Full Landscape of Supersymmetric Pati-Salam Models from Intersection Brane Worlds (IBWs)

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KIAS

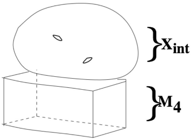
String Phenomenology 2023

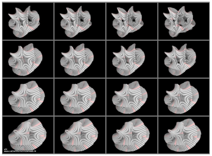
- The Complete Search for the Supersymmetric Pati-Salam Models from Intersecting D6-Branes, *He, Li,* **RS**, JHEP **2208**, 044 (2022)
- String-scale Gauge Coupling Unification in Pati-Salam Models,

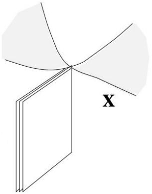
  Li, **RS**, *Wu* JHEP **2303**, 210 (2023)

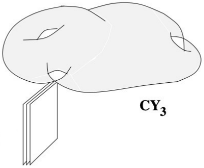
Intersecting Brane Worlds for Standard Model

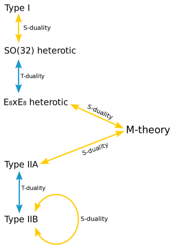
D-branes realize the required gauge symmetry groups via brane intersecting, *with* gauge symmetries from $U(n)$ branes. [Aldazabal, Franco, Ibáñez, Rabadán, Uranga 01; Cvetic, Langacker, Li, Liu 04; Blumenhagen, Cvetic, Langacker, Shiu 05; Douglas, Taylor 07...]

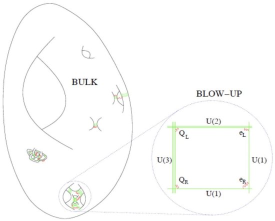


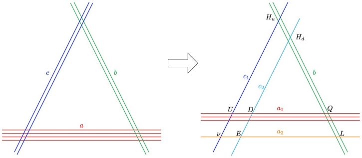












Intersecting Brane Worlds for Standard Model

- $N = 1$ supersymmetric Pati-Salam models on Type IIA $T^6/(Z_2 \times Z_2)$ orientifolds with D6-branes intersecting at generic angles
- Three generations of particles constructed with D-brane wrapping on the $T^6 = T^2 \times T^2 \times T^2$ with a, b, c stacks of branes.
- Chains for symmetry breaking via D6-brane splittings and Higgs Mechanism [Cvetic, Langacker, Wang, Blumenhagen, Lüst, Li, ...'03]

$$
\begin{aligned}
& SU(4) \times SU(2)_L \times SU(2)_R \\
& \xrightarrow{a \to a_1 + a_2} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
& \xrightarrow{c \to c_1 + c_2} SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \\
& \xrightarrow{\text{Higgs Mechanism}} SU(3)_C \times SU(2)_L \times U(1)_Y
\end{aligned}
$$

- Goal: constructing beyond Standard Model from IBWs

Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifolds with Intersecting D6-Branes

Four components of the $T^6/(Z_2 \times Z_2)$ orientifold, namely the image of T^6 under the action of ΩR, $\Omega R\theta$, $\Omega R\omega$ and $\Omega R\theta\omega$ bearing RR charges, and thus $D6$-branes are introduced to cancel their RR charges

Orientifold Action	O6-Plane	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$
ΩR	1	$(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$
$\Omega R\omega$	2	$(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$
$\Omega R\theta\omega$	3	$(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$
$\Omega R\theta$	4	$(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$

Sector	**Representation**
aa	$U(N_a/2)$ vector multiplet 3 adjoint chiral multiplets
$ab+ba$	I_{ab} $(\square_a, \overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}$ $(\square_a, \square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'} - \frac{1}{2} I_{a,O6})$ $\square\square$ fermions $\frac{1}{2}(I_{aa'} + \frac{1}{2} I_{a,O6})$ $\begin{array}{c}\square\\[-4pt]\square\end{array}$ fermions

Table: Massless particle spectrum for intersecting D6-branes

Three Generation for for Standard Model Particles

D-brane wrapping number n_x^i and l_x^i, x a, b, c stacks of branes, i refers to $1, 2, 3$ for different wrapping directions. Intersection numbers between different stacks

$$I_{ab} = 2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i - n_b^i l_a^i) \ , \ \ I_{ab'} = -2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i + n_b^i l_a^i) \ , \ \ I_{aa'} = -2^{3-k} \prod_{i=1}^{3} (n_a^i l_a^i)$$

$$I_{aO6} = 2^{3-k} (-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3)$$

Three generation conditions: to have three families of fermions, we require the intersection numbers to satisfy

$$I_{ab} + I_{ab'} \ = \ 3 \ , \ I_{ac} \ = \ -3 \ , \ I_{ac'} \ = \ 0$$

Tadpole Cancellation Conditions

- **The RR Tadpole Cancellation Conditions**: sum of the RR charges of D6-branes and O6-planes must be zero
- Filler branes wrapping along the orientifold planes are introduced for RR tadpole cancellation

$$-2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a =$$
$$-2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16,$$

where $2N^{(i)}$ is the number of these filler branes wrapping along the i-th O6-plane, and A_a, B_a, C_a, D_a products of wrapping numbers n^i, l^i

$$\begin{aligned} A_a &\equiv -n_a^1 n_a^2 n_a^3, & B_a &\equiv n_a^1 l_a^2 l_a^3, & C_a &\equiv l_a^1 n_a^2 l_a^3, & D_a &\equiv l_a^1 l_a^2 n_a^3, \\ \tilde{A}_a &\equiv -l_a^1 l_a^2 l_a^3, & \tilde{B}_a &\equiv l_a^1 n_a^2 n_a^3, & \tilde{C}_a &\equiv n_a^1 l_a^2 n_a^3, & \tilde{D}_a &\equiv n_a^1 n_a^2 l_a^3 \end{aligned}$$

Supersymmetry Condition

- In $4d$ $\mathcal{N}=1$ supersymmetric models, $1/4$ supercharges remain preserved under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientation projection
- Survive from the orientation projection $\Rightarrow$ rotation angle θ_i of D6-brane with respect to the orientifold plane must be an element of $SU(3)$ [Berkooz, Douglas, Leigh, 96'].
  $\Rightarrow \theta_1 + \theta_2 + \theta_3 = 0 \text{ mod } 2\pi$
- **Supersymmetry condition**: The $4d$ $\mathcal{N}=1$ supersymmetry automatically survive in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0,$$

$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0,$$

where $x_A = \lambda,\ x_B = \lambda 2^{\beta_2+\beta 3}/\chi_2\chi_3,\ x_C = \lambda 2^{\beta_1+\beta 3}/\chi_1\chi_3,\ x_D = \lambda 2^{\beta_1+\beta 2}/\chi_1\chi_2$, in which $\chi_i = R_i^2/R_i^1$ complex structure moduli for i-th two-torus

Mathematical Search for Pati-Salam Models

- Random Scanning, Machine Learning [Cvetic, Li, 04; Halverson, Nelson, Ruehle,19; Li, RS, 19; Loges, Shiu, 21...]
- **Deterministic Algorithm**: common solution of RR tadpole cancellation conditions, supersymmetry conditions, and three generation conditions [He, Li, RS, 22]
- susy equality condition, (x_A, x_B, x_C, x_D) is solution to the linear system

$$\begin{cases} x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0, \\ x_A \tilde{A}_b + x_B \tilde{B}_b + x_C \tilde{C}_b + x_D \tilde{D}_b = 0, \\ x_A \tilde{A}_c + x_B \tilde{B}_c + x_C \tilde{C}_c + x_D \tilde{D}_c = 0. \end{cases}$$

  Combine with the susy inequality condition, three generation condition, tadpole cancellation condition $\rightarrow$ Diophantine equation

- Integer solutions to the system: **Diophantine equation**

  $\Rightarrow$ solving the diophantine equation with explicit models

Strategy of Deterministic Algorithm

- **First step:** list all the possible combinations of the signs of the twelve wrapping number products $A_a, B_a, \ldots, C_c, D_c$.
- **Second step:** for each possibility listed in the first step, append the twelve corresponding inequalities to our system and solve the new system
- Look for three generation conditions for Subsystem 1, 2 and 3, and tadpole condition for Subsystem 4.

  Subsystem 1: Equation of the form $v_1 \cdots v_j = 0$ where $v_1, \ldots, v_j$ are variables. This includes for example $0 = 2I_{ac} = \prod_{i=1}^{3}(n_a^i l_c^i - l_a^i n_c^i)$ where we view $n_a^i l_c^i - l_a^i n_c^i$, $i = 1, 2, 3$ as variables. An equation of this form, obviously, can be solved as $v_1 = 0$ or $v_2 = 0$ or ...or $v_j = 0$.

  Subsystem 2: Equation of the form $v_1 \cdots v_j = p$ where $v_1, \ldots, v_j$ are variables and p is an nonzero integer. Since p has only finitely many factors, an equation of this form can also be solved, leading to finitely many choices for the variables $(v_1, \ldots, v_j)$.

Strategy of Deterministic Algorithm

Subsystem 3: A system of linear equations of full rank. This includes for example

$$\begin{cases} n_a^1 l_c^1 - l_a^1 n_c^1 = 0, \\ n_a^1 l_c^1 + l_a^1 n_c^1 = 6, \end{cases}$$

where $n_a^1 l_c^1$ and $l_a^1 n_c^1$ are viewed as variables. A system of this kind has unique solution.
Subsystem 4: A system of linear inequalities according to A_a, A_b, A_c which has finitely many integer solutions. e.g., subsystem

$$\begin{cases} 4 + 2A_a + A_b + A_c \geq 0, \\ A_a < 0, \\ A_b < 0, \\ A_c = 0. \end{cases}$$

A subsystem of linear inequalities over the real numbers corresponds to a polyhedron, and there are well-known algorithms (e.g. Fourier-Motzkin elimination) that determine whether the polyhedron volume is finite.

- Finite volume $\rightarrow$ Finite many integer solutions
- Infinite volume $\rightarrow$ Infinite integer solutions

We only deal with the first case in this Subsystem, and consider the case with infinitely many solutions in the next step.

Strategy of Deterministic Algorithm

Example: 1st step, categorize with: $A_a < 0, B_a < 0, C_a < 0, D_a > 0, A_b < 0, B_b < 0, C_b = 0, D_b = 0, A_c < 0, B_c > 0, C_c < 0, D_c < 0$. 2nd & 3rd steps, equality solution: $n_a^1 = 1, n_a^2 = -1, n_a^3 = 1, n_b^1 = -1, n_b^2 = -1, n_b^3 = 1, n_c^1 = -1, n_c^2 = -1, n_c^3 = 1, l_a^1 = -1, l_a^3 = -1, l_b^1 = 0, l_c^1 = 1, l_c^3 = 2$ as partial solution. Remaining variables are l_a^2, l_b^2, l_b^3 and l_c^2. Then $B_a < 0, B_b < 0, B_c > 0$ implies that $l_a^2 < 0$, $l_c^2 < 0$ and $l_b^2 l_b^3 > 0$. It follows that

$$\frac{x_A}{x_C} = \frac{-3l_b^2 + l_a^2 l_b^3 + l_b^3 l_c^2}{-l_b^3(l_a^2 + 2l_c^2)} < 0.$$

Exclusion: note x_A and x_C have opposite signs and one must be negative, which contradicts with the susy equality condition that x_A, x_B, x_C, x_D are required to be all positive with positive overall factor λ.

After running the whole algorithm, we find that intriguingly all the cases left from the third step are eliminated in the fourth step, and thus we **complete the landscape** with 202752 models and 33 gauge coupling relations $\triangle$

Landscape of IBWs

- All the possible 202752 supersymmetric Pati-Salam Models, constituting the supersymmetric Pati-Salam landscape[1]
- 33 physical independent models with 33 different gauge coupling relations after modding out the equivalent relations
- Consider String Dualities: $3 \times 4 \times 4^3 \times 4 \times 2 = 6144$ equivalent relations
- Largest allowed wrapping number is 5

model	$U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$									
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	$n_{\square\square}$	$n_{\substack{\square\\\square}}$	b	b'	c	c'	1	4
a	8	$(1, -1) \times (-1, 1) \times (1, -1)$	0	4	0	3	0	-3	-1	1
b	4	$(0, 1) \times (-2, 1) \times (-1, 1)$	-1	1	-	-	0	-1	-1	0
c	4	$(-1, 0) \times (5, 2) \times (-1, 1)$	3	-3	-	-	-	-	0	-5
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$x_A = 2x_B = \frac{14}{5}x_C = 7x_D$							
4	2	$(0, -1) \times (0, 1) \times (2, 0)$	$\beta^g_1 = -3,\ \beta^g_4 = 1$							
			$\chi_1 = \frac{7}{\sqrt{5}},\ \chi_2 = \sqrt{5},\ \chi_3 = \frac{4}{\sqrt{5}}$							

[1] The full data of wrapping numbers (n^i_x, l^i_x) with $x = a, b, c$ and $i = 1, 2, 3$ for 202752 models are listed in http://newton.kias.re.kr/ sunrui/files/finaldata.csv.

Phenomenological Studies

Model 1	$SU(4) \times SU(2)_L$ $\times SU(2)_R \times USp(2)^4$	Q_4	Q_{2L}	Q_{2R}	Q_{em}	$B-L$	Field
ab'	$3 \times (4,2,1,1,1,1,1)$	1	1	0	$-\frac{1}{3}, \frac{2}{3}, -1, 0$	$\frac{1}{3}, -1$	Q_L, L_L
ac'	$3 \times (\overline{4},1,\overline{2},1,1,1,1)$	-1	0	-1	$\frac{1}{3}, -\frac{2}{3}, 1, 0$	$-\frac{1}{3}, 1$	Q_R, L_R
$a2$	$1 \times (4,1,1,1,2,1,1)$	1	0	0	$\frac{1}{6}, -\frac{1}{2}$	$\frac{1}{3}, -1$	
$a3$	$1 \times (4,1,1,1,1,2,1)$	1	0	0	$\frac{1}{6}, -\frac{1}{2}$	$\frac{1}{3}, -1$	
$b1$	$3 \times (1,2,1,2,1,1,1)$	0	1	0	$\pm\frac{1}{2}$	0	
$b3$	$1 \times (1,2,1,1,1,2,1)$	0	1	0	$\pm\frac{1}{2}$	0	
$c2$	$1 \times (1,1,2,1,2,1,1)$	0	0	1	$\pm\frac{1}{2}$	0	
$c4$	$3 \times (1,1,2,1,1,1,2)$	0	0	-1	$\pm\frac{1}{2}$	0	
$b_{\square\square}$	$2 \times (1,\overline{3},1,1,1,1,1)$	0	-2	0	$0, \pm 1$	0	
$b_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$2 \times (1,1,1,1,1,1,1)$	0	2	0	0	0	
$c_{\overline{\square\square}}$	$2 \times (1,1,3,1,1,1,1)$	0	0	2	$0, \pm 1$	0	
$c_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$2 \times (1,1,1,1,1,1,1)$	0	0	-2	0	0	

Chiral spectrum in the open string sector with composite states

Model		$SU(4) \times SU(2)_L \times SU(2)_R \times USp(2)^4$	
Confining Force	Intersection	Exotic Particle Spectrum	Confined Particle Spectrum
$USp(2)_1$	$b1$	$3 \times (1,2,1,2,1,1,1)$	$6 \times (1,2^2,1,1,1,1,1)$
$USp(2)_2$	$a2$	$1 \times (4,1,1,1,2,1,1)$	$1 \times (4^2,1,1,1,1,1,1)$, $1 \times (4,1,2,1,1,1,1)$
	$c2$	$1 \times (1,1,2,1,2,1,1)$	$1 \times (1,1,2^2,1,1,1,1)$
$USp(2)_3$	$a3$	$1 \times (4,1,1,1,1,2,1)$	$1 \times (4^2,1,1,1,1,1,1)$, $1 \times (4,2,1,1,1,1,1)$
	$b3$	$1 \times (1,2,1,1,1,2,1)$	$1 \times (1,2^2,1,1,1,1,1)$
$USp(2)_4$	$c4$	$3 \times (1,1,2,1,1,1,2)$	$6 \times (1,1,2^2,1,1,1,1)$

formed due to the strong forces from hidden sector

String-scale Gauge Coupling Relation

String-scale Gauge Coupling Unification in Pati-Salam Models, *Li*, **RS**, *Wu*, JHEP **2303**, 210 (2023)

- $SU(5)$ and $SO(10)$ GUT theories, additional vector-like matter push GUT-scale unification up to Planck-scale [Blumenhagen, Lüst, Stieberger, 03']
- Instead, we first introduce the vector-like particles whose quantum numbers are the same as those of the SM fermions and their Hermitian conjugate [Barger, Jiang, Langacker, Li, Deshpande, 05', 07'...]
- Their quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y$ and their contributions to one-loop beta functions, $\Delta b \equiv (\Delta b_1, \Delta b_2, \Delta b_3)$ as complete supermultiplets are

$$
\begin{aligned}
& XQ + \overline{XQ} = (\mathbf{3}, \mathbf{2}, \mathbf{\tfrac{1}{6}}) + (\mathbf{\bar{3}}, \mathbf{2}, -\mathbf{\tfrac{1}{6}})\,, \Delta b = (\tfrac{1}{5}, 3, 2)\,; \quad XD + \overline{XD} = (\mathbf{3}, \mathbf{1}, -\mathbf{\tfrac{1}{3}}) + (\mathbf{\bar{3}}, \mathbf{1}, \mathbf{\tfrac{1}{3}})\,, \quad \Delta b = (\tfrac{2}{5}, 0, 1)\,; \\
& XU + \overline{XU} = (\mathbf{3}, \mathbf{1}, \mathbf{\tfrac{2}{3}}) + (\mathbf{\bar{3}}, \mathbf{1}, -\mathbf{\tfrac{2}{3}})\,, \Delta b = (\tfrac{8}{5}, 0, 1)\,; \quad XL + \overline{XL} = (\mathbf{1}, \mathbf{2}, \mathbf{\tfrac{1}{2}}) + (\mathbf{1}, \mathbf{2}, -\mathbf{\tfrac{1}{2}})\,, \quad \Delta b = (\tfrac{3}{5}, 1, 0)\,; \\
& XE + \overline{XE} = (\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, -\mathbf{1})\,, \Delta b = (\tfrac{6}{5}, 0, 0)\,; \quad XT + \overline{XT} = (\mathbf{1}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}, -\mathbf{1})\,, \quad \Delta b = (\tfrac{18}{5}, 4, 0)\,; \\
& XG = (\mathbf{8}, \mathbf{1}, \mathbf{0})\,, \Delta b = (0, 0, 3)\,; \qquad XW = (\mathbf{1}, \mathbf{3}, \mathbf{0})\,, \Delta b = (0, 2, 0)\,.
\end{aligned}
$$

String-scale Gauge Coupling Relation

Table: D6-brane configurations and intersection numbers, with gauge coupling relation is $g_a^2 = \frac{7}{6}g_b^2 = \frac{5}{6}g_c^2 = \frac{25}{28}(\frac{5}{3}g_Y^2) = \frac{8}{27}5^{3/4}\sqrt{7}\pi e^{\phi^4}$

Model 2	$U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$										
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	n_S	n_A	b	b'	c	c'	1	4
a	8	$(1,-1) \times (-1,1) \times (1,-1)$	0	4	0	3	0	-3	-1	1
b	4	$(0,1) \times (-2,1) \times (-1,1)$	-1	1	-	-	0	-1	-1	0
c	4	$(-1,0) \times (5,2) \times (-1,1)$	3	-3	-	-	-	-	0	-5
1	2	$(1,0) \times (1,0) \times (2,0)$	$x_A = 2x_B = \frac{14}{5}x_C = 7x_D$							
4	2	$(0,-1) \times (0,1) \times (2,0)$	$\beta_1^g = -3,\ \beta_4^g = 1$							
			$\chi_1 = \frac{7}{\sqrt{5}},\ \chi_2 = \sqrt{5},\ \chi_3 = \frac{4}{\sqrt{5}}$							

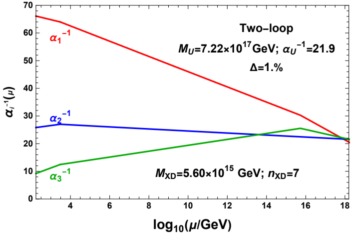
- Using electroweak data, set $M_{\text{SUSY}} = 3$ TeV, study the gauge coupling relations at string scale (around 5×10^{17} GeV) by solving two-loop renormalization group equations (RGEs) [Barger, Jiang, Langacker, Li, Deshpande, 05', 07'...], i.e., $M_U \simeq M_{\text{string}}$.
- The RGEs for the gauge couplings at the two-loop level given by [Barger, Gogoladze..07',10']

$$\frac{d}{d\ln\mu}g_i = \frac{b_i}{(4\pi)^2}g_i^3 + \frac{g_i^3}{(4\pi)^4}\left[\sum_{j=1}^{3}B_{ij}g_j^2 - \sum_{\alpha=u,d,e}d_i^\alpha \mathrm{Tr}\left(h^{\alpha\dagger}h^\alpha\right)\right],$$

SM gauge couplings $g_i(i = 1,2,3)$; Yukawa couplings $h^\alpha(\alpha = u,d,e)$

- Particle number fully determined by brane intersection

Ex: Introduce vector-like particle $XD + \overline{XD} = (\mathbf{3}, \mathbf{1}, -\frac{1}{3}) + (\mathbf{3}, \mathbf{1}, \frac{1}{3})$



- $I_{ac} = 2^{-k}\prod_{i=1}^{2}(n_a^i l_c^i - n_c^i l_a^i) = 7$: By introducing 7 pairs of vector-like particle (XD, $\overline{XD}$) for Model 2 with $k_Y = 125/84,\ k_2 = 7/6$, via two-loop RGEs method, we indeed obtain string scale gauge coupling unification at around 7.22×10^{17} GeV as presented
- To define the unification scale, evolution under the conditions $\alpha_U^{-1} \equiv \alpha_1^{-1} = (\alpha_2^{-1} + \alpha_3^{-1})/2$ and $\Delta = |\alpha_1^{-1} - \alpha_2^{-1}|/\alpha_1^{-1}$ [Chen, Li, 17', 18'...]. difference between $\alpha_{\text{string}}^{-1}$ and α_2^{-1} or α_3^{-1} is limited to be less than 1.0%
- String Theory: realized by setting the minimal distance squared $Z^2_{(ac')}$(in $1/M_s$ units) between parallel D6-branes along the third torus is small (N=2)

Conclusion and Outlook

String Phenomenology - IBWs

- Complete landscape for one type of IBWs with Pati-Salam
  $\rightarrow$ exhaustive list($202752 = 33 \times 6144$) with gauge coupling unification at string scale realized from two-loop RGEs
- Landscape for IBWs with fluxes to be drawn
- Physics beyond Standard Model from exotic particles
- IBWs with generalized fluxes(geo & non-geo)

Thank you very much!

EXTRA

The coefficients of beta functions in SM and supersymmetric models are represented by [Machacek, Cvetic, Martin, Barger..98']

$$b_{\mathrm{SM}} = \left(\frac{41}{6}\frac{1}{k_Y}, -\frac{19}{6}\frac{1}{k_2}, -7\right), \; B_{\mathrm{SM}} = \begin{pmatrix} \frac{199}{18}\frac{1}{k_Y^2} & \frac{27}{6}\frac{1}{k_Y k_2} & \frac{44}{3}\frac{1}{k_Y} \\ \frac{3}{2}\frac{1}{k_Y k_2} & \frac{35}{6}\frac{1}{k_2^2} & 12\frac{1}{k_2} \\ \frac{11}{6}\frac{1}{k_Y} & \frac{9}{2}\frac{1}{k_2} & -26 \end{pmatrix},$$

$$d^u_{\mathrm{SM}} = \left(\frac{17}{6}\frac{1}{k_Y}, \frac{3}{2}\frac{1}{k_2}, 2\right), \; d^d_{\mathrm{SM}} = 0, \; d^e_{\mathrm{SM}} = 0,$$

$$b_{\mathrm{SUSY}} = \left(11\frac{1}{k_Y}, \frac{1}{k_2}, -3\right), \; B_{\mathrm{SUSY}} = \begin{pmatrix} \frac{199}{9}\frac{1}{k_Y^2} & 9\frac{1}{k_Y k_2} & \frac{88}{3}\frac{1}{k_Y} \\ 3\frac{1}{k_Y k_2} & 25\frac{1}{k_2^2} & 24\frac{1}{k_2} \\ \frac{11}{3}\frac{1}{k_Y} & 9\frac{1}{k_2} & 14 \end{pmatrix},$$

$$d^u_{\mathrm{SUSY}} = \left(\frac{26}{3}\frac{1}{k_Y}, 6\frac{1}{k_2}, 4\right), \; d^d_{\mathrm{SUSY}} = 0, \; d^e_{\mathrm{SUSY}} = 0,$$

where k_Y and k_2 are general normalization factors. By solving the two-loop RGEs for SM gauge couplings, we perform numerically calculations including the one-loop RGEs for Yukawa couplings and taking into account the new physics contributions and threshold.