

Flavor, CP and Metaplectic Modular Symmetries in Type IIB Chiral Flux Vacua

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Introduction

Type IIB compactifications on $T^6 / (\mathbb{Z}_2 \times \mathbb{Z}'_2)$

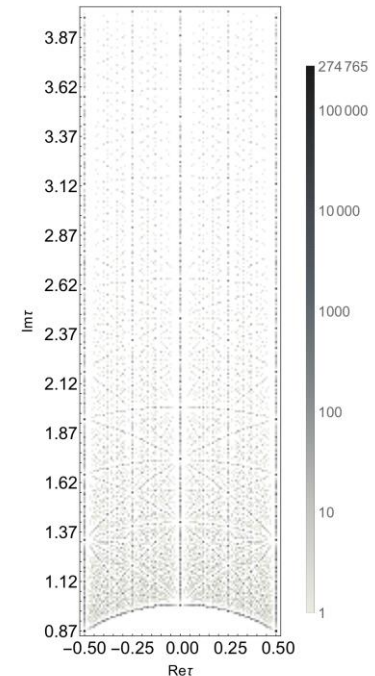
Fluxes $\{a^0, a^i, b_i, b_0\}$

\Rightarrow many VEVs of complex structure (τ)

The distribution of **complex structure moduli**
VEVs clusters at **fixed points of $SL(2, \mathbb{Z})$**
modular symmetry of torus.

$$(\tau = i, \omega, i\infty \text{ with } \omega = \frac{-1+\sqrt{3}i}{2}, \mathbb{Z}_3 \text{ fixed point : } \tau = \omega)$$

Remarkably, **\mathbb{Z}_3 fixed point** has the largest part



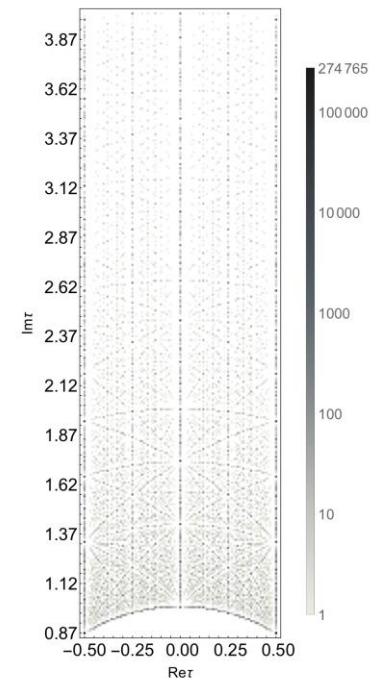
\mathbb{Z}_3 : 40.3%

The numbers of stable flux vacua
on the fundamental domain of τ
Ishiguro, Kobayashi, Otsuka 2011.09154

Introduction

These moduli values determine not only the flavor structure of quarks and leptons but also the CP-violating phases through the Yukawa couplings written by the modular forms.

1. The distribution of 3-generation models in the flux landscape
2. The unification of flavor, CP and modular symmetries in Type IIB
3. Symmetry in 4D EFT



The numbers of stable flux vacua on the fundamental domain of τ

Ishiguro, Kobayashi, Otsuka 2011.09154

Outline

- Introduction
- **Moduli distribution with SM spectra**
- Eclectic Flavor Symmetry
- Summary

MSSM

D-brane configurations leading to left-right symmetric **Minimal Supersymmetric Standard Model** (MSSM).

| N_α | Gauge group | (n_α^1, m_α^1) | (n_α^2, m_α^2) | (n_α^3, m_α^3) |
|------------|-------------|----------------------------|----------------------------|----------------------------|
| $N_a = 6$ | $SU(3)_C$ | $(1, 0)$ | $(g, 1)$ | $(g, -1)$ |
| $N_b = 2$ | $USp(2)_L$ | $(0, 1)$ | $(1, 0)$ | $(0, -1)$ |
| $N_c = 2$ | $USp(2)_R$ | $(0, 1)$ | $(0, -1)$ | $(1, 0)$ |
| $N_d = 2$ | $U(1)_d$ | $(1, 0)$ | $(g, 1)$ | $(g, -1)$ |

m_a^i : wrapping number on $(T^2)_i$
 n_a^i : units of magnetic flux on $(T^2)_i$

Marchesano, Shiu hep-th/0409132

The magnetic flux g determines the generations of quark and lepton chiral multiplets in the visible sector

MSSM

- Tadpole cancellation condition (D3-brane charge)

$$D3 : \sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2} N_{\text{flux}} = 16,$$

- The existence of magnetized D9-branes in the hidden sector

$$8g^2 = -Q_{D3}^{\text{hid}} + 16 - \frac{N_{\text{flux}}}{2},$$

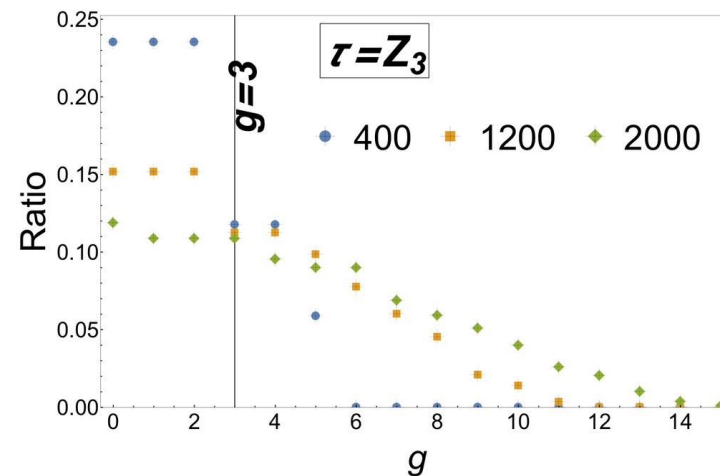
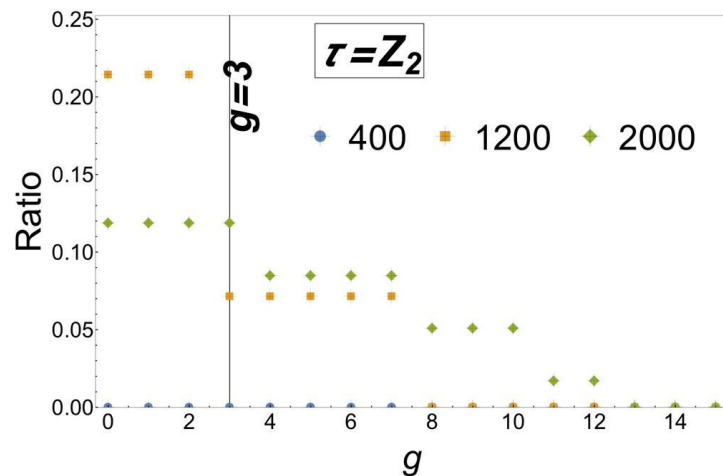
(Q_{D3}^{hid} : D3-brane charge induced by the magnetic flux on D9-branes)

We freely change the value of Q_{D3}^{hid} to reveal **the mutual relation** between **the generation number g** and **the flux quanta N_{flux}** .

MSSM (result)

The numbers of flux vacua as a function of the generation number g at $\tau = i$ and $\tau = \omega$ respectively.

We change the maximum value of Q_{D3}^{hid} as $|Q_{D3}^{\text{hid}}| = 400, 1200, 2000$



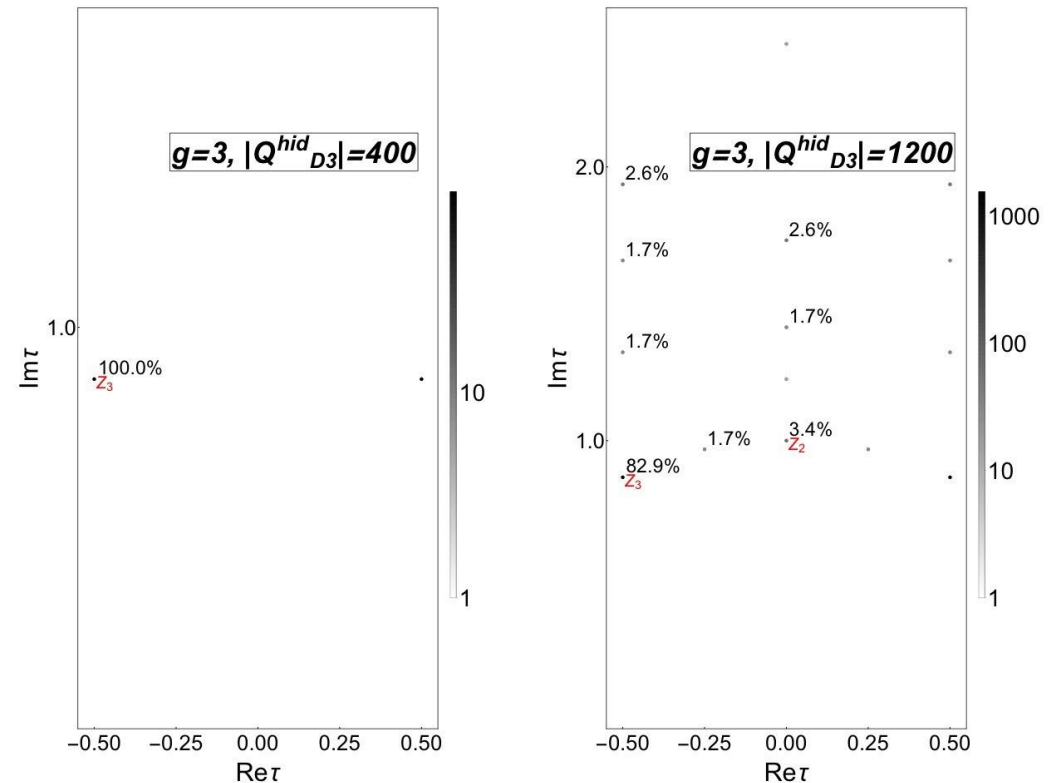
The small generation number is favored in the string landscape

MSSM (result)

The numbers of stable flux vacua with $g = 3$ generation of quarks/leptons on the fundamental domain of τ .

MSSM-like models are still peaked at the \mathbb{Z}_3 fixed point

Similar results are obtained for Pati-Salam-like model



Outline

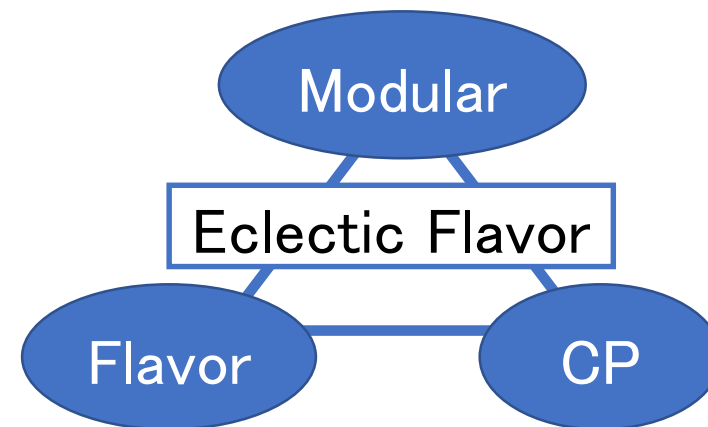
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Eclectic Flavor Symmetry

- Eclectic flavor symmetry

A hybrid picture where **the traditional flavor group** and **the finite modular group** combine to **a generalized flavor group** (and CP)

Nilles, Ramos-Sánchez, Vaundrevange 2001.01736



Top-down model building motivated by string theory

This symmetry potentially incorporates **a different flavor structure** for quark- and lepton-sector of the Standard Model

Eclectic Flavor Symmetry (models)

- Traditional flavor group $G_{\text{flavor}} \equiv \mathbb{Z}_4 \times \mathbb{Z}_2^P \times \mathbb{Z}_2^C \times \mathbb{Z}_2^Z$

Generators : $\{Z', P, C, Z\}$

Abe, Choi, Kobayashi, Ohki 0904.2631

- Generalized CP group $G_{\text{CP}} \equiv \mathbb{Z}_2^{\text{CP}}$

Generator : $\{\text{CP}\}$

The CP transformation

$$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{e}_2 \\ \bar{e}_1 \end{pmatrix}.$$

- Finite metaplectic modular group $G_{\text{modular}} \equiv \tilde{\Gamma}_8$

In order to discuss the action of the full modular group on the half-integral modular forms (such as Yukawa coupling and wavefunction on T^2)

Metaplectic modular symmetry

Metaplectic modular group $Mp(2, \mathbb{Z}) : \tilde{\Gamma}$

$$Mp(2, \mathbb{Z}) = \left\{ \tilde{\gamma} = (\gamma, \varphi(\gamma, \tau)) \mid \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}), \quad \varphi(\gamma, \tau)^2 = (c\tau + d) \right\}$$

Liu, Yao, Qu, Ding, 2007.13706

The principal congruence subgroup

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid a \equiv d \equiv 1, \quad b \equiv c \equiv 0 \pmod{N} \right\},$$

$$\tilde{\Gamma}(4N) = \{ \tilde{\gamma} = (\gamma, v(\gamma)J_{1/2}(\gamma, \tau) \mid \gamma \in \Gamma(4N) \},$$

$$(v(\gamma) = \left(\frac{c}{d}\right) : \text{the Kronecker symbol}, \quad \varphi(\gamma, \tau) = \pm J_{1/2}(\gamma, \tau))$$

The finite metaplectic modular groups are given by $\tilde{\Gamma}_{4N} \equiv \tilde{\Gamma} / \tilde{\Gamma}(4N)$.

($\tilde{\Gamma}_8 : N = 2$)

Eclectic Flavor Symmetry (results)

Traditional flavor and Modular flavor

$$\tilde{S}_{\text{even}} C_{\text{even}} \tilde{S}_{\text{even}}^{-1} = Z_{\text{even}}, \quad \tilde{S}_{\text{even}} Z_{\text{even}} \tilde{S}_{\text{even}}^{-1} = C_{\text{even}},$$

$$\tilde{T}_{\text{even}} C_{\text{even}} \tilde{T}_{\text{even}}^{-1} = C_{\text{even}} Z_{\text{even}} (Z'_{\text{even}})^2, \quad \tilde{T}_{\text{even}} Z_{\text{even}} \tilde{T}_{\text{even}}^{-1} = Z_{\text{even}},$$

The modular transformation is regarded as

the outer automorphism of the traditional flavor group.

$$\rightarrow G_{\text{flavor}} \rtimes G_{\text{modular}} \quad (\text{Semi-direct product})$$

Eclectic Flavor Symmetry (results)

Traditional flavor, Modular flavor and CP

$$\widetilde{CP}Z'_{\text{even}}\widetilde{CP}^{-1} = (Z'_{\text{even}})^{-1}, \quad \widetilde{CP}Z'_{\text{odd}}\widetilde{CP}^{-1} = (Z'_{\text{odd}})^{-1},$$

$$\widetilde{CP}\tilde{S}_{\text{even}}\widetilde{CP}^{-1} = (\tilde{S}_{\text{even}})^{-1}, \quad \widetilde{CP}\tilde{T}_{\text{even}}\widetilde{CP}^{-1} = \tilde{T}_{\text{even}}^{-1}$$

$$\widetilde{CP}\tilde{S}_{\text{odd}}\widetilde{CP}^{-1} = (\tilde{S}_{\text{odd}})^{-1}, \quad \widetilde{CP}\tilde{T}_{\text{odd}}\widetilde{CP}^{-1} = \tilde{T}_{\text{odd}}^{-1}$$

We can construct **the outer automorphism**

$$u_{\text{CP}} : G_{\text{CP}} \rightarrow \text{Aut}(G_{\text{flavor}} \rtimes G_{\text{modular}})$$

$$\rightarrow (G_{\text{flavor}} \rtimes G_{\text{modular}}) \rtimes G_{\text{CP}} \quad (\text{Semi-direct product})$$

Eclectic Flavor Symmetry (\mathbb{Z}_3 fixed point)

\mathbb{Z}_3 modular symmetry generated by $\{1, ST, (ST)^2\}$

$$(\tilde{S}\tilde{T})C_{\text{even}}(\tilde{S}\tilde{T})^{-1} = C_{\text{even}}Z_{\text{even}}(Z'_{\text{even}})^2, \quad (\tilde{S}\tilde{T})Z_{\text{even}}(\tilde{S}\tilde{T})^{-1} = Z_{\text{even}},$$

The discrete non-abelian symmetry $(G_{\text{flavor}} \rtimes \mathbb{Z}_3) \rtimes G_{\text{CP}}$ remains in the low-energy 4D effective action.

Eclectic Flavor Symmetry (\mathbb{Z}_3 fixed point)

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The discrete non-abelian symmetry $(G_{\text{flavor}} \rtimes \mathbb{Z}_3) \rtimes G_{\text{CP}}$ remains in the low-energy 4D effective action.

The coefficient of 4D higher-dimensional operators will be described by the product of modular forms with half-integral modular weights.

→ The eclectic flavor symmetry would control the flavor structure of higher-dimensional operators.

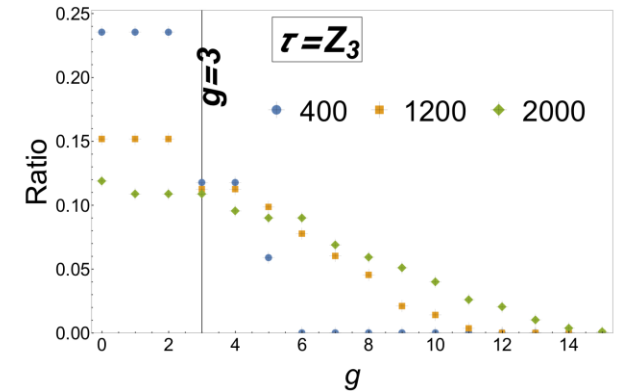
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Summary

By tadpole cancellation conditions, the moduli distribution are related with the generation number.

→ the string landscape leads to small generation number



The traditional flavor, modular flavor and CP symmetries are uniformly described in the context of eclectic flavor symmetry

$$(G_{\text{flavor}} \rtimes G_{\text{modular}}) \rtimes G_{\text{CP}}$$

A part of eclectic flavor symmetry would control the flavor structure of 4D higher-dimensional operators. (at \mathbb{Z}_3 fixed point)