Gravitational positivity bounds: implications for the swampland program

Junsei Tokuda (IBS, CTPU)

based on: [JHEP11(2020)054 **JT**, K. Aoki, S. Hirano]
[PRL127,091602(2021), K. Aoki, T.Q. Loc. T. Noumi, **JT**]
[PRD104,066022(2021) T. Noumi, **JT**]
[arXiv: 2205.12835 T. Noumi, S. Sato, **JT**]
[JHEP06(2023)032 T. Noumi, **JT**]
[arXiv: 2305.10058
K. Aoki, T. Noumi, R. Saito, S. Sato, S. Shirai, **JT**, M. Yamazaki]

2023.07.04 String Pheno ‘23 @ IBS, Daejeon

Why Swampland?

Q. What is **the quantum gravity theory?**

- **We need predictions which can be tested** experimentally.
  - ✓ **Very difficult.** $M_{\rm pl} \sim 10^{18}$ GeV: Very high.

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  - ✓ **Very difficult.** $M_{\text{pl}} \sim 10^{18}$ GeV: Very high.

- But, **hidden predictions may exist.** → **Swampland program:**

[C. Vafa ('05)]

"Not all consistent-looking EFTs are consistent with quantum gravity."

* Inconsistent EFTs are said to be in the Swampland.

Hidden predictions exist! (without gravity)

- Hidden predictions exist! (without gravity): **Positivity bounds**

[Adams – Arkani-Hamed – Dubovsky – Nicolis – Rattazzi ('06)]

[Pham –Truong ('85)]

… based on S-matrix **Unitarity** + **Analyticity** etc.

e.g.) EFT for photon: $\mathcal{L} \sim -F^2 + \frac{c_2}{m_e^4} F^4 + \cdots$ ⟶ $\boldsymbol{c_2 > 0}$

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- *Can we extend positivity bound to the gravitational context?*
- *If so, does it give any interesting Swampland Conditions?*

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➢ Talk plan:

1. **A review of** positivity bounds without gravity.
2. **Formulation** of gravitational positivity bounds.
3. **Implications** for the Standard Model.

Positivity bound (without gravity) (1/3)

- Let's consider $\gamma\gamma \to \gamma\gamma$ amplitude $\mathcal{M}(s,t)$.

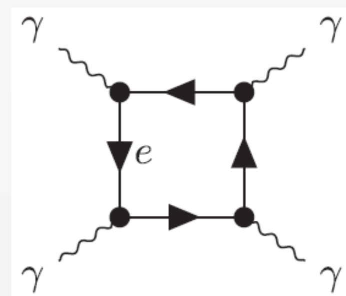
  * $s \sim (\text{CM energy})^2$
  * $t \sim \begin{pmatrix}\text{momentum}\\ \text{transfer}\end{pmatrix}^2$

- UV complete theory: Local, Unitary, Lorentz invariant, Causal.

  ⟹ $\boldsymbol{\mathcal{M}(s,t)}$ **behaves well** at high energies.

QED

$\mathcal{L}_{\text{QED}}[\gamma, e^-]$

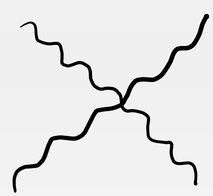


$\sim \alpha^2 \log^2(m_e^2/s)$

Embedding is possible **if and only if $c_2 > 0$.**

EFT for photon

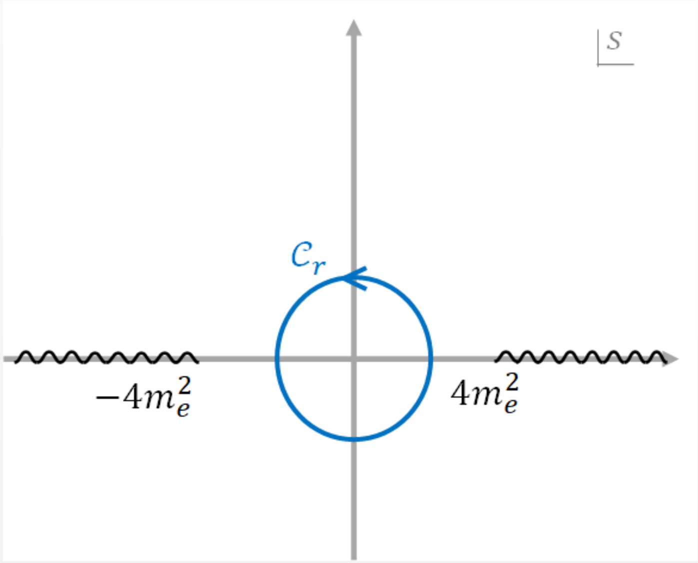
$\mathcal{L}_{\text{EFT}}[\gamma] \ni \dfrac{c_2}{m_e^4} F^4$



$\sim \dfrac{c_2}{m_e^4} s^2$

Positivity bound (without gravity) (2/3)

- We assume the analyticity of $\mathcal{M}(s,0)$ except for usual poles and cuts. We consider a complex integral of $\mathcal{M}(s,0)/s^3$:



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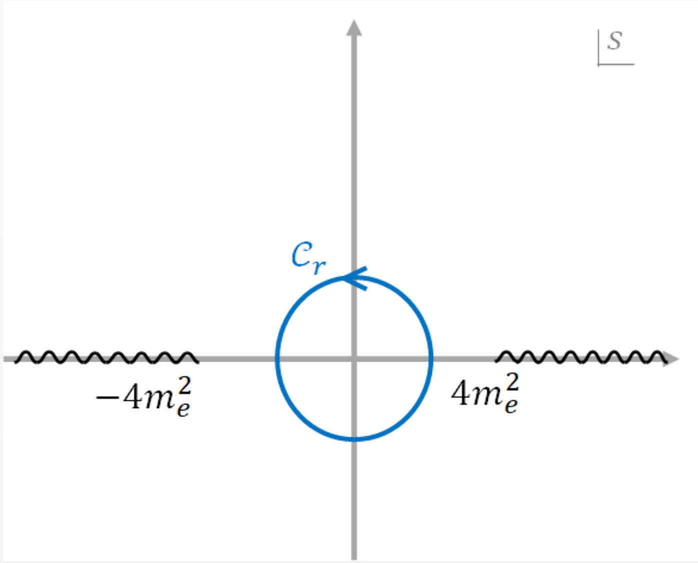

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$= c_2$, EFT calculable

$$\mathcal{M}(s,0) \sim c_0 s^0 + \boldsymbol{c_2} s^2 + \cdots$$

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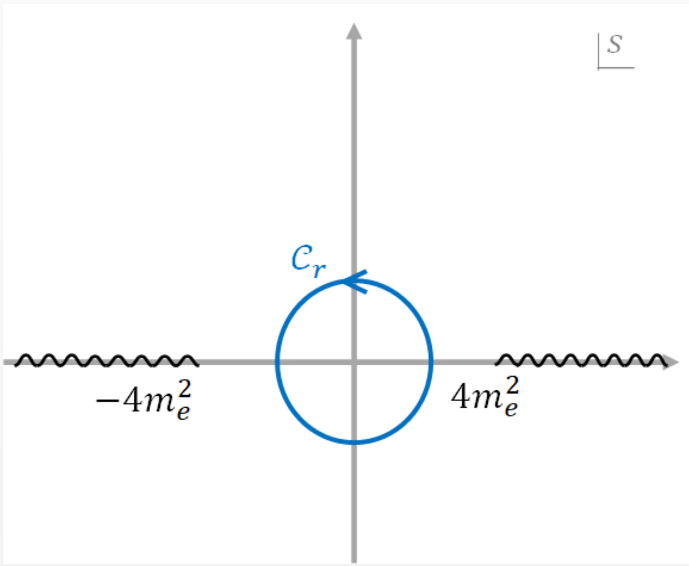
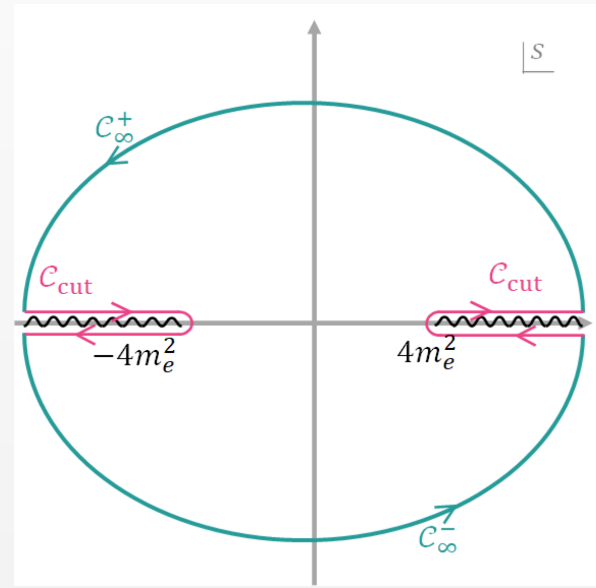


Contour deformation



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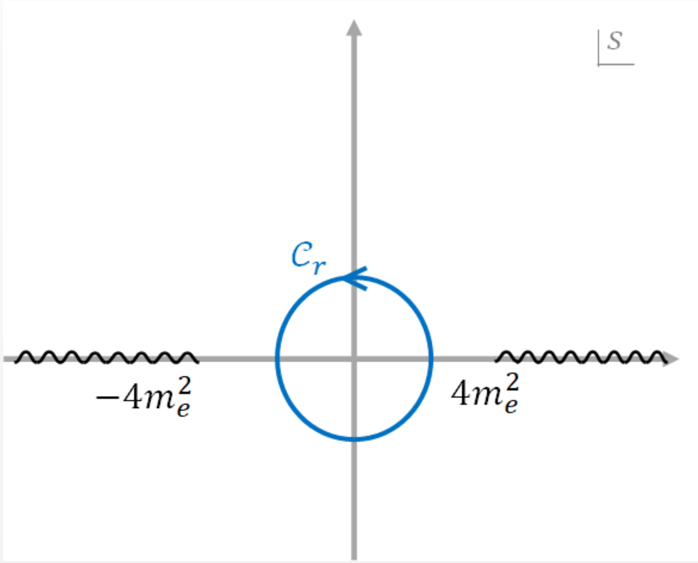

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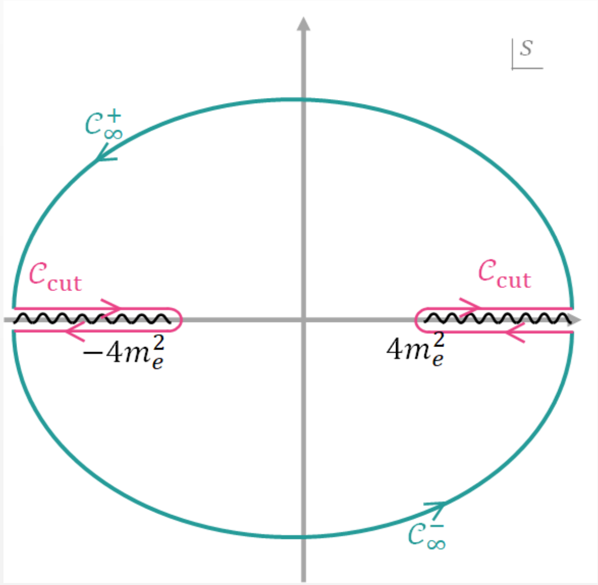

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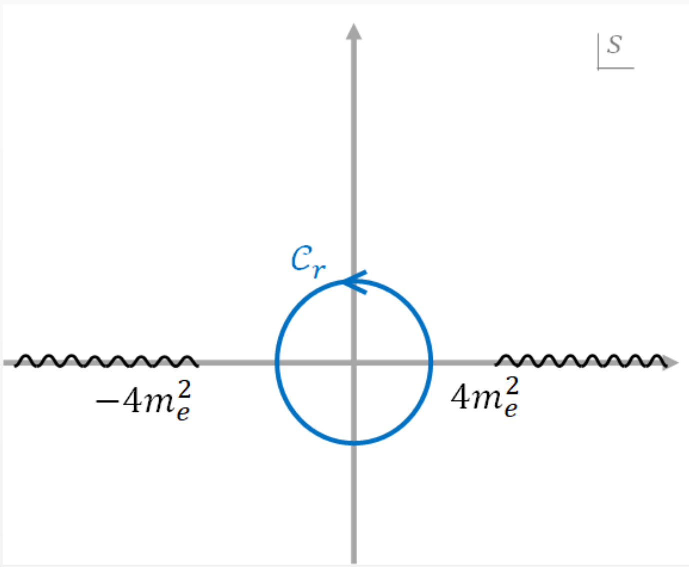

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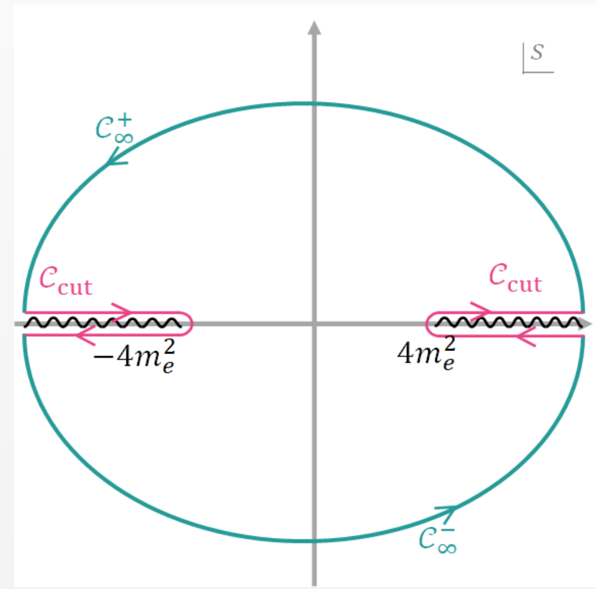

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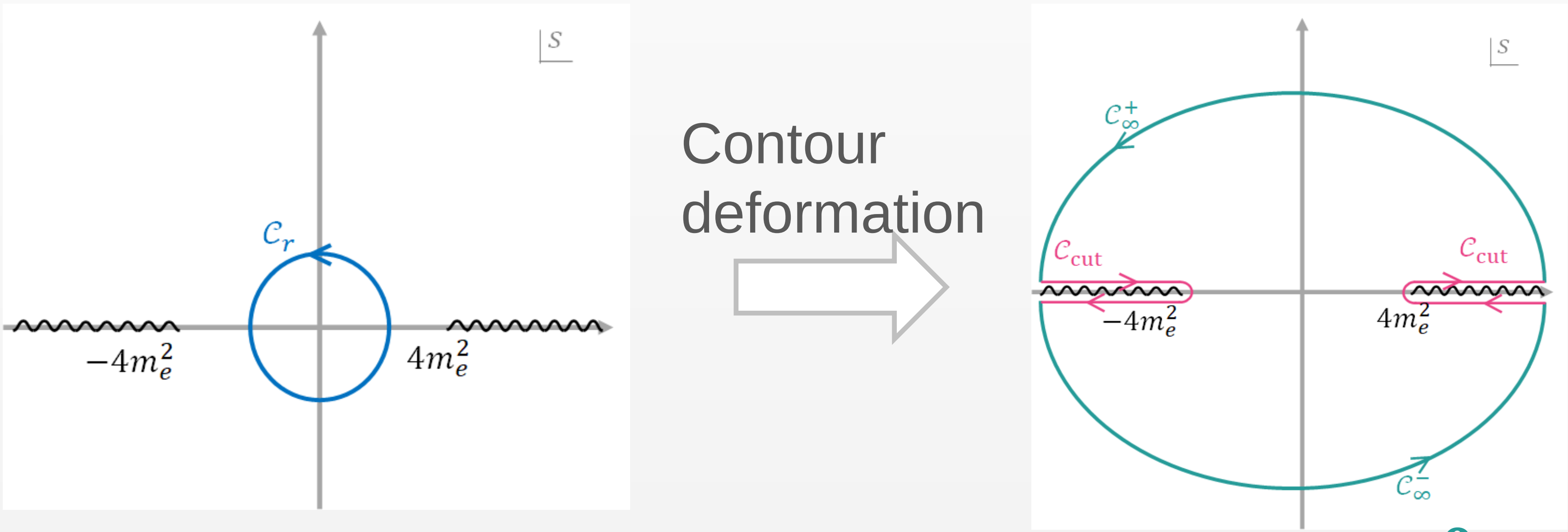

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$= c_2$, EFT calculable > 0 (Unitarity) → **0**

$\lim_{|s|\to\infty} \left|\mathcal{M}(s,0)/s^2\right| = 0\,.$

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"Locality"(Froissart-Martin)

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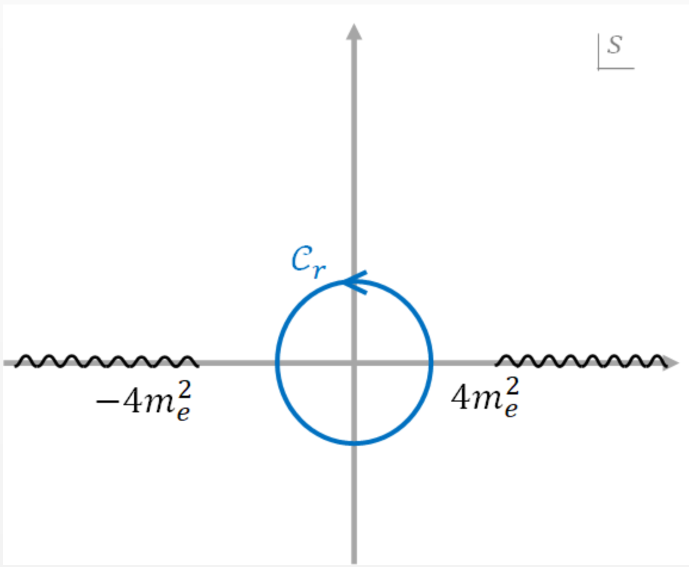

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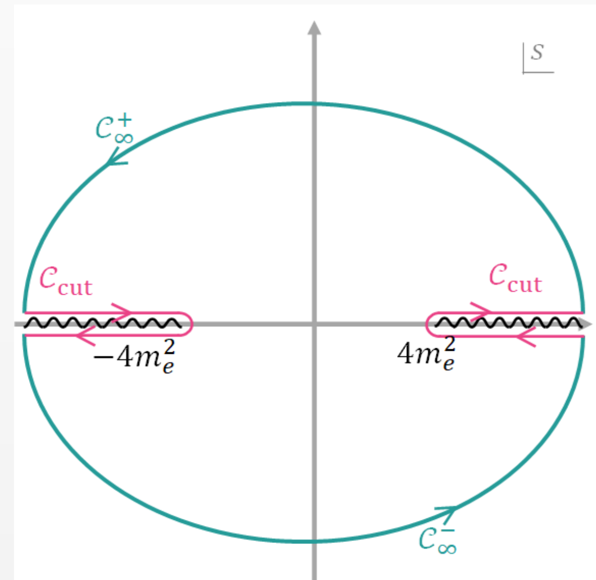

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$= c_2$, EFT calculable　　> 0 (Unitarity)　　$\lim_{|s|\to\infty}\left|\mathcal{M}(s,0)/s^2\right| = 0\,.$

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Positivity bound (without gravity) (3/3)

- **“Positivity bounds” (without gravity)**

$$\boldsymbol{c_2} = \frac{2}{\pi}\int_{4m_e^2}^{\infty} \mathrm{d}s \, \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{s^3} > \mathbf{0}.$$

[Pham+(‘85), Adams+(‘06)]

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[Pham+('85), Adams+('06)]

- Separate the EFT piece and high-energy piece:

$$c_2(\Lambda) \coloneqq c_2 - \frac{2}{\pi}\int_{4m_e^2}^{\Lambda^2} \mathrm{d}s \, \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{s^3}$$

* Λ= cutoff scale of EFT.

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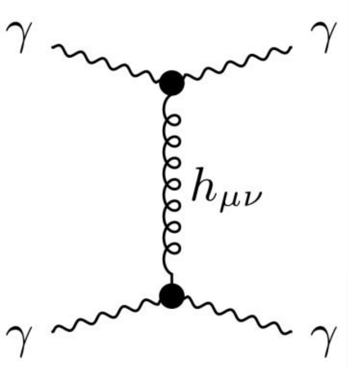
"Improved positivity bounds"

[Bellazzini('16), de Rham+('17)]

Extension to Gravity?

- Quantum gravity S-matrix: **not fully understood.** c.f.) [Häring+('22)]
- Feature: t-channel graviton exchange **grows as fast as s^2**,

$\mathcal{M}(s,t) \ni$  $\sim \dfrac{s^2}{M_{\rm pl}^2 t}$

- **Assume the mild UV behavior**: $\lim_{|s|\to\infty} \left|\mathcal{M}(s,t)s^{-2}\right| = 0$ for $t < 0$

c.f.) tree-level string: $\mathcal{M} \sim s^{2+\alpha' t}/M_{\rm pl}^2 t$

- We can then derive a sum rule for $c_2(\Lambda)$.

* We ignore graviton loops and work up to $\mathcal{O}\left(M_{\rm pl}^{-2}\right)$.

Positivity bounds with Gravity (1/2)

- The sum rule for $c_2(\Lambda)$ contains **graviton t-channel pole:**

$$c_2(\Lambda) = \lim_{t\to 0^-}\left\{\int_{\Lambda^2}^{\infty} \mathrm{d}s\,\frac{\mathrm{Im}\,\mathcal{M}(s,t)}{s^3} + \frac{\mathbf{1}}{\boldsymbol{M_{\mathrm{pl}}^2 t}}\right\} = \text{"}\infty - \infty\text{"} \overset{?}{>} \mathbf{0}$$

$$\mathcal{M}(s,t) \sim (s,t,u\ \mathrm{poles}) + c_2\, s^2 + \cdots \quad \ni \left[\frac{-1}{M_{\mathrm{pl}}^2 t} + c_2\right] s^2$$

- One solution: consider sum rules **away from $t = 0$ limit.**

[Caron-Huot+ ('21)]

  ✓ Perform smearing in t to localize the sum rule in the domain $t \gtrsim -4m_e^2$. Equivalent to work in impact parameter $b \lesssim (2m_e)^{-1}$.

  ✓ IR div in D=4: $c_2 > -25\left(2M_{\mathrm{pl}}m_e\right)^{-2} \log(0.6 m_e b_{\mathrm{IR}})$.

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[JT-Aoki-Hirano ('20)]

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$$\int_{s_* \gg \Lambda^2}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{s^3} \sim f(t) \int_{s_* \gg \Lambda^2}^{\infty} \mathrm{d}s \frac{s^{2+\alpha' t+\cdots}}{s^3}$$

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Positivity bounds **with** Gravity (2/2)

(Related discussions:
[Hamada+('18 '23)]
[Herrero-Valea+('20)]
[Bellazzini+('19)]
[Alberte+('20,'21)])

- **Gravitational positivity bound:** [JT-Aoki-Hirano ('20)]

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- **Scale of M: depends on details of the Regge behavior.**
- **$c_2(\Lambda) = 0$ is allowed.** ✓ Type-II superstring amplitude: $c_2 = 0$.

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- **Scale of M: depends on details of the Regge behavior.**
- **$c_2(\Lambda) = 0$ is allowed.** ✓ Type-II superstring amplitude: $c_2 = 0$.
- **An explicit relation between EFT and quantum gravity!**
  - Violation of $c_2(\Lambda) \geq 0$ **constrains UV properties of S-matrix** (e.g. details of Regge behavior).
  - Today, we discuss **implications of $c_2(\Lambda) \geq 0$ for models.**

Application: Standard Model + GR

[Aoki-Loc-Noumi-JT ('21)]

- We focus on the $\gamma\gamma \to \gamma\gamma$ process in the Standard Model + GR.

$$\mathcal{L} = \frac{M_{\mathrm{pl}}^2}{2} R + \mathcal{L}_{\mathrm{SM}} + \cdots$$

* We ignore higher-derivative terms for simplicity.

* For earlier analysis in QED(-like)+GR, see [Cheung+('14), Andriolo+('18), Chen+('19), Alberte+('20)]

- We compute $c_2(\Lambda)$ and then discuss implications of $c_2(\Lambda) \geq 0$.
  - ➢ We decompose $c_2(\Lambda)$ into two pieces:

$$c_2(\Lambda) = (\mathbf{nongrav}) + (\mathbf{grav})$$

$(\mathbf{nongrav})$: contributions from processes without graviton.

$(\mathbf{grav})$: contributions from processes with graviton.

Evaluate (nongrav)

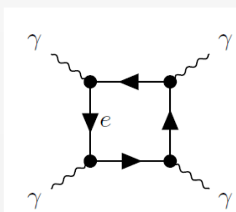
[Aoki-Loc-Noumi-JT ('21)]

- $\mathcal{M}_{\text{nongrav}}$ satisfies the twice-subtracted dispersion relation. Hence,

$$(\textbf{nongrav}) = \int_{\Lambda^2}^{\infty} \mathrm{d}s \, \frac{\mathrm{Im}\, \mathcal{M}_{\text{nongrav}}(s, 0)}{s^3} \simeq \int_{\Lambda^2}^{\infty} \mathrm{d}s \, \frac{\sigma_{\text{tot}}^{\text{nongrav}}(s)}{s^2} > 0.$$

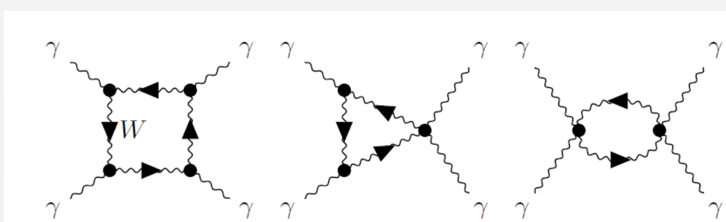
* $\sigma_{\text{tot}}^{\text{nongrav}} \simeq \frac{1}{s} \mathrm{Im}\, \mathcal{M}_{\text{nongrav}}(s, 0)$: total cross section

QED:



$$\sigma_{\text{tot}}^{\text{nongrav}}(s) \sim \frac{e^4}{s}$$

Weak:



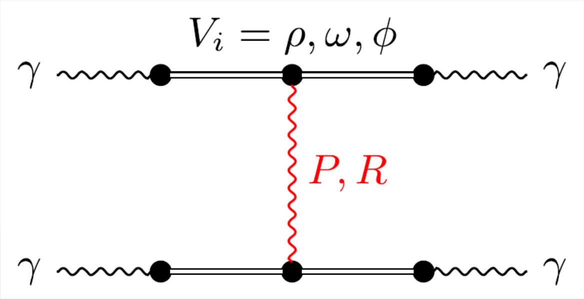
$$\sigma_{\text{tot}}^{\text{nongrav}}(s) \sim \frac{e^4}{m_{\mathrm{W}}^2}$$

Evaluate (nongrav)

[Aoki-Loc-Noumi-JT ('21)]

- Since we consider physics at $t = 0$, IR physics can play a role.
- To calculate $\sigma_{\rm tot}(s)$ at $s \gg \Lambda_{\rm QCD}^2$, the Regge theory is useful.
- QCD gives dominant contributions via ***t*-ch. Pomeron exchange:**

QCD:



$$\sigma_{\rm tot}^{\rm nongrav}(s) \sim e^4 \left(\frac{s}{\Lambda_{\rm QCD}^2}\right)^{0.08}$$

* [Kłusek-Gawenda+ ('16)] evaluates QCD contributions using the vector meson dominance model. We extrapolated their results up to high energies.

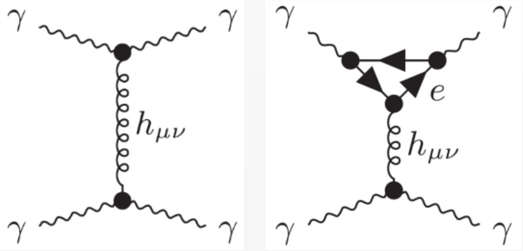
$$\therefore \ (\textbf{nongrav}) \sim \frac{e^4}{\Lambda^4} + \frac{e^4}{\Lambda^2 m_{\rm W}^2} + \frac{e^4}{\Lambda^2 \Lambda_{\rm QCD}^2}\left(\frac{\Lambda^2}{\Lambda_{\rm QCD}^2}\right)^{0.08}$$

Evaluate (grav) & Results

[Aoki-Loc-Noumi-JT ('21)]

- For gravitational piece, electron one-loop correction to $\gamma\gamma h$ couplings gives a dominant contribution, because electron is light:

(grav):



$$\mathcal{M}_{\rm grav}(s,t) \sim -\frac{s^2}{M_{\rm pl}^2 t} - \frac{e^2 s^2}{M_{\rm pl}^2 m_e^2} + \mathcal{O}\left(t/m_e^2\right)$$

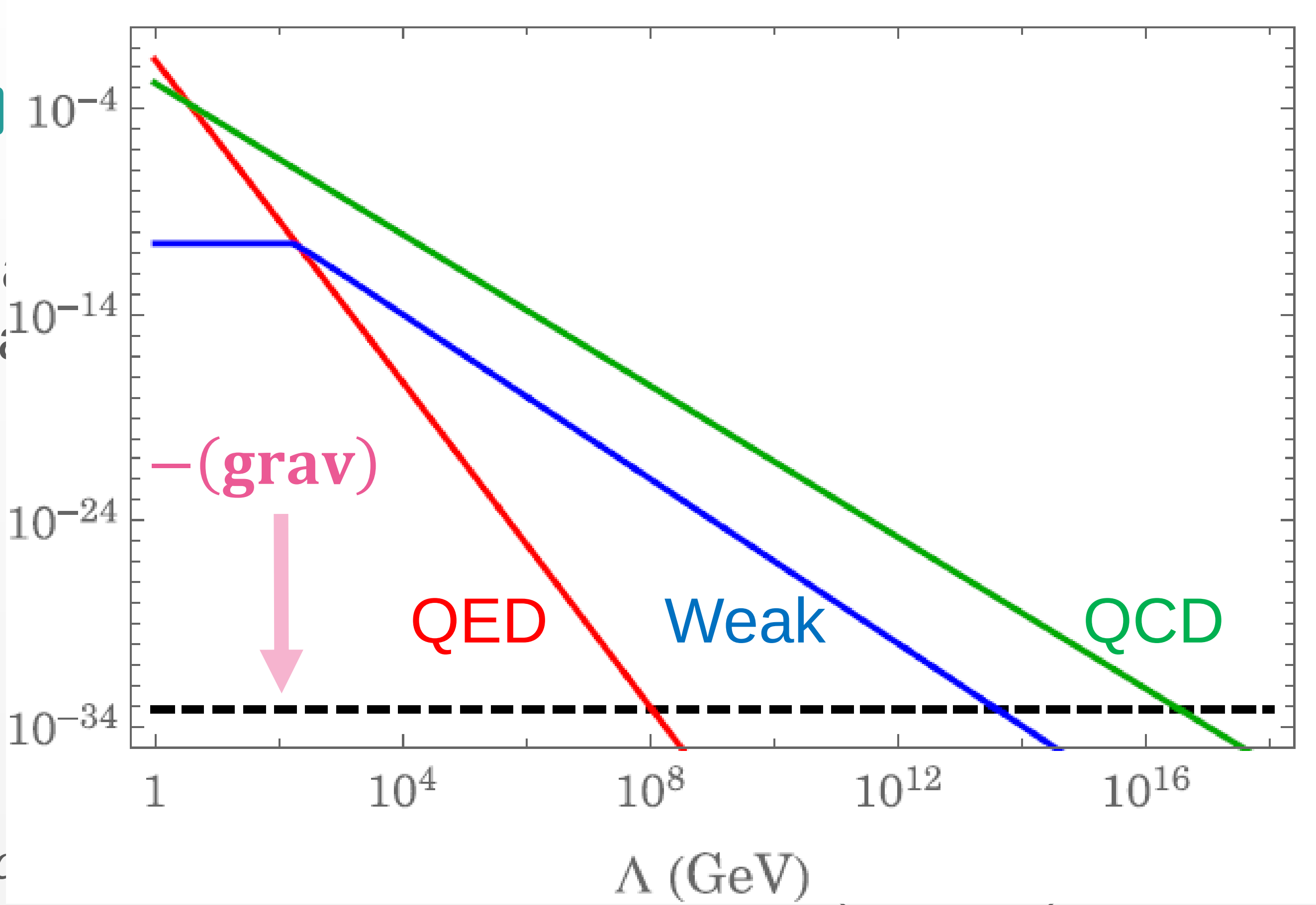
$$\therefore\ (\textbf{grav}) \sim -\frac{e^2}{M_{\rm pl}^2 m_e^2} < 0$$

- Now, $c_2(\Lambda) = (\textbf{nongrav}) + (\textbf{grav}) \geq 0$ reads

$$\frac{e^4}{\Lambda^4} + \frac{e^4}{\Lambda^2 m_{\rm w}^2} + \frac{e^4}{\Lambda^2 \Lambda_{\rm QCD}^2}\left(\frac{\Lambda^2}{\Lambda_{\rm QCD}^2}\right)^{0.08} - \frac{e^2}{M_{\rm pl}^2 m_e^2} \geq 0$$

Evalu … ni-JT ('21)]

- For gr… h couplings gives a…



- Now, …

$$\frac{e^4}{\Lambda^4}+\frac{e^4}{\Lambda^2 m_{\rm w}^2}+\frac{e^4}{\Lambda^2\Lambda_{\rm QCD}^2}\left(\frac{\Lambda^2}{\Lambda_{\rm QCD}^2}\right)^{0.08}-\frac{e^2}{M_{\rm pl}^2 m_e^2}\geq 0$$

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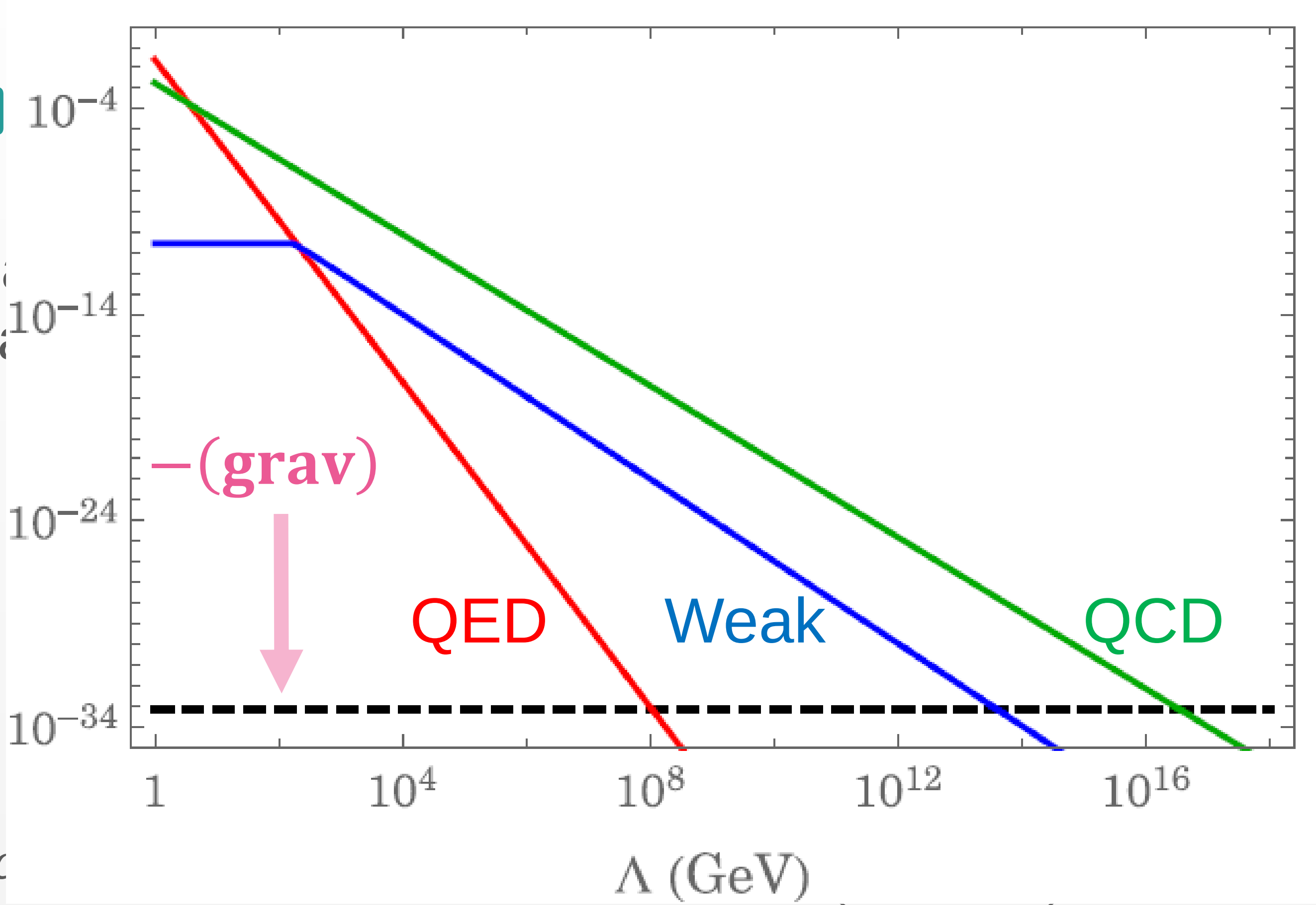


- Now, ...

$$\frac{e^4}{\Lambda^4}+\frac{e^4}{\Lambda^2 m_{\rm w}^2}+\frac{e^4}{\Lambda^2\Lambda_{\rm QCD}^2}\left(\frac{\Lambda^2}{\Lambda_{\rm QCD}^2}\right)^{0.08}-\frac{e^2}{M_{\rm pl}^2 m_e^2}\geq 0$$

$\Rightarrow$ $\boldsymbol{\Lambda \lesssim \mathcal{O}(10^{15-16})}$**GeV.**

- **The Standard Model can easily satisfy our bound!**

Summary

- Gravitational positivity bounds reveal UV-IR correlations.

  Approximate positivity: $c_2(\Lambda) \geq \pm 1/M_{\rm pl}^2 M^2$

  * M: depends on details of UV theory (e.g. Regge behavior)

  ✓ Violation of $\boldsymbol{c_2(\Lambda) \geq 0}$ **constrains UV properties of S-matrix**

- We discuss implications of $c_2(\Lambda) \geq 0$ for models.

  ・The Standard Model ➡ Easily satisfies the bound ☺.

  ・Nontrivial for generic models. ➡ Sota Sato's talk! (Next talk)

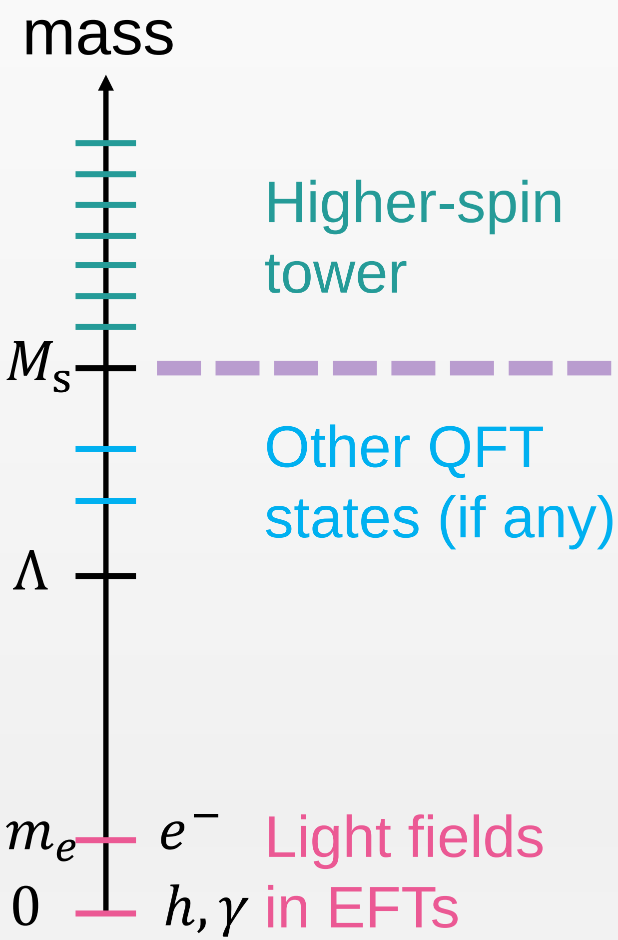
  e.g.) dark sector physics

- **Further study of quantum gravity S-matrix will be important for phenomenology, and vice versa.**

backup

The setup we consider

- We ***assume*** the Reggeization of graviton exchange at UV.



UV completion with Reggeization

$$\mathcal{M} \sim \frac{f(t)}{M_{\rm pl}^2 t} s^{2+\alpha' t} < s^2$$

Any correlations?

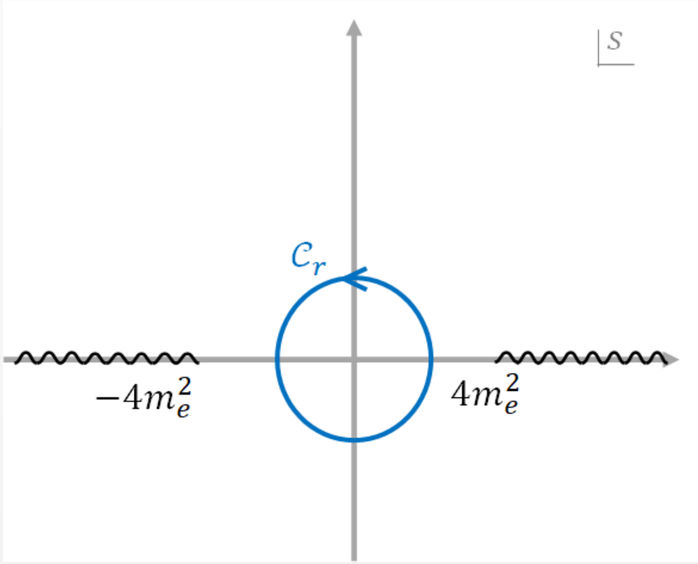
* up to $\mathcal{O}\left(M_{\rm pl}^{-2}\right)$.

EFT (QFT + gravity in 4D)

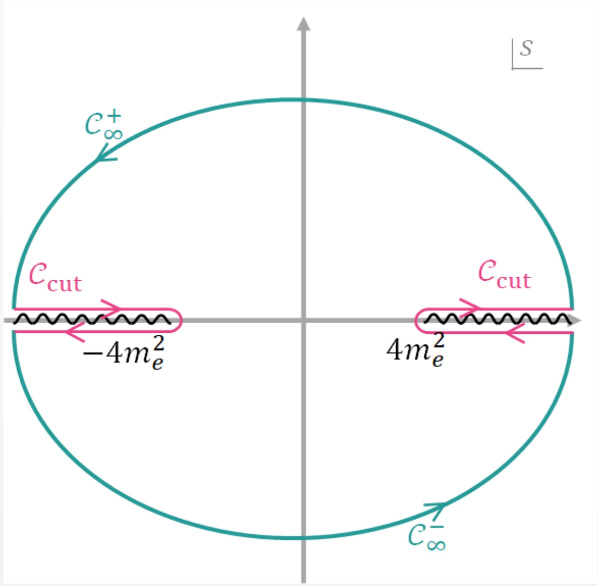
e.g.) $\mathcal{L} \sim M_{\rm pl}^2 R + \mathcal{L}_{\rm matter}[\gamma, e^-, \ldots]$

Basic properties of S-matrix

- How can we understand good properties of S-matrix?



Contour deformation



➢ **Analyticity** of $\mathcal{M}(s,0)$ in the s-plane ⟺ **Causality**

➢ **Mild behavior** $\lim_{|s|\to\infty}\left|\frac{\mathcal{M}(s,0)}{s^2}\right| = 0$ ⟺ **Locality** & **Unitarity**

Analyticity & Causality

c.f.) [Camanho-Edelstein-Maldacena-Zhiboedov+('14)]

- To get some intuition, let's consider **a signal model.**
- We have an initial signal $f_{\text{in}}(t)$ and an out-signal $f_{\text{out}}(t)$ with

$$f_{\text{out}}(t) = \int_{-\infty}^{\infty} \mathrm{d}t' \, S(t-t') f_{\text{in}}(t').$$

$$\Leftrightarrow \tilde{f}_{\text{out}}(\omega) = \tilde{S}(\omega)\tilde{f}_{\text{in}}(\omega), \qquad S(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \tilde{S}(\omega) e^{-i\omega t}.$$

- $\tilde{S}(\omega)$: S-matrix element.
- **Causality** implies: $S(t) = 0$ for $t < 0$.

$$\Rightarrow \tilde{S}(\omega) = \int_{0}^{\infty} \mathrm{d}t \, S(t) e^{i\omega t}$$ **Analytic** in the upper half plane.

Mild behavior from Locality & Unitarity

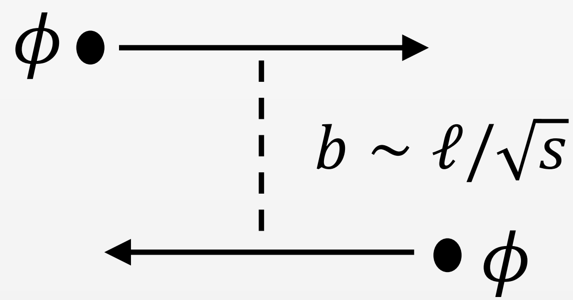
- Key: **polynomial boundedness** (PB)

$$\boldsymbol{\mathcal{M}(s,t) < s^N} \quad \text{as } s \to \infty, t\text{: fixed}.$$

$s^N \sim \partial^\# \sim$ **Locality**

- Consider the partial-wave expansion

$$\mathcal{M} \sim \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(s) P_\ell(\cos\theta)$$



  - **Partial-wave unitarity:** $|f_\ell(s)| \leq 1$
  - Short-range force: $|f_\ell(s)| < s^N \exp\left(-m_e \ell/\sqrt{s}\right)$ @ large ℓ

$$|\mathcal{M}(s,0)| < \sum_{\ell=0}^{\sqrt{s}\ln s} (2\ell+1) \sim s(\ln s)^2 \quad \text{as } s \to \infty.$$

Froissart-Martin bound.

Why “s^2”- bound ? (Naïve)

- The following *may be* useful. (But, just my naïve guess)

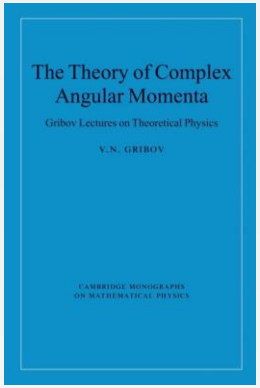
- Suppose that we have an elementary particle with spin-$\mathcal{J}$ in our Lagrangian. t-channel exchange of spin-$\mathcal{J}$ particle:

$$\mathcal{M}(s,t) \sim \frac{s^{\mathcal{J}}}{m_{\mathcal{J}}^2 - t}.$$

- **This violates the condition** $\lim_{|s|\to\infty} \left|\frac{\mathcal{M}(s,0)}{s^2}\right| = 0$ **when $\mathcal{J} \geq 2$.**

- Usually renormalizable theories contain elementary particles with spin **less than two.**

c.f.) 4.3 of Gribov’s textbook



Remark

- Recall that $(\mathbf{nongrav})$ is evaluated as

$$(\mathbf{nongrav}) \simeq \int_{\Lambda^2}^{\infty} \mathrm{d}s \, \frac{\sigma_{\mathrm{tot}}^{\mathrm{nongrav}}(s)}{s^2} > 0.$$

* $\sigma_{\mathrm{tot}}^{\mathrm{nongrav}} \simeq \frac{1}{s} \mathrm{Im}\, \mathcal{M}_{\mathrm{nongrav}}(s, 0)$: total cross section

- Typically, physics with the mass scale M gives

$$\sigma_{\mathrm{tot}}^{\mathrm{nongrav}}(s) \lesssim M^{-2} \quad \left(@\, \mathrm{s} \gtrsim M^2\right).$$

- If $\sigma_{\mathrm{tot}}^{\mathrm{nongrav}}(s)$ decays at $s \gg M^2$, its contribution to $(\mathbf{nongrav})$ also decays.
- **Light particles with non-decaying $\sigma_{\mathrm{tot}}^{\mathrm{nongrav}}$ are important.**

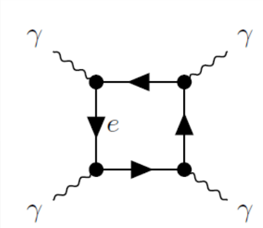
QED + GR

[Alberte-de Rham-Jaitly-Tolley. ('20)]
(Earlier works: [Cheung+('14), Andriolo+('18), Chen+('19)])
(see also: [Hamada+('23)])

- We focus on the $\gamma\gamma \to \gamma\gamma$ process.

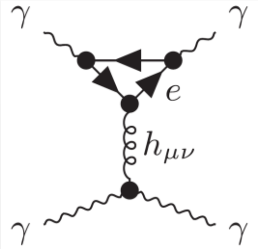
$$c_2(\Lambda) = (\textbf{nongrav}) + (\textbf{grav}) > -\mathcal{O}\left(M_{\rm pl}^{-2} M_s^{-2}\right).$$

$(\textbf{nongrav})$:



$$\sim \frac{e^4}{\Lambda^4} > 0$$

$(\textbf{grav})$:



$$\sim -\frac{e^2}{M_{\rm pl}^2 m_e^2} < 0$$

$$\boldsymbol{\Lambda \lesssim \sqrt{e M_{\rm pl} m_e}}$$
$$\boldsymbol{\approx \mathcal{O}(10^8)}\textbf{GeV.}$$

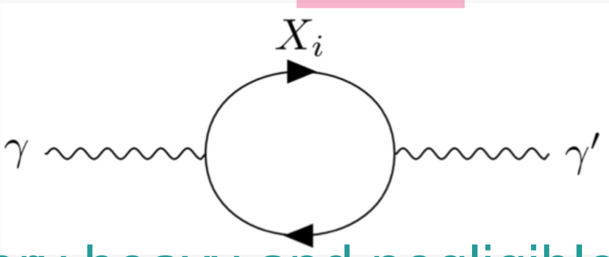
- **Renormalizable couplings can be constrained!**
- A cutoff is too low ? $m_e \to 0$ is prohibited?
- It is conjectured that $c_2(\Lambda) > -\mathcal{O}\left(M_{\rm pl}^{-2} m_e^{-2}\right)$.

Setup: dark photon model

[Noumi-Sato-JT ('22)]

- A hidden U(1) massive gauge field A' with **kinetic mixing:**

$$\mathcal{L} \supset \frac{M_{\rm pl}^2}{2} R - \frac{1}{4} F^2 + e J_{\rm SM}^{\mu} A_\mu - \frac{1}{4} F'^2 - \frac{m_{A'}^2}{2} A'^2 - \frac{\epsilon}{2} F F'$$



* F': field strength of hidden U(1)

* For a while, we assume $\{X_i\}$'s are very heavy and negligible.

Setup: dark photon model

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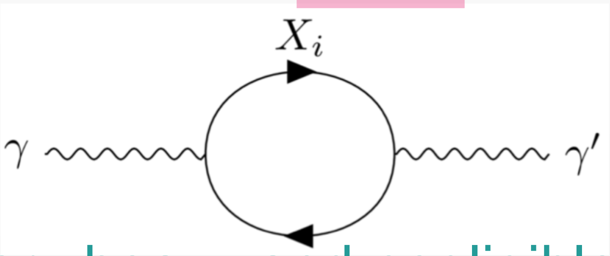


* F': field strength of hidden U(1)

* For a while, we assume $\{X_i\}$'s are very heavy and negligible.

- Diagonalizing the kinetic term,

$$\mathcal{L} \supset \frac{M_{\rm pl}^2}{2}R - \frac{1}{4}F^2 - \frac{1}{4}F'^2 - \frac{m_{A'}^2}{2}A'^2 + eJ_{\rm SM}^{\mu}(A_{\mu} - \epsilon A'_{\mu})$$

- The dark photon A' is **coupled to the SM sector.**
- We consider $\boldsymbol{\gamma\gamma' \to \gamma\gamma'}$ **process.** We ignore QCD sector.

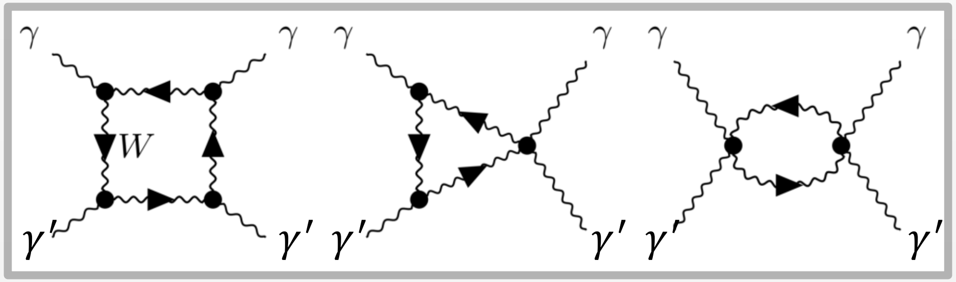
Results (1/2)

[Noumi-Sato-JT ('22)]

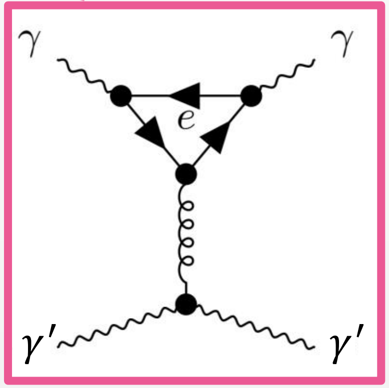
- For transverse modes:

$$c_2(\Lambda) \simeq \frac{32\alpha^2\epsilon^2}{m_w^2\Lambda^2} - \frac{11\alpha}{45 m_e^2 M_{\rm pl}^2} \geq 0.$$

α: fine structure const
$M_{\rm pl}$: Planck mass
m_w: mass of W-boson
m_e: mass of electron



[Non-grav pieces]



[Grav pieces]

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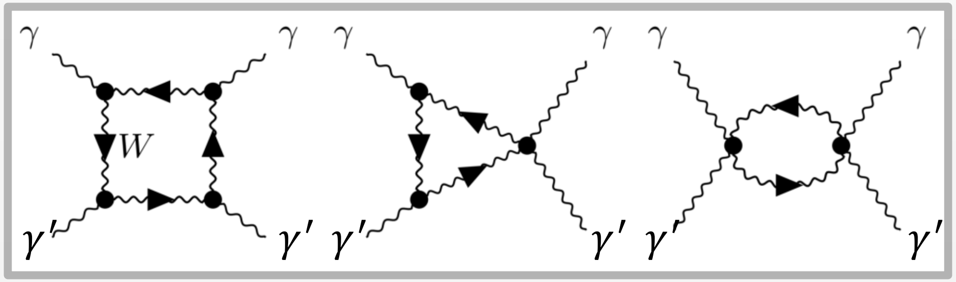

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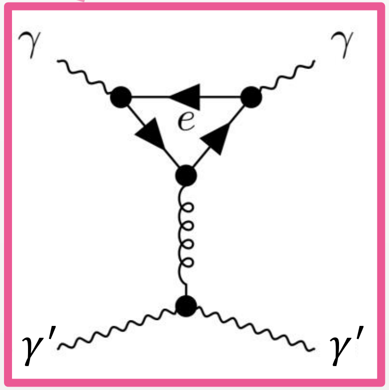

[Grav pieces]

- For longitudinal modes:

$$c_2(\Lambda) = \frac{8\alpha^2\epsilon^2}{m_w^2\Lambda^2} \cdot \left(\frac{m_{A'}}{m_w}\right)^2 - \frac{11\alpha}{45 m_e^2 M_{\rm pl}^2} \geq 0.$$

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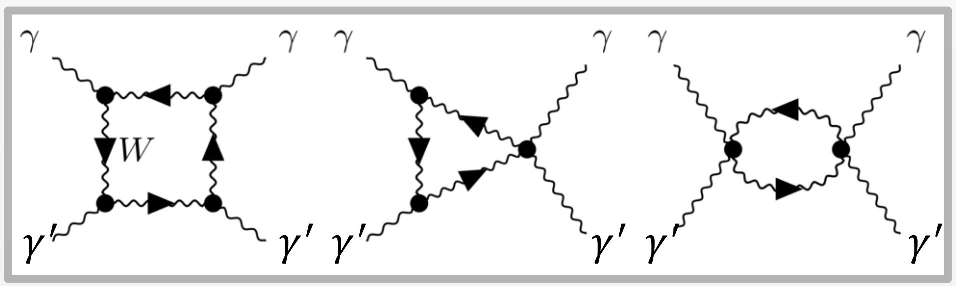
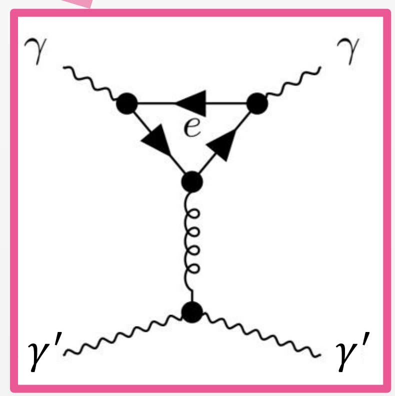


[Non-grav pieces]



[Grav pieces]

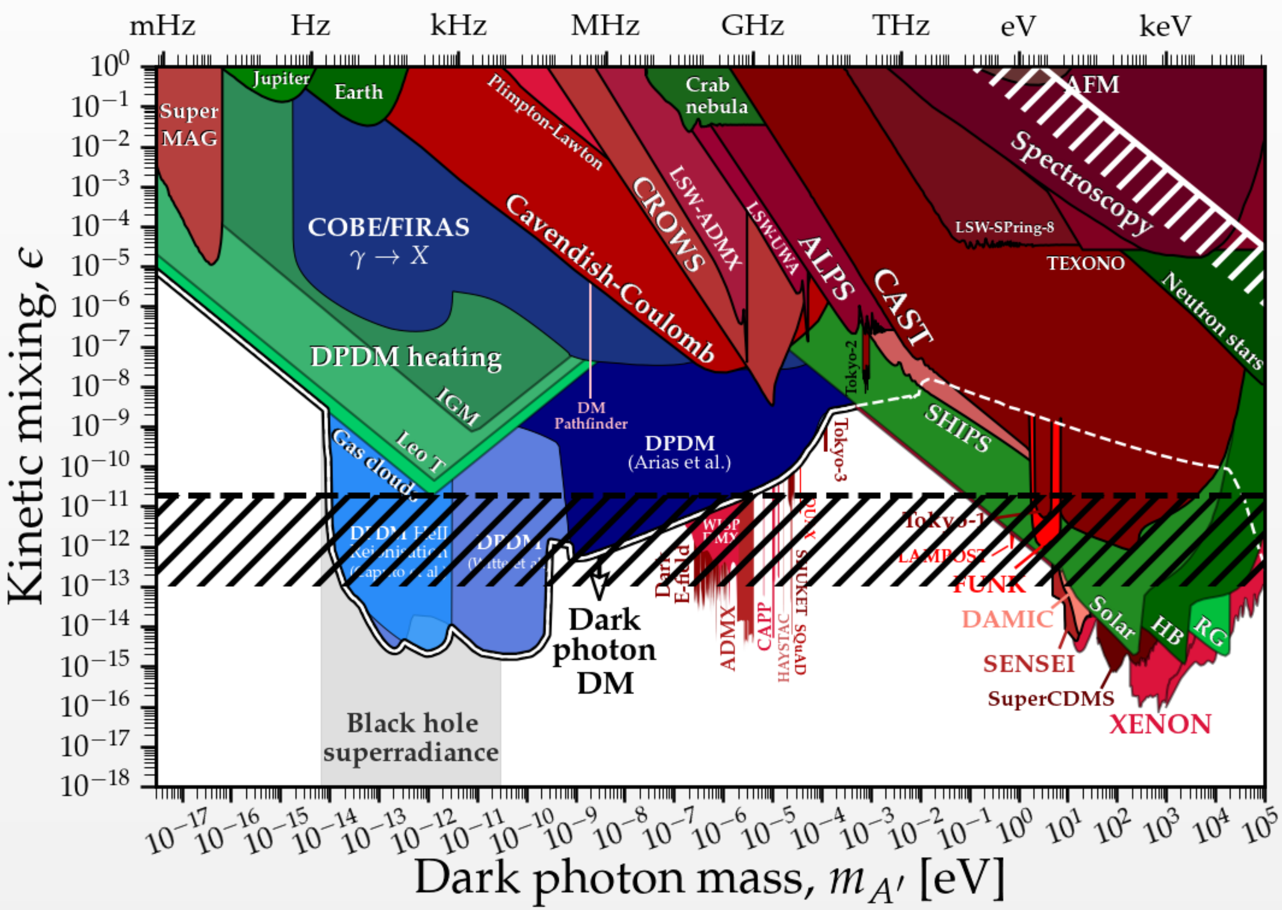
- For longitudinal modes:

Decoupling of longitudinal modes as $m_{A'} \to 0$.

$$c_2(\Lambda) = \frac{8\alpha^2\epsilon^2}{m_w^2\Lambda^2} \cdot \left(\frac{m_{A'}}{m_w}\right)^2 - \frac{11\alpha}{45m_e^2M_{\rm pl}^2} \geq 0.$$

Results (2/2)

[Noumi-Sato-JT ('22)]



Bound from longitudinal modes

Bound from transverse modes

* We take $\Lambda = 1$ TeV

- Testable **LOWER** bound on ϵ. **Complementary to experimental search.**
- **Lower bound on $m_{A'}$** is suggested.
- New particles mediating $\gamma\gamma' \to \gamma\gamma'$ will change the results (next page).

Adding new particles

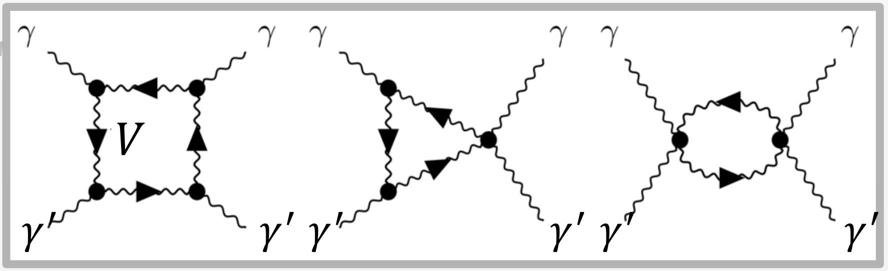
[Noumi-Sato-JT ('22)]

- We consider **vector bosons V charged under both U(1)s.**
- For **longitudinal** modes:

$\tilde{\alpha}$: fine structure const for dark U(1)

m_V: mass of bi-charge vector bosons

$$c_2(\Lambda) = \frac{16\alpha\tilde{\alpha}}{m_V^2\Lambda^2}\cdot\left(\frac{m_{A'}}{m_V}\right)^2 - \frac{11\alpha}{45m_e^2M_{\rm pl}^2} \geq 0.$$



* We set $\epsilon \ll 1$ so that we can ignore the contributions from kinetic mixing.

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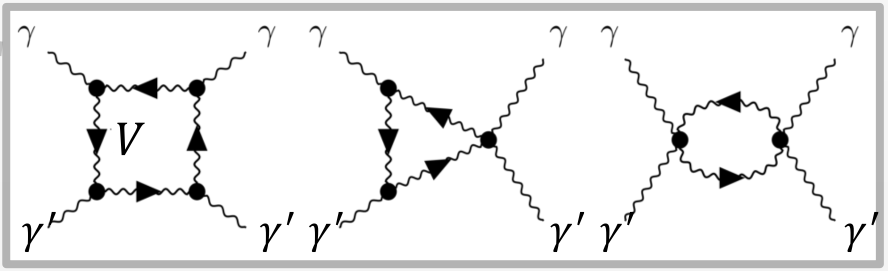


* We set $\epsilon \ll 1$ so that we can ignore the contributions from kinetic mixing.

$$\Rightarrow \quad \Lambda \lesssim 10\ \mathrm{TeV} \times \sqrt{4\pi\tilde{\alpha}}\left(\frac{m_{A'}}{1\ \mathrm{keV}}\right)\left(\frac{1\ \mathrm{TeV}}{m_V}\right)$$

- Again, **very tiny mass $m_{A'}$ is disfavored.** This is because, non-gravitational piece vanishes in the limit $m_{A'} \to 0$.

(c.f. [Aoki-Noumi-Saito-Sato-Shirai-JT-Yamazaki ('23)])

Generalization

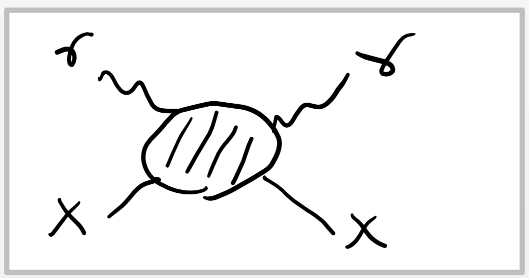
[Noumi-Sato-JT ('22)]

- Consider the GR+SM+dark sector model in 4D.

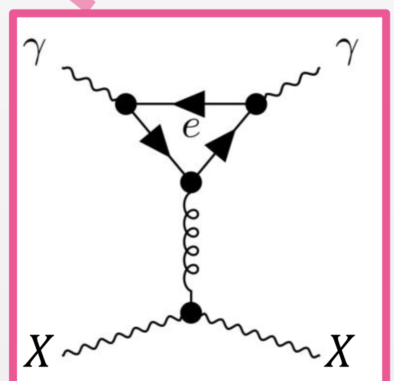
$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} R + \mathcal{L}_{\rm SM}[A_\mu, \cdots] + \mathcal{L}_{\rm dark-sector}[X, \ldots]$$

- Focus on the process $\gamma X \to \gamma X$. (We've discussed $X = \gamma'$ case.)

$$c_2(\Lambda) \simeq c_2(\Lambda)\Big|_{\rm non-grav} - \frac{11\alpha}{m_e^2 M_{\rm pl}^2} \geq 0.$$



[Non-grav pieces]



[Grav pieces]

- **"Dark sector cannot be too dark."**
- It would be interesting to consider various models! (e.g. axion)

Regge behavior: higher-spin states [Gribov's textbook]

- Regge behavior ~ Regge pole $f_\ell(t) \sim \left(\ell - \alpha(t)\right)^{-1}$:

$$\mathcal{M}(s,t) \sim \sum_\ell f_\ell(t)\, P_\ell\left(1+\frac{2s}{t-4m^2}\right) \sim P_{\alpha(t)}\left(1+\frac{2s}{t-4m^2}\right) \sim s^{\alpha(t)}.$$

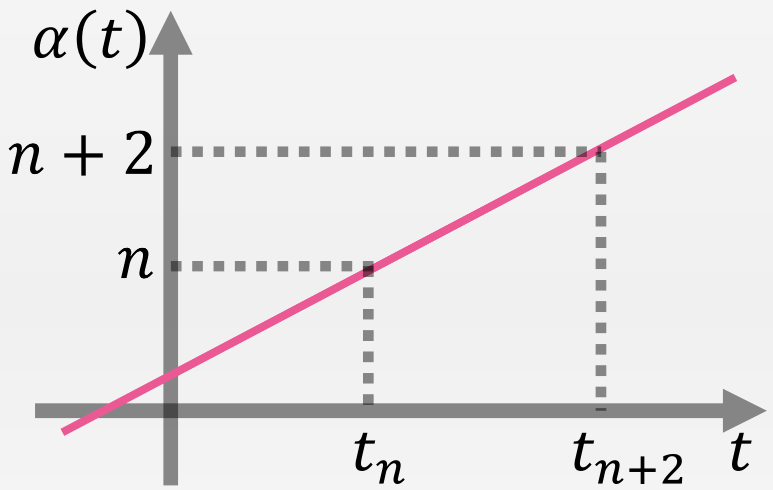
* Spin-J particle exchange (t-ch): $f_\ell(t) \sim \dfrac{\delta_{\ell J}}{t-M^2}$ ➡ $\mathcal{M}(s,t) \sim s^J$.

- **The Regge trajectory $\alpha(t)$ ~ higher-spin spectra.**

$$\alpha(t=t_n) = n \in \text{integers.}$$

$$f_{\ell=n}(t\sim t_n) \sim \frac{\text{const.}}{\alpha'(t_n)(t-t_n)}$$

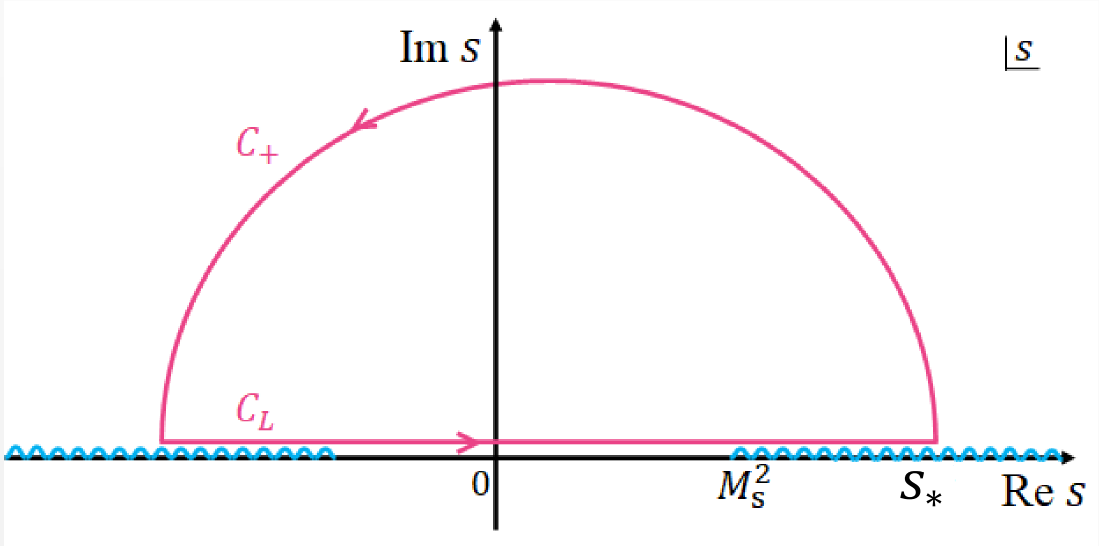
➡ **Spin-n state with mass $\sqrt{t_n}$.**



Finite energy sum rules

[Noumi-JT ('22)]

- We consider the scattering of massless identical scalar.
- We have $\int_{C_+ + C_L} \frac{\mathrm{d}s}{2\pi i} (s + t/2)^{2n+1} \mathcal{M}(s,t) = 0 \quad (n = 0,1,2, \ldots)$.



* We ignored s, u-channel poles of light particles.

* We ignored loops of light particles (for a while).

- We assume $\int_{C_+} \frac{\mathrm{d}s}{2\pi i} (\cdots) \mathcal{M}(s,t) \simeq \int_{C_+} \frac{\mathrm{d}s}{2\pi i} (\cdots) \mathcal{M}_{\mathrm{R}}(s,t)$,

$$\mathcal{M}_{\mathrm{R}} = \frac{-f(t)\left[e^{-i\pi\alpha(t)} + 1\right]}{\sin \pi\alpha(t)} (s/s_*)^{\alpha(t)}, \quad \alpha(t) = 2 + \alpha' t + \alpha'' t^2/2 + \cdots.$$

Finite energy sum rules

[Noumi-JT ('22)]

- **Finite energy sum rules (FESRs)** [for $n = 0,1,2,\ldots$]

$$\frac{f(t)}{\alpha(t)+2n+2} = \underbrace{\frac{1}{(s_* + t/2)^{2n+2}} \int_{M_s^2}^{s_*} \mathrm{d}s\, (s+t/2)^{2n+1}\, \mathrm{Im}\, \mathcal{M}(s,t)}_{=:\, S_{2n+1}(t)}\,.$$

- FESRs directly connect Regge parameters with "infrared" physics $s \le s_*$!

✓ We can derive FESRs for $f(t)$ and $\alpha(t)$. For instance,

$$f'(0) = \frac{2}{n-m}\left[(n+2)^2 S'_{2n+1}(0) - (m+2)^2 S'_{2m+1}(0)\right] \quad (n, m = 0,1,2,\ldots).$$

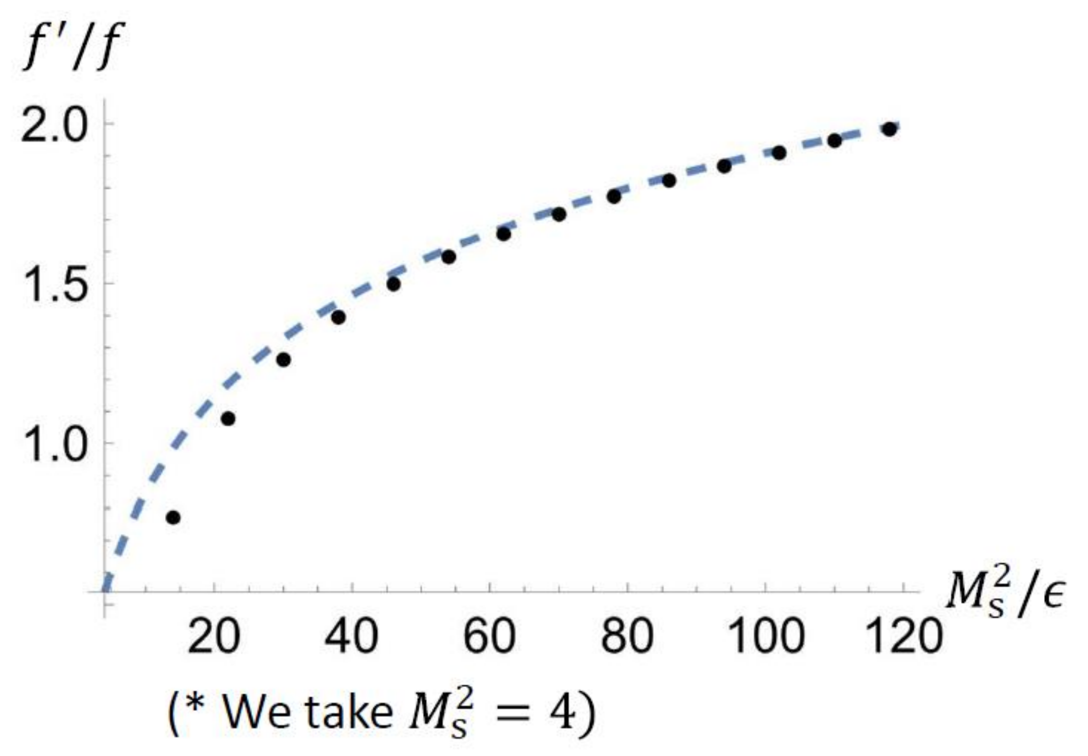
FESR test: examples

[Noumi-JT ('22)]

$$\mathrm{Im}\,\mathcal{M}_{\text{type II}}(s,t)|_{s\gg 4,\,t\sim 0} \simeq \frac{256}{[\Gamma(1+t/4)]^2}\left(\frac{s+t/2}{4}\right)^{2+t/2}.$$

$$f(t) = \frac{256}{[\Gamma(1+t/4)]^2}\left(\frac{1}{\epsilon}+\frac{t}{8}\right)^{\alpha(t)}$$

$$\alpha(t) = 2 + t/2\,.$$



$* \ \epsilon \coloneqq M_s^2/s_* \ll 1.$

Key Idea

[Noumi-JT ('22)]

$$\frac{f(t)}{\alpha(t)+2n+2}=\frac{1}{(s_*+t/2)^{2n+2}}\int_{M_S^2}^{s_*}\mathrm{d}s\,(s+t/2)^{2n+1}\,\mathrm{Im}\,\mathcal{M}(s,t)\,.$$

- FESRs were useful in the context of strong interactions.

[Igi (1962), Dolen+(1967,68), Ademollo+(1967,68)…]

e.g.) ・Experimental input for the RHS -> Constraints on LHS.

・Veneziano amplitude = solution of "FESR bootstrap" for $\pi\pi\to\pi\omega$.

- Our case: no experimental input for $\mathrm{Im}\,\mathcal{M}(s,0)$ with $s>M_S^2$.

- But, we have a theoretical input !! **"Null constraints"**

… implied by **crossing symmetry.**

[Arkani-Hamed+('19, '21), Bellazzini+ ('20), Tolley+ ('20), Caron-Huot+('20)]

Results

[Noumi-JT ('22)]

- We confirm that the Regge parameters **$f(t)$ and $\alpha(t)$ are governed by the scales of higher-spin tower** M_s and α', ignoring loops of light particles.

$$f'/f < 9.1 \times 10^2 M_s^{-2} \qquad \alpha''/\alpha' > -2f'/f - 2.4 \times 10^5 M_s^{-4}/\alpha'$$

$$c_2 > -M_{\rm pl}^{-2} M_s^{-2} \left[3.7 \times 10^3 + 2.4 \times 10^5 \left(M_s^2 \alpha'\right)^{-1}\right]$$

* We choose $s_* = 10 M_s^2$ as a benchmark point in this talk.

- **IR finite grav. positivity bounds in D=4** dimensions!
- **New bounds** on gravitational Regge parameters.
- An extension to higher-D is straightforward.

Comparisons

[Noumi-JT ('22)]

- Our bounds are easily satisfied by string amplitude.

$$\frac{f'}{f} < \frac{10^2}{M_s^2} \times \{3.0, 2.2, 1.8, 1.5, 1.4, 1.3, 1.2\} \qquad (D = 4,5,6,\cdots,10)$$

$$\left.\frac{f'}{f}\right|_{\text{type-II}} \simeq \frac{5.86}{M_s^2}$$

- In general, we expect $c_2 > \frac{-\mathcal{O}(1)}{M_{\rm pl}^2 M_s^2}$ from power counting. This is proven in [Caron-Huot+('21)] in higher dimensions $D > 4$.

- However, the finite bound on c_2 in $D = 4$ was not known.

- We expect the presence of theoretical bound on c_2 which is much stronger than ours: ***future work.***

Application 1: 4D scalar QFT + GR

- We consider 4D scalar QFT +GR.

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + (\text{counterterms}) \qquad V(\phi) = \frac{m^2\phi^2}{2} + \frac{g\phi^3}{3!} + \frac{\lambda\phi^4}{4!}$$

- We compute $c_2(\Lambda)$ and then discuss implications of $c_2(\Lambda) > \frac{-O(1)}{M_{\rm pl}^2 M_s^2}$.

* $c_2(\Lambda)$: calculable within EFT

$$c_2(\Lambda) \coloneqq c_2 - \int_{4m^2}^{\Lambda^2} \mathrm{d}s \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{(s-2m^2)^3}$$

$$\mathcal{M}(s,t) \sim (s,t,u \text{ poles}) + c_2\left(s-2m^2\right)^2 + \cdots$$

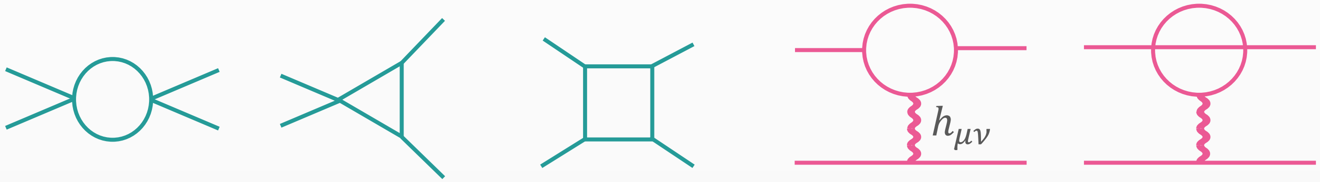
*Λ: EFT cutoff

$* \; c_2 \sim$ "s^2 coefficient"

- In this setup, loop diagrams give leading contributions to c_2. Then, we also need to compute $\mathrm{Im}\,\mathcal{M}(s,0)$ to get $c_2(\Lambda)$. Actually, the "improvement procedure" is practically important. [Alberte, de Rham, Jaitly, Tolley ('20,'21)]

Results (1/2)

[T. Noumi, JT ('21)]



$$c_2(\Lambda) = c_{\text{non-grav}} + c_{\text{grav}} > -O\left(M_{\text{pl}}^{-2} M_s^{-2}\right).$$

$$c_{\text{non-grav}} = \left\{\frac{\lambda^2}{16\pi^2\Lambda^4} - \frac{\lambda g^2}{6\pi^2\Lambda^6}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) - \frac{1}{6}\right] + \frac{g^4}{12\pi^2 m^2\Lambda^6}\right\} > 0$$

$$c_{\text{grav}} = -\frac{1}{M_{\text{pl}}^2}\left(\frac{45 - 8\pi\sqrt{3}}{1296\pi^2}\frac{g^2}{m^4} + \frac{10 - \pi^2}{4608\pi^4}\frac{\lambda^2}{m^2}\right) < 0$$

✓This inequality is nontrivial when **the negative c_{grav}** dominates over "$-O\left(M_{\text{pl}}^{-2} M_s^{-2}\right)$" on the RHS.

✓In the limit $m \to 0$ while keeping $(\lambda, g/m)$ fixed, $c_{\text{grav}} \to -\infty$, leading to the violation of inequality.

→ **$V(\phi)$ cannot be arbitrarily flat.**

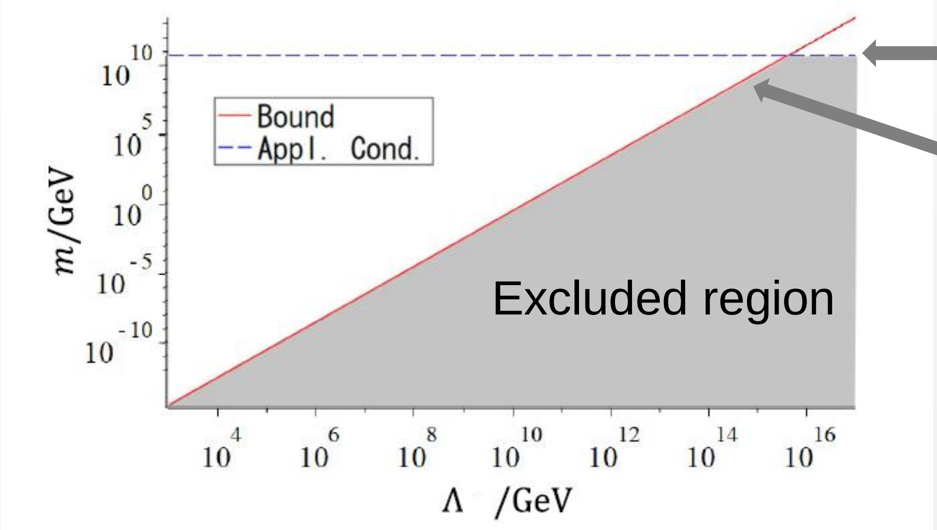
Results (2/2)

[T. Noumi, JT ('21)]

- When having a scaling $g^2 \leq |\lambda| m^2$, the bound reads

$$m \gtrsim \frac{\Lambda^2}{M_{\rm pl}} \left[\frac{1.8 \times 10^{-2} \, (g/m)^2}{\lambda^2} + 4.6 \times 10^{-5} \right]^{1/2}$$

- In $\lambda\phi^4$ theory, the bound becomes a lower bound on m^2



$c_{\rm grav} = M_{\rm pl}^{-2} M_s^{-2} \Leftrightarrow m = 5.4 \times 10^{-4} |\lambda| M_s$

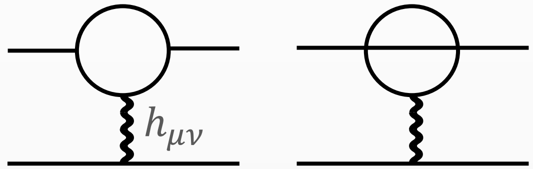
$m = \sqrt{4.6 \times 10^{-5}} \, \frac{\Lambda^2}{M_{\rm pl}}$

$\approx 2.8 \times 10^9 \left(\frac{\Lambda}{10^{15}\,\mathrm{GeV}} \right)^2$ GeV

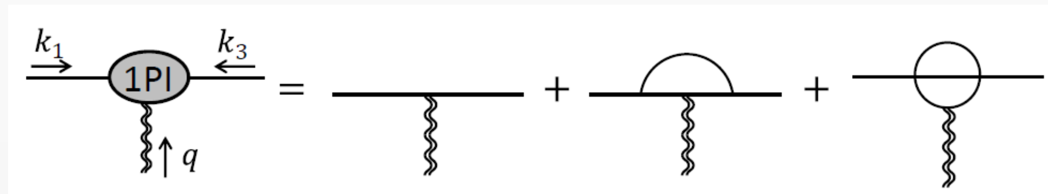
*$\lambda = 10^{-2}$, $M_s = 10^{16}$ GeV is used in this plot.

- **Our bound provides quantitative swampland criteria.**
- **Typically, tiny mass $m \ll \Lambda$ is disfavored.**

Computation of c_{grav}

- $c_{\mathrm{grav}} =$ 

$+\cdots$

- 1PI vertices $V^{\mu\nu}(k_1, k_3) =$ 

$$V^{\mu\nu}(k_1, k_3)\big|_{k_1^2=k_3^2=-m^2} \ni R(q^2)(k_1 - k_3)^{\mu}(k_1 - k_3)^{\nu}, \qquad R_{\mathrm{tree}}(q^2) = 1/2.$$

$$\Rightarrow \mathcal{M}(s,t)\Big|_{\mathrm{grav}} \sim \frac{4R^2(-t)su}{M_{\mathrm{pl}}^2 t} \sim \frac{4R'(0)}{M_{\mathrm{pl}}^2} s^2$$

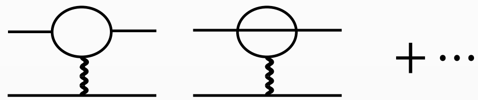
$$\Rightarrow c_{\mathrm{grav}} \simeq \frac{8R'(0)}{M_{\mathrm{pl}}^2} \simeq -\frac{1}{M_{\mathrm{pl}}^2}\left(\frac{45 - 8\pi\sqrt{3}}{1296\pi^2}\frac{g^2}{m^4} + \frac{10 - \pi^2}{4608\pi^4}\frac{\lambda^2}{m^2}\right) < 0.$$

Negative!!

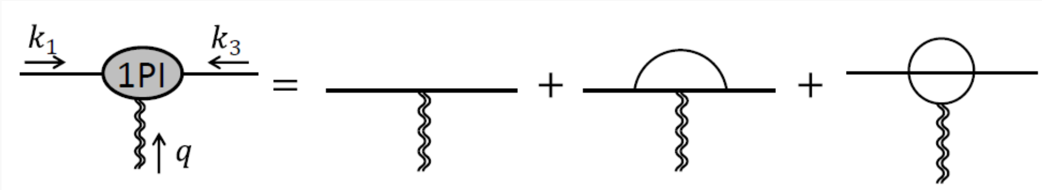
- Negative term arises as a result of expanding $R(q^2)$ around $q^2 = 0$.

Sign of $c_{\rm grav}$ and superluminality

- Consider scalar theory. $c_{\rm grav} =$



- Relevant 1PI vertices:



$$\mathcal{L} = M_{\rm pl}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \alpha R_{\mu\nu}(\partial^\mu\phi)(\partial^\nu\phi)$$

$\alpha < 0$

$\Leftrightarrow c_{\rm grav} < 0$

Effective metric for ϕ: $\tilde{g}_{\mu\nu} = g_{\mu\nu} - 2\alpha R_{\mu\nu}$

e.g.) FLRW metric with $\dot{H} < 0$

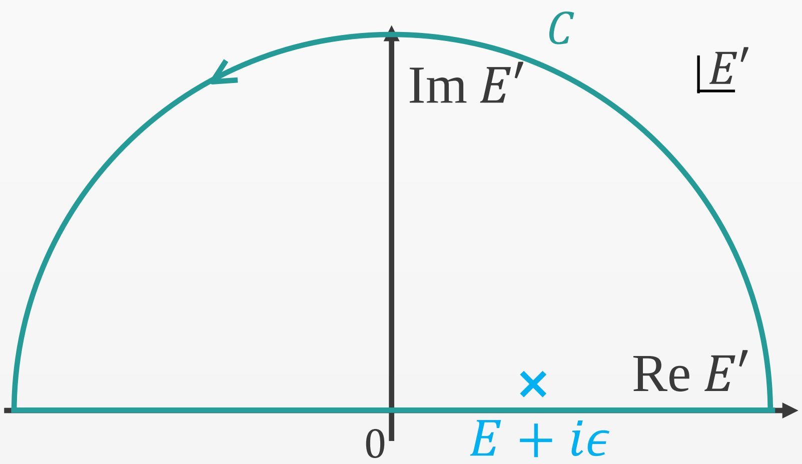
Dispersion relation: $\omega^2 \simeq \left(1 + 4\alpha\dot{H}\right)k^2 > k^2$

Superluminal relative to the speed of GW!

$c_{\rm grav} < 0 \sim$ Superluminal propagation in b.g. satisfying null-E condition.

Analyticity and Causality (4/5)

- Let us assume the analyticity and poly. boundedness of $S(E)$



$$S(E) = \frac{g(E)}{2\pi \mathrm{i}} \oint_C \frac{\mathrm{d}E'}{g(E')} \frac{S(E')}{E' - E - i\epsilon}$$

where $g(E)$ ensures $\lim_{|E|\to\infty} \left| \frac{S_{\beta\alpha}(E)}{g(E)} \right| = 0.$

$g(E)$: polynomial

$$\Rightarrow \quad S(E) = \frac{g(E)}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}E'}{g(E')} \frac{-iS(E')}{E' - E - i\epsilon}$$

$$\Rightarrow \quad S(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{2\pi} S(E) e^{-iEt} = \cdots \text{ (next page)} \cdots$$

Analyticity and Causality (5/5)

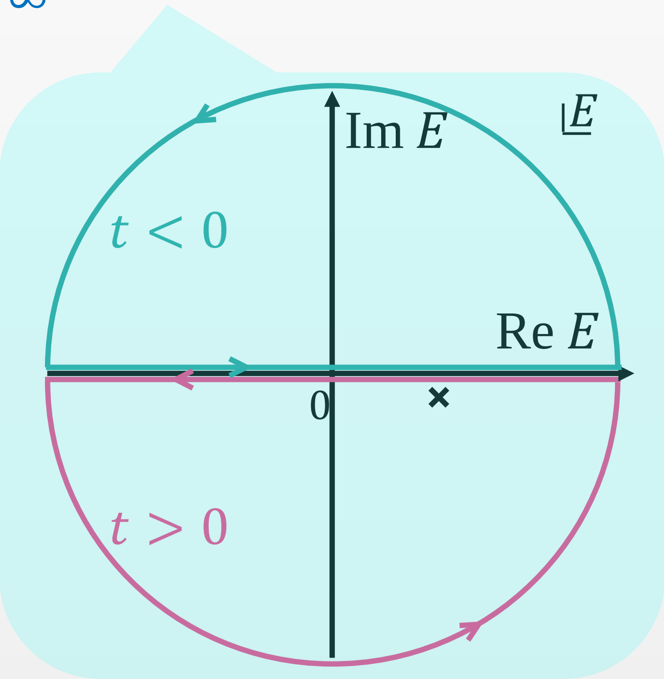
- Analyticity of $S_{\beta\alpha}(E)$ gives rise to

$$S(t) = g(i\partial_t)\int_{-\infty}^{\infty}\frac{\mathrm{d}E'}{2\pi}\frac{iS_{\beta\alpha}(E')}{g(E')}e^{-iE't}\int_{-\infty}^{\infty}\frac{\mathrm{d}E}{2\pi}\frac{e^{-i(E-E')t}}{E-E'+i\epsilon}$$

$$= g(i\partial_t)[\Theta(t)f(t)]$$

where

$$f(t) \equiv \int_{-\infty}^{\infty}\frac{\mathrm{d}E'}{2\pi}\frac{iS(E')}{g(E')}e^{-iE't}$$



- Polynomial boundedness of $S(E)$

➡ $g(i\partial_t)$ contains finite number of derivatives

➡ $S(t) = 0$ for t < 0 : micro-causality.

Results (1/2)

[T. Noumi, JT ('21)]

- For transverse modes:

$$c_2(\Lambda) \simeq \frac{32\alpha^2\epsilon^2}{m_w^2\Lambda^2} - \frac{11\alpha}{m_e^2 M_{\rm pl}^2} > -O\left(M_{\rm pl}^{-2} M_s^{-2}\right).$$

$$\longrightarrow \quad \epsilon \geq \sqrt{\frac{11}{5760\pi\alpha}\frac{m_w\Lambda}{m_e M_{\rm pl}}} \simeq 1.9 \times 10^{-11}\left(\frac{\Lambda}{1\ \rm TeV}\right)$$

α: fine structure const
$M_{\rm pl}$: Planck mass
m_w: mass of W-boson
m_e: mass of electron

- For longitudinal modes:

$$c_2(\Lambda) = \frac{8\alpha^2\epsilon^2}{m_w^2\Lambda^2} \cdot \left(\frac{m_{A'}}{m_w}\right)^2 - \frac{11\alpha}{m_e^2 M_{\rm pl}^2} > -O\left(M_{\rm pl}^{-2} M_s^{-2}\right).$$

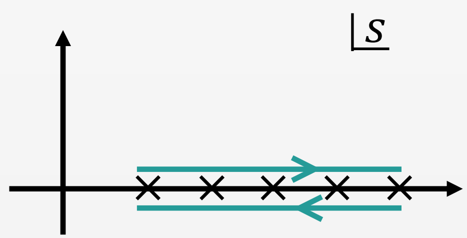
$$\longrightarrow \quad \epsilon \geq \sqrt{\frac{11}{1440\pi\alpha}\frac{m_w\Lambda}{m_e M_{\rm pl}} \cdot \left(\frac{m_w}{m_{A'}}\right)} \simeq 3.0 \times 10^{-3}\left(\frac{\Lambda}{1\ \rm TeV}\right)\left(\frac{1\ \rm keV}{m_{A'}}\right)$$

Example of positivity violation

- e.g) type-Ⅱ superstring amplitude of identical massless boson

$$\mathcal{M}(s,t) = -A\left(s^2u^2 + t^2u^2 + s^2t^2\right)\frac{\Gamma\left(-\frac{\alpha' s}{4}\right)\Gamma\left(-\frac{\alpha' t}{4}\right)\Gamma\left(-\frac{\alpha' u}{4}\right)}{\Gamma\left(1+\frac{\alpha' s}{4}\right)\Gamma\left(1+\frac{\alpha' t}{4}\right)\Gamma\left(1+\frac{\alpha' u}{4}\right)} \quad (A > 0)$$

Higher-spin tower states Reggeize the amplitude.



$$\mathrm{Im}\,\mathcal{M}\left(se^{i\varepsilon},t\right) \approx f(t)\left(\frac{\alpha' s}{4}\right)^{2+\alpha' t/2}$$ for $s \gg \alpha'^{-1}, 0 < \varepsilon \ll 1.$

$$f(0) = 256A/\alpha'^4,\ \partial_t f(t)|_{t=0} = 128A/\alpha'^3.$$

$s = 4N/\alpha'\ (N = 1,2,\cdots)$

spin $J = 2N + 2$

$$c_2(\Lambda) = \lim_{t\to -0}\left\{\frac{4}{\pi}\int_{\Lambda^2}^{\infty} \mathrm{d}s' \frac{\mathrm{Im}\,\mathcal{M}(s',t)}{(s'+t/2)^3} + \frac{2}{M_{\mathrm{pl}}^2 t}\right\} = 0$$

✓ Strict positivity is **violated,** due to **the exact cancellation:**

(Regge states) – (graviton t-pole) = 0