Gravitational Positivity Bounds on Dark Gauge Bosons

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[hep-th]

Introduction

- Unitarity of scattering amplitudes impose strong swampland-like constraints on gravitational theories(Gravitational Positivity Bounds)

- Application of gravitational positivity bounds to gauge boson models

- We find that gravitational positivity bounds put constraints on gauge coupling and gauge boson mass

Outline

- Formulation of positivity bound

- Application to U(1) gauge boson
  - Bounds on Abelian Higgs model
  - Bounds on Stueckelberg model

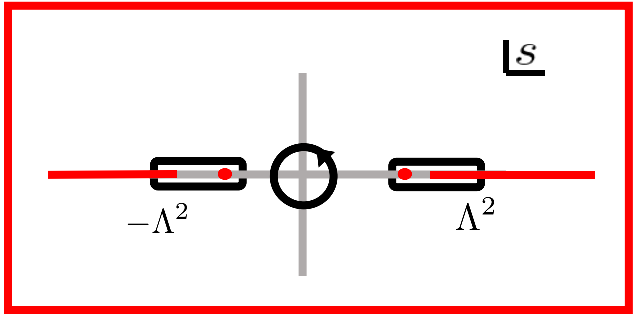
Positivity Bound

Positivity bounds w/o gravity

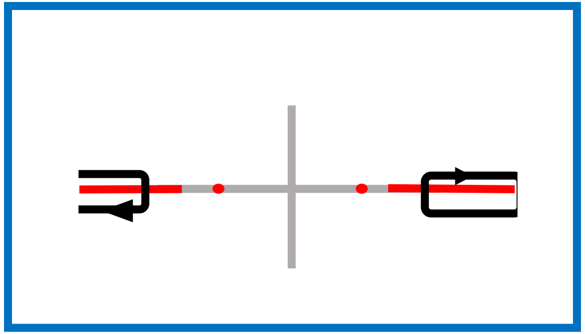
- Constraints on EFT by using unitarity of scattering amplitude
- $\boxed{B^{(2)}(\Lambda)}$ (calculated in a given EFT) = $\boxed{\text{integral of } \mathrm{Im}\mathcal{M}}$ (positive by unitarity)

Improved positivity bound Bellazini '16, de Rham+ '17

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s'^3} = \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s'^3} > 0$$



Deform contour



Gravitational positivity bound

- Additional assumptions for removing the divergence in the forward limit

Assumption(1) $\mathrm{Im}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)} \quad \text{for } s > M_*^2$

Regge behavior at the high energy
Cancel out the divergent term

Assumption(2) $\left|\frac{f'}{f}\right|, \left|\frac{j''}{j'}\right|, |j'| \ll \frac{1}{\Lambda^2}$

The remaining term is small

- Positivity bound holds approximately:

Gravitational positivity bound

Tokuda, Aoki, Hirano '20

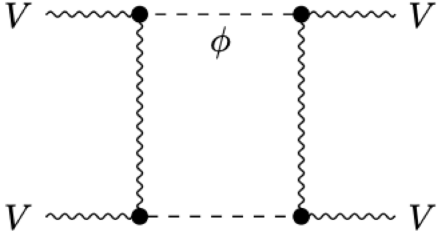
$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\left(s',0\right)}{s^3} \gtrsim 0$$

Structure of $B^{(2)}(\Lambda)$

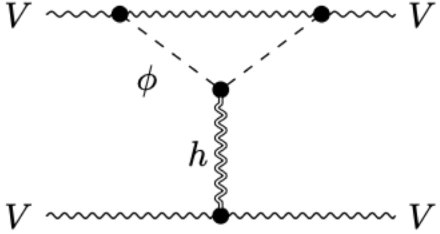
- Focus on scattering of photon and some particle X in gravitational theory

$$B^{(2)}(\Lambda) = B^{(2)}_{\text{non-grav}}(\Lambda) - \left| B^{(2)}_{\text{grav}}(\Lambda) \right|$$

non-gravitational part, positive



graviton-exchange part, negative



Implication of gravitational positivity bound

- Non-gravitational interaction is bounded below by gravitational interaction

→ gravity should be weak!

$$\boxed{B^{(2)}_{\text{non-grav}}(\Lambda) > \left|B^{(2)}_{\text{grav}}(\Lambda)\right|}$$

- Relation with weak gravity conjecture Tolley+ '20 (See also Cheung+ '14, Hamada+ '18)

Application to U(1) Gauge Boson

- Application to Higgs gauge theory
- Application to Stueckelberg gauge theory

Abelian Higgs model

- Lagrangian: $\mathcal{L}_{AH} = -\frac{1}{4}F^2 + |D_\mu \Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$

$$F = \partial_\mu V_\nu - \partial_\nu V_\mu \qquad D_\mu = \partial_\mu - i g_\Phi V_\mu$$

- Three independent parameters: m_V , m_Φ , g_Φ

$$\Rightarrow \quad v = \frac{m_V}{\sqrt{2} g_\Phi} \qquad \lambda = \frac{2 g_\Phi^2 m_\Phi^2}{m_V^2}$$

- We consider the simple model: Abelian Higgs + gravity

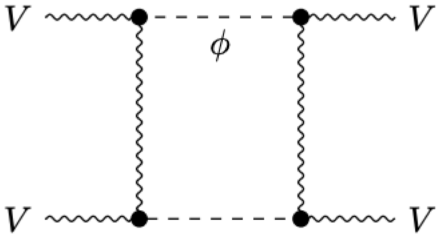
$$S = S_{\text{AH}} + S_{\text{Gravity}}$$

Application of positivity

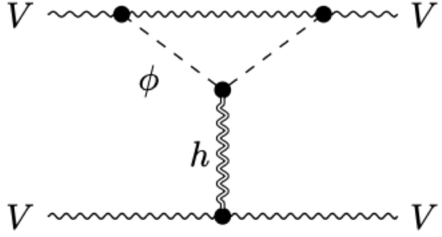
- Calculate scattering of gauge bosons @ 1-loop
- Gauge bosons have Transverse mode & Longitudinal mode

  → Three helicity amplitudes: TT , TL , LL
- Calculate $B^{(2)}(\Lambda) = B^{(2)}_{\text{non-grav}}(\Lambda) - \left| B^{(2)}_{\text{grav}}(\Lambda) \right|$

non-gravitational part



gravitational part



Bounds on Abelian Higgs model

TT $$\frac{g_\Phi^4}{4\pi^2\Lambda^4} - \frac{g_\Phi^2}{72\pi^2 M_{Pl}^2 m_\Phi^2} > 0 \quad \Rightarrow \quad m_\Phi > \frac{\Lambda^2}{3\sqrt{2}g_\Phi M_{\mathrm{Pl}}}$$

Lower bound on Higgs mass

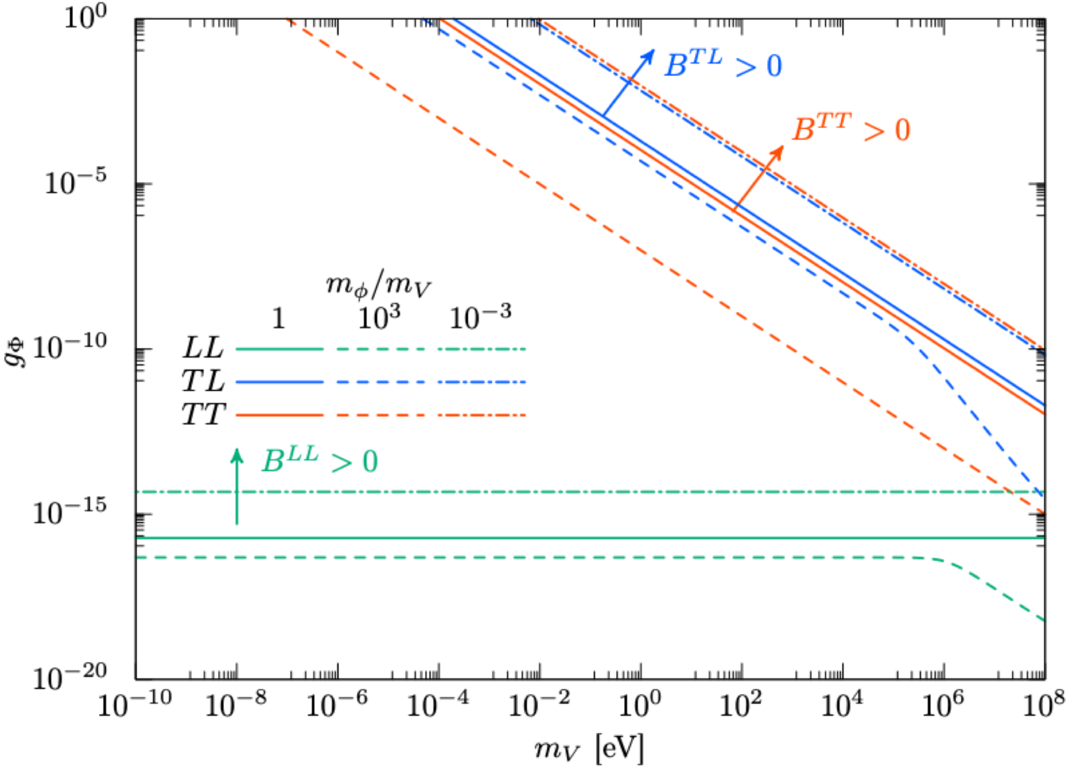
TL $$\frac{g_\Phi^4}{2\pi^2\Lambda^4} - \frac{g_\Phi^2}{144\pi^2 M_{Pl}^2 m_V^2} > 0 \quad \Rightarrow \quad m_V > \frac{\Lambda^2}{6\sqrt{2}g_\Phi M_{\mathrm{Pl}}}$$

Lower bound on gauge boson mass

LL $$\underbrace{\frac{g_\Phi^4}{\pi^2\Lambda^2 m_V^2}}_{\boxed{\text{non-gravitational}}} - \underbrace{\frac{g_\Phi^2}{72\pi^2 M_{Pl}^2 m_V^2}}_{\boxed{\text{gravitational}}} > 0 \quad \Rightarrow \quad g_\Phi > \frac{\Lambda}{6\sqrt{2}M_{\mathrm{Pl}}}$$

Lower bound on gauge coupling

Bounds on Abelian Higgs model



$\Lambda = 1\text{TeV}$

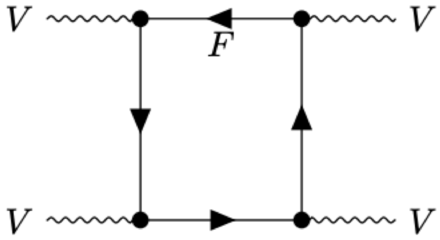
Stueckelberg + Fermion

- Lagrangian: $\mathcal{L} = -\frac{1}{4}F^2 - \frac{1}{2}m^2V^2 + i\bar{\psi}\gamma^\mu D_\mu\psi \ - m_F\bar{\psi}\psi$

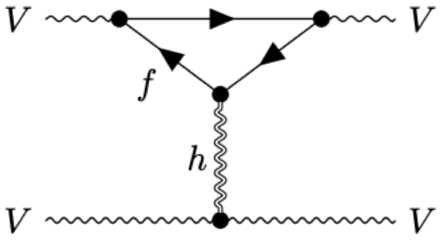
$$F = \partial_\mu V_\nu - \partial_\nu V_\mu \quad D_\mu = \partial_\mu - ig_F V_\mu$$

- Gauge boson acquires mass by Stueckelberg mechanism

non-gravitational part



gravitational part



Bounds on Stuckelberg + Fermion

TT $$\frac{g_F^4(2\log\frac{\Lambda^2}{m_F^2}+1)}{4\pi^2\Lambda^4}-\frac{11g_F^2}{360\pi^2M_{Pl}^2m_F^2}>0 \;\Rightarrow\; g_F>0.2\frac{\Lambda^2}{m_FM_{\mathrm{Pl}}\sqrt{\log(\Lambda m_F^{-1})}}$$

Lower bound on gauge coupling

TL $$\frac{4g_F^4m_V^2}{3\pi^2\Lambda^6}-\frac{11g_F^2}{720\pi^2M_{Pl}^2m_F^2}>0 \;\Rightarrow\; m_V>0.1\frac{\Lambda^3}{g_Fm_FM_{\mathrm{Pl}}}$$

Lower bound on gauge boson mass

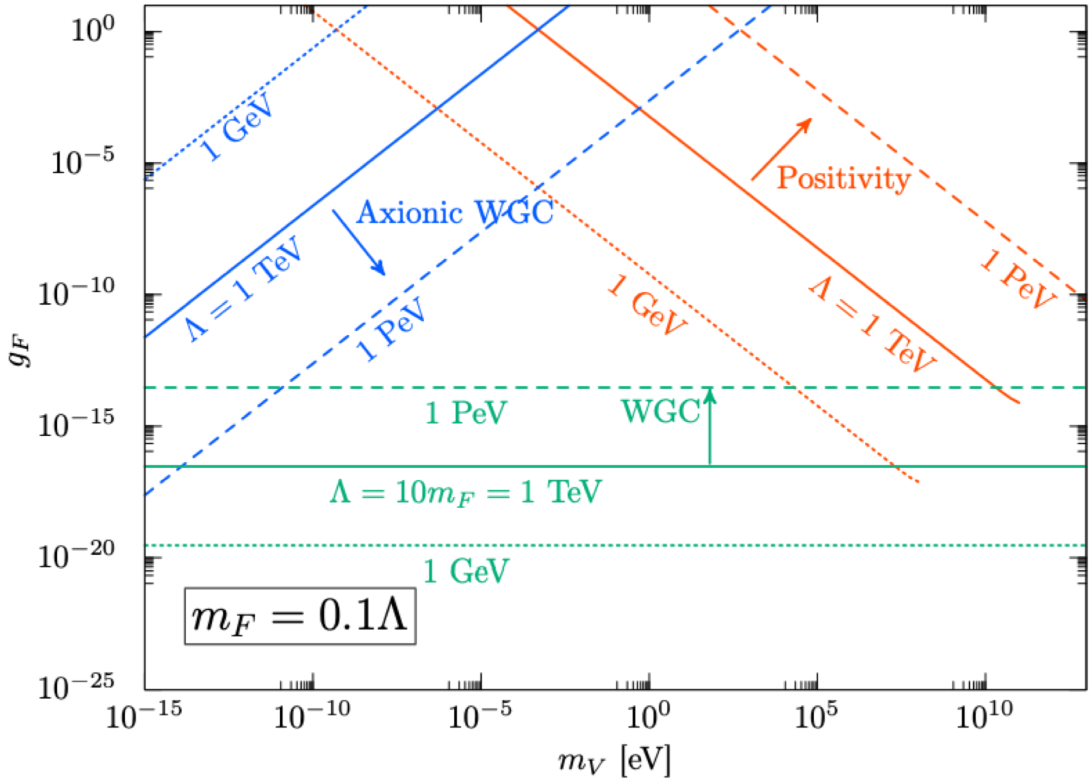
LL $$\frac{g_F^4m_V^4(4\log\frac{\Lambda^2}{m_F^2}+7)}{\pi^2\Lambda^8}-\frac{g_\Phi^2m_V^2}{420\pi^2M_{Pl}^2m_F^4}>0 \;\Rightarrow\; m_V>0.02\frac{\Lambda^4}{g_Fm_F^2M_{\mathrm{Pl}}\sqrt{\log(\Lambda m_F^{-1})}}$$

Lower bound on gauge boson mass

non-gravitational

gravitational

Comparison with other swampland constraints



Positivity(This work)

$$g_F m_V > 0.02 \frac{\Lambda^4}{m_F^2 M_{\mathrm{Pl}} \sqrt{\log(\Lambda m_F^{-1})}}$$

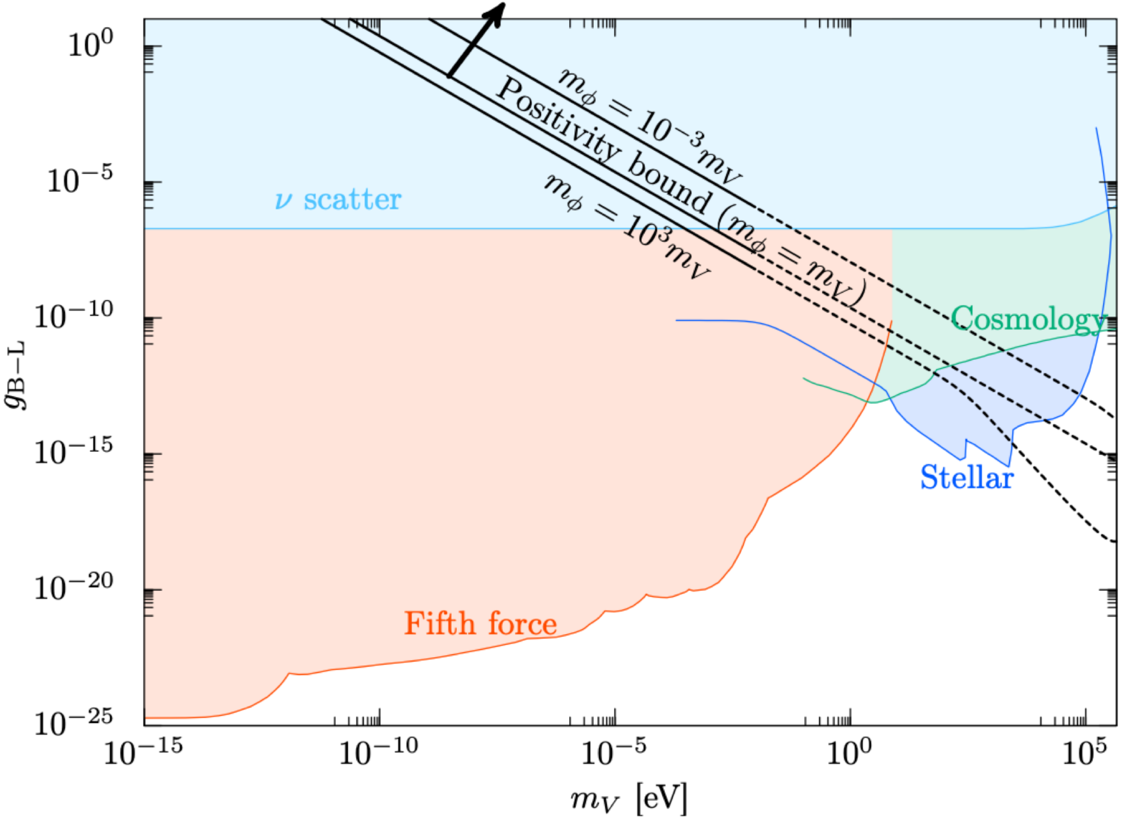
WGC

$$g_F > \frac{m_F}{M_{\mathrm{Pl}}}$$

Axionic WGC [Reece '18]

$$\frac{m_V}{g_F} > \frac{\Lambda^2}{M_{\mathrm{Pl}}}$$

Implication to $U(1)_{B-L}$ gauge boson



Positivity bound on Abelian Higgs Model (from TL scattering) gives strong constraint

Assumptions

- SM particles are neglected
- $g_{B-L} = g_{\Phi}$

$\Lambda = 1\mathrm{GeV}$

Summary

- Gravitational positivity bound: Unitarity of scattering amplitudes impose swampland-like constraints on gravitational theories

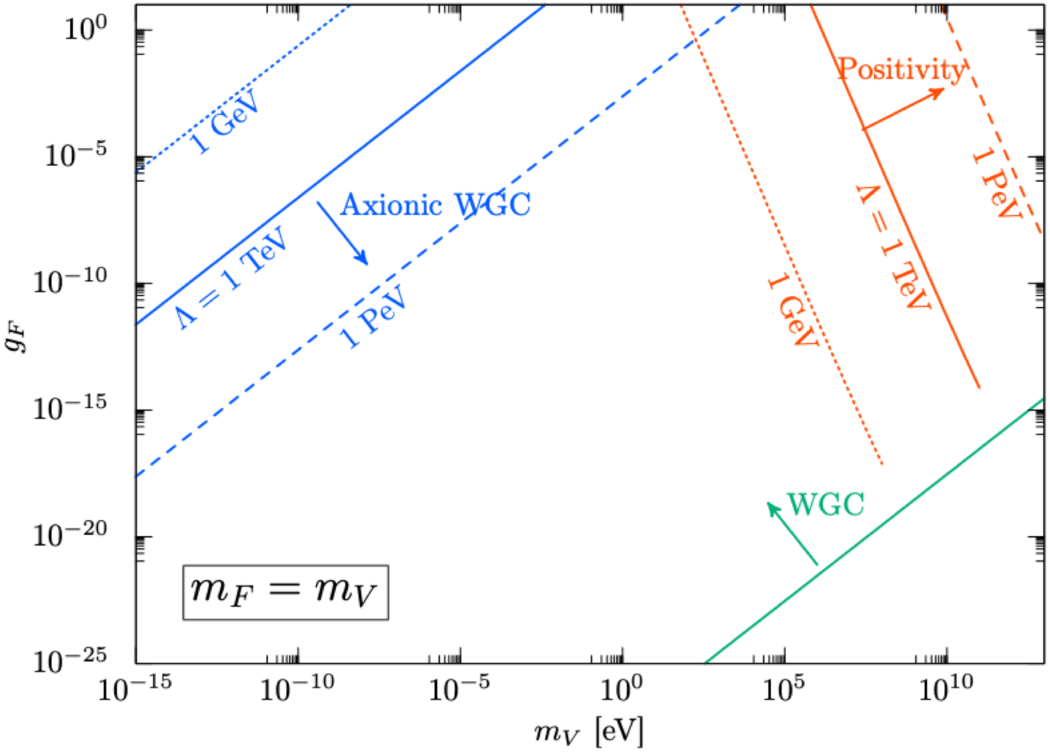
- Application to U(1) gauge boson

  → Lower bound on gauge coupling and gauge boson mass

- Gravitational positivity bounds Potentially put stringent constraints on phenomenological models

Backup Slides

Comparison with other swampland constraints



Positivity(This work)

$$g_F m_V > 0.02 \frac{\Lambda^4}{m_F^2 M_{\rm Pl} \sqrt{\log(\Lambda m_F^{-1})}}$$

WGC

$$g_F > \frac{m_F}{M_{\rm Pl}}$$

Axionic WGC [Reece '18]

$$\frac{m_V}{g_F} > \frac{\Lambda^2}{M_{\rm Pl}}$$

Toward phenomenological constraints

- Contribution of Standard model particle (especially QCD sector)

- Handling unstable particles
  - Generally, gauge bosons have finite decay width → What is positivity constraints on scatterings of unstable particles?[Aoki ‘22]

Gravitational positivity bound

- Additional assumptions to remove the divergence in the forward limit

Assumption(1) $\mathrm{Im}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)} \quad \text{for } s > M_*^2$

Regge behavior at the high energy
Cancel out the divergent term

Assumption(2) $\left|\frac{f'}{f}\right|, \left|\frac{j''}{j'}\right|, |j'| \ll \frac{1}{\Lambda^2}$

- Positivity bound holds approximately:

Gravitational Positivity Bound

Tokuda, Aoki, Hirano '20

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s^3} \gtrsim 0$$

Gravitational positivity bound

- Additional assumptions to remove the divergence in the forward limit

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Regge behavior at the high energy
Cancel out the divergent term

$$B^{(2)}(\Lambda) > \underbrace{\frac{1}{M_{\mathrm{Pl}}^2}\left[\frac{f'}{f} + j' \ln\left(\frac{\alpha' M_*^2}{4}\right) - \frac{j''}{j'}\right]}_{\text{The remaining term}}$$

Gravitational positivity bound

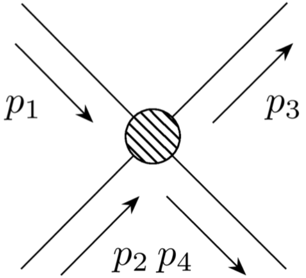
Tokuda, Aoki, Hirano ‘20

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s^3} \gtrsim 0$$

Positivity bound

- Non-trivial consistency condition on low energy EFT Adams+ '06
- Consider 2 to 2 scattering in some EFT

$$\mathcal{M}(s,t) =$$



$s = -(p_1+p_2)^2 \sim (\text{CM energy })^2$

$t = -(p_1-p_3)^2 \sim$ scattering angle

$t = 0:$ forward scattering

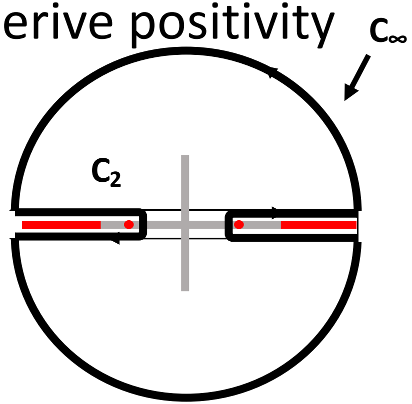
- Low energy expansion of amplitude: $\mathcal{M}(s,0) = c_0 + c_1 s + c_2 s^2 + \cdots$
- Positivity bound: If UV completion of EFT is "standard" theory (Unitary, Lorentz invariant , Analytic, Local), $\boxed{c_2 > 0}$

e.g. $-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \lambda(\partial^\mu\phi\partial_\mu\phi)^2 + \cdots \Rightarrow \lambda > 0$

s^2 bound

- s^2 bound: $\lim_{s\to\infty} |\mathcal{M}(s,0)| < s^2$ should be satisfied to derive positivity

$$c_2 = \frac{1}{2\pi i}\int_{C_1} ds' \frac{\mathcal{M}(s',0)}{s'^3} = \frac{1}{2\pi i}\left(\int_{C_2} + \int_{C_\infty}\right)\frac{\mathcal{M}(s,0)}{s'^3}ds'$$



- s^2 bound is guaranteed by Froissart bound for gapped theory

Froissart, ’61

Azimov, ’11

- For theory with gravity, see Caron-Huot+, ’21, Zhiboedov+, ’22

Technical problem with gravity

$$c_2 = \lim_{t\to 0}\left(\frac{2}{\pi}\int ds' \frac{\mathrm{Disc}\,\mathcal{M}\left(s',t\right)}{s'^3} + \frac{1}{M_{\mathrm{Pl}}^2 t}\right)$$

- Assume Regge behavior in the high-energy limit (Realized in string theory)

$$\boxed{\mathrm{Disc}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)} = f(t)\left(\frac{\alpha' s}{4}\right)^{2+j' t+\cdots} \quad \text{for } s > M_*^2}$$

$$c_2 = \lim_{t\to 0}\left(\overset{>0}{\boxed{\frac{2}{\pi}\int^{M_*^2} ds' \frac{\mathrm{Disc}\,\mathcal{M}\left(s',t\right)}{s'^3}}} + \overset{{}^{\prime\prime}\infty - \infty{}^{\prime\prime}}{\boxed{\frac{2}{\pi}\int_{M_*^2}^{\infty} ds' \frac{\mathrm{Disc}\,\mathcal{M}\left(s',t\right)}{s'^3} + \frac{1}{M_{\mathrm{Pl}}^2 t}}}\right)$$

$$> \frac{1}{M_{\mathrm{Pl}}^2}\underbrace{\left[\frac{f'}{f} + j'\ln\left(\frac{\alpha' M_*^2}{4}\right) - \frac{j''}{j'}\right]}_{\text{the remaining part, } \mathcal{O}\left(\frac{1}{M^2}\right)} = \pm\mathcal{O}\left(\frac{1}{M_{\mathrm{Pl}}^2 M^2}\right)$$

Tokuda, Aoki, Hirano ‘20

- The implication depends on the M

Our assumption

$$\sim \mathcal{O}\left(\frac{1}{M_{\mathrm{Pl}}^2 m_e^2}\right)$$

$$B^{(2)}_{\text{non-grav}}(\Lambda) > \left|B^{(2)}_{\text{grav}}(\Lambda)\right| \pm \mathcal{O}\left(\frac{1}{M_{\mathrm{Pl}}^2 M^2}\right)$$

- the sign of third term and the scale M is determined by the high-energy behavior of the amplitude
- different possible implication
  1. $M \sim m_e$

     Non-trivial high energy behavior of the scattering amplitude
  2. $M \gg m_e$

     $$B^{(2)}_{\text{non-grav}}(\Lambda) > \left|B^{(2)}_{\text{grav}}(\Lambda)\right|$$
- In our work, we assume 2: justifying this assumption is future work

Decoupling of the longitudinal mode

- A polarization vector of the longitudinal mode is proportional to the momentum in the $m_{A'} \to 0$ limit

$$\epsilon_\mu = (k, 0, 0, \sqrt{k^2 + m^2})/m = k_\mu + \mathcal{O}\left(\frac{m_{A'}}{k}\right)$$

- $k_\mu \mathcal{M}^\mu = 0$ by the Ward identity → $\mathcal{M} \propto m_{A'}$

$$\mathcal{M} = \epsilon_\mu \mathcal{M}^\mu = \left(k_\mu + \mathcal{O}\left(\frac{m_{A'}}{k}\right)\right) \mathcal{M}^\mu \to \mathcal{O}\left(\frac{m_{A'}}{k}\right)$$

Implication of gravitational positivity bound

- Cutoff scale of gravitational theory

$$\boxed{B^{(2)}_{\text{non-grav}}(\Lambda) > \left|B^{(2)}_{\text{grav}}(\Lambda)\right|}$$