



3-7 July 2023

STRING PHENO 2023

*Institute for Basic Science
Daejeon, South Korea*

Gravitino in nonlinear supergravity and the Swampland

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[BONNEFOY, CASAGRANDE, DUDAS 2206.13451](#)

& ongoing work with E. Dudas, M. Peloso and Q. Bonnefoy

Plan of the talk

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I. Nonlinear supergravity

- Nonlinear SUSY and the goldstino
- Superfield constraints
- $\mathcal{N} = 1$ orthogonal SUGRA

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- The sound speed c_s
- Potential superluminality: $c_s > 1$
- Unbounded particle production: $c_s = 0$

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III. Causality constraints in the rigid limit

- The equivalence theorem & positivity bounds
- Improved setup & full causality

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IV. Remarks on $c_s = 0$

- EFT breakdown
- Backreaction & QFT in curved spacetime

Nonlinear supergravity

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- The goldstino couplings are such that *supersymmetry is realised nonlinearly*: $\delta_\epsilon G = f \epsilon + \frac{i}{f} (G \sigma^\mu \bar{\epsilon} - \epsilon \sigma^\mu \bar{G}) \partial_\mu G$

[VOLKOV, AKULOV '73]

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EFT construction

Nonlinear supergravity

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SUGRA multiplet

Spectrum in the unitary gauge $G = 0$
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minimal inflation in SUGRA

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Einstein-Hilbert term

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inflaton sector

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scalar potential $V(\varphi)$

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field-dependent mass parameter

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gravitino sector

Gravitino in FRW

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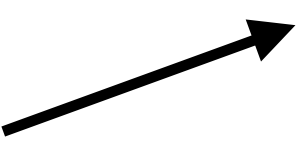
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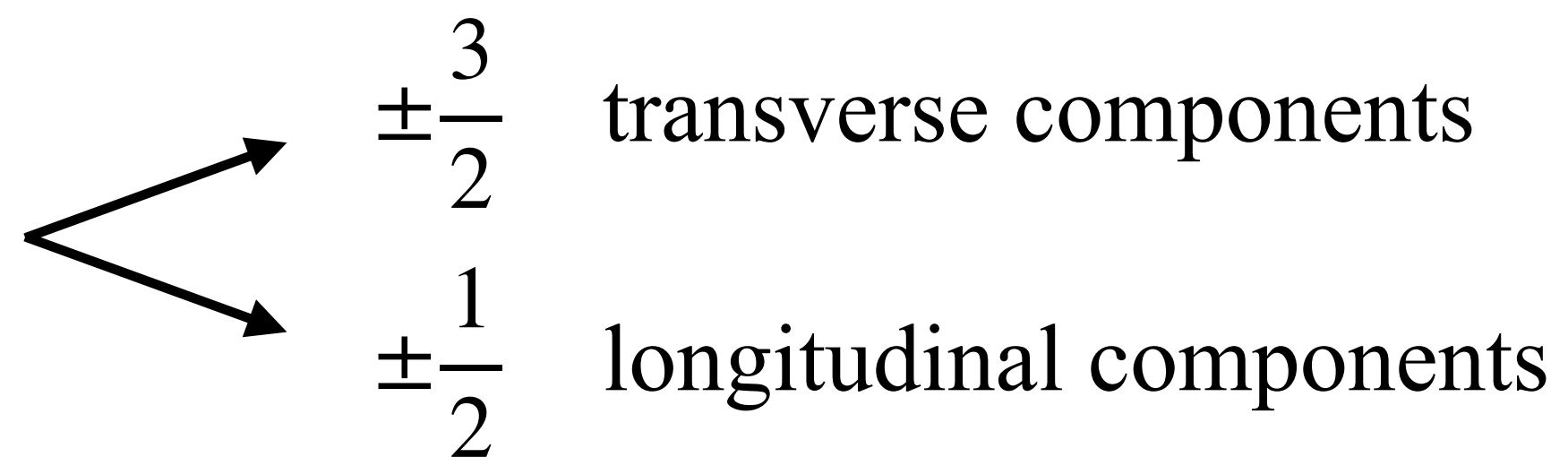
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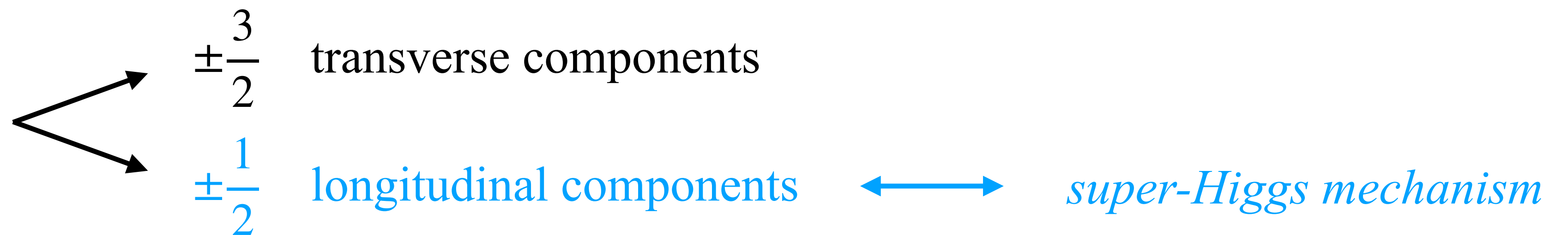


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[KALLOSH, KOFMAN, LINDE, VAN PROEYEN '00]

[NILLES, PELOSO, SORBO, '01]

$$\omega_k^2 = c_s^2 k^2 + a^2 m^2$$

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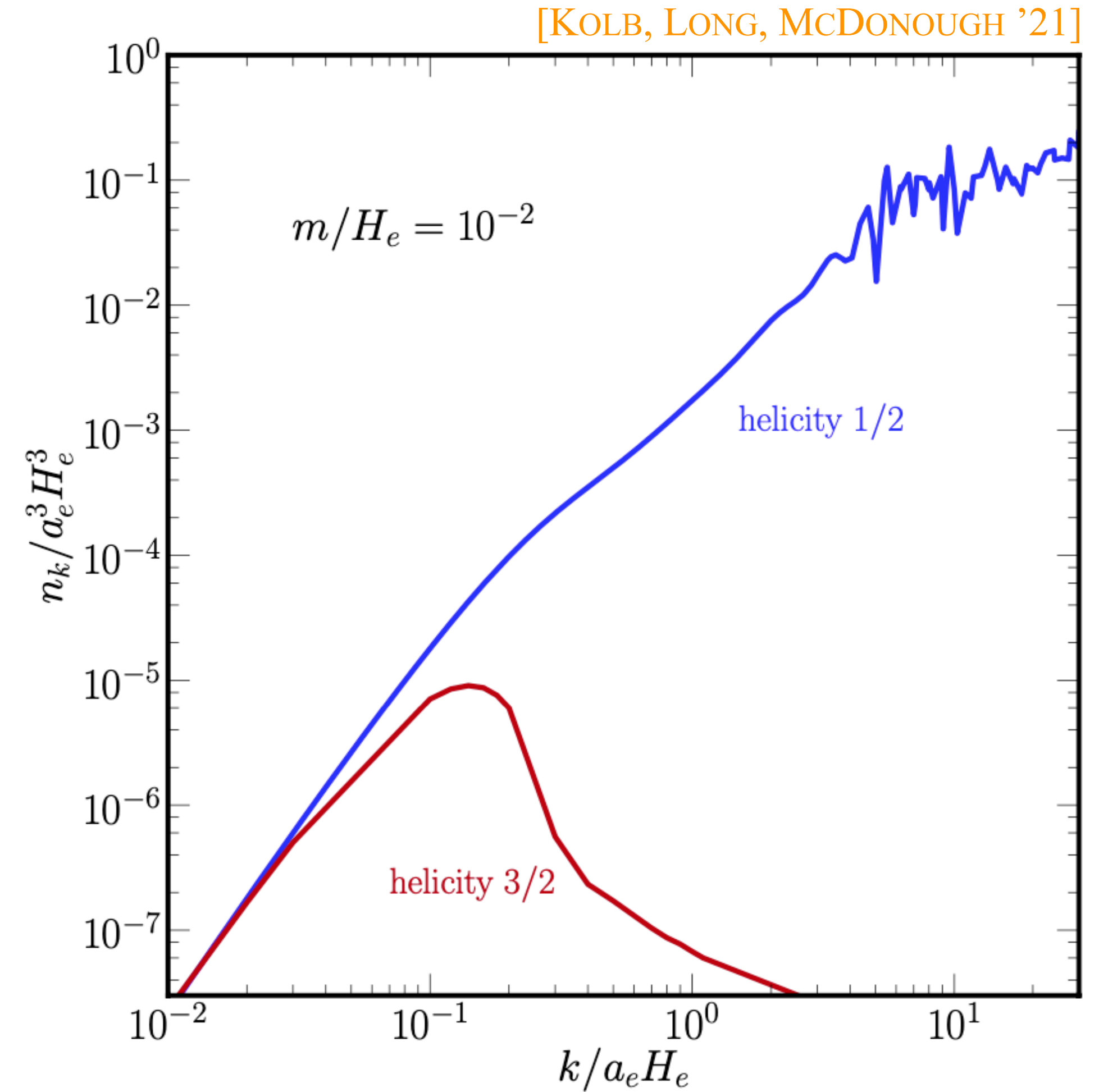
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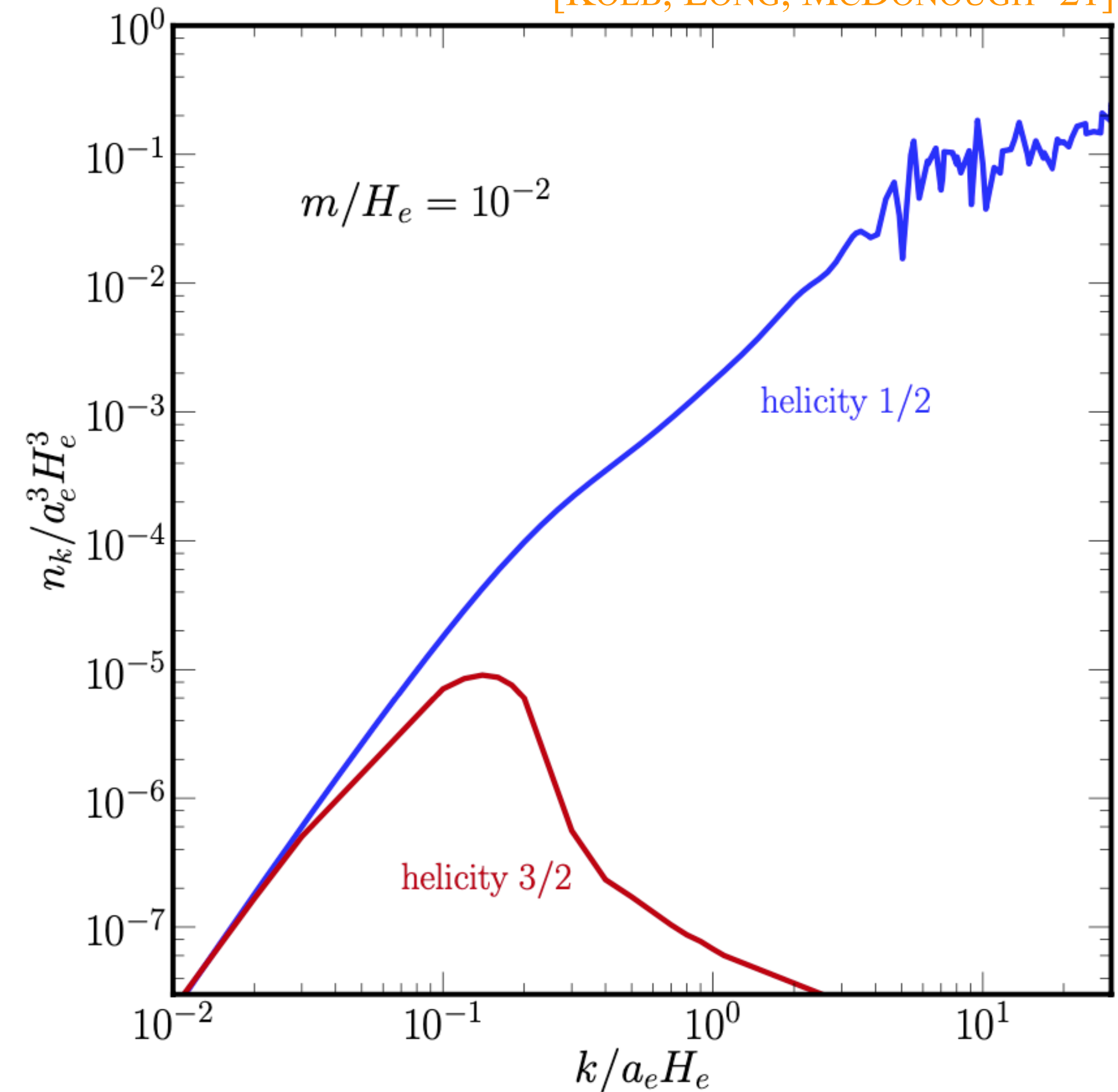
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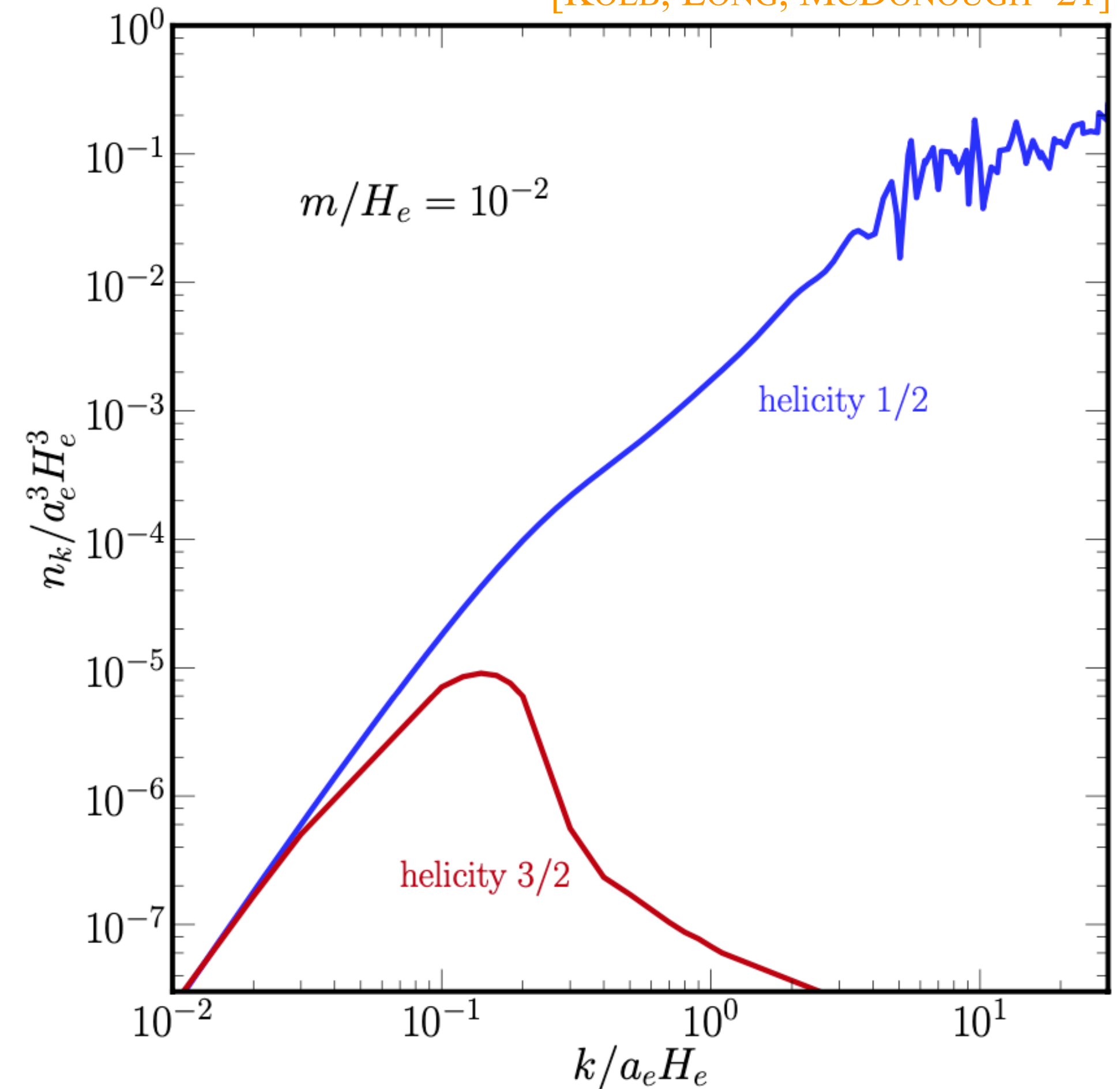
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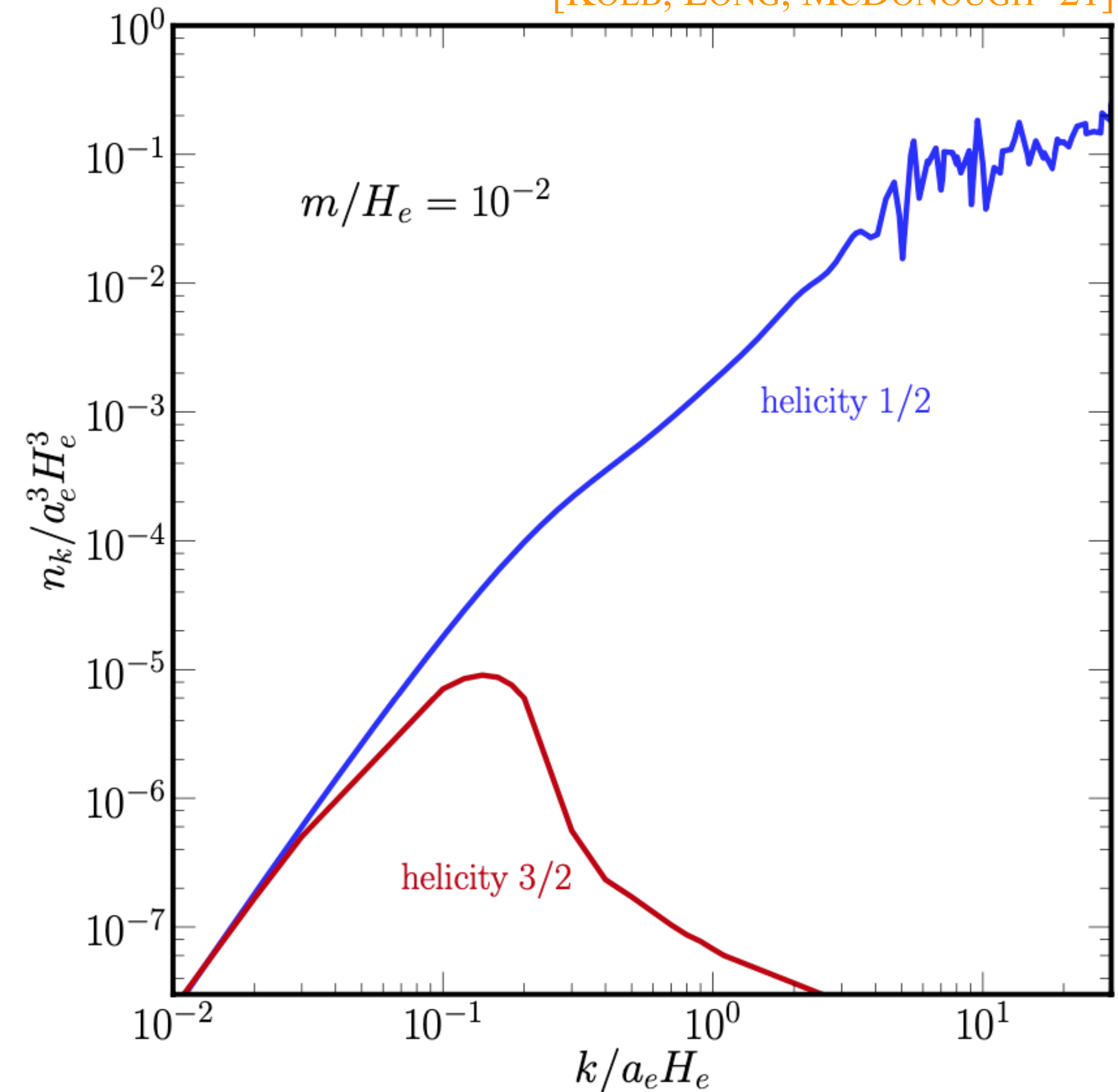
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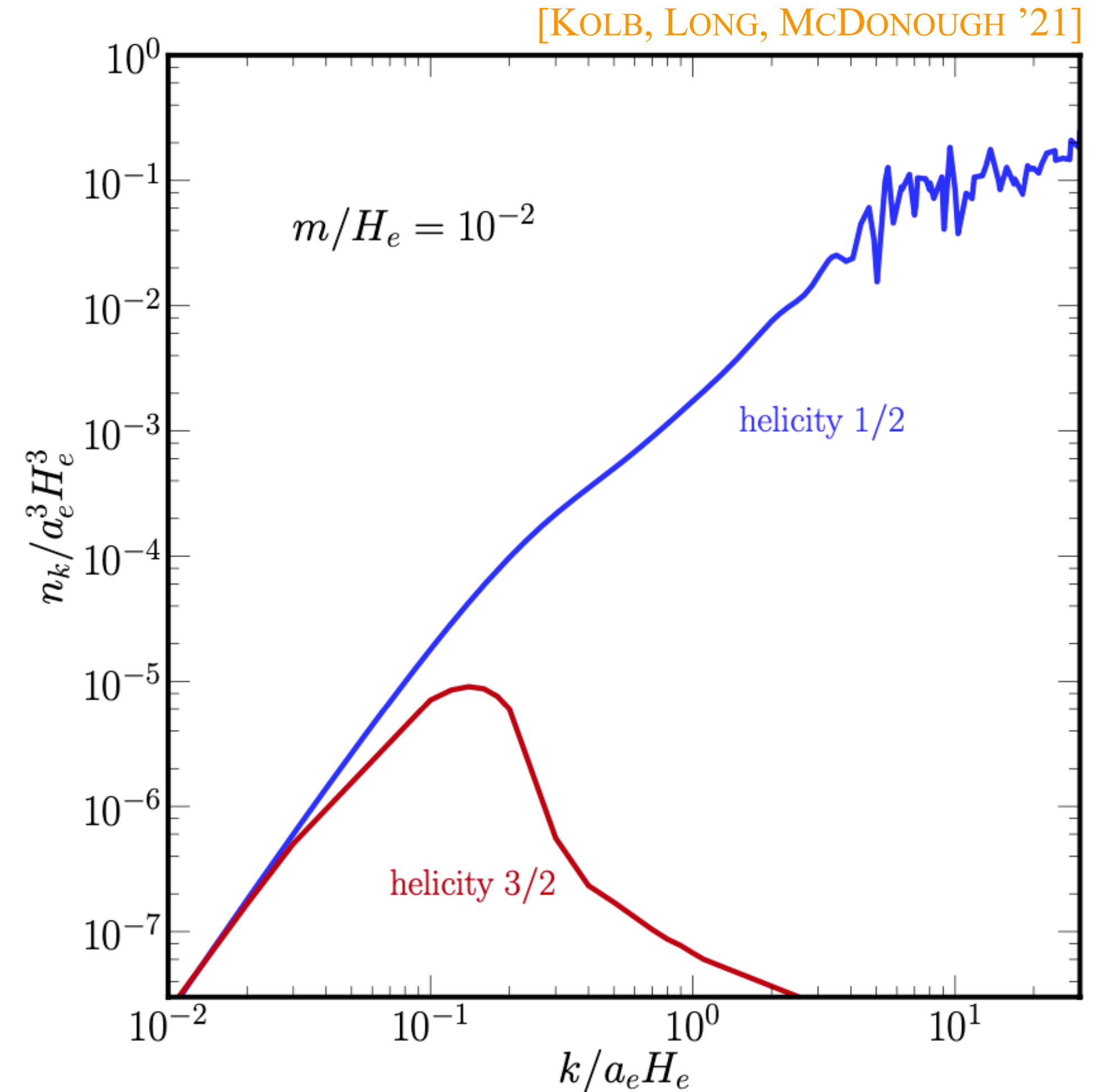
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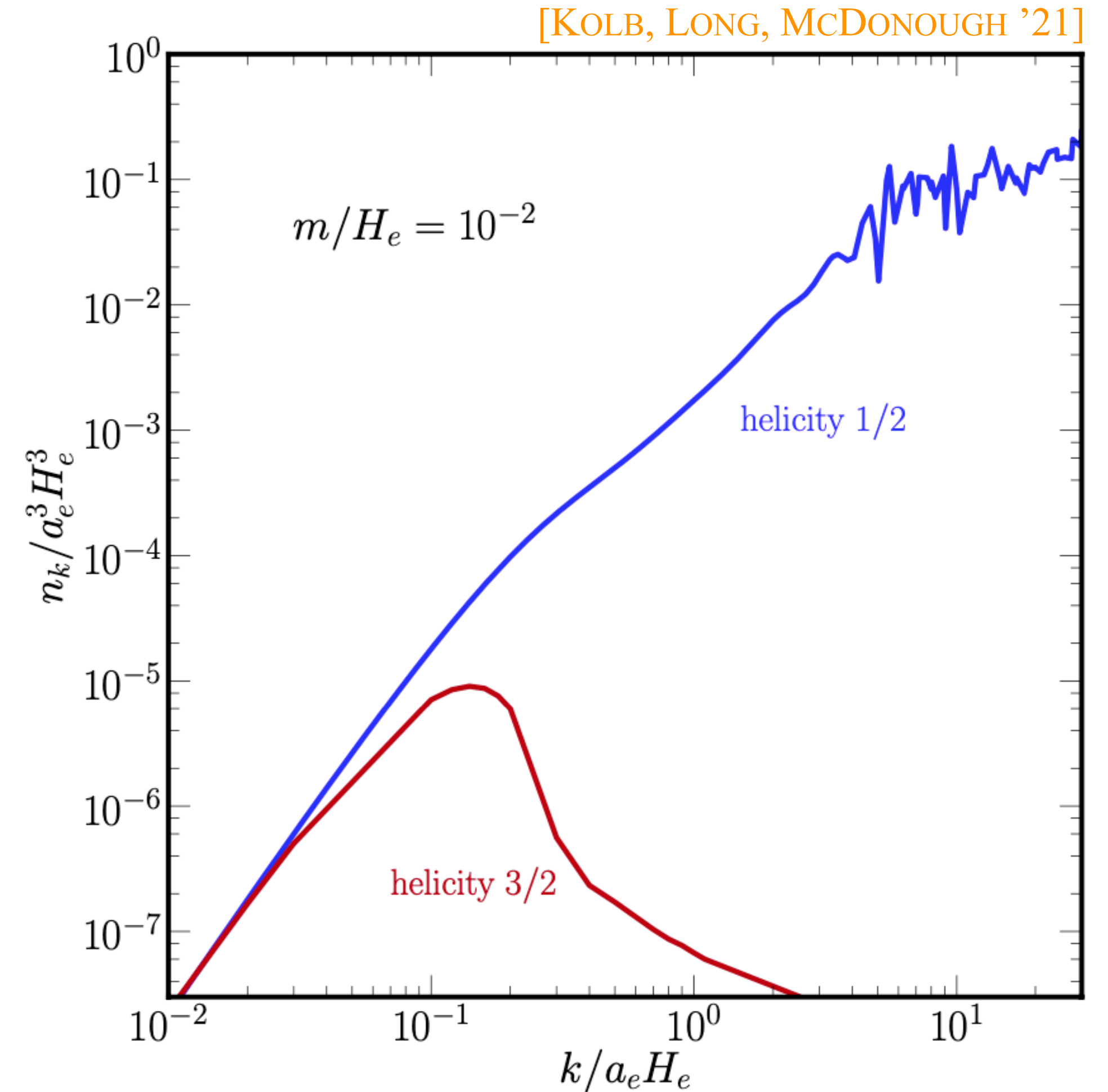
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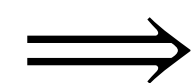
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We can address the superluminality problem in terms of positivity bounds on the operators of the low-energy, SUSY goldstino theory.

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- Goldstino - real scalar SUSY lagrangian:

$$\begin{aligned} \mathcal{L} = & -f(\varphi)^2 + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + i\partial_\mu G\sigma^\mu\bar{G} + (\dots) \\ & + \frac{1}{2f(\varphi)^2} \left(1 - \frac{2g'(\varphi)^2}{f(\varphi)^2} \right) \left(i\partial^\mu G\sigma^\nu\bar{G}\partial_\mu\varphi\partial_\nu\varphi + h.c. \right) - \frac{1}{4f(\varphi)^2} \left(1 - \frac{g'(\varphi)^2}{f(\varphi)^2} \right) \bar{G}^2 \square G^2 \end{aligned}$$

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it coincides with the $c_s < 1$ condition

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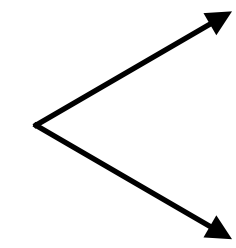
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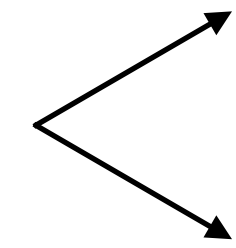
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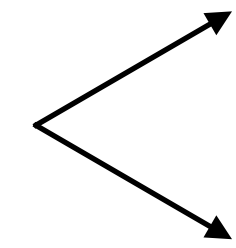
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right setup to build minimal inflation in Supergravity

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Gravitino Swampland Conjecture: $c_s = 0 \in \text{Swampland}$

[KOLB, LONG, MCDONOUGH '21]

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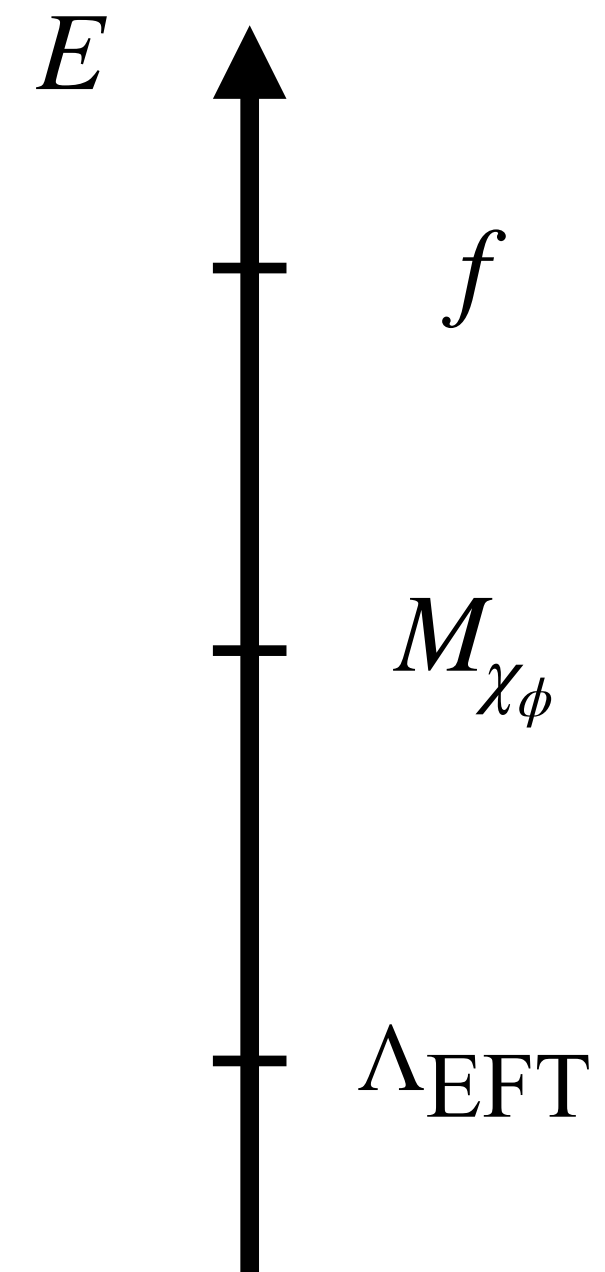
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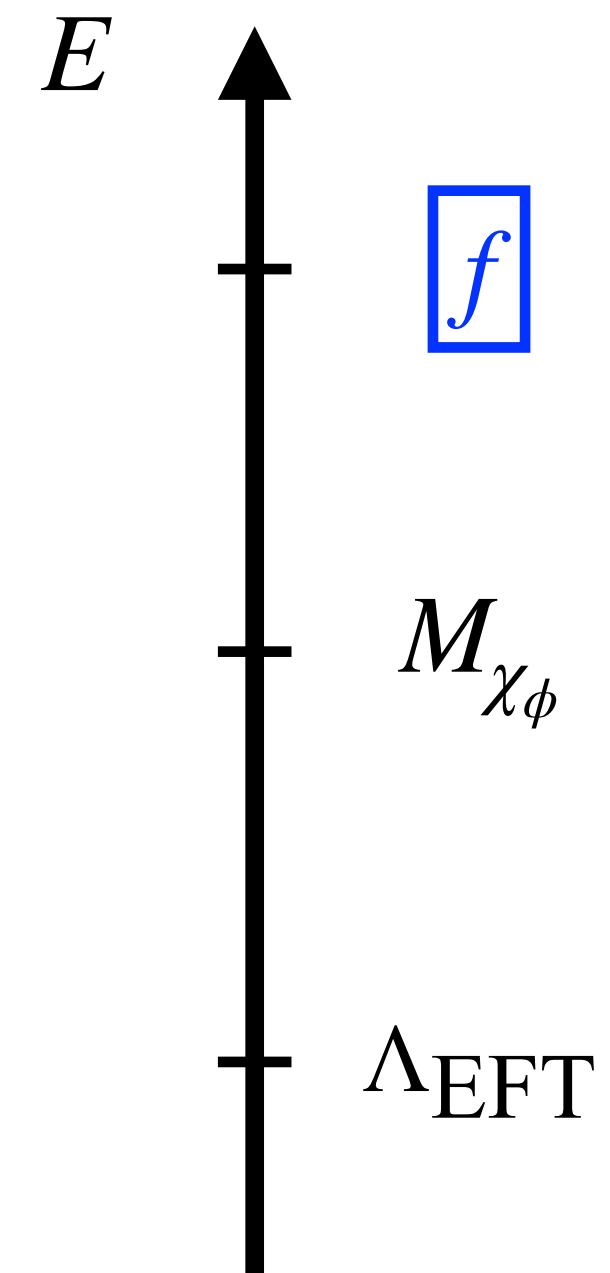
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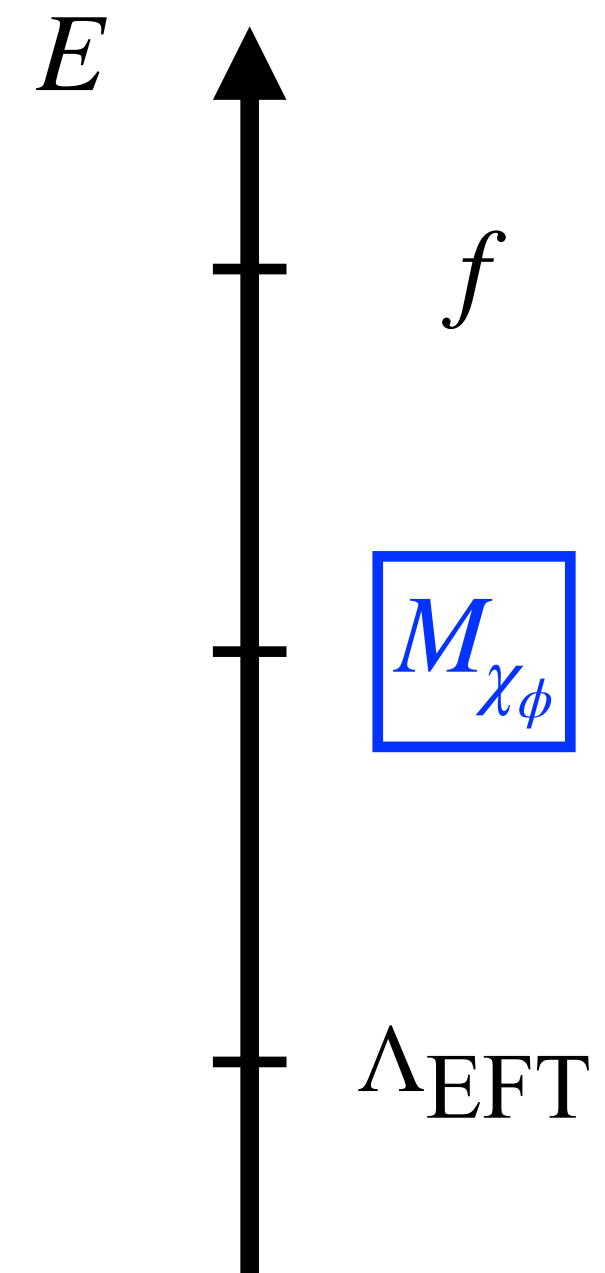
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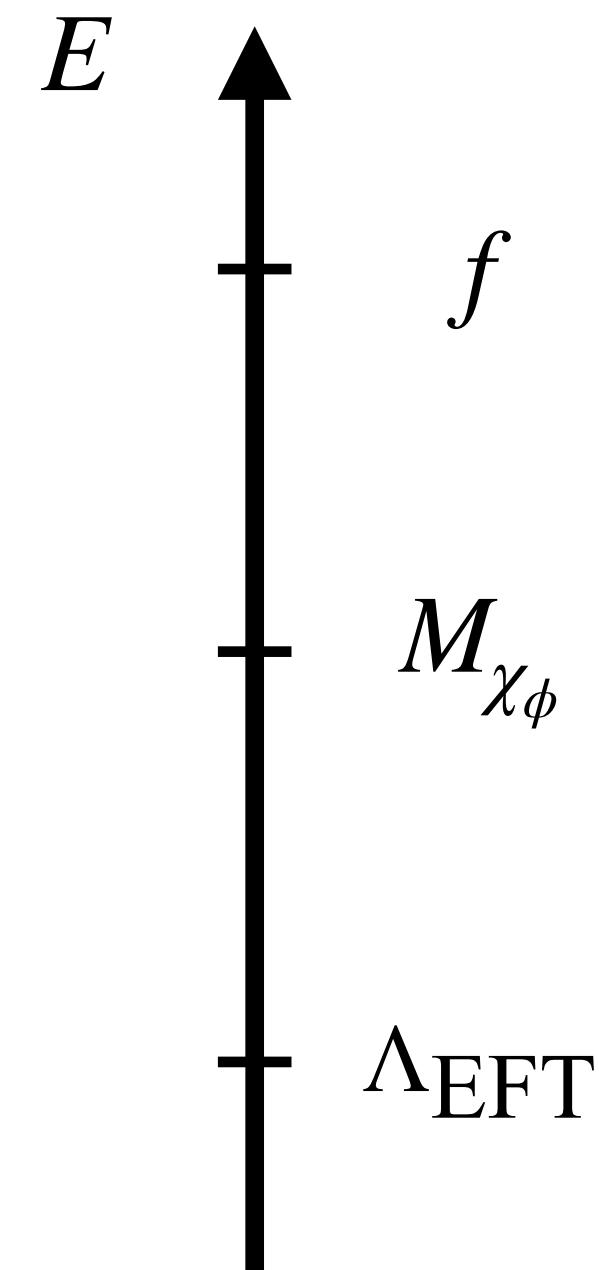
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the goldstino-inflatino mixing prevents the overproduction

[DUDAS, GARCIA, MAMBRINI, OLIVE, PELOSO, VERNER '21]



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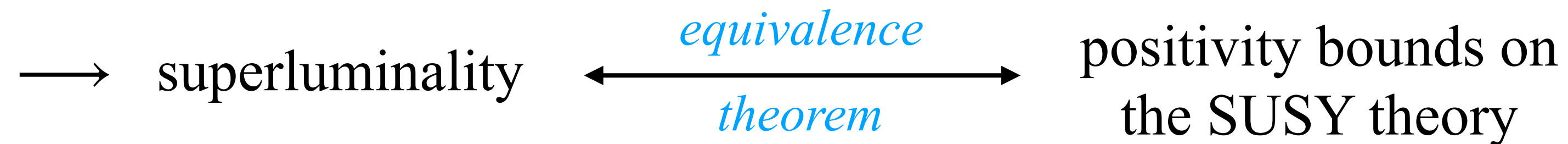
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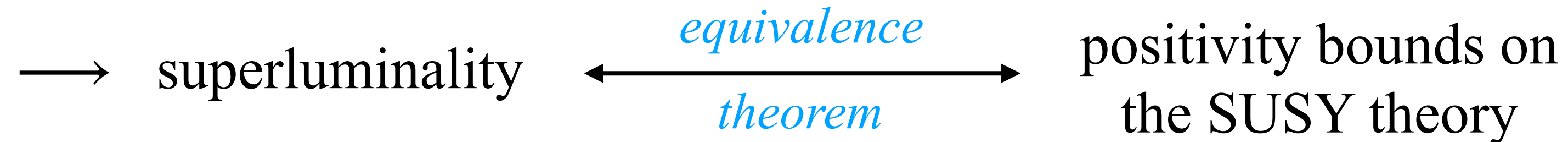
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→ superluminality $\xleftrightarrow[\textit{theorem}]{\textit{equivalence}}$ positivity bounds on the SUSY theory

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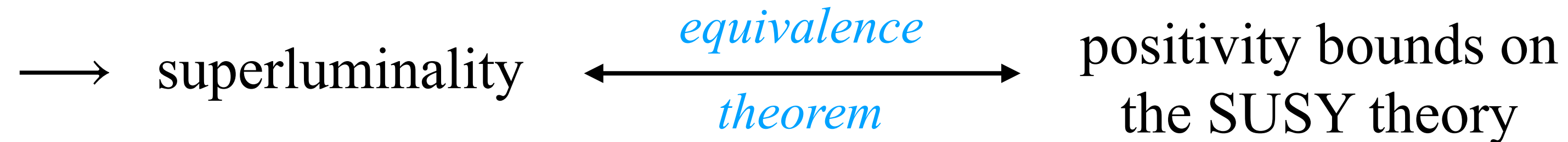
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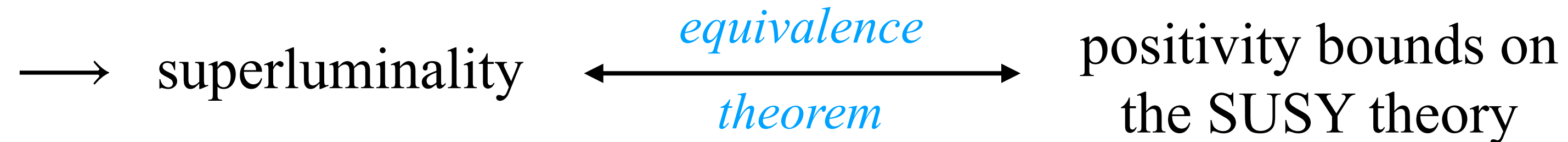
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$$c_s^2 < 1 \quad \forall \varphi$$

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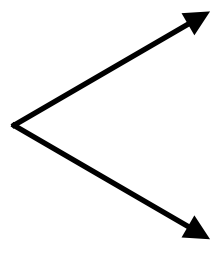
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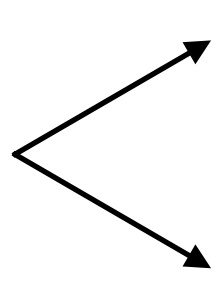
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the fermion mixing prevents the overproduction

Thank you!