String Pheno @ IBS Daejeon

Quark and lepton hierarchies from S4’ modular flavor symmetry

Junichiro Kawamura

Institute for Basic Science, CTPU

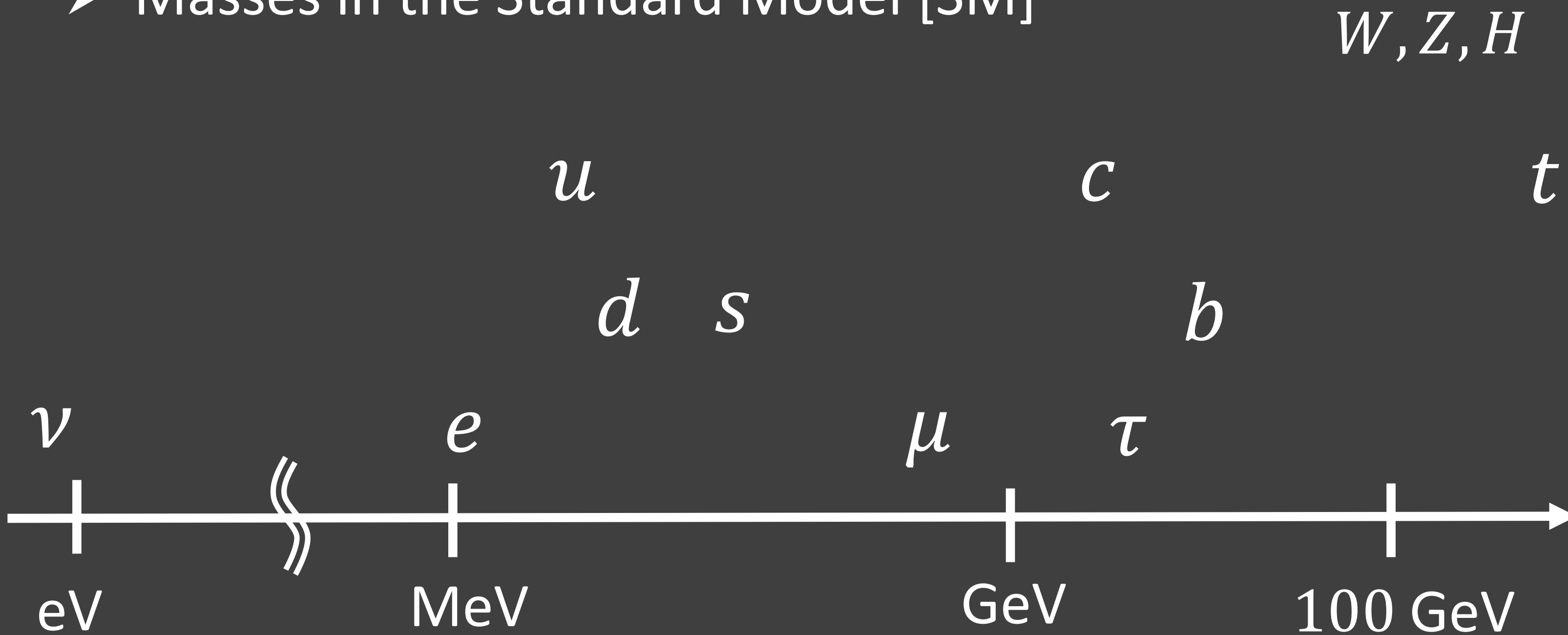
based on arXiv:2301.07439, 2302.11183 [PLB], 2307.today (hep-ph)

in collaboration with

Y.Abe (U. of Wisconsin), T.Higaki (Keio U.), T.Kobayashi (Hokkaido U.)

Mass hierarchies

- Masses in the Standard Model [SM]



Why so hierarchical ?

Quark hierarchies

- Cabibbo-Kobayashi-Maskawa [CKM] matrix

$$M_u \qquad\qquad U_L^\dagger M_u U_R = \mathrm{diag}(m_u, m_c, m_t)$$

$$M_d \quad \rightarrow \quad V_L^\dagger M_d V_R = \mathrm{diag}(m_d, m_s, m_b)$$

$$\frac{\mathrm{g}}{\sqrt{2}} W_\mu^+ \overline{u_i}\, \delta_{ij} d_j \qquad\qquad \frac{\mathrm{g}}{\sqrt{2}} W_\mu^+ \overline{u_i}\, V_{ij}^{CKM} d_j$$

diagonal W-couplings — diagonal masses

CKM matrix $$V^{CKM} = U_L^\dagger V_L \sim \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.041 \\ 0.009 & 0.040 & 0.99 \end{pmatrix}$$

CKM also has hierarchical structure

Lepton hierarchies

- charged lepton masses are hierarchical

$$\left(m_e, m_\mu, m_\tau\right) \sim (0.5, 106, 1776)\ \mathrm{MeV}$$

- neutrino masses are not so hierarchical

$$\Delta m_{21}^2 \sim 10^{-5}\mathrm{eV}^2,\ \Delta m_{32}^2 \sim 10^{-3}\mathrm{eV}^2$$

$\rightarrow$ $m_2 \sim 10^{-2.5}, m_3 \sim 10^{-1.5}$ eV for $m_1 < m_{2,3}$

- PMNS matrix has large mixing angles

$$V^{PMNS} \sim \begin{pmatrix} 0.8 & 0.6 & 0.2 \\ 0.3 & 0.6 & 0.7 \\ 0.5 & 0.6 & 0.7 \end{pmatrix}$$

Aim of this work

Understand the hierarchies in quark and lepton hierarchies

- Modular flavor symmetry

what if Yukawa couplings (masses) are modular form ?

Altarelli, Feruglio, 2010

$$Y = Y(\tau) \to (c\tau + d)^k \rho(r) Y(\tau)$$

- non-Abelian discrete flavor symmetry

→ • Froggatt-Nielsen [FN] mechanism by residual symmetry

- modular symmetry appears in string models

e.g. talks by H.P.Niles, T.Kobayashi (Tue)

T.Kobayashi, S.Nagamoto et.al. ‘17 ’18 ‘20
J.Lauer, J.Mas, H.P.Nilless ‘89, ‘91,
S.Ferrara, D.Lust, S.Theisen, ‘89
A.Baur, H.P.Nilles, A.Trautner,
PKS.Vaudrevange S.Ramos-Sanches, ‘19, ‘20

→ explain the quark and lepton hierarchies

Outline

1. Introduction
2. Modular flavor symmetry
3. S_4' model for quark and lepton hierarchies
4. Summary

Modular group

- modular group $\Gamma \Leftrightarrow$ special linear group

$$\Gamma := SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \ a,b,c,d \in \mathbb{Z}\, , ad-bc=1 \right\}$$

generators

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT$$

- action to modulus τ : complex scalar with $\mathrm{Im}\,\tau > 0$

$$\tau \xrightarrow{\Gamma} \frac{a\tau + b}{c\tau + d} \qquad \tau \xrightarrow{S} -1/\tau \qquad \tau \xrightarrow{T} \tau + 1 \qquad \tau \xrightarrow{R} \tau$$

Finite modular group Γ_N

- Congruence group $\Gamma(N)$ level $N \in \mathbb{N}$

$$\Gamma(N) \coloneqq \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma := \mathrm{SL}(2,\mathbb{Z}) \;\middle|\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \bmod N \right\}$$

ex) $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \to \; T^N = \begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in \Gamma(N)$

- finite modular group $\Gamma_N \coloneqq \Gamma/\Gamma(N)$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT, \quad T^N = 1$$

→ isomorphic to non-Abelian discrete symmetries for $N \leq 5$

$$\Gamma_2 \simeq S_3', \quad \Gamma_3 \simeq A_4', \quad \Gamma_4 \simeq S_4', \quad \Gamma_5 \simeq A_5'$$

*e.g. $\Gamma_4/\mathbb{Z}_2^R \simeq S_4$

$\Gamma_4 \simeq S_4'$ modular symmetry Novichkov, Penedo, Petkov, Titov, 18'

- Representations under $S_4 = S_4'/\mathbb{Z}_2^R$
  - singlets $1, 1'$, doublet 2 and triplets $3, 3'$
  - there are $\mathbb{Z}_2^R$-odd representations denoted by $\hat{r}$ under S_4'

- Modular form of rep. r and modular weight $k \in \mathbb{N}$

  is a holomorphic function of τ transforms as

$$Y_r^{(k)} = Y_r^{(k)}(\tau) \to (c\tau + d)^k \rho(r) Y_r^{(k)}(\tau) \qquad \text{modulus } \tau \xrightarrow{\Gamma} \frac{a\tau+b}{c\tau+d}$$

$\rho(r)$: representation matrix of r

  - the number of rep. is fixed for a given weight k
  - one $\hat{3}$ at $k = 1$, one 2 and one $3'$ at $k = 2$, and so on

Residual $\mathbb{Z}_4^T$ symmetry

Novichkov, Penedo, Petkov, 21'

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad T^4 = 1$$

- At $\tau \sim i\infty$

  τ is insensitive to $\tau \xrightarrow{T} \tau + 1$ $\Rightarrow$ $\mathbb{Z}_4^T$ symmetry is unbroken

- Modular forms at $\mathrm{Im}\tau \gg 1$

$$Y_{\hat{3}}^{(1)}(\tau) \sim \begin{array}{cc} & \mathbb{Z}_4^T\text{-charge} \\ \begin{pmatrix} \sqrt{2}\epsilon(\tau) \\ \epsilon(\tau)^2 \\ -1 \end{pmatrix} & \begin{array}{c} 1 \\ 2 \\ 0 \end{array} \end{array} \qquad \epsilon(\tau) \sim 2\exp\left(\frac{2\pi i \tau}{4}\right) \ll 1$$

powers of $\epsilon \ll 1$ is controlled by $\mathbb{Z}_4^T$ charge

$\Rightarrow$ Froggatt-Nielsen [FN] mechanism $\quad \left(\frac{\langle\phi\rangle}{\Lambda}\right)^n \Leftrightarrow \epsilon(\tau)^n$

natural and predictive realization of FN mech.

Outline

1. Introduction
2. Modular flavor symmetry
3. S_4' model for quark and lepton hierarchies
4. Summary

Representations of S_4'

1st example for both Q and L from single modular sym.

- Quark sector

$$\overset{0}{} \quad \overset{0}{} \quad \overset{1}{}$$

$$u^c = 1 \oplus 1 \oplus \hat{1}'$$

RH up quark

$$\overset{0}{} \quad \overset{0}{} \quad \overset{0}{}$$

$$d^c = 1 \oplus 1 \oplus 1$$

RH down quark

(2,3,1)

$$Q = 3$$

LH doublet quark

$\mathbb{Z}_4^T$-charge

small angles and mass hierarchy mainly from Q

- Lepton sector

0 0 0

$$L = 1 \oplus 1 \oplus 1$$

LH doublet lepton

(2,3,1)

$$e^c = 3$$

RH charged lepton

$\mathbb{Z}_4^T$-charge

large angles from L, while mass hierarchy from e^c

Quark and lepton hierarchies

- masses and CKM /PMNS matrix

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t$$
$$(m_d, m_s, m_b) \sim \left(m_e, m_\mu, m_\tau\right) \sim (\epsilon^3, \epsilon^2, \epsilon)\, m_t/t_\beta$$

$$V^{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \qquad V^{PMNS} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$N = 4$ is minimal to have ϵ^3

*see also models based on A_4 ('22 Petcov&Tanimoto), Γ_6 , A_4^3 ('23 S.Kikuchi, T.Kobayashi, S.Takada et.al) for Q

- Additional factors
  - powers of $(2\mathrm{Im}\tau\,)^k$ from canonical normalization
  - numerical factors from modular forms

Fit results

$$\tan\beta = 3.7, \mathrm{Im}\,\tau = 2.8, |\alpha_3^1| = 0.0012$$

$$\frac{1}{|\alpha_3^1|}\begin{pmatrix}\alpha_1\\ \alpha_2^1\\ \alpha_2^2\\ \alpha_3^1\\ \alpha_3^2\end{pmatrix} = \begin{pmatrix}-0.69\\ -1.8\\ 0.84\\ e^{-1.6i}\\ -1.1\end{pmatrix} \quad \frac{1}{|\alpha_3^1|}\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3^1\\ \beta_3^2\end{pmatrix} = \begin{pmatrix}1.6\\ -1.8\\ 0.6\\ -0.62\end{pmatrix} \quad \frac{1}{|\alpha_3^1|}\begin{pmatrix}\gamma_1\\ \gamma_2\\ \gamma_3^1\\ \gamma_3^2\end{pmatrix} = \begin{pmatrix}-1.7\\ -2.8\\ 0.80\, e^{0.93i}\\ 2.75\end{pmatrix}$$

$$\frac{1}{|\alpha_3^1|}\begin{pmatrix}c_{11}\\ c_{22}\\ c_{33}\\ c_{13}\\ c_{23}\end{pmatrix} = \begin{pmatrix}1.7\\ -1.9\\ -0.67\\ -5.4\\ -2.0\end{pmatrix}$$

- coefficients are in [0.62,5.4]
- two phases are introduced

→

obs.	value	center	error
$y_u/10^{-6}$	4.44	2.85	0.88
$y_c/10^{-3}$	1.481	1.479	0.052
y_t	0.5322	0.5320	0.0053
$y_d/10^{-5}$	1.94	1.93	0.21
$y_s/10^{-4}$	3.88	3.82	0.21
$y_b/10^{-2}$	2.097	2.100	0.021
$y_e/10^{-6}$	7.816	7.816	0.047
$y_\mu/10^{-3}$	1.6496	1.6500	0.0099
$y_\tau/10^{-2}$	2.808	2.805	0.028

s_{12}	0.22520	0.22541	0.00072
$s_{23}/10^{-2}$	4.007	3.998	0.064
$s_{13}/10^{-3}$	3.43	3.48	0.13
δ_{CKM}	1.2395	1.2080	0.0540
$R_{32}^{21}/10^{-2}$	3.053	3.070	0.084
s_{12}^2	0.302	0.307	0.013
s_{23}^2	0.547	0.546	0.021
$s_{13}^2/10^{-2}$	2.203	2.200	0.070
δ_{PMNS}	-0.85	-2.01	0.63

Well-fit ($< 1.8\sigma$) to the data by $\mathcal{O}(1)$ coefficients

Summary

- Modular flavor symmetry realizes
  - generalized non-Abelian discrete symmetry
  - hierarchical Yukawa matrix via Froggatt-Nielsen mechanisms
- S_4' model
  - maybe minimal possibility to realize the Q and L hierarchy
  - successfully explains the quark and lepton hierarchies
  - can not be embedded into e.g. SU(5) GUT

can be done in Γ_6' 2307.today

Thank you

backups

Modular weights

2302.11183

$$\left(k_{u_1}, k_{u_2}, k_{u_3}\right) = (0,4,3), \quad \left(k_{d_1}, k_{d_2}, k_{d_3}\right) = (0,2,4), \ k_Q = 4$$

$$\left(L_1, k_{L_2}, k_{L_3}\right) = (0,2,4), \qquad k_e = 4$$

→ Modular weights of Yukawa couplings

$$k_{Y_u} = (4,8,7), \qquad k_{Y_d} = k_{Y_e} = (4,6,8), \qquad k_{c_{ij}} = 2i + 2j - 4$$

- minimal combinations for rank-3 Yukawa matrices up to $\mathcal{O}(\epsilon^3)$
- neutrino mass hierarchy from $2\mathrm{Im}\,\tau \sim 5$

➢ Modular forms:

ex) $Y_{\hat{3}}^{(7)} \sim \epsilon\,(1, \sqrt{2}\epsilon, 7\sqrt{2}\epsilon^2)$

→ Cabbibo angle is enhanced as $s_C \sim 7\sqrt{2}\epsilon \sim 0.2$

Yukawa couplings 2302.11183

$$W_u = H_u \left\{ \sum_{a=1}^{2} \alpha_a \left(Q Y_3^{(k_{u_a}+k_Q)} \right)_1 u_i^c + \alpha_3 \left(Q Y_{\hat{3}}^{(k_{u_3}+k_Q)} u_3^c \right)_1 \right\}$$

$$W_d = H_d \left\{ \sum_{i=13} \beta_i \left(Q Y_3^{(k_{d_i}+k_Q)} \right)_1 d_i^c + \gamma_i \left(L_i Y_3^{(k_e+k_{L_i})} e^c \right)_1 \right\}$$

$$W_\nu = \sum_{i,j=1}^{3} \frac{c_{ij}}{\Lambda} Y_1^{(k_{L_i}+k_{L_j})} L_i H_u L_j H_u$$

$(\cdots)_1$: singlet combination

$\alpha_i, \beta_i, \gamma_i, c_{ij} : \mathcal{O}(1)$ coefficients

For neutrinos, we assume

- Majorana masses from Weinberg operator
- L_i is singlet, s.t. Yukawa is singlet ($L_i = \hat{1} \rightarrow$ Y is $1'$)

Toy model   2 flavor (1st and 3rd) quark model

- representations

Z_4^T-charge

$Q = 2 \quad \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad \bar{u} = \hat{1}' \oplus 1' \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \bar{d} = 1' \oplus 1' \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\rightarrow \quad Y_u \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 \\ \epsilon & 1 \end{pmatrix} \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 \\ 1 & 1 \end{pmatrix}$

$\rightarrow \quad (y_u, y_d, y_b) \sim (\epsilon^3, \epsilon^2, 1) \qquad V_{\text{CKM}} \sim \begin{pmatrix} 1 & \epsilon^2 \\ \epsilon^2 & 1 \end{pmatrix}$

- bad example 1: $Q = 1' \oplus 1' \qquad \bar{d} = 2$

$\rightarrow \quad Y_d \sim \begin{pmatrix} \epsilon^2 & 1 \\ \epsilon^2 & 1 \end{pmatrix} \quad \rightarrow \quad (y_d, y_b) \sim (\epsilon^2, 1) \quad V_{\text{CKM}} \sim \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

hierarchical down masses, but no hierarchy in CKM (good for PMNS)

Toy model   2 flavor (1st and 3rd) quark model

- representations Z_4^T-charge

$Q = 2 \quad \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad \bar{u} = \hat{1}' \oplus 1' \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \bar{d} = 1' \oplus 1' \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\rightarrow \quad Y_u \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 \\ \epsilon & 1 \end{pmatrix} \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 \\ 1 & 1 \end{pmatrix}$

$\rightarrow \quad (y_u, y_d, y_b) \sim (\epsilon^3, \epsilon^2, 1) \qquad V_{\mathrm{CKM}} \sim \begin{pmatrix} 1 & \epsilon^2 \\ \epsilon^2 & 1 \end{pmatrix}$

- bad example 2: $\bar{d} = 1 \oplus 1 \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\rightarrow \quad Y_d \sim \begin{pmatrix} 1 & 1 \\ \epsilon^2 & \epsilon^2 \end{pmatrix} \rightarrow (y_d, y_b) \sim (\epsilon^2, 1) \quad V_{\mathrm{CKM}} \sim \begin{pmatrix} \epsilon^2 & 1 \\ 1 & \epsilon^2 \end{pmatrix}$

CKM is not identity for $\epsilon \to 0$

Toy model 2 flavor (1st and 3rd) quark model

- representations and modular weights

Z_4^T-charge

$Q = 2 \quad \binom{0}{2} \qquad \bar{u} = \hat{1}' \oplus 1' \quad \binom{1}{2} \qquad \bar{d} = 1' \oplus 1' \quad \binom{2}{2}$

mod. weights: $k_Q = 0 \qquad k_u = (5, 6) \qquad k_d = (2,4)$

wo/ loss of generality

- modular forms

$Y_u \sim \left(Y_{\hat{2}}^{(k_u^1)} \quad Y_2^{(k_u^2)} \right) \qquad Y_d \sim \left(Y_2^{(k_d^1)} \quad Y_2^{(k_d^2)} \right)$

$\hat{2}$ exists for $k \geq 5$ 2 exists for $k \geq 2$

Why not e.g. $k_u = (5,2), k_d = (2,2)$?

Toy model 2 flavor (1st and 3rd) quark model

- representations and modular weights

Z_4^T-charge

$$Q = 2 \quad \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad \bar{u} = \hat{1}' \oplus 1' \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \bar{d} = 1' \oplus 1' \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

mod. weights: $k_Q = 0$ $\quad k_u = (5,6)$ $\quad k_d = (2,4)$

wo/ loss of generality

- bad example 1: $k_d = (2,2)$

→ $Y_d \sim \begin{pmatrix} \beta_1 Y_2^{(2)} & \beta_2 Y_2^{(2)} \end{pmatrix}$ same modular form in two columns

→ Y_d is rank-1, so down quark is massless

→ $k_d = (2,4)$ is the minimal possibility for $y_d \neq 0$

Toy model 2 flavor (1st and 3rd) quark model

- representations and modular weights Z_4^T-charge

$$Q = 2 \quad \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad \bar{u} = \hat{1}' \oplus 1' \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \bar{d} = 1' \oplus 1' \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

mod. weights: $k_Q = 0$ $\qquad k_u = (5, 6) \qquad k_d = (2,4)$

wo/ loss of generality

- bad example 3: $k_u = (5, 2)$ $\qquad t := \sqrt{2\mathrm{Im}\,\tau} \sim \sqrt{5}$

can. norm.

$\rightarrow$ $$Y_u \sim t^2 \begin{pmatrix} \epsilon^3 t^3 & \epsilon^2 \\ \epsilon\, t^3 & 1 \end{pmatrix} \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 t^2 \\ 1 & t^2 \end{pmatrix}$$

$\rightarrow$ $(y_u, y_d, y_b) \sim (\epsilon^3 t^3, \epsilon^2, t^2)$ $\qquad y_b, y_u$ are too large

$\rightarrow$ $k_u^2 = 6$ is minimal possibility

Toy model 2 flavor (1^{st} and 3^{rd}) quark model

- representations

Z_4^T-charge

$Q = 2 \quad \binom{0}{2}$ $\qquad \bar{u} = \hat{1}' \oplus 1' \quad \binom{1}{2}$ $\qquad \bar{d} = 1' \oplus 1' \quad \binom{2}{2}$

$k_u = (5, 6)$ $\qquad k_d = (2,4)$

$\rightarrow$ $Y_u \sim t^6 \begin{pmatrix} \epsilon^3 t^{-1} & \epsilon^2 \\ \epsilon t^{-1} & 1 \end{pmatrix}$ $\quad Y_d \sim t^6 \begin{pmatrix} \epsilon^2 t^{-4} & \epsilon^2 t^{-2} \\ t^{-4} & t^{-2} \end{pmatrix}$

$\rightarrow$ $(y_u, y_d, y_b) \sim (\epsilon^3 t^{-1}, \epsilon^2 t^{-4}, t^{-2})$ $\qquad V_{\mathrm{CKM}} \sim \begin{pmatrix} 1 & \epsilon^2 \\ \epsilon^2 & 1 \end{pmatrix}$

$\rightarrow$ this may be the minimal possibility

S_3 symmetry

the coefficients have the “hierarchy” structure

$$\alpha_1 \ll \alpha_2, \alpha_3 \qquad\qquad \beta_{11} \gg \beta_{21}, \beta_{22}, \beta_{23}$$

first low is smaller (larger) than others in up (down) quarks

- S_3 model with another modulus τ_2

d^c, q_1: singlet 1 $\qquad$ u^c, q_2: non-trivial singlet $1'$

$$\rightarrow \quad Y_u \propto \begin{array}{cc} \begin{pmatrix} \epsilon_2 & \epsilon_2 & \epsilon_2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} & \begin{array}{l} \alpha_1 \\ \\ \alpha_2, \alpha_3 \end{array} \end{array} \qquad Y_d \propto \begin{array}{cc} \begin{pmatrix} 1 & 1 & 1 \\ \epsilon_2 & \epsilon_2 & \epsilon_2 \\ \epsilon_2 & \epsilon_2 & \epsilon_2 \end{pmatrix} & \begin{array}{l} \beta_{11} \\ \\ \beta_{21}, \beta_{22}, \beta_{23} \end{array} \end{array}$$

the hierarchy is explained by $\epsilon_2(\tau_2) \sim 0.1$

Canonical normalization

- modular invariant kinetic term

kinetic term $\dfrac{\overline{Q}Q}{(-i\tau + i\bar{\tau})^{k_q}} \rightarrow \overline{Q}Q$

canonical basis

Yukawa coup. $Y^{(k_Y)}(\tau) \rightarrow (2\mathrm{Im}\tau)^{k_Y/2}\, Y^{(k_Y)}$

- When $\epsilon(\tau) \ll 1$

$\epsilon(\tau) \sim 0.05 \rightarrow t \coloneqq 2\mathrm{Im}\,\tau \sim 5$ gives additional structure

another FN-like mechanism controlled by modular weights

Spontaneous CP violation

* Thank to Tanimoto-san for comments

- from S_4'

$\epsilon(\tau) \sim 2\exp\left(\frac{2\pi\tau i}{4}\right)$ is a complex parameter

However, it induces only unphysical phases in CKM matrix up to ϵ^3

$\rightarrow$ spontaneous CP violation is too small $\sim \epsilon^4$

- from S_3

CPV from $\epsilon_2 \sim 0.1$ does not physical phase up to ϵ_2

However, $\epsilon_2^2 \sim 0.01$ ($\mathbb{Z}_2^T$ neutral) is enough for CKM phase

$\rightarrow$ moderate CP violation from S_3

Quark hierarchies

see also models based on A_4 (2212. 13336), Γ_6 (2301.03737), $A_4 \times A_4 \times A_4$ (2302.03326)

- masses and CKM matrix

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t$$

$$(m_d, m_s, m_b) \sim \epsilon^p (\epsilon^2, \epsilon^2, 1)\, m_t / t_\beta$$

$$V^{CKM} \sim \begin{pmatrix} 1 & 1 & \epsilon^2 \\ 1 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$$

$\epsilon \sim 0.05$
$p = 0{,}1$
$t_\beta = v_u / v_d$

- $N = 4$ is the minimal number for the hierarchy with ϵ^3
- $\mathcal{O}(0.1)$ deviations could be explained by modular forms

- representations of quarks for $p = 1$

c.f. Novichkov, Penedo, Petkov, 21'

there is only one combination of reps. for the quark hierarchy *

* assume no coexistence of $\mathbb{Z}_2^R$-even and –odd states in same quark

$\mathbb{Z}_4^T$-charge : (2,3,1) | 2 2 2 | 0 (0, 2)

$u^c = 3$ $d^c = 1' \oplus 1' \oplus 1'$ $Q = 1 \oplus 2$

RH up quark   RH down quark   LH doublet quark

Representation matrices in S_4'

$$\rho_S(2) = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}, \quad \rho_T(2) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and

$$\rho_S(3) = -\frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix}, \quad \rho_T(3) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix}.$$

The primed and hatted representations are related as

$$\begin{aligned} \rho_S(r) &= \; -\rho_S(r') = -i\rho_S(\hat{r}) = i\rho_S(\hat{r}'), \\ \rho_T(r) &= \; -\rho_T(r') = i\rho_T(\hat{r}) = -i\rho_T(\hat{r}'), \\ \mathbb{I} = \rho_R(r) &= \rho_R(r') = -\rho_R(\hat{r}) = -\rho_R(\hat{r}'). \end{aligned}$$

$\tau \sim i\infty$

$\rightarrow$

$$Y_1 \sim 1, \quad Y_{1'} \sim \epsilon^2, \quad Y_{\hat{1}} \sim \epsilon^3, \quad Y_{\hat{1}'} \sim \epsilon, \quad Y_2 \sim \begin{pmatrix} 1 \\ \epsilon^2 \end{pmatrix}, \quad Y_{\hat{2}} \sim \begin{pmatrix} \epsilon^3 \\ \epsilon \end{pmatrix},$$

$$Y_3 \sim \begin{pmatrix} \epsilon^2 \\ \epsilon^3 \\ \epsilon \end{pmatrix}, \quad Y_{3'} \sim \begin{pmatrix} 1 \\ \epsilon \\ \epsilon^3 \end{pmatrix}, \quad Y_{\hat{3}} \sim \begin{pmatrix} \epsilon \\ \epsilon^2 \\ 1 \end{pmatrix}, \quad Y_{\hat{3}'} \sim \begin{pmatrix} \epsilon^3 \\ 1 \\ \epsilon^2 \end{pmatrix},$$

Yukawa matrices

- model for $p = 1$

$$Y_u = \begin{pmatrix} \alpha_1[Y_3^{(4)}]_1 & \alpha_1[Y_3^{(4)}]_3 & \alpha_1[Y_3^{(4)}]_2 \\ -2\alpha_2[Y_3^{(6)}]_1 & \alpha_2[Y_3^{(6)}]_3 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_2 & \alpha_2[Y_3^{(6)}]_2 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_3 \\ -2\alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_1 & \alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_3 - \sqrt{3}\alpha_2[Y_3^{(6)}]_2 & \alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_2 - \sqrt{3}\alpha_2[Y_3^{(6)}]_3 \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \beta_{11}Y_{1'}^{(6)} & 0 & 0 \\ -\beta_{21}^{iY}[Y_2^{iY\,(8)}]_2 & -\beta_{22}[Y_2^{(6)}]_2 & -\beta_{23}[Y_2^{(4)}]_2 \\ \beta_{21}^{iY}[Y_2^{iY\,(8)}]_1 & \beta_{22}[Y_2^{(6)}]_1 & \beta_{23}[Y_2^{(4)}]_1 \end{pmatrix},$$

- model for $p = 0$

$$Y_u = \begin{pmatrix} \alpha_1[Y_3^{(6)}]_1 & \alpha_1[Y_3^{(6)}]_3 & \alpha_1[Y_3^{(6)}]_2 \\ -2\alpha_2[Y_3^{(6)}]_1 & \alpha_2[Y_3^{(6)}]_3 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_2 & \alpha_2[Y_3^{(6)}]_2 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_3 \\ -2\alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_1 & \alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_3 - \sqrt{3}\alpha_2[Y_3^{(6)}]_2 & \alpha_3^{iY}[Y_{3'}^{iY\,(6)}]_2 - \sqrt{3}\alpha_2[Y_3^{(6)}]_3 \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \beta_{11}Y_{\hat{1}}^{(9)} & 0 & 0 \\ \beta_{21}[Y_{\hat{2}}^{(9)}]_1 & \beta_{22}[Y_{\hat{2}}^{(7)}]_1 & \beta_{23}[Y_{\hat{2}}^{(5)}]_1 \\ \beta_{21}[Y_{\hat{2}}^{(9)}]_2 & \beta_{22}[Y_{\hat{2}}^{(7)}]_2 & \beta_{23}[Y_{\hat{2}}^{(5)}]_2 \end{pmatrix}.$$

Model for $p = 0$

$u^c = 3 \quad d^c = \hat{1}' \oplus \hat{1}' \oplus \hat{1}' \quad Q = 1 \oplus 2$

$d_1^c \quad d_2^c \quad d_3^c \qquad q_1 \quad q_2$

$$k_u = 2, \ \left(k_{d_1}, k_{d_2}, k_{d_3}\right) = (5\,3, 1), \quad \left(k_{q_1}, k_{q_2}\right) = (4, 4)$$

$$\tan\beta = 1.6, \tau = 1.5 + 2.7i, |\alpha_3| = 0.0013$$

at GUT scale

S.Antusch, V.Maurer 1306.6879

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_3'\end{pmatrix} = \begin{pmatrix}-0.27\\ -1.7\\ e^{-3.1i}\\ -1.4\end{pmatrix}$$

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\beta_{11}\\ \beta_{21}\\ \beta_{22}\\ \beta_{23}\end{pmatrix} = \begin{pmatrix}-6.9\\ 0.13\\ 0.28\\ 0.41\end{pmatrix}$$

$\rightarrow$

obs.	value	center	error
$y_u/10^6$	2.9	2.9	1.3
$y_c/10^3$	1.560	1.508	0.095
y_t	0.5464	0.5464	0.0084
$y_d/10^6$	9.00	9.06	0.87
$y_s/10^4$	1.73	1.79	0.14
$y_b/10^2$	1.011	0.994	0.013
s_{12}	0.2274	0.2274	0.0007
$s_{23}/10^2$	3.991	3.989	0.065
$s_{13}/10^3$	3.47	3.47	0.13
$\delta_{\rm CP}$	1.204	1.208	0.054

our model exp. error

- similar to the model for $p = 1$, but with $\tan\beta \sim 1$
- the sizes of coefficients are in $[0.13, 6.9]$, ratio is 50

Rank condition

- e.g. if $k_d = (4,2,2)$, $k_Q = (2,0)$

$$Y_d = \begin{pmatrix} \beta_{11} Y_{1'}^{(6)} & 0 & 0 \\ \beta_{21} Y_2^{(4)} & \beta_{22} Y_2^{(2)} & \beta_{23} Y_2^{(2)} \end{pmatrix}$$ is "rank-2" up to ϵ^3 $\rightarrow$ need larger weights

- # of representations * reps. are hatted for odd weights

weight	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	1	0	1	0	1	1	1	0
$1'$	0	0	1	0	0	1	1	0	1	1	1
2	0	1	0	1	1	1	1	2	1	2	2
3	1	0	1	1	2	1	2	2	3	2	3
$3'$	0	1	1	1	1	2	2	2	2	3	3

there are $2k+1$ independent modular forms at a weight k

of representations under S_4'

weight	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	1	0	1	0	1	1	1	0
$1'$	0	0	1	0	0	1	1	0	1	1	1
2	0	1	0	1	1	1	1	2	1	2	2
3	1	0	1	1	2	1	2	2	3	2	3
$3'$	0	1	1	1	1	2	2	2	2	3	3

* reps. for odd weights are hatted ones

there are $2k+1$ independent modular forms at a weight k

Bad case for CKM hierarchy

- Quark masses

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t \qquad (m_d, m_s, m_b) \sim \epsilon\, (\epsilon^2, \epsilon, 1)\, m_t / t_\beta$$

is also realized if

c.f. Novichkov, Penedo, Petkov, 21’

$$\overset{(2,3,1)}{Q} = 3 \qquad u^c = \overset{2}{1'} \oplus \overset{0}{1} \oplus \overset{0}{1} \qquad d^c = \overset{0}{1} \oplus \overset{0}{1} \oplus \overset{0}{1}$$

- Yukawa hierarchies

$$Y_u \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ \epsilon & \epsilon & \epsilon^3 \\ \epsilon^3 & \epsilon^3 & \epsilon \end{pmatrix} \begin{pmatrix} t \\ c \\ u \end{pmatrix}_L \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ \epsilon^3 & \epsilon^3 & \epsilon^3 \end{pmatrix} \begin{pmatrix} s \\ b \\ d \end{pmatrix}_L$$

$$\rightarrow \quad V^{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \mathcal{O}(\epsilon)$$

not identity at LO