On the origin of species thermodynamics

Niccolò Cribiori

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

String Phenomenology 2023,

IBS Daejeon, 4th July 2023

Based on 2212.10286, 2305.10489, 2306.08673,
with D. Lüst, C. Montella, G. Staudt

When gravity is strongly coupled?

The scale at which gravity becomes strongly coupled is called **species scale**. For d-dimensional EFTs with N species [Veneziano '01; Dvali, (Redi) '07,. . . Castellano, Herraez, Ibanez '22,. . .]

$$\Lambda_{sp} = \frac{M_P}{N^{\frac{1}{d-2}}} < M_P$$

It gives an upper bound on UV cutoff of EFTs with gravity.

Note: $N \gg 1$ for hierarchy to exist.
Is there underlying "statistical" interpretation?

Species scale and BH entropy

Species scale set by the size of the smallest possible black hole in the EFT. [Brustein, Dvali, Veneziano '09; Brustein, Medved '10]

A black hole (BH) of Planckian size leads to

$$\mathcal{S}_{BH} \sim (M_P R_{BH})^{d-2} \sim (M_P L_P)^{d-2} \sim 1$$

However, in a theory with N species, the BH should at least account for all of them. Thus, we should rather find

$$\mathcal{S}_{BH,min} \simeq N$$

This happens if the smallest possible BH has radius

$$R_{BH} = \Lambda_{sp}^{-1} > L_P.$$

[Bonnefoy, Ciambelli, Lüst, Lüst '19] studied limits of large/small BH entropy on moduli space. Same technique can be used to find N.

- Dilatonic BHs in heterotic string

$$S_{BH} \simeq pq \simeq p^2 g_s^{-2} \quad \Longrightarrow \quad S_{BH,min} \simeq g_s^{-2} \simeq N$$

reproducing [Dvali, Lüst '09]

- BPS BHs in type IIA on CY_3 ...

$$S_{BH} \simeq \sqrt{qp^3} \simeq p^3 \mathcal{V}_6^{\frac{1}{3}} \quad \Longrightarrow \quad S_{BH,min} \simeq \mathcal{V}_6^{\frac{1}{3}} \simeq N$$

- $\ldots + R^2$ corrections [NC, Lüst, Staudt '22]

$$S_{BH} \simeq \sqrt{q(p^3 + c_2 p)} \quad \Longrightarrow \quad S_{BH,min} \simeq \sqrt{qc_2 p} \simeq F_1 \simeq N$$

reproducing [Van de Heisteeg, Vafa, Wiesner, Wu '22]

Evidence that number of species has to be understood as entropy

$$N = \mathcal{S}_{sp}$$

Can we take this analogy any further?

Can we define energy and temperature?

Is there a thermodynamical picture behind? [NC, Lüst, Montella '23]

See talk by D. Lüst

Temperature and energy of species

[NC, Lüst, Montella '23]

- Schwarzschild BH has $\mathcal{S}_{BH} T_{BH}^{d-2} = 1$. Since $\mathcal{S}_{BH} \simeq N$, then

$$T_{sp} = \mathcal{S}_{sp}^{-\frac{1}{d-2}} \equiv \Lambda_{sp}$$

- As for E_{sp}, consider tower of N species with $\Delta E = \Lambda_{sp}/N$. k-*th* level has energy $E_k = k\Delta E$. Total energy of the tower is

$$E_{sp} = \sum_{k=1}^{N} E_k \simeq \Lambda_{sp}^{3-d} = (\mathcal{S}_{sp})^{\frac{d-3}{d-2}}$$

- T_{sp}, E_{sp} and $\mathcal{S}_{sp}$ obey the usual thermodynamic relation

$$\frac{1}{T} = \frac{\partial \mathcal{S}}{\partial E}$$

Laws of species thermodynamics

[NC, Lüst, Montella '23]

- **Zero-th law**: If two points in moduli space have same $\Lambda_{sp}(\phi)$, then they have same $T_{sp}(\phi)$

- **First law**: Two neighbouring stationary species towers are related by

$$\delta E_{sp} = T_{sp} \delta \mathcal{S}_{sp} + \dots$$

- **Second law**: The species entropy does not decrease when moving adiabatically towards boundary of moduli space

$$\delta \Lambda_{sp}(\phi) \leq 0, \qquad \delta \mathcal{S}_{sp}(\phi) \geq 0$$

  Naturally compatible with distance conjecture(s).

- **Third law**: It is impossible to reach $T_{sp} = 0$ with a finite sequence of steps.

Conclusions and future directions

- Gravitational EFTs with N species valid at most up to $\Lambda_{sp} = M_P / N^{\frac{1}{d-2}}$
- The function $N(\phi)$ can be calculated minimizing BH entropy over moduli space, see e.g. [NC, Lüst, Staudt '22]

  $$N = (\text{min})\text{ entropy.}$$

  Besides, an expression for $N(\phi)$ valid over all moduli space can be found with dualities [NC, Lüst '23]
- Laws of species thermodynamics from Schwarzschild BH formulated in [NC, Lüst, Montella '23]
- Extension to Reissner-Nordstrom BHs with charge Q?
  How to interprete Q, pressure, ...?

Thank you!

Extra slides

Black holes and attractors

We work within 4d N=2 SUGRA, with a prepotential $F = F(X)$. BH solutions are further specified by a central charge

$$Z = q_\Lambda X^\Lambda - p^\Lambda \partial_\Lambda F$$

with $\Lambda = 0, 1, \ldots, n_V$. X^Λ are scalars in vector multiplets, $\partial_\Lambda F$ their symplectic duals and q_Λ, p^Λ electric, magnetic charges.

The attractor mechanism [Ferrara, Kallosh, Strominger '95; Ferrara, Kallosh '96] states that at the horizon

$$p^\Lambda = -2\mathrm{Im} X^\Lambda, \qquad q_\Lambda = -2\mathrm{Im}\partial_\Lambda F$$

and the entropy is

$$\mathcal{S}_{BH} = \frac{Area}{4} = \pi Z \bar{Z}$$

Probing the moduli space with BHs

[Bonnefoy, Ciambelli, Lüst, Lüst, '19] proposed a general method to study moduli spaces via BH solutions of the same compactification.

- At the horizon, moduli are functions of charges, $\phi = \phi(q, p)$. Consider e.g. the volume modulus

$$\mathcal{V} = \mathcal{V}(q, p)$$

Then, we can express $\mathcal{S}_{BH} = \mathcal{S}_{BH}(q, p)$ as

$$\mathcal{S}_{BH} = \mathcal{S}_{BH}(q, \mathcal{V}), \qquad \text{or} \qquad \mathcal{S}_{BH} = \mathcal{S}_{BH}(\mathcal{V}, p)$$

- Large/small $\mathcal{S}_{BH}$ induced by large/small $\mathcal{V}$ (or any other modulus) and viceversa. Direct connection to **swampland distance conjecture**.

Application: Find smallest possible BH by tuning charges. Then, read moduli dependence of the species scale from the entropy.

Extremal BHs in heterotic on $K3 \times T^2$

Consider extremal dilatonic BHs in heterotic string on $K3 \times T^2$.

The entropy is given by [Ferrara, Kallosh '96]

$$\mathcal{S}_{BH} = \pi Z \bar{Z} = \pi p q$$

The attractor equations fix $g_s^{-2} = \frac{q}{p}$ and thus

$$\mathcal{S}_{BH} = \pi p^2 g_s^{-2}$$

The minimal entropy is reached for $p = 1$, giving

$$\mathcal{S}_{BH,min} \simeq g_s^{-2} \simeq N_{sp}.$$

It reproduces [Dvali, Lüst '10] for species = string states.

Extremal BHs in IIA on CY_3 (1/2)

A slightly more involved example is given by IIA on a CY_3.

At large volume, 4d N=2 SUGRA is described by

$$F(X) = \frac{1}{6} C_{ijk} \frac{X^i X^j X^k}{X^0}$$

where

$$z^i = \frac{X^i}{X^0}, \qquad i = 1, \dots, h^{11} \equiv n_V$$

are complexified Kähler moduli (actual Kähler moduli: $t^i = \mathrm{Im} z^i$).

BHs can be supported by charges $q, p^i > 0$, giving an entropy

$$\mathcal{S}_{BH} = \pi Z \bar{Z} = 2\pi \sqrt{\frac{q}{6} C_{ijk} p^i p^j p^k}$$

Extremal BHs in IIA on CY_3 (2/2)

The attractor equations fix the CY volume modulus

$$\mathcal{V}_6 = \frac{1}{6} C_{ijk} t^i t^j t^k = \sqrt{\frac{q^3}{\frac{1}{6} C_{ijk} p^i p^j p^k}}$$

and thus

$$\mathcal{S}_{BH} = 2\pi \mathcal{V}_6^{\frac{1}{3}} \left(\frac{1}{6} C_{ijk} p^i p^j p^k \right)^{\frac{2}{3}}$$

The minimal entropy is reached for $\frac{1}{6} C_{ijk} p^i p^j p^k = 1$, giving

$$\mathcal{S}_{BH,min} \simeq \mathcal{V}_6^{\frac{1}{3}} \simeq N_{sp}$$

Species = KK modes of decompactification of a 2-cycle [Lee, Lerche, Weigand '19; Fierro Cota, Mininno, Weigand, Wiesner '22].
Smallest possible cycle on a simply connected CY_3.

Extremal BHs with R^2 corrections

Certain higher derivative corrections to supergravity are known. For example, we have

$$S_{corr} = \frac{1}{96\pi} \int \underbrace{c_{2i} t^i}_{F_1} R \wedge *R, \qquad c_{2i} = \int c_2(CY_3) \wedge \omega_i$$

from R^4 term in 11D [Antoniadis, Ferrara, Minasian, Narain '97].
It is one-loop and non-renormalized [Green, Gutperle '97].

There exist BPS black hole solutions with this correction. These are MSW black holes [Maldacena, Strominger, Witten '97].

The interaction S_{corr} can be supersymmetrized on a graviphoton background A [Cardoso, de Wit, Mohaupt '98]

$$F = \frac{1}{6} C_{ijk} \frac{X^i X^j X^k}{X^0} + c_{2i} \frac{X^i}{X^0} A \equiv F_0 + F_1 A$$

The modified entropy

In practice, one deals with 4d N=2 SUGRA with (X^Λ, A), $\Lambda = 0, 1, \ldots, n_V$. The attractor mechanism works the same [Behrndt, Cardoso, de Wit, Kallosh, Lüst, Mohaupt '96]

$$p^\Lambda = -2\,\mathrm{Im} X^\Lambda, \qquad q_\Lambda = -2\,\mathrm{Im}\partial_\Lambda F(X, A), \qquad A = -64.$$

Entropy is obtained with Wald formula [Wald '91]

$$\mathcal{S}_{BH} = 2\pi \int_{S^2} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$

giving [Cardoso, de Wit, Mohaupt '98]

$$\mathcal{S}_{BH} = \pi \left[Z\bar{Z} + 4\mathrm{Im}\left(A\,\partial_A F(X, A)\right)\right]$$

Model-independent and valid for any $F(X, A)$.

The species scale and F_1

For black holes in IIA on CY_3, we find

$$\mathcal{S}_{BH} = \sqrt{\frac{q}{6}\left(C_{ijk}p^i p^j p^k + c_{2i}p^i\right)}$$

$$F_0 \qquad\qquad F_1$$

Entropy receives additive correction due to R^2 term.
Minimal entropy is now for $\frac{1}{6}C_{ijk}p^i p^j p^k = 0$ but $c_{2i}p^i \neq 0$, giving

$$\mathcal{S}_{BH,min} \simeq \sqrt{qc_{2i}p^i}$$

From the solution of the attractor equations, one finds

$$\sqrt{qc_{2i}p^i} \sim c_{2i}t^i \sim F_1$$

and thus we recover [Vafa, Van de Heisteeg, Wiesner, Wu '22]

$$N_{sp} \sim S_{BH,min} \sim F_1$$

A shortcut

One can arrive at the same result by using special geometry with coordinates (X^Λ, A) to rewrite the entropy as

$$\mathcal{S}_{BH} = \pi \left[e^{-K} + \frac{1}{6} c_{2i} \mathrm{Im} \frac{X^i}{X^0} \right]$$

Then, one can check that for the minimal charge configuration described above the two terms compete

$$e^{-K} \sim \frac{1}{6} c_{2i} \mathrm{Im} \frac{X^i}{X^0} \sim F_1$$

and thus

$$\mathcal{S}_{BH} = \pi \left[e^{-K} + \frac{1}{6} c_{2i} \mathrm{Im} \frac{X^i}{X^0} \right] \gtrsim F_1 \simeq \mathcal{S}_{BH,min}$$

Comments

- Our black hole argument employs SUGRA and can only detect large volume limit of F_1

$$F_1 \sim c_{2i} t^i .$$

To go beyond, one can use dualities [NC, Lüst to appear].

- A priori, the species scale we calculated in SUGRA is **unrelated** to the topological string and thus to [Vafa, Van de Heisteeg, Wiesner, Wu '22].
Enforcing that the two scales are instead one and the same amounts to impose

$$e^{\mathcal{F}_{BH}} = \mathcal{Z}_{BH} \equiv |\mathcal{Z}_{top}|^2 = e^{\mathcal{F}_{top} + \bar{\mathcal{F}}_{top}}$$

This is the conjecture of [Ooguri, Strominger, Vafa '04], here recovered from the species scale.

Black hole thermodynamics

For the seminal work of Bekenstein and Hawking, we know that BHs can be described with the language of thermodynamics.

Consider Schwarzschild BH with radius R_{BH}. We have ($M_P = 1$)

$$M_{BH} = (R_{BH})^{d-3}$$
$$T_{BH} = (R_{BH})^{-1}$$
$$\mathcal{S}_{BH} = (R_{BH})^{d-2}$$

from which

$$\mathcal{S}_{BH} T_{BH}^{d-2} = 1$$

Comments

- The 2nd law selects a preferred direction over moduli space, along which the number of species cannot decrease

$$\delta N \geq 0$$

For KK towers, it is towards large volume $\delta\mathcal{V} \geq 0$. For string towers, it is towards weak coupling $\delta g_s \leq 0$. Expected from **swampland conjectures** [Lee, Lerche, Weigand '19].

- The point $T_{sp} = 0$ is at infinite distance in moduli space.

- If two towers of species coalesce, the final species scale is always less then the minimum of the initial ones

$$\Lambda_{sp_1+sp_2} \leq \min(\Lambda_{sp_1}, \Lambda_{sp_2})$$