

# The Quantum Gravity Cut-Off in a (convex) Nutshell

String Pheno '23  
Jul 2023

*Based on [arXiv:2306.16450] and [arXiv: 2307.XXXXXX]*

*Work with L. Ibáñez, A. Herráez and J. Calderón  
and WIP with I. Valenzuela and I. Ruiz*



Residencia de Estudiantes

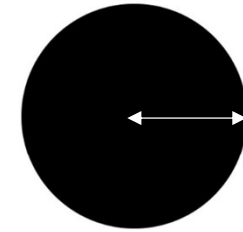
# The Species Scale

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• (Einstein) Gravity leads to some nice (and strange) solutions: **Black Holes**

• E.g. **Schwarzschild** solution

$$ds^2 = - \left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$



$$R_{\text{BH}} \gg \ell_{\text{Pl}}$$

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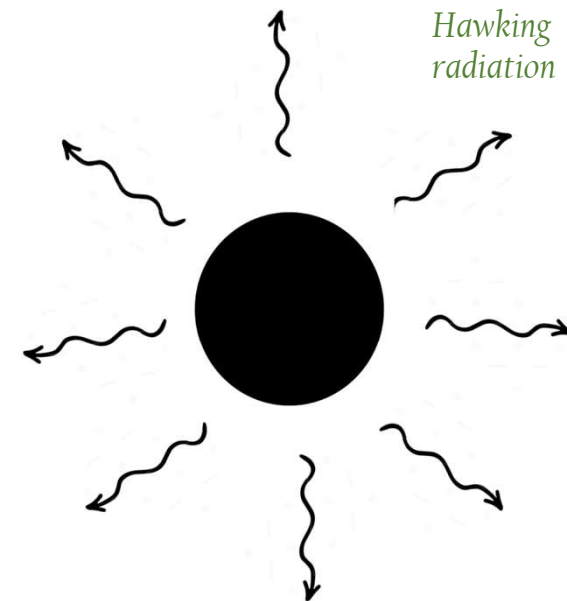
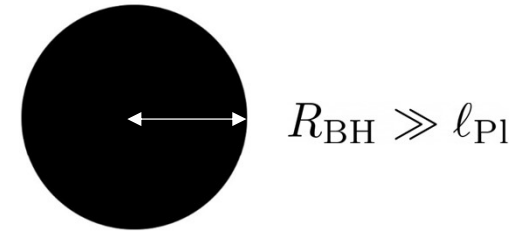
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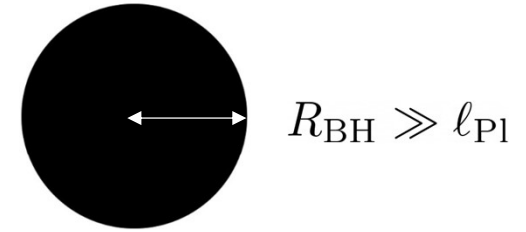
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$$T_{\text{BH}} \sim \frac{1}{T_{\text{BH}}} \sim \left(\frac{M_{\text{Pl}}^{d-2}}{M_{\text{BH}}}\right)^{\frac{1}{d-3}}$$



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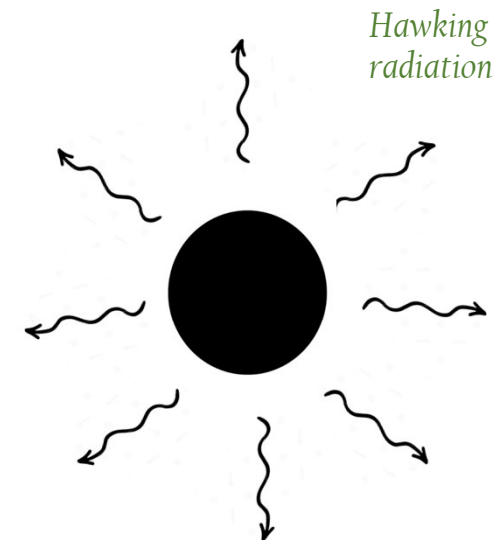


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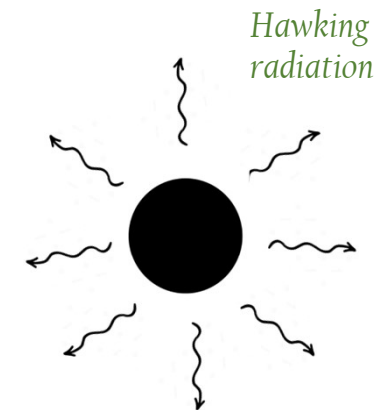
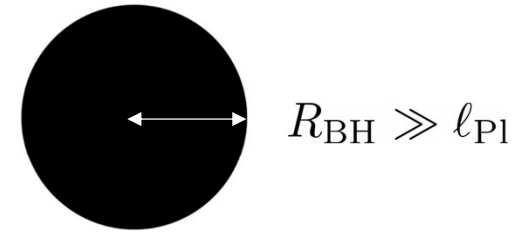
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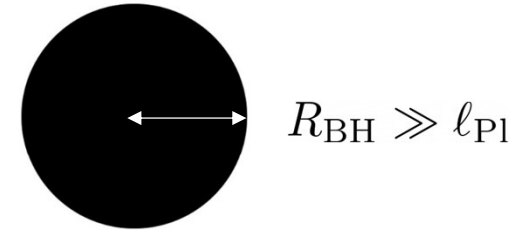
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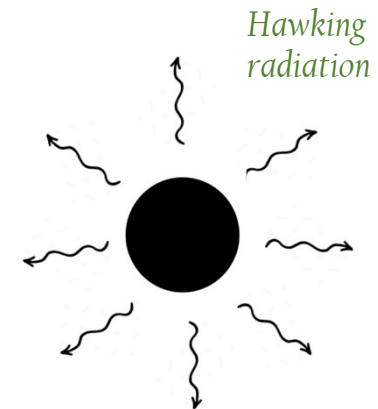


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• What is the **minimum** sized BH describable by our EFT?

# The Species Scale

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- A natural guess is to take it to be approx. of Planckian size



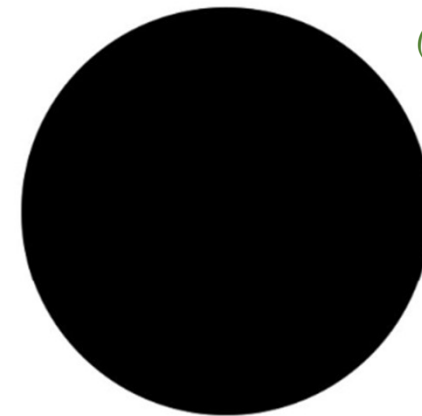
$$R_{\text{BH}} \sim \mathcal{O}(\ell_{\text{Pl}})$$

- It turns out that for theories with large number of species, this is an overestimate!  
(Problems with entropy, BH decay, ...)

- The correct QG scale is the Species Scale [Dvali, Redi, '07]

$$\ell_{\text{sp}} \sim \ell_{\text{Pl}} N^{\frac{1}{d-2}}$$

- Notice that for  $N \gg 1$  one finds that  $\ell_{\text{sp}} \gg \ell_{\text{Pl}}$

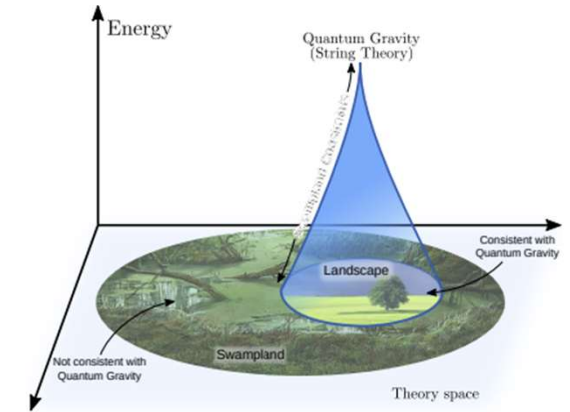


(in Planck units)

# The Species Scale

- The species scale plays a starring role in the **Swampland Program**

Max. Validity of any Eff. Field Theory Description!!



[van Beest, Calderón, Mirfendereski, Valenzuela, '21]

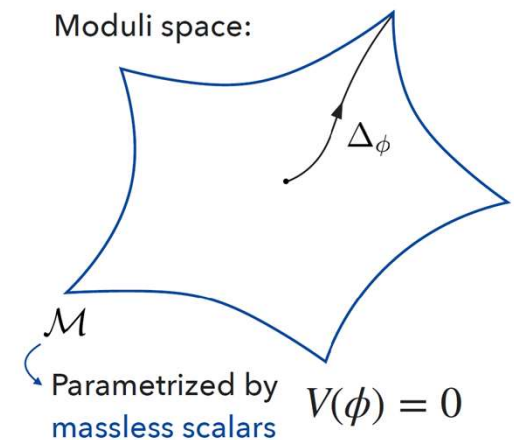
- Natural question:** Can we parametrically decouple such scale from the Planck mass? How low should it be when approaching infinite distance/weak coupling limits?

- Based on **String Theory** evidence and **entropy bounds** we propose:

$$\lambda_{\text{sp}} \geq \frac{1}{\sqrt{(d-2)(d-1)}}$$

See A. Herráez  
Talk!!

- Where  $\Lambda_{\text{sp}} \sim e^{-\lambda_{\text{sp}} \Delta\phi}$





# Outline

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## I. THE SPECIES SCALE DISTANCE CONJECTURE

The bound and its **convex hull** formulation

## II. A SIMPLE EXAMPLE IN 9D

M-theory on  $T^2$

## III. A 'DUALITY' IN DISGUISE

A cute **pattern** between light towers and the QG cut-off

## IV. SUMMARY & OUTLOOK

**Open Questions.** Future Directions

# A Convex Hull condition

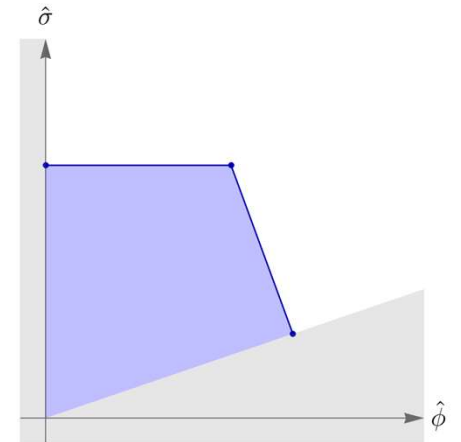
- In [Calderón, AC, Herráez, Ibáñez '23] we propose a bound for  $\lambda_{sp}$  in **any** inf. dist. limit
- In **multi-moduli** cases, what one defines is the charge-to-mass and species vectors

$$\zeta_t^i \equiv -\partial^i \log m_{\text{tower}} \quad \mathcal{Z}_{sp}^i \equiv -\partial^i \log \Lambda_{sp}$$

- In analogy with the (Sharpened) Distance Conjecture, it is useful to define a **convex hull** [Calderón, Uranga, Valenzuela '20]  
→ It gives us **information** about the **nature** of the inf. dist. limit

- This allows us to quickly **visualize** whether the SSDC holds or not for a given theory

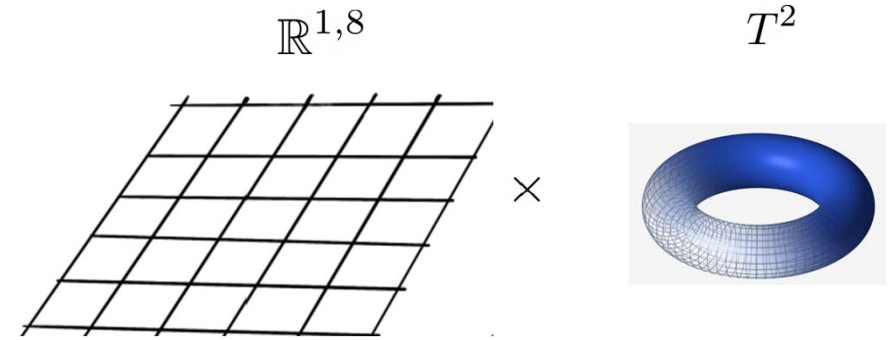
$$\lambda_{sp} = \max \left\{ \hat{n} \cdot \vec{Z}_{eff} \right\} \quad \rightarrow \quad \text{SSDC: } \lambda_{sp} \geq \lambda_{sp,min}$$



# A Simple Example in 9d

• Take M-theory on  $T^2$   $\longrightarrow$  9d N=2 SUGRA theory

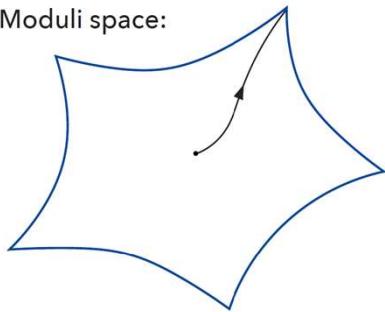
• The theory has a classically exact moduli space



$$\mathcal{M}_{9d} = \underbrace{SL(2, \mathbb{Z}) \backslash SL(2, \mathbb{R}) / U(1)}_{\text{Complex Structure}} \times \underbrace{\mathbb{R}}_{\text{Volume}}$$

• There are lots of towers and infinite distance limits... Each one of them characterized by some  $\Lambda_{sp}$

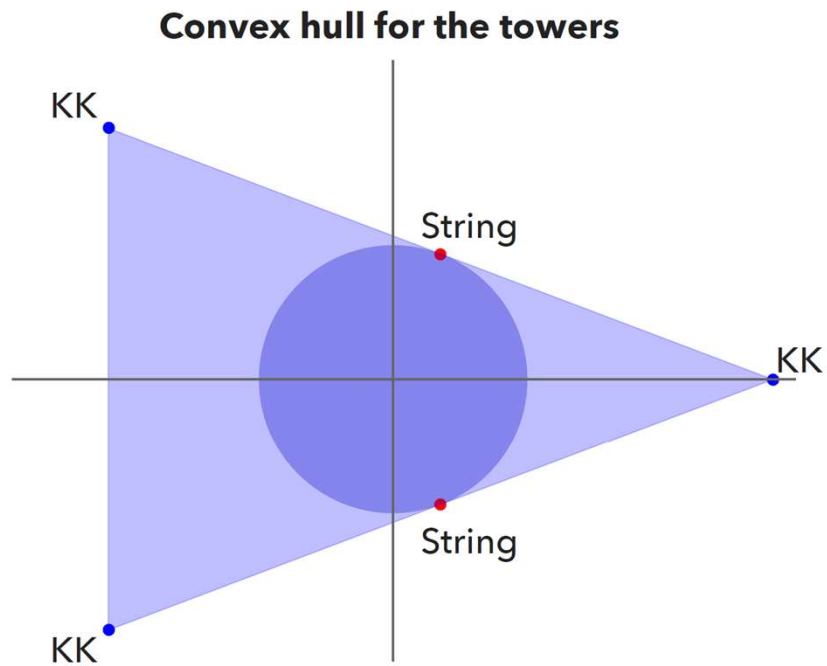
Moduli space:



# A Simple Example in 9d

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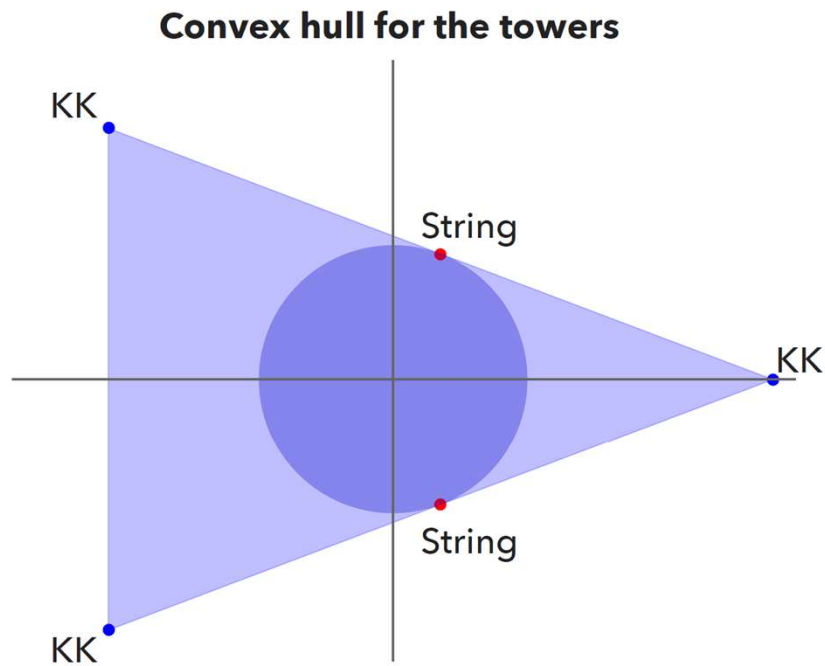
- We compute the **charge-to-mass vectors** of the towers and from them we extract those of the species scale



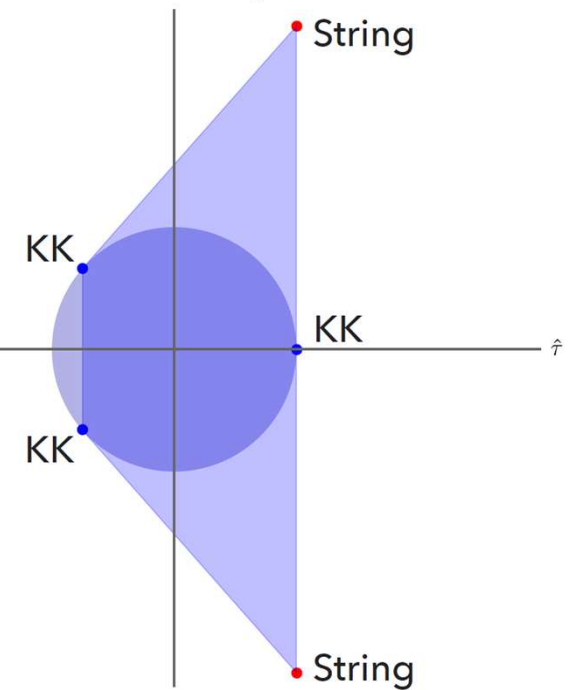
[Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]

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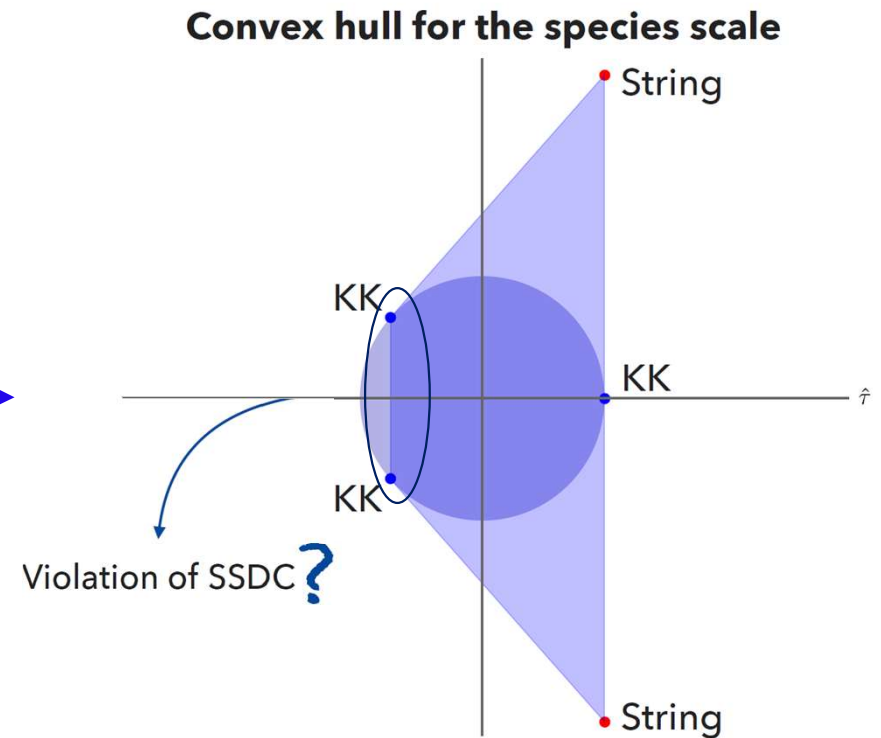
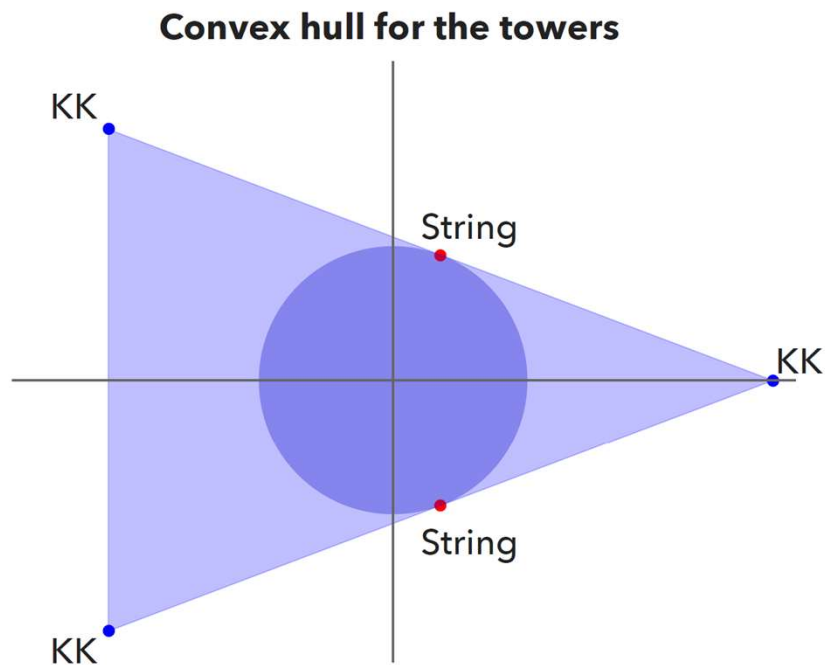
**Convex hull for the species scale**



[Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]

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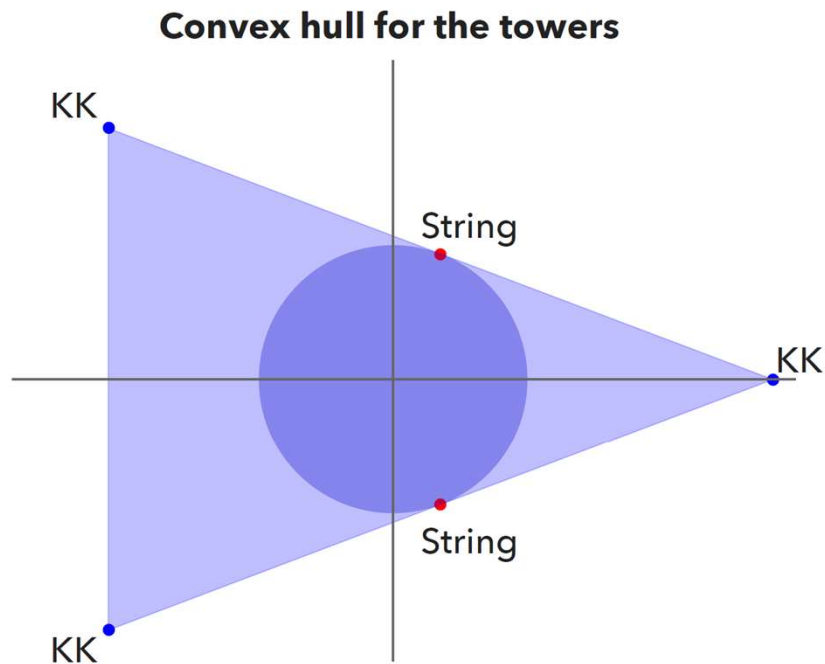
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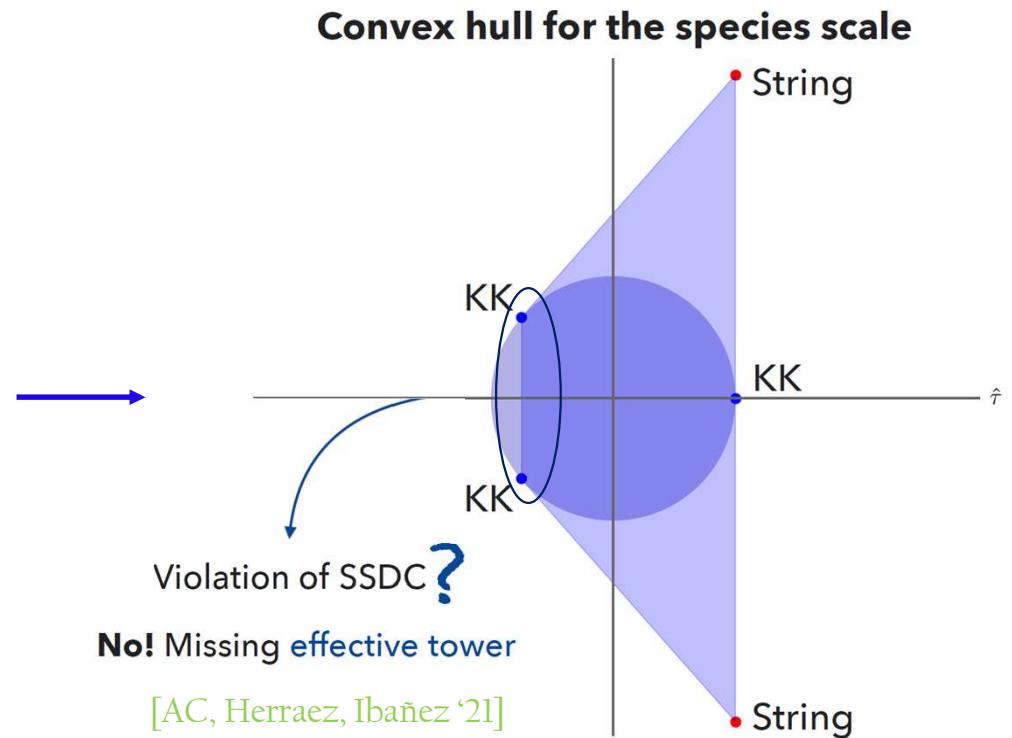
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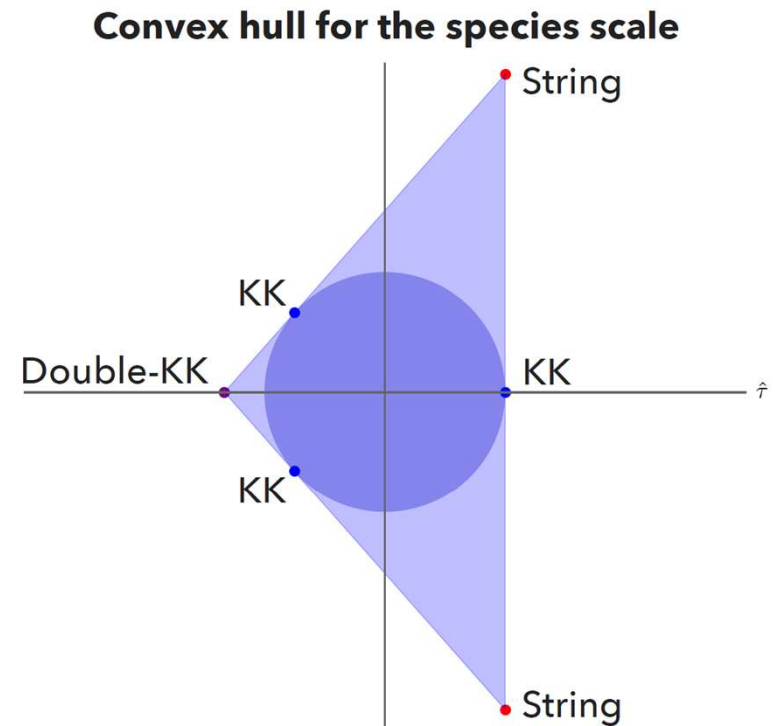
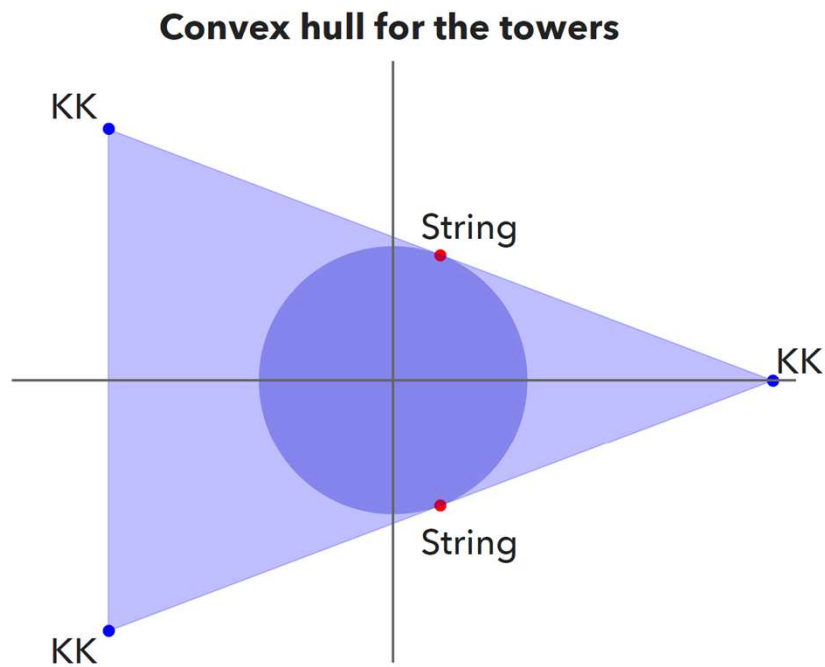


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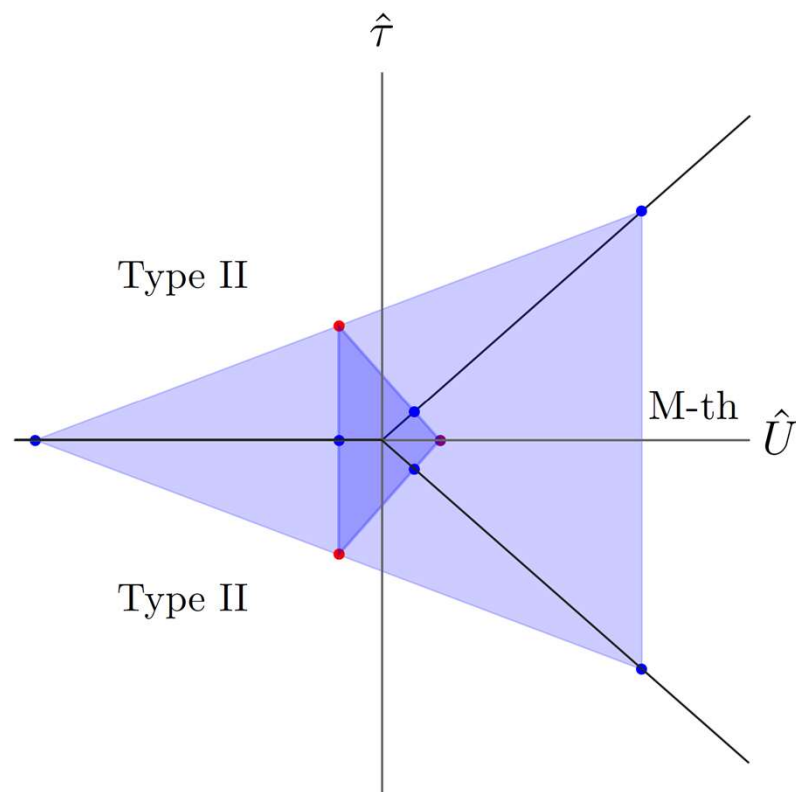


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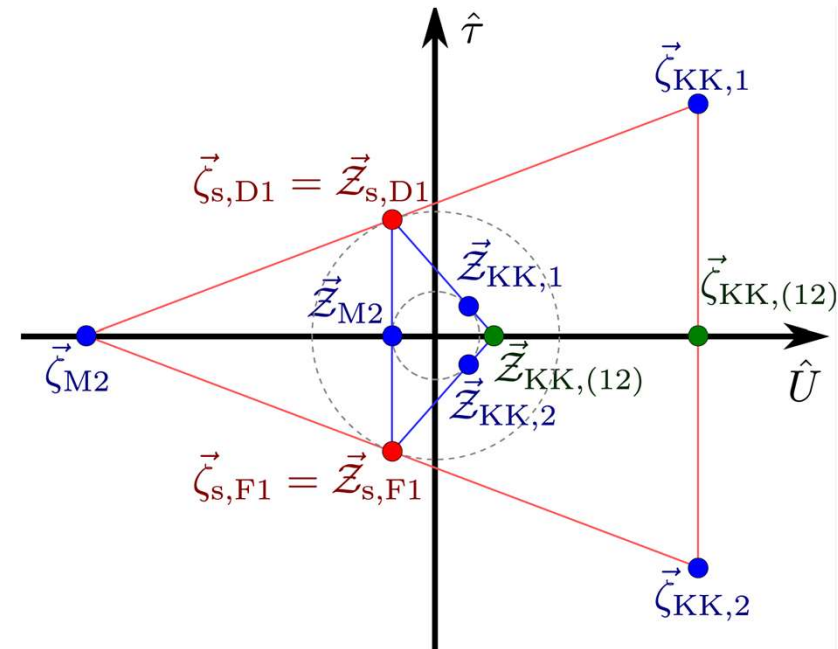
- The Species Scale provides **useful** information: it tells us about **duality frames**!



# A 'Duality' in disguise

- Let's have a closer look at the previous diagram
- Notice that the role between **generating/saturating** towers gets **reversed!**
- Moreover, strings are **fixed** under going to one convex hull to the other
- In addition, the faces of one hull are **orthogonal** to the vertices of the other

$$\vec{\zeta}_t \cdot \vec{z}_{sp} = G^{ij} (\partial_i \log m_{\text{tower}}) (\partial_j \log \Lambda_{sp}) = \frac{1}{d-2}$$



# A 'Duality' in disguise

- The proposal is that this **pattern**, namely

See I. Valenzuela's Talk!!

$$\vec{\zeta}_t \cdot \vec{Z}_{\text{sp}} = G^{ij} (\partial_i \log m_{\text{tower}}) (\partial_j \log \Lambda_{\text{sp}}) = \frac{1}{d-2} \longrightarrow G^{ij} (\partial_i \log m_{\text{tower}}) (\partial_j \log N) = -1$$

where one should take the **lightest** tower

- This seems to be **non-trivially** verified in all (to our knowledge) String Theory examples

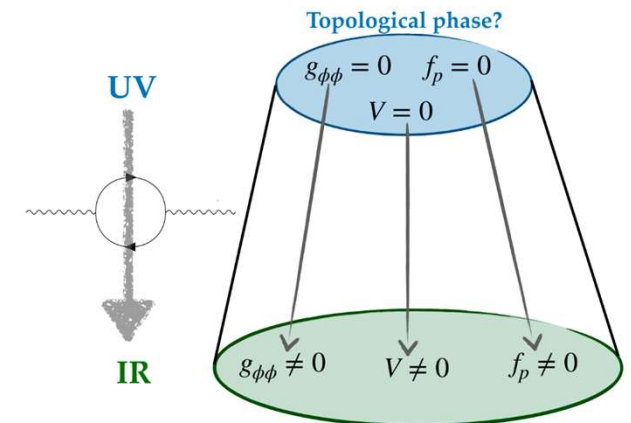
- It implies the **Sharpened Distance Conjecture** and is related to the **SSDC**

- Underlying **reason**? The Emergence Proposal? The Emergent

String Conjecture?

[Blumenhagen, AC, Corvilain, Gligovich, Grimm, Heidenreich, Herraez, Heidenreich, Kawamura, Marchesano, Melotti, Paraskevopoulou, Rudelius, Seo Valenzuela]

See L. Ibáñez Talk!!



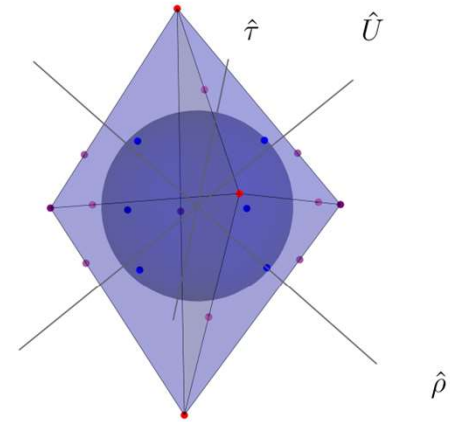
# Summary & Outlook

- There seems to be a lower **universal bound** for the exponential rate of the QG cut-off

$$\lambda_{\text{sp}} \geq \frac{1}{\sqrt{(d-2)(d-1)}}$$

- This is **satisfied** in many string theory examples in a non-trivial way

The concept of **effective tower** seems to be crucial



- There is moreover a **pattern** in string theory constructions relating the lightest towers of states to the

QG cut-off

$$\vec{\zeta}_t \cdot \vec{Z}_{\text{sp}} = \frac{1}{d-2}$$

- Is there any **simple explanation** for it? Perhaps Emergence?

Stay Tuned!!!

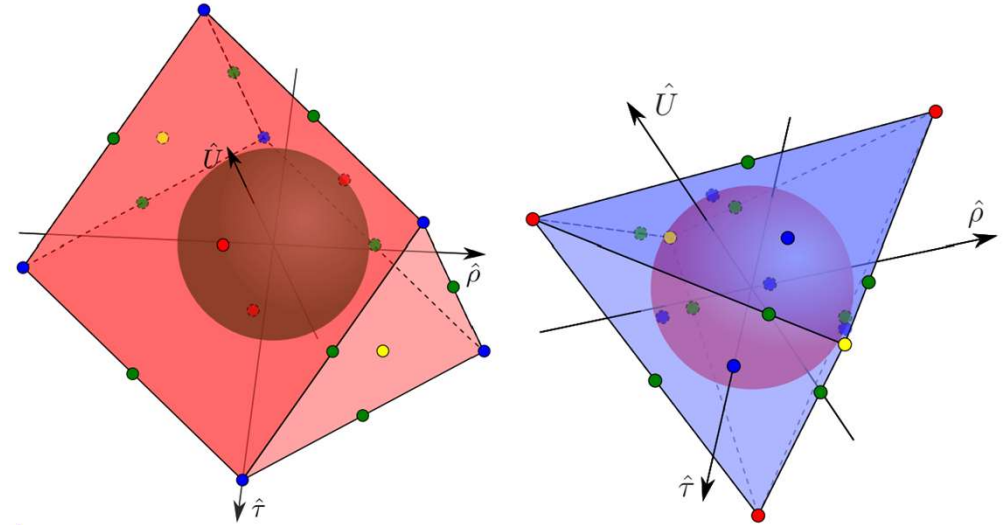
# Thank you for your attention!

QUESTIONS?

 Contact: [alberto.castellano@csic.es](mailto:alberto.castellano@csic.es)

# 8d Maximal SUGRA

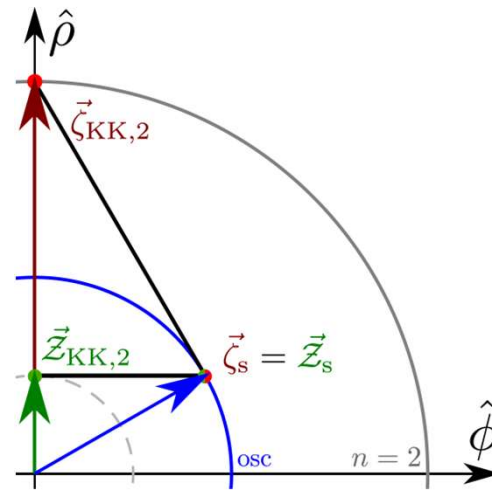
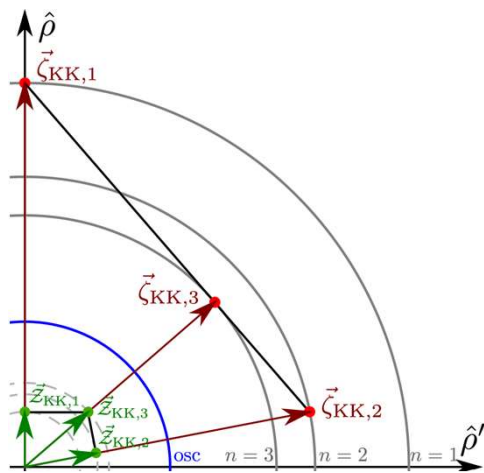
Again, BPS states generate both convex hulls



One can choose a 'fundamental domain' and check the pattern

There are essentially two possibilities: **Decompactification** or **emergent strings**

[Lee, Lerche, Weigand '19]



# Examples

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• In the papers we discuss several **examples** of the SSDC and the pattern:

○ **4d N=2** and **4d N=1** vacua

- i. Hypermultiplet moduli space
- ii. Vector multiplet moduli space

○ **Theories with 16 supercharges**

- i. Heterotic string theory on a circle
- ii. M-theory on K3

○ **Theories with maximal supersymmetry in  $d \geq 4$**

