EFT strings and emergence

Based on: - 2211.01409 with Fernando Marchesano
- work in progress

Luca Melotti, String Pheno, July 2023

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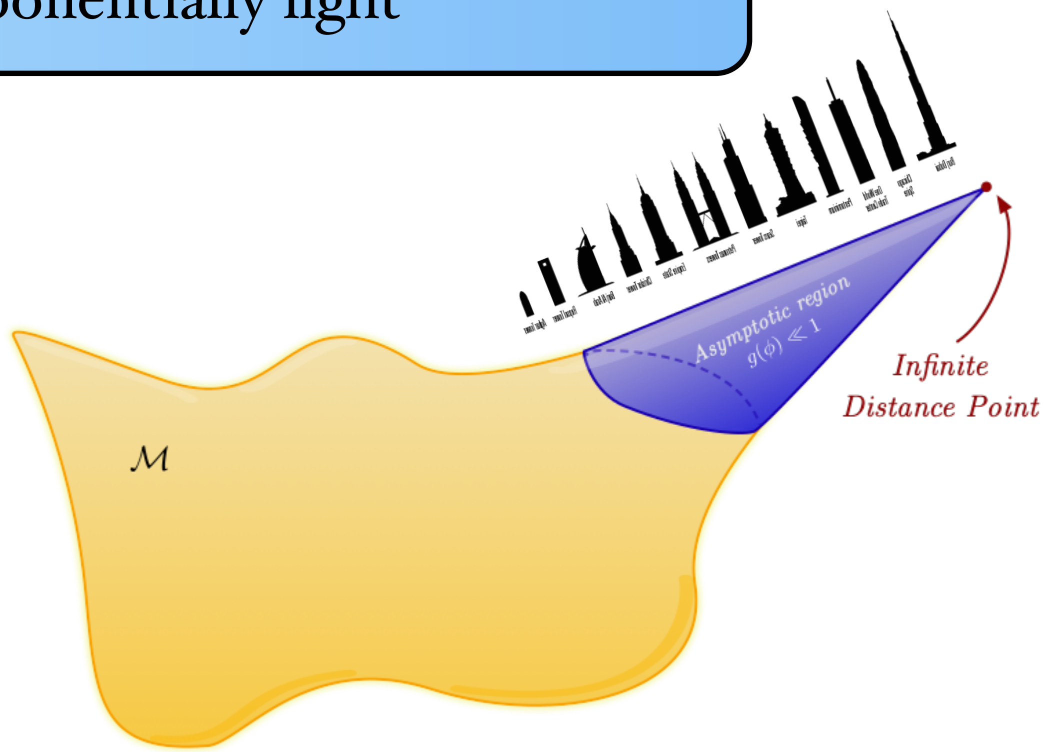
"la Caixa" Foundation

Emergence Proposal

Swampland Distance Conjecture: Ooguri & Vafa '06

Along geodesic paths of infinite distance there is an infinite tower of states which, asymptotically, becomes exponentially light



Emergence Proposal

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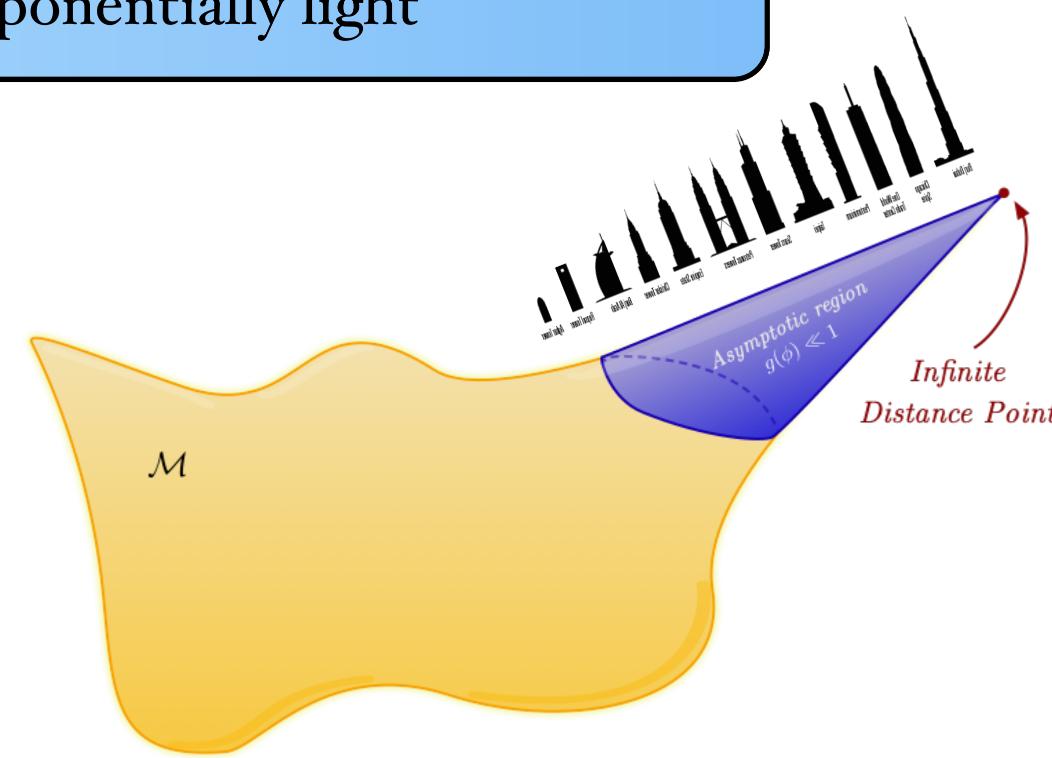
The Emergence Proposal "reverses the logic" of the Swampland Distance Conjecture



Harlow '15
Grimm, Palti, Valenzuela '18
Heidenreich, Reece, Rudelius '18

Emergence Proposal

> Kinetic terms in the IR emerge from integrating out towers of states down from the species scale $\Lambda_{\rm sp} = M_{\rm P}^{4d}/\sqrt{S}$

Emergence Proposal

$\Longrightarrow$ infinite distances and small gauge coupling limits in moduli space arise from integrating out the infinite towers of light states predicted by the SDC

We focus on gauge kinetic functions in 4d EFTs, which receive 1-loop corrections

$$I_{AB}^{\mathrm{IR}} = I_{AB}^{\mathrm{UV}} - \sum_{k=1}^{S} q_{k,A} q_{k,B} \log \frac{\Lambda_{\mathrm{sp}}}{m_k}$$

I_{AB}^{UV} ↓ finite!

$q_{k,A} q_{k,B}$ ↓ divergent!

Emergence Proposal

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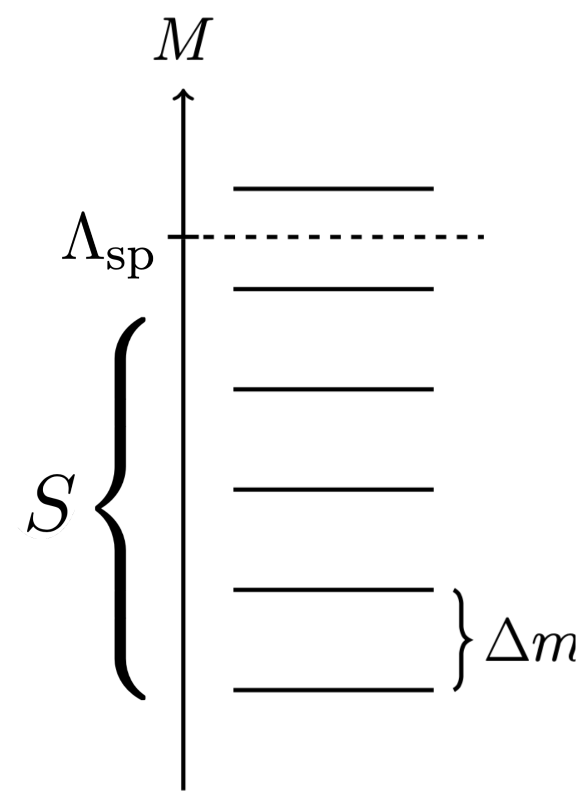
Corrections from states up to the species scale

Dvali '07

$$\Lambda_{sp} = \frac{M_P^{4d}}{\sqrt{S}}$$



Setup

Goal: test the Emergence Proposal along EFT string limits

Setup: type IIA compactifications on CY_3, vector multiplet moduli space

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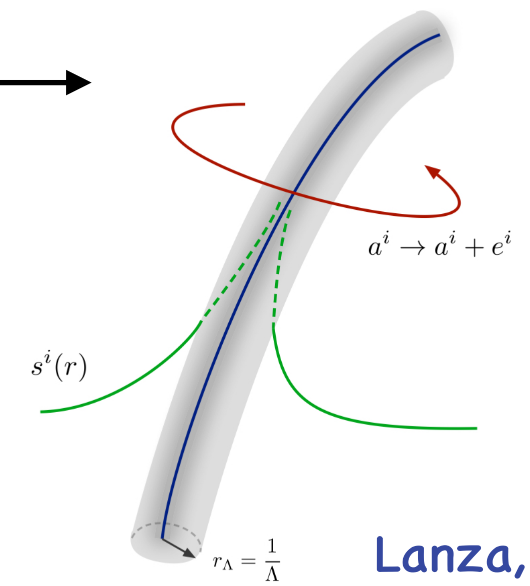
- EFT string = NS5-brane on Nef divisor $D = e^a D_a$ →
- complex scalars $T^a = b^a + it^a$
- $U(1)$ gauge fields F^0, F^a

↓

gauge kinetic terms

$$I = V_X \begin{pmatrix} 1 & 0 \\ 0 & 4g_{ab} \end{pmatrix}$$



Lanza, Marchesano, Martucci, Valenzuela '21

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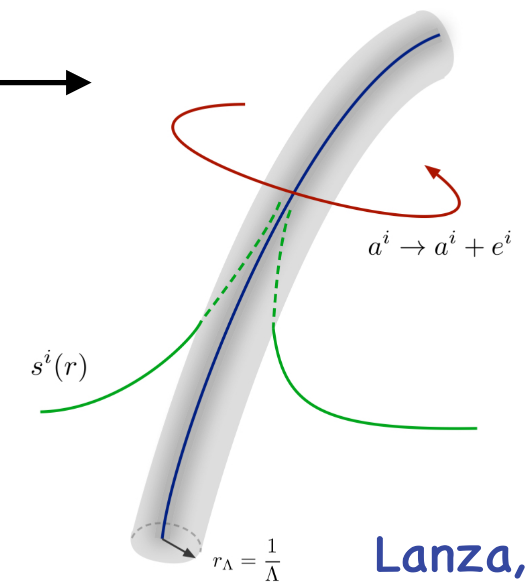

Lanza, Marchesano, Martucci, Valenzuela '21

Large volume and strong 10d coupling

$$\begin{cases} t^a = e^a \phi \quad \text{with} \quad \phi \to \infty \\ g_s(\phi) \sim \sqrt{V_X(\phi)} \to \infty \end{cases}$$

Emergence in type II CY

Grimm, Palti, Valenzuela '18

One can classify infinite distance limits in terms of the scaling weight w (or equivalently $K \sim -n \log \phi$)

$$m_*^2 \sim M_{\rm P}^2 \left(\frac{T_{\rm EFT}}{M_{\rm P}^2}\right)^w \qquad w = 1,2,3$$

Lanza, Marchesano, Martucci, Valenzuela '21

and try to reproduce the gauge kinetic terms via the Emergence Proposal

$$I_{AB}^{\rm IR} = I_{AB}^{\rm UV} - \sum_{k=1}^{S} q_{k,A} q_{k,B} \log \frac{\Lambda_{\rm sp}}{m_k}$$

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Classification of limits:

Corvilain, Grimm, Valenzuela '18   Lee, Lerche, Weigand '19

n=w	Description	Corvilain et al.	Lee et al.	
3	$k \neq 0$	IV_d	M-theory limit	$k = \kappa_{abc} e^a e^b e^c$
2	$k=0$, $k_a \neq 0$	III_c	J-Class A: T^2	$k_a = \kappa_{abc} e^b e^c$
1	$k_a=0$, $k_{ab} \neq 0$	II_b	J-Class B: T^4 or K3	$k_{ab} = \kappa_{abc} e^c$

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Result for some components:

n=w	I_{00}	I_{aa}	$(I_{00})^{EP}$	$(I_{aa})^{EP}$	ESC
3	ϕ^3	ϕ	ϕ^3	ϕ	KK-like tower
2	ϕ^2	ϕ^2	ϕ^2	$\phi^{2/3}$	KK-like tower
1	ϕ	ϕ	ϕ	ϕ	emergent (EFT) string

Emergent String Conjecture

Lee, Lerche, Weigand '19

→ string modes dominate!

Towers in EFT string limits

Mass scales along the limits

n=w	m_{KK}	$m^* = m_{D0}$	m_{D2}	$\Lambda_{NS5} = T^{1/2}$	Λ_{sp} (D0's)
3	$\phi^{-1/2}$	$\phi^{-3/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$
2	$\phi^{-1/2}$	ϕ^{-1}	ϕ^{-1}	$\phi^{-1/2}$	$\phi^{-1/3}$
1	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/6}$

→ Only considering D0's!

- Lightest tower of states: D0-branes
- Further tower: D2/D0 bound states charged under U(1)a

Key point: one needs to take into account all the leading towers in order to properly compute the species scale

Example

$X =$ elliptic fibration over $\mathbb{P}^2$, two moduli t^1, t^2

- $t^1 \equiv \phi \to \infty, \quad w = 3$ emergence computation = effective theory
- $t^2 \equiv \phi \to \infty, \quad w = 2$ emergence computation $\neq$ effective theory

$I_{11} \sim \phi^{2/3}$ $\qquad$ $I_{11} \sim \phi^2$

↓

light BPS towers of D2+D0 with fixed D2 charges and growing D0 charge $\qquad$ Grimm, Li, Palti '18

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light BPS towers of D2+D0 with fixed D2 charges and growing D0 charge

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Key difference between the two limits: $\begin{cases} w = 3 \to \text{only } m_{D0} \ll \Lambda_{sp} \\ w = 2 \to \text{both } m_{D0}, m_{D2} \ll \Lambda_{sp} \end{cases}$

n=w	m_{KK}	m_{D0}	m_{D2}	$T^{1/2}$	Λ_{sp} (D0's)
3	$\phi^{-1/2}$	$\phi^{-3/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$
2	$\phi^{-1/2}$	ϕ^{-1}	ϕ^{-1}	$\phi^{-1/2}$	$\phi^{-1/3}$

Example

⇒ in $w = 2$ double tower of states with arbitrary D2 and D0 charges

↳ D2 wrapping multiple times an elliptic fibre

⇒ correct the species scale

n=w	m_{KK}	m_{D0}	m_{D2}	$T^{1/2}$	Λ_{sp}	Limit
3	$\phi^{-1/2}$	$\phi^{-3/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	5d M-theory
2	$\phi^{-1/2}$	ϕ^{-1}	ϕ^{-1}	$\phi^{-1/2}$	$\phi^{-1/3} \to \phi^{-1/2}$	6d F-theory

No other tower below Λ_{sp}!

Example

$\Rightarrow$ in $w = 2$ double tower of states with arbitrary D2 and D0 charges

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$\Rightarrow$ correct the species scale

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No other tower below $\Lambda_{\rm sp}$!

Considering the double tower of D2-D0 states in the w = 2 limit lowers the species scale but increases the number of species

$\Rightarrow$ there is a compensating effect

$$I_{00} \sim I_{11} \sim \sum_{j,k=-S_{\rm D\mathit{p}}}^{S_{\rm D\mathit{p}}} k^2 \log \frac{\Lambda_{\rm sp}}{m_{(j,k)}} \sim S_{\rm D\mathit{p}} \cdot S_{\rm D\mathit{p}}^3 \sim \phi^2 \longrightarrow$$

matches with the EFT scaling!

↓ sum over both D2 and D0 charges!

w=1 limits

Plenty of leading towers, including that of the EFT string oscillations

n=w	m_{KK}	$m^* = m_{D0}$	m_{D2}	$m_{D4,\text{fibre}}$	$T^{1/2}$
1	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$

The leading contribution to Λ_{sp} depends on whether the EFT string is:

- non-critical → D4/D2/D0 bound states dominate and set Λ_{sp}
- critical → string modes dominate and set Λ_{sp}

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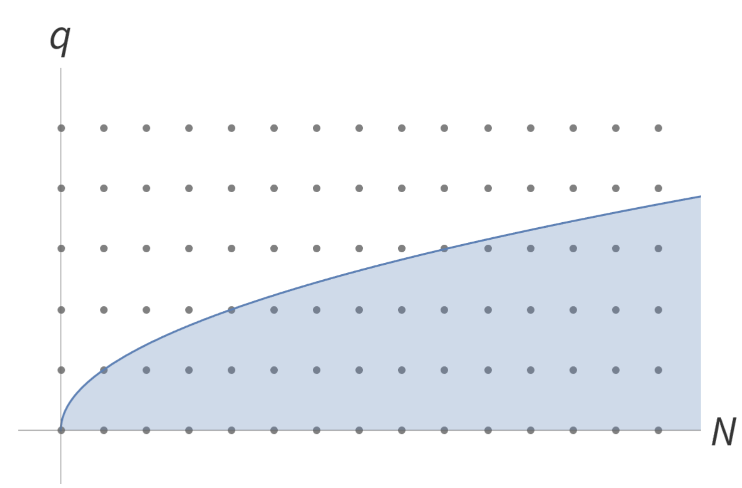
- non-critical → D4/D2/D0 bound states dominate and set Λ_{sp}
- critical → string modes dominate and set Λ_{sp}

$$I_{AB} \sim \phi\,, \quad \Lambda_{\text{sp}} \sim \sqrt{T}$$

- $m_N^2 = TN$, exponential degeneracy in N Castellano, Herraez, Ibañez '22

- spectrum populated by charged states with $|\mathbf{q}|^2_{\text{max}} \sim N$ (saturate the BPS bound)



motivated by anomaly inflow on axionic strings Heidenreich, Reece, Rudelius '21

Remarks

- It is crucial to sum over all the towers below the species scale

- The towers that set Λ_{sp} must enter the 1-loop corrections
  $\Rightarrow$ charged states

- In $w = 1$ limits we recover the correct scaling for both critical and non-critical EFT strings

Geometry of the scaling weight

- Focus on $w = 2,3$ $\longrightarrow$ decompactification limits
- Space-time $M_D = M_4 \times X_d$ with metric $ds_D^2 = e^{2A} ds_4^2 + M_D^2\, ds_X^2$
- X characterized by length scales L_I $\longrightarrow$ $V(X) = V_X^0 \prod_I L_I^{d_I}\,, \quad d = \sum_I d_I$

  $\downarrow$ associated to KK towers

- EFT string = higher dim brane wrapping $\Sigma \subset X$

  $\hookrightarrow$ $V(\Sigma) = V_\Sigma^0 \prod_I L_I^{\hat{d}_I}\,, \quad \hat{d} = \sum_I \hat{d}_I$

  $V(\Sigma)$ controls the EFT string tension

Geometry of the scaling weight

- Restricting to single scale L_0 $\longrightarrow$ scaling weight $w = \dfrac{2-d}{d-\hat{d}}$

- Restrict possible decompactifications:

w	$\hat{d}$	d
3	0	1
	2	4
	4	7
2	0	2
	1	4
	2	6

- Generalized version for many scales L_I

- Tested in examples (type IIA, M-theory,...)

Conclusions

- EFT strings are a useful tool to explore infinite distance limits of 4d EFTs. They allow to classify such limits in terms of the scaling weight w, which carries information about the mass of the leading tower m_*.

- In type IIA CY_3 compactifications EFT strings are given by NS5-branes wrapping Nef divisors. The other relevant towers are bound states of Dp-branes wrapping internal cycles.

- We tested the Emergence Proposal in this context, focusing on the vector multiplet moduli space and the gauge kinetic function. In $w = 2$ limits the relevant tower is a 2-dim D2-D0 bound states tower. In general the leading tower must dominate both the computation of the species scale and the 1-loop corrections.

- $w = 1$ limits contain emergent EFT string limits, in which the oscillations of the critical string dominate the spectrum and should be charged. It would be important to look at explicit EFT string spectra to test these assumptions.

- In decompactification limits the scaling weight w encodes geometrical information about the decompactification.

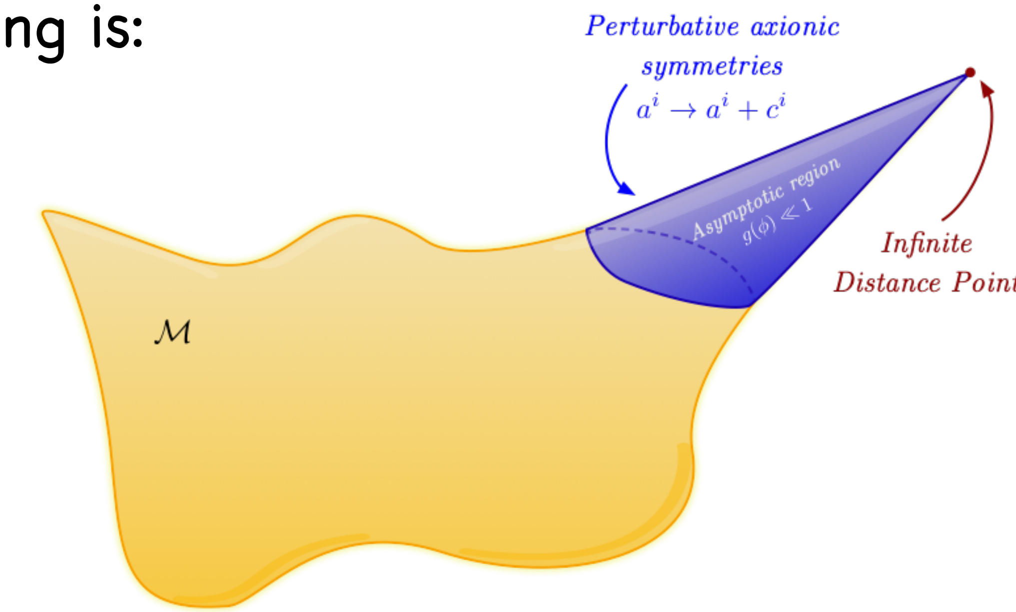
Thank you!

Backup slides

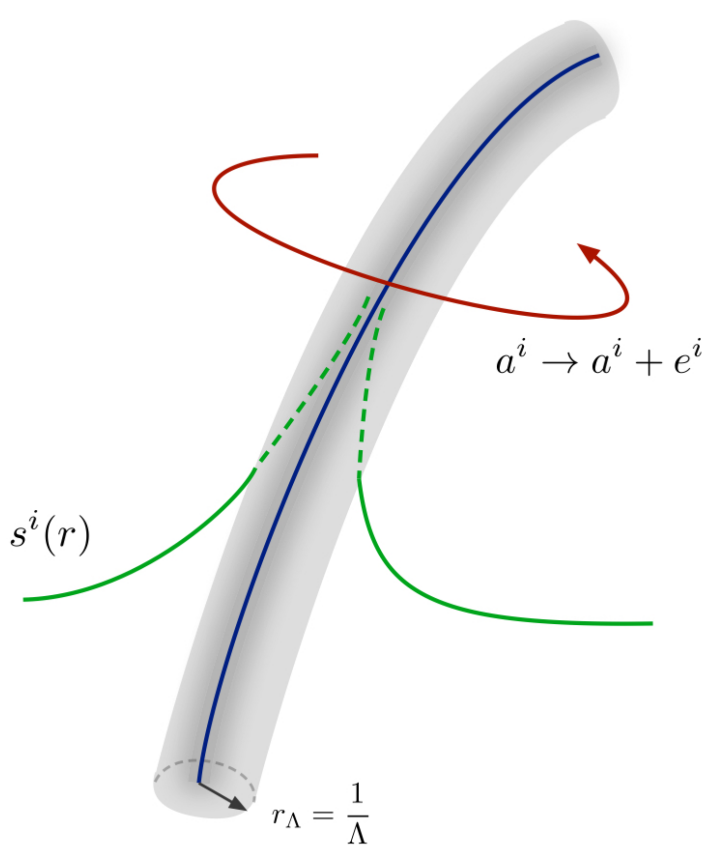
EFT strings

Lanza et al. '21

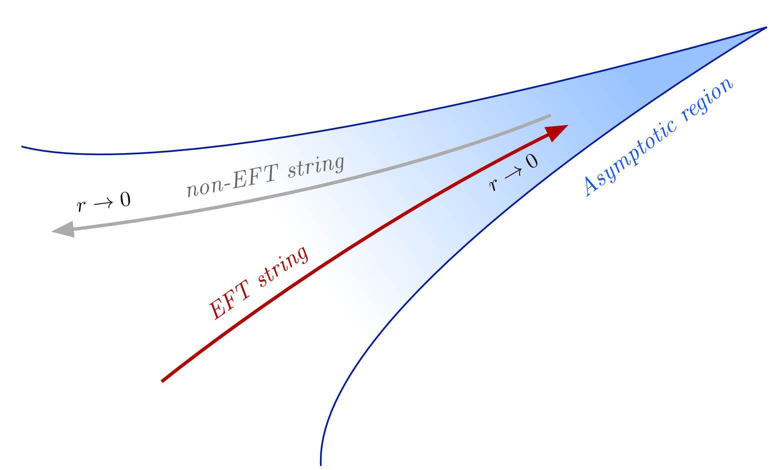
In a 4d EFT coupled to gravity, in an asymptotic region of the moduli space with a perturbative axionic shift symmetry, an EFT string is:



- a fundamental string, $\Lambda^2 < T < 2\pi M_P^2$
- an axionic string, $a^i \to a^i + e^i$, $e^i \in \mathbb{Z}$ around its core



- approaching its core all the non-perturbative effects that break the shift symmetry are suppressed



EFT string flows

Lanza et al. '21

Backreaction on the moduli:

- axion monodromy $a^i \to a^i + e^i$ for $\theta \to \theta + 2\pi$
- saxions $s^i \to e^i \cdot \infty$ for $r \to 0$

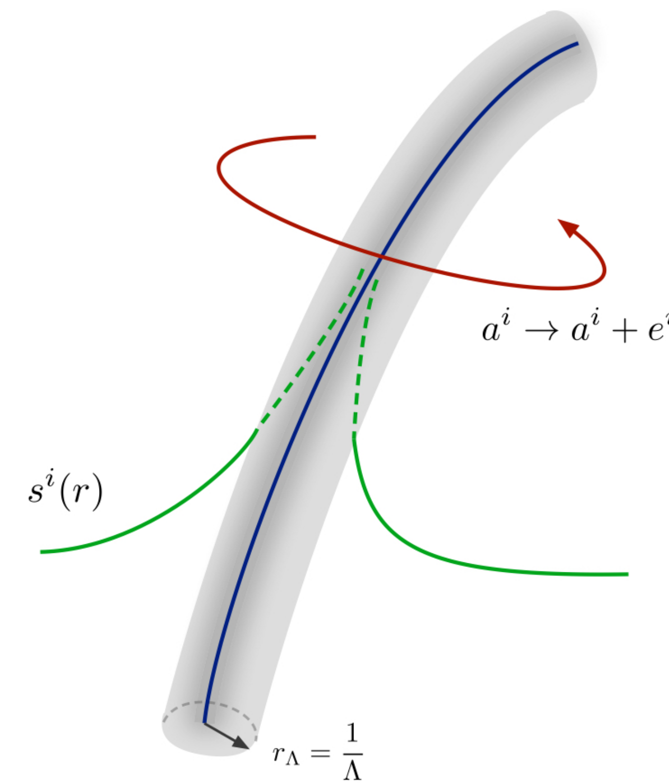
$\updownarrow$

$s^i \sim e^i \phi$ for $\phi \to \infty$

EFT string flow

$\downarrow$

EFT string becomes tensionless, but there can be lighter towers



Integral Scaling Conjecture

In an asymptotic limit specified by an EFT string flow, the leading tower mass m_* scales as

$$m_*^2 \sim M_{\rm P}^2 \left(\frac{T}{M_{\rm P}^2}\right)^w \qquad w = 1,2,3$$

Example

$X =$ elliptic fibration over $\mathbb{P}^2$, two moduli t^1, t^2

Effective theory

- $t^1 \equiv \phi \to \infty, \quad w = 3$ $\qquad I_{00} \sim \phi^3, \quad I_{ab} \sim \phi$
- $t^2 \equiv \phi \to \infty, \quad w = 2$ $\qquad I_{00} \sim \phi^2, \quad I_{11} \sim \phi^2, \quad I_{12}, I_{22}$ const.

Emergence computation

- $t^1 \equiv \phi \to \infty, \quad w = 3$

  $\mathbf{q} = (0,0,0,9,3,-9k) \longrightarrow I_{00} \sim \phi^3, \quad I_{ab} \sim \phi$

- $t^2 \equiv \phi \to \infty, \quad w = 2$

  $\mathbf{q} = (0,0,0,1,0,-k) \longrightarrow I_{00} \sim \phi^2, \quad I_{11} \sim \phi^{2/3}, \quad I_{12}, I_{22}$ const.

light D2/D0 towers $\mathbf{q} = (D6, D4_1, D4_2, D2_1, D2_2, D0)$ Grimm, Li, Palti '18

Example

n=w	m_{KK}	m_{D0}	m_{D2}	$T^{1/2}$	Λ_{sp} (D0's)
3	$\phi^{-1/2}$	$\phi^{-3/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$
2	$\phi^{-1/2}$	ϕ^{-1}	ϕ^{-1}	$\phi^{-1/2}$	$\phi^{-1/3}$

Key difference between the two limits: $\begin{cases} w = 3 \to \text{only } m_{D0} \ll \Lambda_{sp} \\ w = 2 \to \text{both } m_{D0}, m_{D2} \ll \Lambda_{sp} \end{cases}$

$\Longrightarrow$ in $w = 2$ double tower of states: $\mathbf{q} = (0,0,0,1,0,-k) \;\to\; \mathbf{q} = (0,0,0,j,0,-k)$

D2 wrapping j times an elliptic fibre

n=w	m_{KK}	m_{D0}	m_{D2}	$T^{1/2}$	Λ_{sp}
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2	$\phi^{-1/2}$	ϕ^{-1}	ϕ^{-1}	$\phi^{-1/2}$	$\phi^{-1/3} \to \phi^{-1/2}$

No other tower below Λ_{sp}!

$\longrightarrow$ Correct species scale:

$$\Lambda_{sp} \sim \frac{M_P}{\sqrt{S_{D0}S_{D2}}} \sim \frac{M_P}{\sqrt{S_{D0}^2}}$$

Example

n=w	m_{KK}	m_{D0}	m_{D2}	$T^{1/2}$	Λ_{sp} (D0's)
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Example

Considering the double tower of D2-D0 states in the w = 2 limit lowers the species scale and changes the number of species:

Before

$$\Lambda_{\rm sp} \sim \phi^{-\frac{1}{3}}$$
$$S \sim S_{\rm D0} \sim \phi^{\frac{2}{3}}$$

After

$$\Lambda_{\rm sp} \sim \phi^{-\frac{1}{2}}$$
$$S_{{\rm D}p} \sim \phi^{\frac{1}{2}}, \;\; S \sim S_{\rm D0} S_{\rm D2} \sim \phi$$

But there is a compensating effect due to the charge degeneracy:

$$\mathbf{q} = (0,0,0,j,0,-k) \rightarrow \quad I_{00} \sim \sum_{j,k=-S_{{\rm D}p}}^{S_{{\rm D}p}} k^2 \log \frac{\Lambda_{\rm sp}}{m_{(j,k)}} \sim S_{{\rm D}p} \cdot S_{{\rm D}p}^3 \sim \phi^2 \qquad \text{(same for } I_{11}\text{)}$$

(sum over j: $S_{{\rm D}p}$; sum over k: $S_{{\rm D}p}^3$)

This effect also occurs for multi-towers of higher dimension, but only if the leading multi-tower is made up of charged states

Example

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  $\mathbf{q} = (0,0,0,j,0,-k) \longrightarrow I_{00} \sim \phi^2, \quad I_{11} \sim \phi^2, \quad I_{12}, I_{22}$ const.

$\longrightarrow$ Generalize for $w = 2$ (III_0 or J-class A) limits using elliptic fibration

w=1 limits

II_b or J-class B limits, $X \to K3$ or T^4 fibration over $\mathbb{P}^1$
EFT string = NS5-brane on the fibre

Plenty of leading towers, including that of the EFT string oscillations

n=w	m_{KK}	$m^* = m_{D0}$	m_{D2}	$m_{D4,\text{fibre}}$	$T^{1/2}$
1	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$	$\phi^{-1/2}$

The species scale computation should include all the leading towers

$$\Lambda_{\text{sp}} = \frac{M_P^2}{\sqrt{S_{\text{KK}} + {S_{\text{D}p}}^r + S_{\text{str}}}}$$

The leading contribution depends on whether the EFT string is:

- critical $\to$ string modes dominate and set Λ_{sp}
- non-critical $\to$ D4/D2/D0 bound states dominate and set Λ_{sp}

Critical case

String oscillation modes dominate the spectrum and give the leading contribution to the species scale

Mass spectrum: $m_N^2 = TN \longrightarrow TN_{\text{max}} \sim \Lambda_{\text{sp}}^2 \sim \frac{M_{\text{P}}^2}{S_{\text{crit}}}$

Degeneracy ansatz:

Castellano, Herraez, Ibañez '22

$$S_{\text{crit}} \sim \sum_{N=0}^{N_{\text{max}}} e^{N^\alpha} N^\gamma \sim e^{N_{\text{max}}^\alpha} N_{\text{max}}^{\gamma+1-\alpha} \qquad \alpha > 0, \quad \gamma \in \mathbb{R}$$

$$N_{\text{max}} \sim \left[\log \frac{M_P^2}{T}\right]^{\frac{1}{\alpha}} \sim (\log \phi)^{\frac{1}{\alpha}}$$

$$\Longrightarrow \quad \Lambda_{\text{sp}} \sim \sqrt{T} \left[\log \frac{M_{\text{P}}^2}{T}\right]^{\frac{1}{2\alpha}} \sim \phi^{-\frac{1}{2}} (\log \phi)^{\frac{1}{2\alpha}} \longrightarrow$$

String scale times log corrections

$$S_{\text{crit}} \sim \frac{M_P^2}{T} \left[\log \frac{M_P^2}{T}\right]^{-\frac{1}{\alpha}} \sim \phi \, (\log \phi)^{-\frac{1}{\alpha}}$$

Critical case

Ansatz for the charge spectrum:

- at each mass level N there are $f_N(\mathbf{q})$ states, with charges

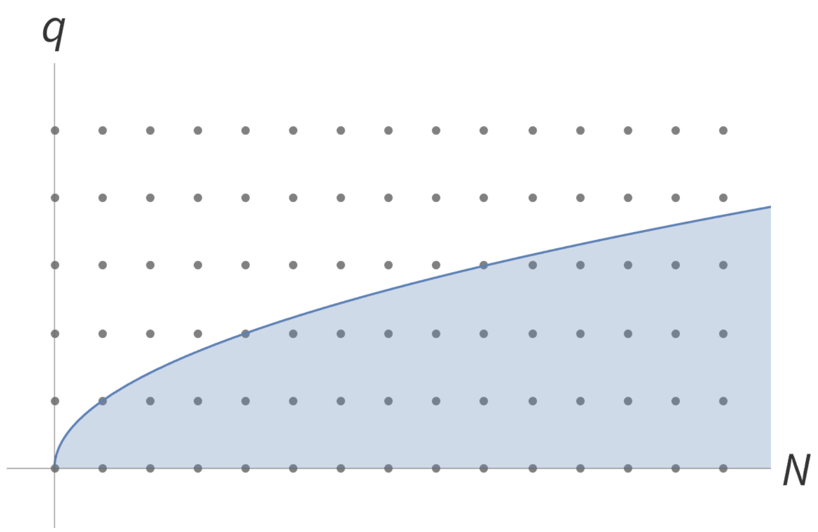
$$\mathbf{q} = \left(0, q_{\mathrm{D4}} e^a, \mathbf{k}_{ab} w^b, q_{\mathrm{D0}}\right) \cdot Q^{t-1}$$

⟶ Motivated by anomaly inflow on axionic strings

Heidenreich, Reece, Rudelius '21

- $f_N(\mathbf{q}) \neq 0$ if $|\mathbf{q}|^2 \leq N$,

  $|\mathbf{q}|^2_{\max} \sim N$ saturates the BPS bound



- charged states populate the light spectrum

$$\sum_{\mathbf{q}} f_N(q) \sim e^{N^\alpha} N^\gamma \sim S^{(N)}_{\mathrm{crit}}$$

Critical case

1-loop corrections to the gauge kinetic function (case with one charge):

$$I \sim \sum_{N=1}^{N_{\max}} \sum_{q=1}^{q_{\max}(N)} f_N(q)\, q^2 \log\left(\frac{\Lambda_{\rm sp}}{m_N} + c\right)$$

$$\sim \sum_{N=1}^{N_{\max}} e^{N^\alpha} N^{\gamma+1} \log\left(\frac{N_{\max}}{N} + c\right)$$

$$\sim e^{N_{\max}^\alpha} N_{\max}^{\gamma+2-\alpha} \sim \frac{M_P^2}{T} \sim \phi$$

$$\sum_{\mathbf{q}} f_N(q) \sim e^{N^\alpha} N^\gamma$$

$$\sum_{n=1}^{N} f(n) \log\left(\frac{N}{n} + c\right) \sim g(n)$$

$$g' = f$$

$$S_{\rm crit} N_{\max} \sim \frac{M_{\rm P}^2}{T}$$

The scaling is recovered independently of α and γ!

Non-critical case

Bound states of D4/D2/D0-branes dominate the spectrum and give the leading contribution to the species scale

$$\mathbf{q} = (\mathrm{D6}, \overrightarrow{\mathrm{D4}}, \overrightarrow{\mathrm{D2}}, \mathrm{D0}) = \left(0, q_{\mathrm{D4}} e^a, \mathbf{k}_{ab} w^b, q_{\mathrm{D0}}\right) \cdot Q^{t-1} \qquad q_{\mathrm{D0}}, q_{\mathrm{D4}}, w^b \in \mathbb{Z}$$

↳ Curvature corrections

$b = \mathrm{rank}\, \mathbf{k_{ab}}$ determines the dimension of the lattice of charges of BPS particles $\rightarrow$ $r = 2 + b$

The lattice is generated by a D4-brane wrapping the $K3/T^4$ fibre, with generic worldvolume fluxes

$$\left.\begin{aligned} \Lambda_{\mathrm{sp}} &\sim \frac{M_{\mathrm{P}}}{\sqrt{S_{\mathrm{D}p}{}^r}} \\ S_{\mathrm{D}p} &\sim \frac{\Lambda_{\mathrm{sp}}}{m_{\mathrm{D}p}} \end{aligned}\right\} \implies \Lambda_{\mathrm{sp}} \sim \phi^{-\frac{r}{2(2+r)}}, \ S_{\mathrm{D}p} \sim \phi^{\frac{1}{2+r}}$$

Non-critical case

The 1-loop corrections to the gauge kinetic function give

$$I_{00} \sim \sum_{\mathbf{q},\,|q_i|\leq S_{\mathrm{D}p}} q_{\mathrm{D0}}^2 \log \frac{\Lambda_{\mathrm{sp}}}{m_{\mathbf{q}}} \sim S_{\mathrm{D}p}^{r-1} \cdot S_{\mathrm{D}p}^{3} \sim \phi$$

sum over q_{D0}

sum over q_{D4}, $\vec{q}_{\mathrm{D2}}$

Remarks

- The precise scaling of $\Lambda_{\rm sp}$ and the number of species does not matter, but it is important that we sum over all the relevant towers

- The tower that sets $\Lambda_{\rm sp}$ must enter the 1-loop corrections

- In $w = 1$ limits we recover the correct scaling for both critical and non-critical EFT strings

- We did not compute the corrections to the moduli space metric, one expects things to work if the spectrum is dominated by BPS states. This could pose further constraints on $f_N(\mathbf{q})$