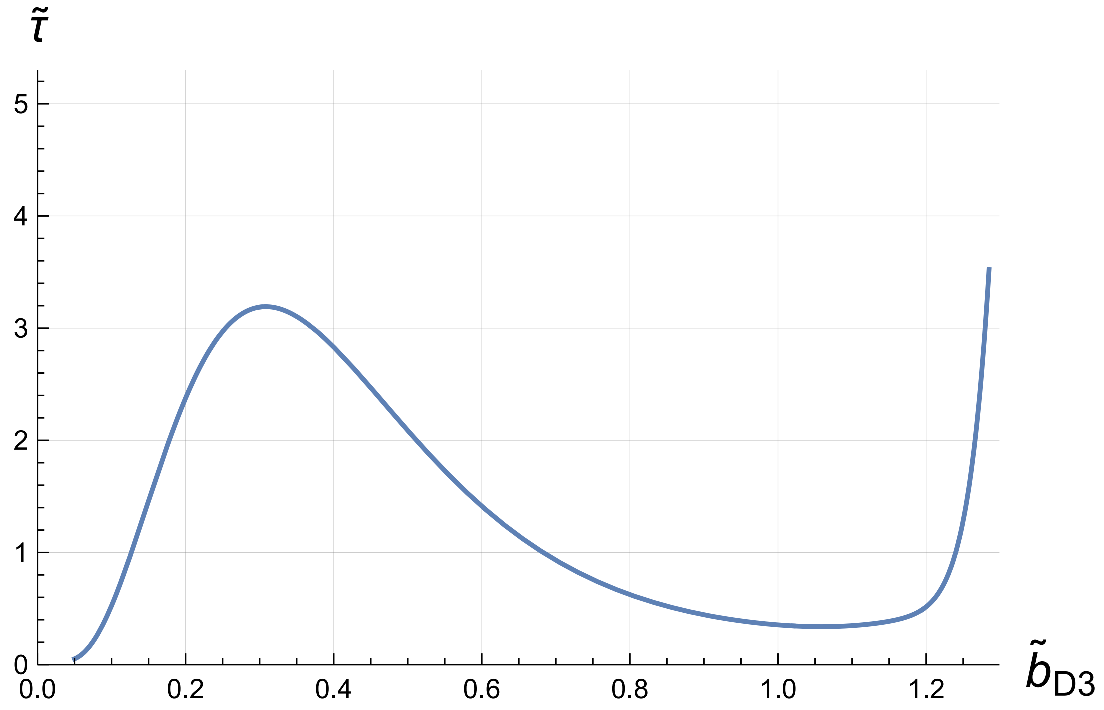
- We get a complete 1-loop result of the lifetime.

$$\tau = \frac{\sqrt{\pi}}{\left(\widetilde{R}_{\max}-\widetilde{R}_{\min}\right)}\frac{P_{\min}^{1/4}}{Q_{\min}^{3/4}}\exp\left[-\sqrt{\frac{Q_{\min}}{P_{\min}}}\int_{\widetilde{R}_{\min}}^{\widetilde{R}_{\max}}\left\{\frac{1}{\sqrt{(\dot{\widetilde{R}}_b)^2}}-\sqrt{\frac{P_{\min}}{Q_{\min}}}\frac{1}{\widetilde{R}-\widetilde{R}_{\min}}\right\}d\widetilde{R}_b\right]e^B\ .$$

Numerical calculation (lifetime)

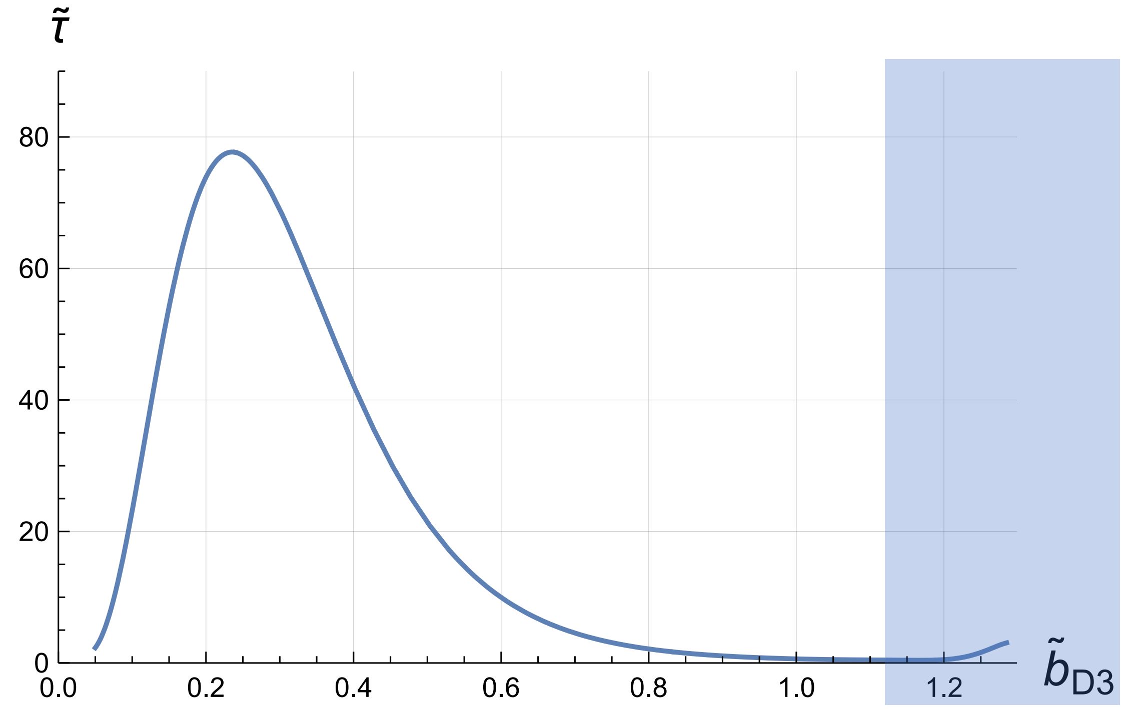


$A = 1.5$

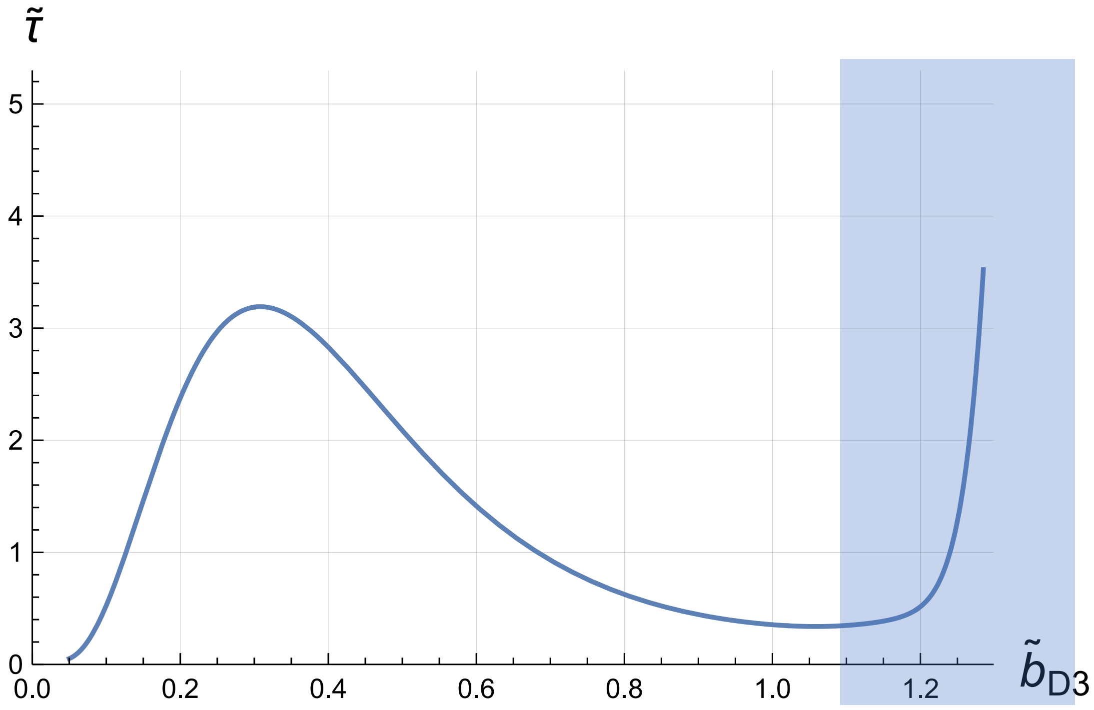


$A = 1.1$

Numerical calculation (lifetime)



$A = 1.5$



$A = 1.1$

Reduction to cubic oscillator

- For nearly flat potential, we can expand our DBI Lagrangian and approximate by the anharmonic oscillator.

$$L_E \simeq A\left[\sqrt{\tilde{R}_v^4+\tilde{b}_{\rm crit}^2}\,\frac{\dot{\tilde{y}}^2}{2}-\alpha(\tilde{R}_v,\tilde{b}_{\rm crit})\tilde{y}-\beta(\tilde{R}_v,\tilde{b}_{\rm crit})\tilde{y}^3+V(\tilde{R}_v,\tilde{b}_{\rm crit})\right]$$

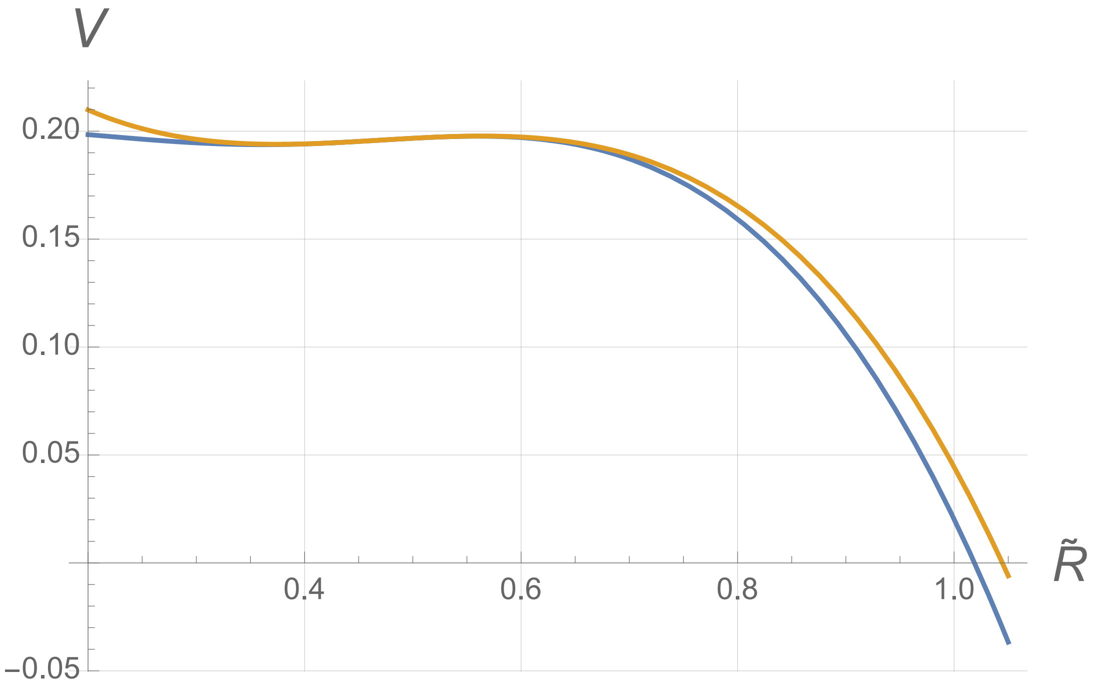
- For the cubic oscillator, the decay rate for low potential barriers is well investigated by variational perturbation method.

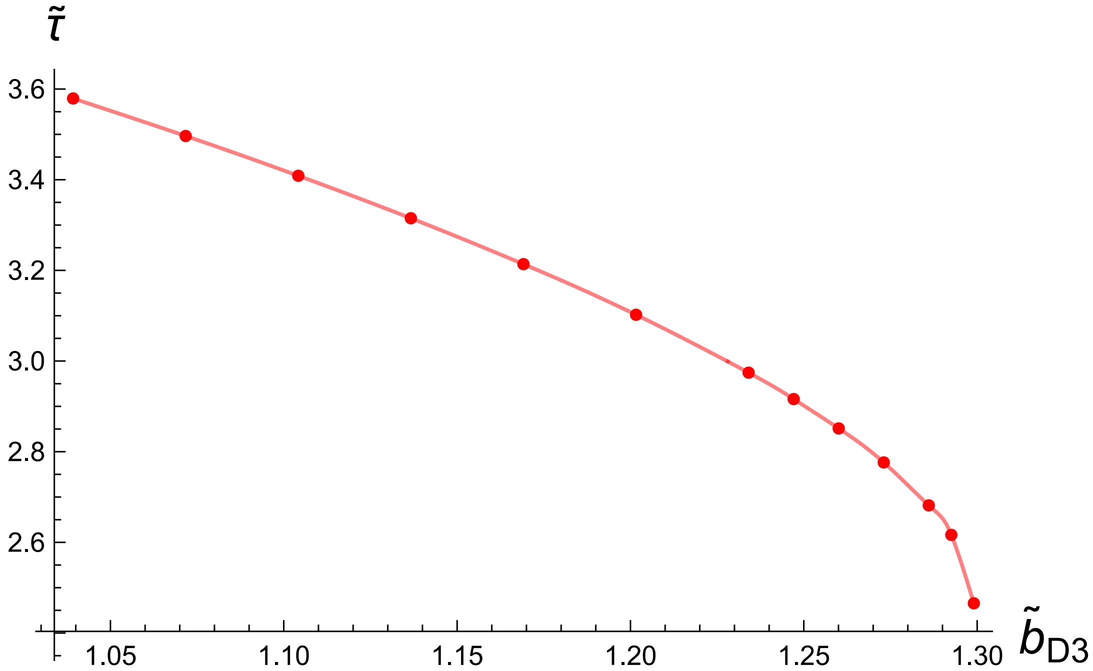
[H.Kleinert and I. Mustapic, *Int.J.Mod.Phys.A* 11 (1996) 4383-4400]

$$L_{KM}=\frac{m}{2}\dot{x}^2+\frac{m\omega^2}{2}x^2-\lambda x^3$$

$$\Gamma=-2{\rm Im}\ E_0\simeq 2\times 0.448\left(\frac{\lambda^2}{m^3}\right)^{1/5}\left(1-0.186\left(\frac{m^2\omega^5}{\lambda^2}\right)^{2/5}\right)$$

Lifetime calculation based on VPM

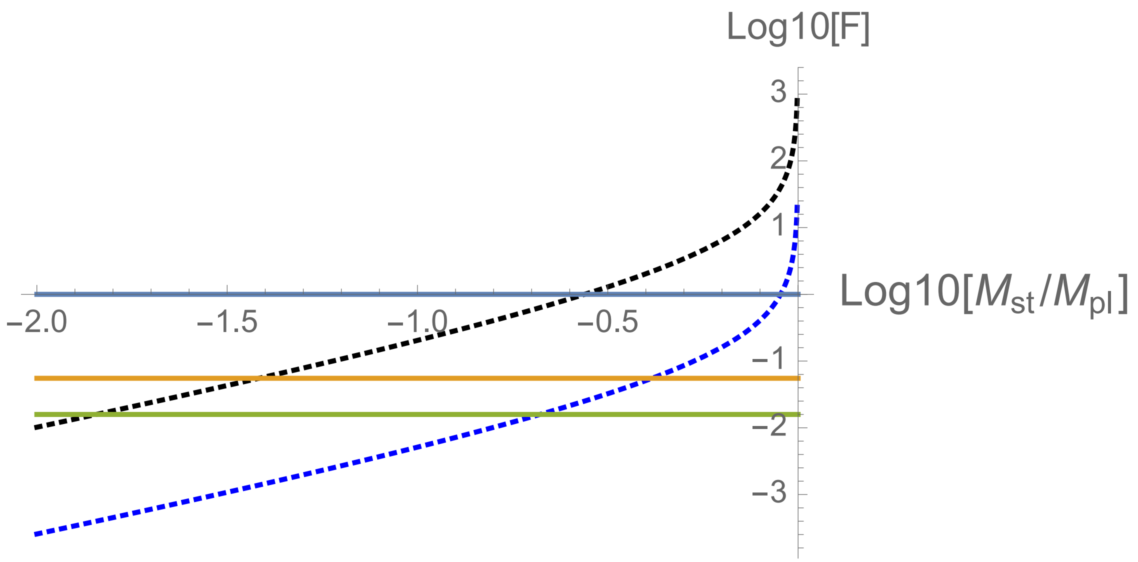




Comparison to TCC bound

- We compare the critical lifetime for completely flat potential to TCC.
- All parameters are rewritten by string scale M_{st}, Planck scale M_{pl} and string coupling constant g_s.

$$\tau_{\rm crit} \simeq \frac{1}{2\times 0.448}\frac{T_{DW}}{2T_{D5}r}A^{1/5}\left(\frac{\beta(\tilde{R}_v,\tilde{b}_{\rm crit})}{(\tilde{R}_v^4+\tilde{b}_{\rm crit}^2)^{3/2}}\right)^{-1/5} \simeq 2.46A^{1/5}\frac{T_{DW}}{2T_{D5}r} \leq \frac{1}{H_I}\log\frac{M_{\rm pl}}{H_I}$$



$$\log F \leq \frac{9}{5}\log g_s$$

$$F = \frac{2.46}{2}\left(\frac{4\pi}{n^8}\right)^{1/5}\left(\frac{M_{\rm st}}{M_{\rm pl}}\right)\left(\log\frac{M_{\rm st}}{M_{\rm pl}}\right)^{-1}$$

Thank you!

BACKUP

Discrepancy between bounce action and normalization constant

$$\text{EoM}: \ \partial_s \left(\sqrt{\frac{\tilde{R}^4 + \tilde{b}^2}{1 + \dot{\tilde{R}}^2}} - \tilde{b}\tilde{R}_2F_1\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}; -\frac{\tilde{R}^4}{\tilde{b}^2}\right)\right) = 0$$

$\tilde{B} = \tilde{S}_{\rm b} - \tilde{S}_{\rm sub}$ ⟵ Difference between the instanton action and the background.

$$= 2\int_{\tilde{R}_{\rm min}}^{\tilde{R}_{\rm max}} d\tilde{R}_{\rm b} \sqrt{\tilde{R}_{\rm b}^4 + \tilde{b}^2 - \left[C + \tilde{b}\tilde{R}_{\rm b}{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\rm b}^4}{\tilde{b}^2}\right)\right]^2}$$

$\tilde{N}_{\rm b} = \int ds \left(\frac{d\tilde{R}_{\rm b}}{ds}\right)^2$ ⟵ Normalization constant of the zeromode.

$$= 2\int_{\tilde{R}_{\rm min}}^{\tilde{R}_{\rm max}} \frac{d\tilde{R}_{\rm b}}{\boxed{C + \tilde{b}\tilde{R}_{\rm b}{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\rm b}^4}{\tilde{b}^2}\right)}} \times \sqrt{\tilde{R}_{\rm b}^4 + \tilde{b}^2 - \left[C + \tilde{b}\tilde{R}_{\rm b}{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{\tilde{R}_{\rm b}^4}{\tilde{b}^2}\right)\right]^2}$$

[I.M.Gelfand and A.M.Yaglom, J. Math. Phys. 1, 48 (1960)]

$$\left[-\frac{d^2}{dx^2}+V(x)\right]\psi_n(x)=\lambda_n\psi_n(x)\ ,\quad M\begin{pmatrix}\psi_n(0)\\ \psi_n'(0)\end{pmatrix}+N\begin{pmatrix}\psi_n(L)\\ \psi_n'(L)\end{pmatrix}=0$$

$$\psi_0^1(0)=1\ ,\quad \psi_0^{1\prime}(0)=0\ ,$$
$$\psi_0^2(0)=0\ ,\quad \psi_0^{2\prime}(0)=1\ .$$

- M and N are 2×2 matrices, which determine boundary conditions.
- ψ_0^i $(i=1,2)$ are independent solutions for zeromode equations.

$$\det\left[-\frac{d^2}{dx^2}+V(x)\right]=\det\left[M+N\begin{pmatrix}\psi_0^1(L) & \psi_0^2(L)\\ \psi_0^{1\prime}(L) & \psi_0^{2\prime}(L)\end{pmatrix}\right]$$