



CSIC

# Strings, D-branes, and supergravities from M2-branes with discrete spectra

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STRING PHENOMENOLOGY 2023

IBS, DAEJEON

In collaboration with M.P Garcia del Moral y A. Restuccia



(Font, Ibañez, Lust, Quevedo '90) (Rabinovici '94) (Witten '95) (Horava, Witten '95) (Hull, Townsend '95)



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(Cremmer, Julia, Scherk '78)(Brink, Howe '80) (Cremmer, Ferrara '80)





M-theory



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- SU(N) regularized model with continuous spectra (De Wit, Luscher, Nicolai '88)
   String configurations = instabilities
- Membranes with "winding" on a torus have the same spectral behavior (De Wit, Peeters, Plefka '97, '98)

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- It has been formulated in:  $\mathbb{R}^{1,8} \times T^2$ ,  $\mathbb{R}^{1,3} \times T^6 \times S^1$  and  $\mathbb{R}^{1,3} \times G_2$ (García del Moral, Peña, Restuccia '08) (Bilhal, García del Moral, Restuccia '09)

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• SU(N) regularized model with **discrete supersymmetric spectra**.

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(García del Moral, CLH '22)

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#### **GOAL:** Non perturbative string theory (and supergravity)



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The worldvolume ( $\Sigma_g \times \mathbb{R}$ ) theory of these sectors is characterized by:

- Embedding of the M2-brane (with g = 1) on  $\mathbb{R}^{1,8} \times T^2$
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- <u>Nontrivial gauge symmetries: symplectic and 3 U(1) (on the same bundle)</u>. (García del Moral, CLH, León, Peña, Restuccia '19, '20)

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The associated (p,q)-strings on a circle, which we called parabolic (p,q)-strings or just q-strings, were conjectured by C. Hull (Hull, '98) as compactification of F-theory on a twisted 3-torus. Their low energy is given by type IIB parabolic gauged supergravity

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#### Type IIA D2-branes with fluxes from M2-branes with discrete spectra How can we obtan these objects from DBI + WZ action? 3 U(1) and symplectic **U(1)** DBI Nontrivial gauge Nontrivial RR and NSNS LCG D2-brane $\mathcal{H}_{10D} =$ ++fluxes (B-field terms) symmetries on a torus

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## Acknowledgments

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# THANK YOU FOR YOUR ATTENTION



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