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Free Fermionic Webs of Heterotic T-folds

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Overview

- Plan for talk:
  - Motivation
  - Heterotic Orbifolds and Bosonisation
  - Fermionic Symmetries
  - Example Free Fermionic Web
  - Moduli
  - Intrinsically Asymmetric
  - Conclusions

Motivation:

- ⋆ Non-geometric spaces (asymmetric orbifolds)- stringy geometry

  [Narain+,Antoniadis+,Kawai+,Plauschinn,Groot Nibbelink+,Dabholkar+,Graña+[1, 2, 3, 4, 5, 6, 7, 8]]

- ⋆ Moduli stabilisation *and* identification

- ⋆ Asymmetric orbifolds Pheno. features:

  - ⋆ Early realistic models [Faraggi+ [9, 10]], Fixing Geometric Moduli, Doublet-Triplet Splitting, Untwisted TQMC, Hierarchical Top-bottom quark mass splitting [Faraggi [11, 12, 13, 14]]

Heterotic Orbifolds

- ⋆ Realise in terms of bosons or fermions via bosonisation.
- ⋆ Bosonic coordinate fields in $d = 10 - D$:

$$\begin{aligned} \text{R:} \quad & x^\mu(z),\ \psi^\mu, \chi^{\ I},\ X_{\text{R}}^{\ I}, \\ \text{L:} \quad & x^\mu(\bar{z}),\ X_{\text{L}}^{\ a=1,\dots D+16} \end{aligned} \tag{1}$$

$I = 1, ..., D, \mu = 2, ..., d - 1$

- ⋆ Torus periodicites

$$X \sim X + 2\pi\, N\,, \qquad N \in \mathbb{Z}^{2D+16}\,. \tag{2}$$

- ⋆ Orbifold action

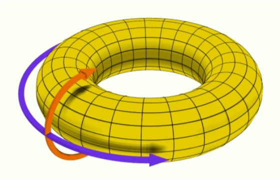
$$X \sim e^{2\pi i\, v}\, X - 2\pi\, V\,, \tag{3}$$

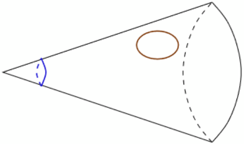
Heterotic Orbifold Input data

- ⋆ (Narain) orbifold inputs are these v (twist) and V (shift) vectors
- ⋆ In free fermionic models define: basis vectors of boundary conditions

$$\boldsymbol{\beta} = \{\beta(f_1), ..., \beta(f_n)\} \tag{4}$$

- ⋆ $\mathbb{Z}_2$ restriction: $\alpha(f) = 0, 1$ and v, V entries real
- ⋆ (GGSO phases/discrete torsion details not needed here)





Right Bosonisation and action

- Worldsheet SUSY requires invariant supercurrent

$$T_F = i\,\psi_\mu \partial x^\mu + i\,\chi^I \partial X_{\mathsf{R}}^I = i\psi_\mu \partial x^\mu + i\chi^I y^I w^I \qquad (5)$$

via R bosonisation $\partial X^i \sim i : y^i w^i :$ s.t.

$$e^{i\,{X_{\mathsf{R}}}^i} \sim y^{\,i} + i w^{\,i} \qquad (6)$$

- R orbifold action

$$v_{\mathsf{R}} = \tfrac{1}{2}\beta(w) - \tfrac{1}{2}\beta(y)\,, \qquad V_{\mathsf{R}} = \tfrac{1}{2}(1^D) - \tfrac{1}{2}\beta(y)\,. \qquad (7)$$

Left bosonisation and action

⋆ L bosonisation, $u \neq v =$1,...,32+2D:

$$e^{i\bar{X}_{\mathsf{L}}{}^{i}} \sim \bar{f}^{\,u} + i\bar{f}^{\,v} \tag{8}$$

⋆ Bosonise via $\bar{\boldsymbol{\beta}} = (\bar{\boldsymbol{\beta}}_{\mathsf{odd}}, \bar{\boldsymbol{\beta}}_{\mathsf{even}})$

$\bar{\boldsymbol{\beta}}_{\mathsf{odd}}$	$\bar{\beta}^{1}$	$\dots$	$\bar{\beta}^{2D+31}$
$\bar{\boldsymbol{\beta}}_{\mathsf{even}}$	$\bar{\beta}^{2}$	$\dots$	$\bar{\beta}^{2D+32}$

L action:

$$v_{\mathsf{L}} = \tfrac{1}{2}\bar{\beta}_{\mathsf{even}} - \tfrac{1}{2}\bar{\beta}_{\mathsf{odd}}\,, \qquad V_{\mathsf{L}} = \tfrac{1}{2}(1^{D+16}) - \tfrac{1}{2}\bar{\beta}_{\mathsf{odd}}\,. \tag{9}$$

⋆ $\Longrightarrow$ A single FF model has many bosonic interpretations!

Fermionic Symmetries

- ⋆ Fermion inversion (u): $\bar{f}^u \to -\bar{f}^u$.
- ⋆ Permutation $(u_1 \ \cdots \ u_p)$ acts as $\bar{f}^{u_1} \to \bar{f}^{u_2} \ldots \to \bar{f}^{u_p} \to \bar{f}^{u_1}$ leaving remaining fermions inert.
- ⋆ Group generated by permutation (uv)
- ⋆ Effect on bosonic data (use eqs. (6,8)) summarised in following tables

Fermionic Symmetries Summary Table

Fermionic symmetry	Action on twist and shift entries
(2a- 1 2b-1)(2a 2b)	$v_{\mathsf{L}}^{a} \leftrightarrow v_{\mathsf{L}}^{b}\,,\; V_{\mathsf{L}}^{a} \leftrightarrow V_{\mathsf{L}}^{b}$
(2a)	$v_{\mathsf{L}}^{a} \to -v_{\mathsf{L}}^{a} + 2\,V_{\mathsf{L}}^{a}\,,\; V_{\mathsf{L}}^{a} \to V_{\mathsf{L}}^{a}$
(2a- 1 2a)	$v_{\mathsf{L}}^{a} \to -v_{\mathsf{L}}^{a}\,,\; V_{\mathsf{L}}^{a} \to V_{\mathsf{L}}^{a} - v_{\mathsf{L}}^{a}$
(2a 2b)	$v_{\mathsf{L}}^{a} \to v_{\mathsf{L}}^{b} + V_{\mathsf{L}}^{a} - V_{\mathsf{L}}^{b}\,,\; V_{\mathsf{L}}^{a} \to V_{\mathsf{L}}^{a}\,,$ $v_{\mathsf{L}}^{b} \to v_{\mathsf{L}}^{a} + V_{\mathsf{L}}^{b} - V_{\mathsf{L}}^{a}\,,\; V_{\mathsf{L}}^{b} \to V_{\mathsf{L}}^{b}$
(2a-1 2b-1)	$v_{\mathsf{L}}^{a} \to v_{\mathsf{L}}^{a} + V_{\mathsf{L}}^{a} - V_{\mathsf{L}}^{b}\,,\; V_{\mathsf{L}}^{a} \to V_{\mathsf{L}}^{b}\,,$ $v_{\mathsf{L}}^{b} \to v_{\mathsf{L}}^{b} + V_{\mathsf{L}}^{b} - V_{\mathsf{L}}^{a}\,,\; V_{\mathsf{L}}^{b} \to V_{\mathsf{L}}^{a}$
(2a-1)	$v_{\mathsf{L}}^{a} \to v_{\mathsf{L}}^{a} - 2\,V_{\mathsf{L}}^{a}\,,\; V_{\mathsf{L}}^{a} \to -V_{\mathsf{L}}^{a}$

Some generate actions (T-dualities) in the bosonic theory; others seemingly not $\Longrightarrow$ "enhanced T-duality"

Example

- Take a simple 6D FF model with single $\mathbb{Z}_2$ twist

$$\begin{aligned} \mathbb{1} &= \{\text{all periodic}\} \quad \text{(R sector)} \\ \boldsymbol{S} &= \{\psi^{\mu=1,2,3,4}, \chi^{12}, \chi^{34}\} \quad \text{(SUSY generator)} \\ \boldsymbol{b} &= \{\chi^{12}.\chi^{34}, y^{1,2,3,4} \mid \bar{f}^{\,1,2,3,4}, \bar{f}^{\,9,\ldots,12}\} \quad (\mathbb{Z}_2 \text{ twist}) \\ \boldsymbol{\xi} &= \{\bar{f}^{\,9,\ldots,40}\} \quad (SO(32) \text{ generator}) \end{aligned} \tag{10}$$

- The twist $\boldsymbol{b}$ has multiple symmetric and asymmetric interpretations

Interpreting $\{\mathbb{1}, \boldsymbol{S}, \boldsymbol{\xi}, \boldsymbol{b}\}$

⋆ Taking $X_L^1 \sim \bar{f}^{\,1} + i\bar{f}^{\,2}$, $X_L^2 \sim \bar{f}^{\,3} + i\bar{f}^{\,4}$, etc, we have

$$v = \frac{1}{2}(1^4|0^{20}) \tag{11}$$

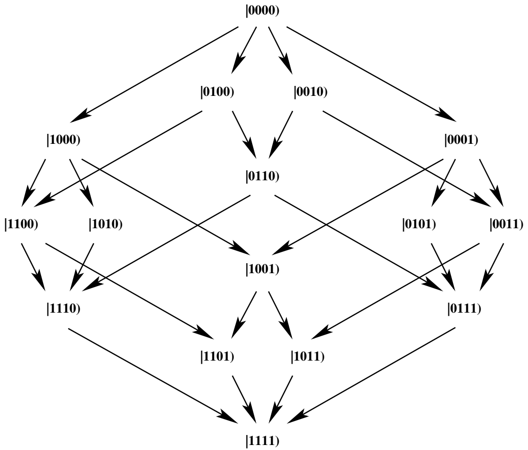
⋆ Permutation: $(2\,6)^{p_1}$, $p_1 = 0, 1$, map twist $v_\mathsf{L} = \frac{1}{2}(0^{20})$ from $\boldsymbol{b}$ to

$$v_L = \tfrac{1}{2}(1, 0, 1, 0, 0^{16})\,, \tag{12}$$

i.e. two of bosonic coordinates now twisted

$\{\mathbb{1}, \boldsymbol{S}, \boldsymbol{\xi}, \boldsymbol{b}\}$- Fermionic Web

Can generate web with other such permutations w.r.t. $\boldsymbol{b}$



(b, ξ)-twist frequencies

b \ ξ	0	2	4	6	8	Sum
0	1	2	3	2	1	**9**
2	2	11	18	12	3	**46**
4	3	18	32	19	6	**78**
6	2	12	19	18	7	**58**
8	1	3	6	7	5	**22**
Sum	**9**	**46**	**78**	**58**	**22**	**213**

The number of bosonisations grows quickly for models with more basis vectors/orbifold actions

Moduli/Thirring Interactions

⋆ The unfixed Narain moduli correspond to operators

$$m_{i\,a}\,\partial X_{\mathsf{R}}{}^{i}\,\bar{\partial} X_{\mathsf{L}}{}^{a}\,, \tag{13}$$

Depends on bosonisation!

⋆ Full set of Thirring Interactions [Chang and Kumar [16]]

$$m_{i\,uv}\,y^{i} w^{i}\,\bar{f}^{\,u}\,\bar{f}^{\,v}\,, \tag{14}$$

total number of massless untwisted scalars is bosonisation independent

Intrinsically Asymmetric Definition

- ⋆ A fermionic model that is an asymmetric orbifold for all bosonisations
- ⋆ $\nexists$ D currents, $\overline{J}^i$, w/ identical twist actions as J^i such that remaining 16 currents $\overline{J}^x$ inert $\forall$ (twist) basis vectors.
- ⋆ Example: purely R twist vector

$$\boldsymbol{b} = \{\chi^{1,2,3,4}, y^{1,2,3,4}\} \tag{15}$$

cannot be interpreted symmetrically

Conclusions

- ⋆ Enhanced T-duality group- generalised T-folds
- ⋆ Applications to Moduli Stabilisation, in particular defining *intrinsically* asymmetric orbifolds
- ⋆ Connections between symmetric and asymmetric orbifolds

Outlook

- ⋆ Enhanced T-duality only at free fermionic points?
- ⋆ Higher order orbifolds?
- ⋆ Type II/Heterotic duality?
- ⋆ Relation to Heterotic T-fold webs from F-theory? [McOrist+ [17]]
- ⋆ Relevance to phenomenological features- intrinsically asymmetric models (truly) reducing moduli space

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