$(p-1)$-Bracket for Dp-branes in Large R-R Field Background

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Large $(p-1)$-Form Field Background

- Lagrangian [Ho and Ma 2013]

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2,$$

where

$$\mathcal{L}_1 \equiv -\frac{1}{2}(\mathcal{D}_\alpha X^I)^2 + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta} + \frac{1}{2}\mathcal{F}^2_{\alpha\dot{\mu}}$$

and

$$\begin{aligned}\mathcal{L}_2 &\equiv -\frac{1}{2}\sum_{n,m,l\in S}\frac{g^{2(p-2-m)}}{(n!)(m!)^2(l!)} \\ &\quad \{X^{\dot{\mu}_1},\cdots,X^{\dot{\mu}_n},a_{\dot{\nu}_1},\cdots,a_{\dot{\nu}_m},y^{\dot{\nu}_1},\cdots,y^{\dot{\nu}_m},X^{I_1},\cdots,X^{I_l}\}^2. \\ S &\equiv \{(n,m,l)\mid n,m,l\geq 0\,;\; n+2m+l=p-1\}\end{aligned}$$

- T-duality for each p; S-duality for $p=3$ [Ho and Ma 2014]

Gauge Symmetry

- gauge symmetry, VPD, $\delta y^{\dot{\mu}} = \hat{\kappa}^{\dot{\mu}}$ preserves the volume-form $dy^{\dot{1}} dy^{\dot{2}} \cdots dy^{\dot{\mu}_{p-1}}$ if $\partial_{\dot{\mu}} \hat{\kappa}^{\dot{\mu}} = 0$
- VPD covariant

$$\hat{\delta}_{\hat{\Lambda}} \Phi = \hat{\kappa}^{\dot{\mu}} \partial_{\dot{\mu}} \Phi \tag{1}$$

- Field Content for R-R D-branes: $\hat{b}$, $\hat{a}$
- gauge transformation is

$$\begin{aligned} \hat{\delta}_{\hat{\Lambda}} \hat{b}^{\dot{\mu}} &= \hat{\kappa}^{\dot{\mu}} + i[\hat{\lambda}, \hat{b}^{\dot{\mu}}] + g \hat{\kappa}^{\dot{\nu}} \partial_{\dot{\nu}} \hat{b}^{\dot{\mu}}; \\ \hat{\delta}_{\hat{\Lambda}} \hat{a}_{\dot{\mu}} &= \partial_{\dot{\mu}} \hat{\lambda} + i[\hat{\lambda}, \hat{a}_{\dot{\mu}}] + g(\hat{\kappa}^{\dot{\nu}} \partial_{\dot{\nu}} \hat{a}_{\dot{\mu}} + \hat{a}_{\dot{\nu}} \partial_{\dot{\mu}} \hat{\kappa}^{\dot{\nu}}); \\ \hat{\delta}_{\hat{\Lambda}} \hat{a}_{\alpha} &= \partial_{\alpha} \hat{\lambda} + i[\hat{\lambda}, \hat{a}_{\alpha}] + g(\hat{\kappa}^{\dot{\nu}} \partial_{\dot{\nu}} \hat{a}_{\alpha} + \hat{a}_{\dot{\nu}} \partial_{\alpha} \hat{\kappa}^{\dot{\nu}}), \end{aligned} \tag{2}$$

  where $g = 1/C_{\dot{\mu}_1 \cdots \dot{\mu}_{p-1}}$
- R-R field background: dominant in the U(1) sector

$(p-1)$-Bracket

- $(p-1)$-Bracket $\{f_1, f_2, \cdots, f_{p-1}\}_{(p-1)}$ $(\partial_{\dot\mu} \to \mathcal{D}_{\dot\mu})$

$$\epsilon^{\dot\mu_1\dot\mu_2\cdots\dot\mu_{p-1}}(\mathcal{D}_{\dot\mu_1} f_1)(\mathcal{D}_{\dot\mu_2 f_2})\cdots(\mathcal{D}_{\dot\mu_{p-1}} f_{p-1}), \qquad (3)$$

where

$$\mathcal{D}_{\dot\mu}\hat{X}^{\dot\nu} \equiv \partial_{\dot\mu}\hat{X}^{\dot\nu} - i[\hat{a}_{\dot\mu}, \hat{X}^{\dot\nu}] \equiv D_{\dot\mu}\hat{X}^{\dot\nu}; \quad \mathcal{D}_{\dot\mu}\hat{a}_{\dot\nu} \equiv (\partial_{\dot\mu} - i\hat{a}_{\dot\mu})\hat{a}_{\dot\nu}$$

- R-R D9-Branes $\mathcal{L}_2^{(p=9)}$

$$\sim \mathrm{Str}\left[\left(\left\{\hat{X}^{\dot\mu_1}, \cdots, \hat{X}^{\dot\mu_n}, \hat{a}_{\dot\nu_1}, \cdots, \hat{a}_{\dot\nu_m}, y^{\dot\nu_1}, \cdots, y^{\dot\nu_m}\right\}_{(p-1)}\right)^2\right]$$

Covariant Field Strength

-

$$
\begin{aligned}
\hat{\mathcal{F}}_{\alpha\dot{\mu}} &\equiv (\hat{V}^{-1})_{\dot{\mu}}{}^{\dot{\nu}}(\hat{F}_{\alpha\dot{\nu}} + g\hat{F}_{\dot{\nu}\dot{\delta}}\hat{B}_{\alpha}{}^{\dot{\delta}});\\
\hat{\mathcal{F}}_{\alpha\beta} &\equiv \hat{F}_{\alpha\beta} - g(\hat{F}_{\alpha\dot{\mu}}\hat{B}_{\beta}{}^{\dot{\mu}} + \hat{F}_{\dot{\mu}\beta}\hat{B}_{\alpha}{}^{\dot{\mu}})\\
&\quad + \frac{g^2}{2}\hat{F}_{\dot{\mu}\dot{\nu}}(\hat{B}_{\alpha}{}^{\dot{\mu}}\hat{B}_{\beta}{}^{\dot{\nu}} + \hat{B}_{\beta}{}^{\dot{\mu}}\hat{B}_{\alpha}{}^{\dot{\nu}}),
\end{aligned}
$$

where

$$
\hat{V}_{\dot{\nu}}{}^{\dot{\mu}} \equiv \delta_{\dot{\nu}}{}^{\dot{\mu}} + gD_{\dot{\nu}}\hat{b}^{\dot{\mu}}, \tag{4}
$$

$$
\hat{V}_{\dot{\mu}}{}^{\dot{\nu}}(D^{\alpha}\hat{b}_{\dot{\nu}} - \hat{V}^{\dot{\rho}}{}_{\dot{\nu}}\hat{B}^{\alpha}{}_{\dot{\rho}}) + \epsilon^{\alpha\beta}\hat{F}_{\beta\dot{\mu}} + g\epsilon^{\alpha\beta}\hat{F}_{\dot{\mu}\dot{\nu}}\hat{B}_{\beta}{}^{\dot{\nu}} = 0 \tag{5}
$$

- generalization provides $\mathcal{L}_1^{(p=9)}$

Discussion and Outlook

- obtain YM theory after integrating out $\hat{b}$
- $(p-1)$-bracket in non-Abelian Theory
- extend to all orders from T-duality (or D3-brane theory)
- consistent with multiple M5-branes [Chu and Ko 2012]

Thank you!