Finiteness of Flux Vacua & Tadpole Conjecture

Based on upcoming work with Thomas Grimm

Jeroen Monnee
Utrecht University

String Pheno 2023
July 6th 2023

Question: *Is the number of 4d EFT's coming from string theory finite?*

[Douglas; 2003 | Douglas, Acharya; 2006]

Question: *Is the number of 4d EFT’s coming from string theory finite?*

[Douglas; 2003 | Douglas, Acharya; 2006]

This talk: Flux compactification of IIB / F-theory on Calabi-Yau.

Question: *Is the number of 4d EFT's coming from string theory finite?*

[Douglas; 2003 | Douglas, Acharya; 2006]

This talk: Flux compactification of IIB / F-theory on Calabi-Yau.

In this setting, finiteness of self-dual vacua has been established using **tame geometry**. [Bakker, Grimm, Schnell, Tsimerman; 2021]

(see also Lorenz' talk!)

In this talk, I will **not** use tame geometry.

In this talk, I will **not** use tame geometry.

Instead, I will use methods from **asymptotic Hodge theory**.

For two reasons:

In this talk, I will **not** use tame geometry.

Instead, I will use methods from **asymptotic Hodge theory**.

For two reasons:

1) Local/intuitive point of view

In this talk, I will **not** use tame geometry.

Instead, I will use methods from **asymptotic Hodge theory**.

For two reasons:

1) Local/intuitive point of view

2) Possible new insights into e.g. tadpole conjecture

Fluxes generate a scalar potential for the **complex structure moduli**

$$V = \frac{1}{\mathcal{V}_4^3}\left(\int_{Y_4} G_4 \wedge \star G_4 - \int_{Y_4} G_4 \wedge G_4\right)$$

Fluxes generate a scalar potential for the **complex structure moduli**

$$V = \frac{1}{\mathcal{V}_4^3}\left(\int_{Y_4} G_4 \wedge \star G_4 - \int_{Y_4} G_4 \wedge G_4\right)$$

Global minima must satisfy three conditions

Fluxes generate a scalar potential for the **complex structure moduli**

$$V = \frac{1}{\mathcal{V}_4^3}\left(\int_{Y_4} G_4 \wedge \star G_4 - \int_{Y_4} G_4 \wedge G_4\right)$$

Global minima must satisfy three conditions

Quantization

$$G_4 \in H^4\left(Y_4, \mathbb{Z}/2\right)$$

Fluxes generate a scalar potential for the **complex structure moduli**

$$V = \frac{1}{\mathcal{V}_4^3}\left(\int_{Y_4} G_4 \wedge \star G_4 - \int_{Y_4} G_4 \wedge G_4\right)$$

Global minima must satisfy three conditions

Quantization

$$G_4 \in H^4\left(Y_4, \mathbb{Z}/2\right)$$

Tadpole constraint

$$\frac{1}{2}\int_{Y_4} G_4 \wedge G_4 \leq \frac{\chi(Y_4)}{24}$$

Fluxes generate a scalar potential for the **complex structure moduli**

$$V = \frac{1}{\mathcal{V}_4^3}\left(\int_{Y_4} G_4 \wedge \star G_4 - \int_{Y_4} G_4 \wedge G_4\right)$$

Global minima must satisfy three conditions

Quantization

$$G_4 \in H^4\left(Y_4, \mathbb{Z}/2\right)$$

Tadpole constraint

$$\frac{1}{2}\int_{Y_4} G_4 \wedge G_4 \leq \frac{\chi(Y_4)}{24}$$

Self-duality

$$\star G_4 = G_4$$

On the **self-dual locus**, tadpole constraint becomes

$$||G_4||^2 := \int_{Y_4} G_4 \wedge \star G_4 < K$$

Hodge norm

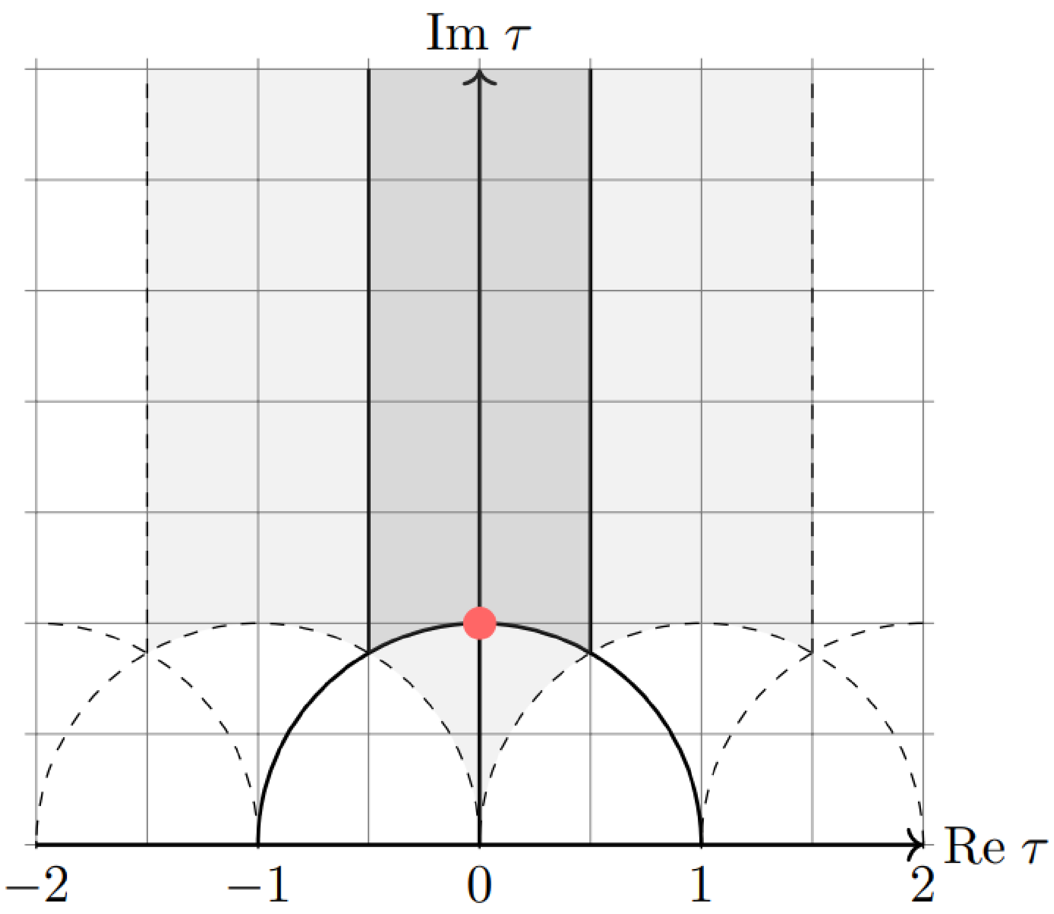
On the **self-dual locus**, tadpole constraint becomes

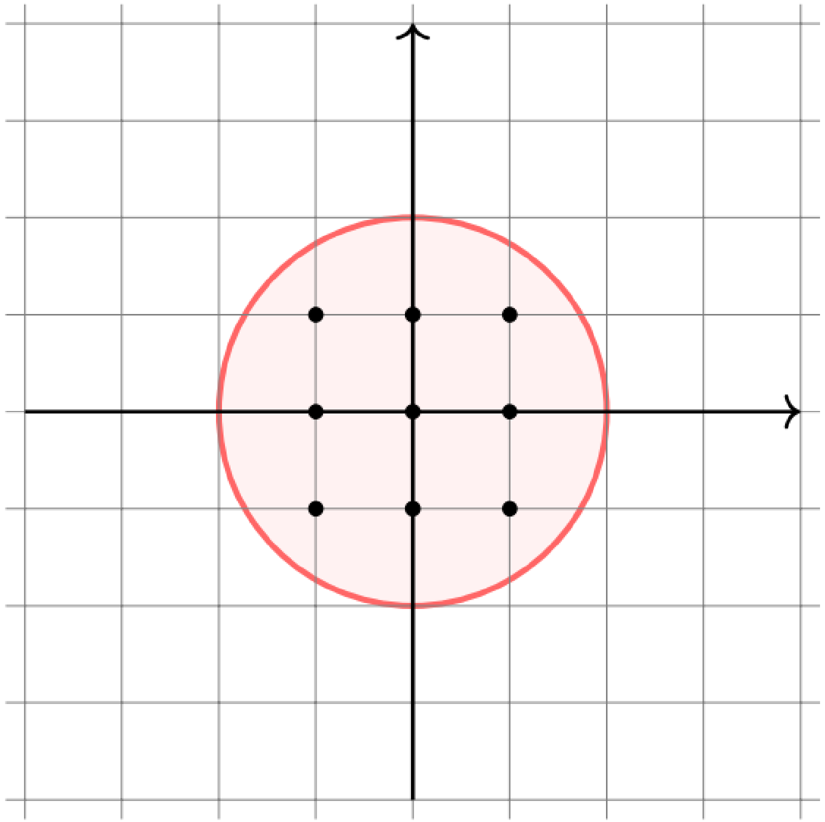
$$||G_4||^2 := \int_{Y_4} G_4 \wedge \star G_4 < K$$

Hodge norm

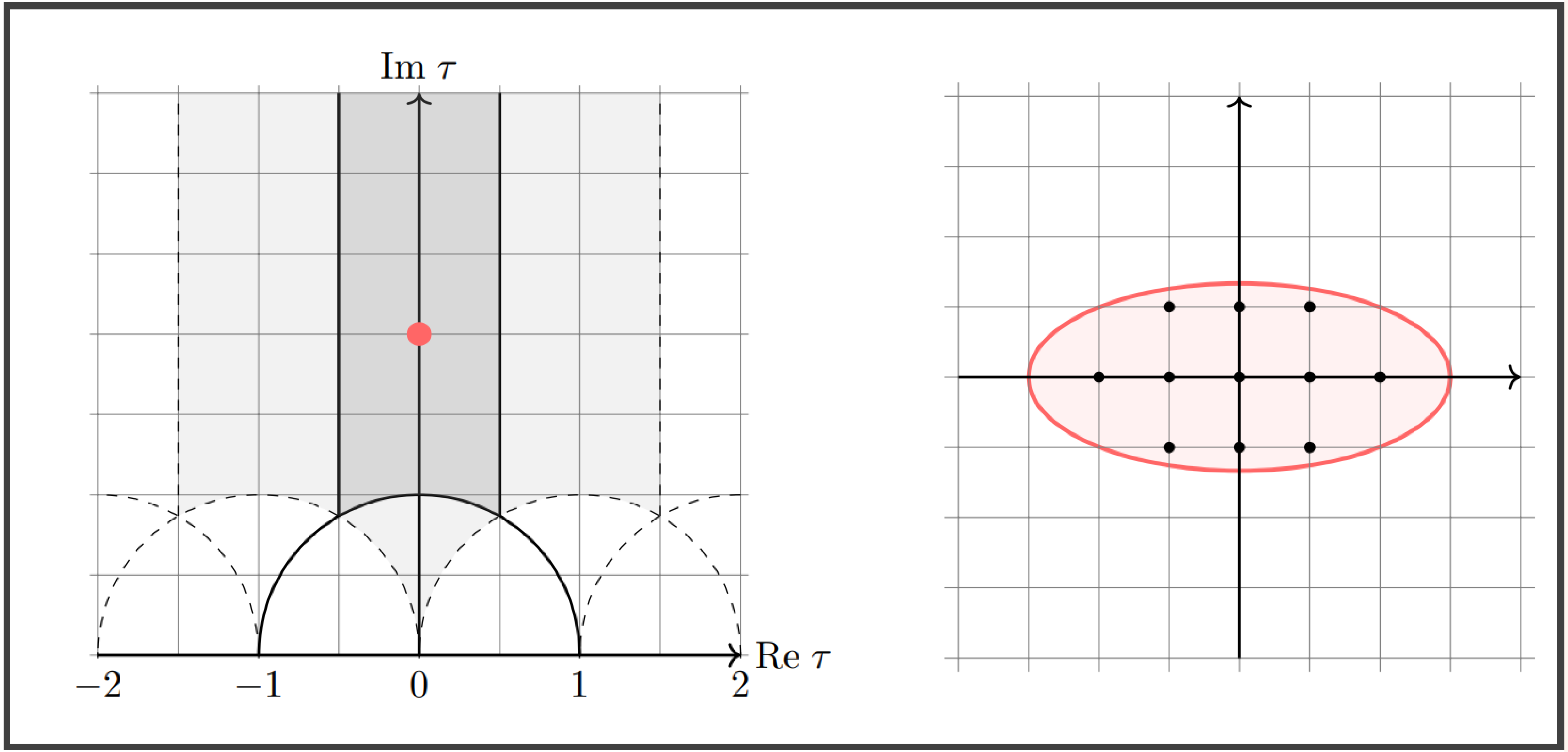
Effectively, search for lattice points within a region whose **shape/size** is controlled by the complex structure moduli.

A Toy Example (Torus)

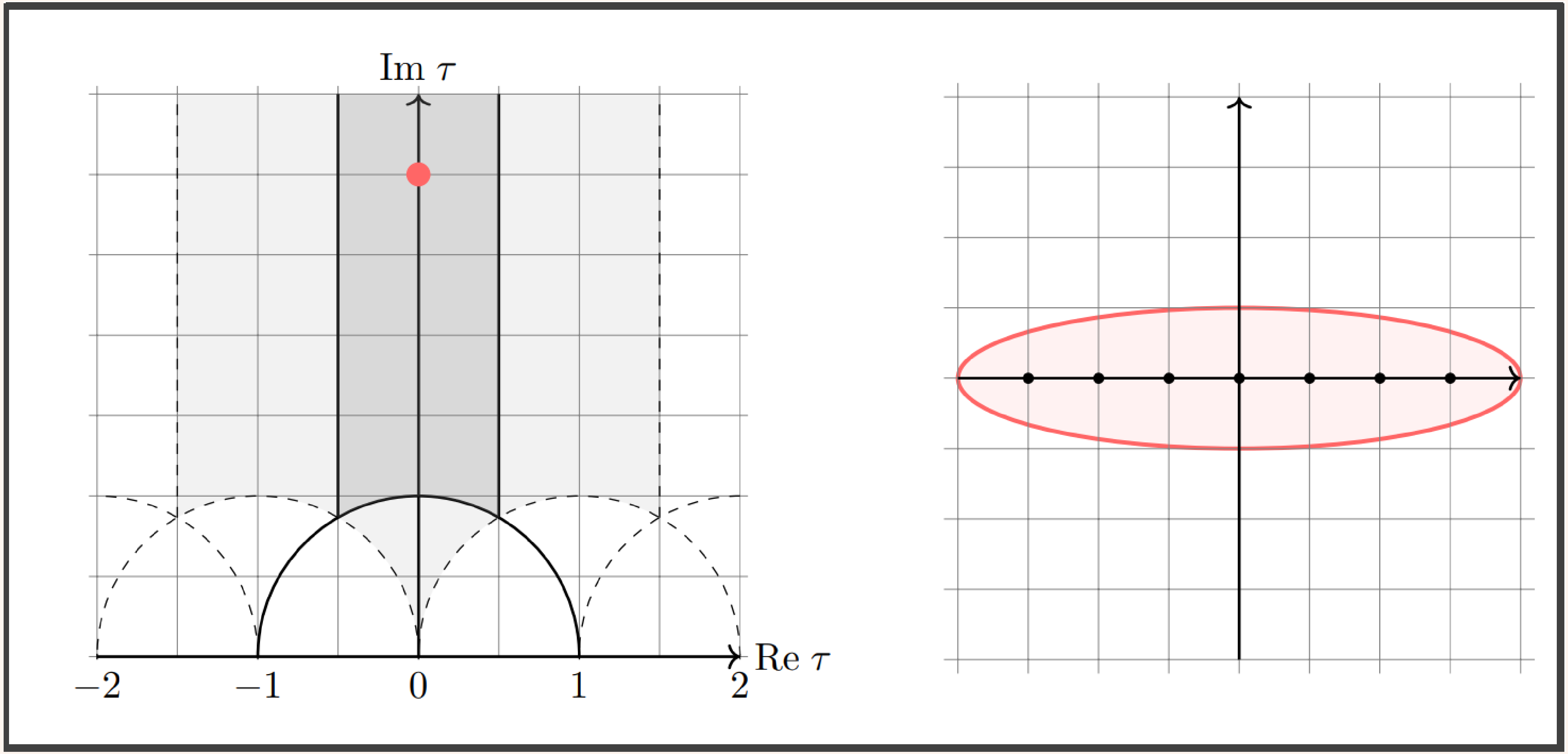




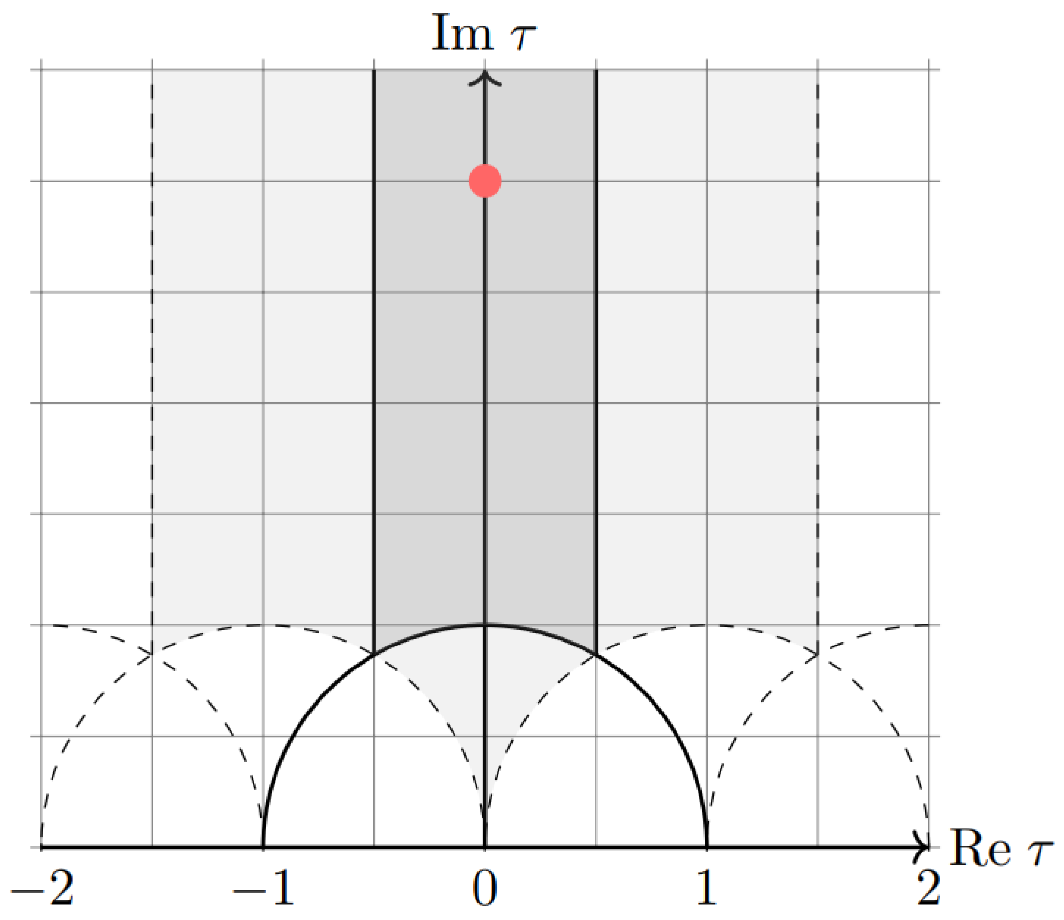
A Toy Example (Torus)

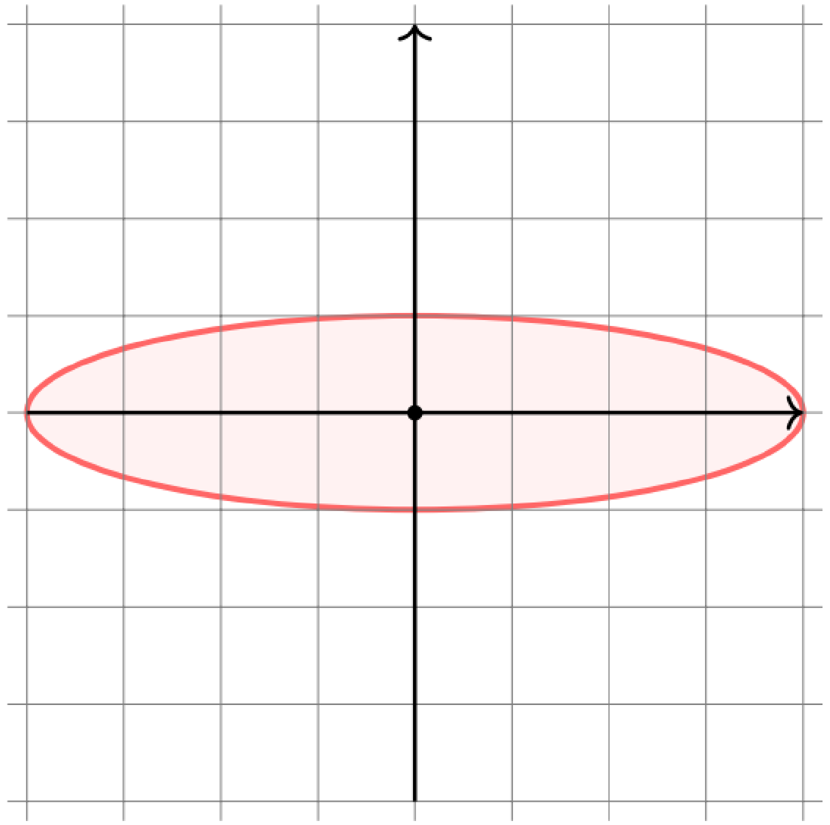


A Toy Example (Torus)

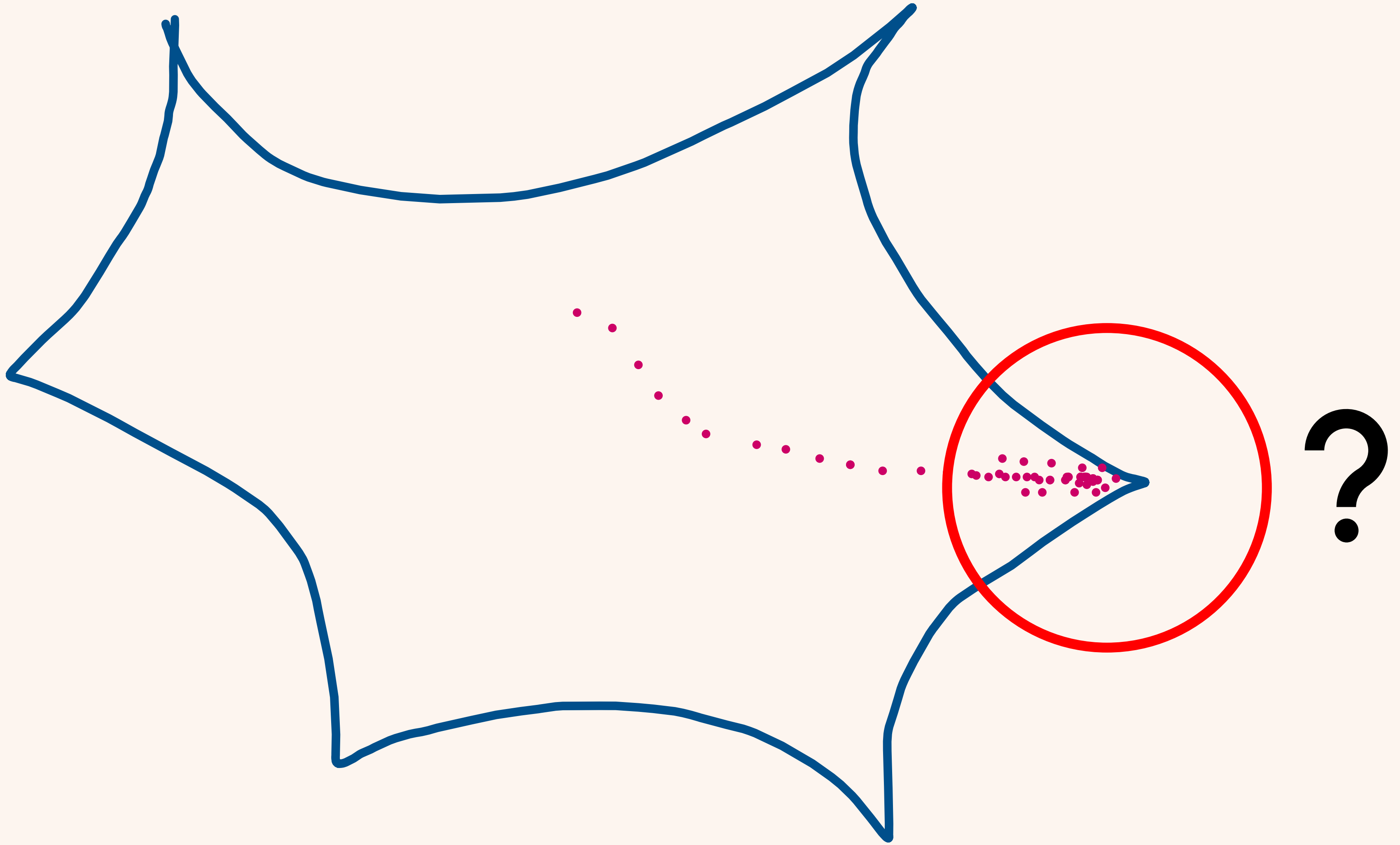


A Toy Example (Torus)

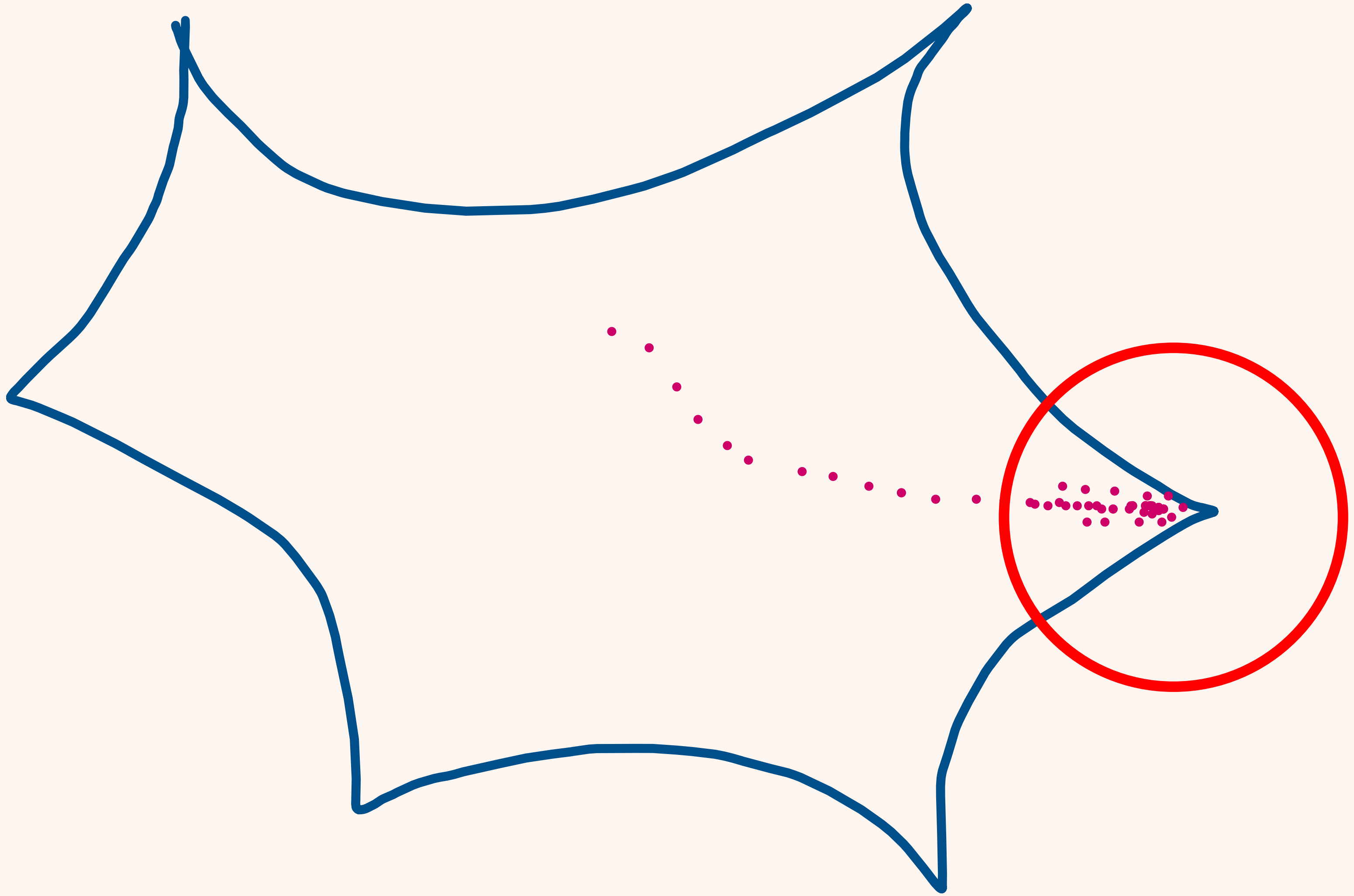




Q: Can there be infinite tails of vacua towards the boundary of the moduli space?



Q: Can there be infinite tails of vacua towards the boundary of the moduli space?



NO!

To answer this question, we study the **asymptotics of the Hodge norm** for

To answer this question, we study the **asymptotics of the Hodge norm** for

1) Arbitrary Calabi-Yau

To answer this question, we study the **asymptotics of the Hodge norm** for

1) Arbitrary Calabi-Yau
2) Any number of moduli

To answer this question, we study the **asymptotics of the Hodge norm** for

1) Arbitrary Calabi-Yau
2) Any number of moduli

Earlier works focused on the **Sl(2)-approximation**

[Grimm, Li, Valenzuela; 2021] [Graña, Grimm, van de Heisteeg, Herraez, Plauschinn; 2022] [Calderón-Infante, Ruiz, Valenzuela; 2022] […]

$$||G_4||^2 \sim \sum_{\ell_1,\dots,\ell_m} \left(\frac{y_1}{y_2}\right)^{\ell_1} \cdots y_m^{\ell_m} \, ||\,(G_4)_{(\ell_1,\dots,\ell_m)}\,||_\infty^2$$

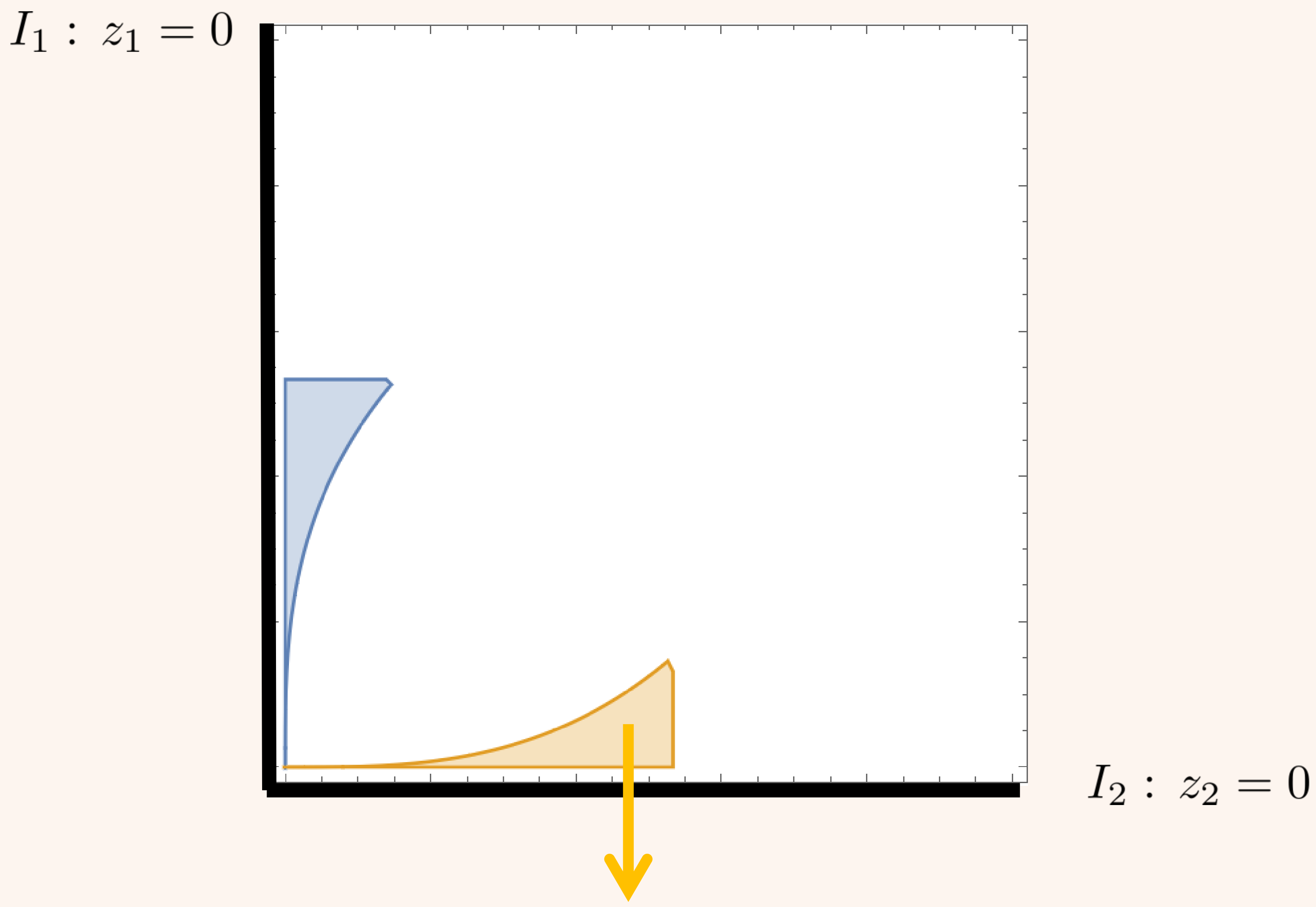
sl(2) weights

saxions

Boundary Hodge norm

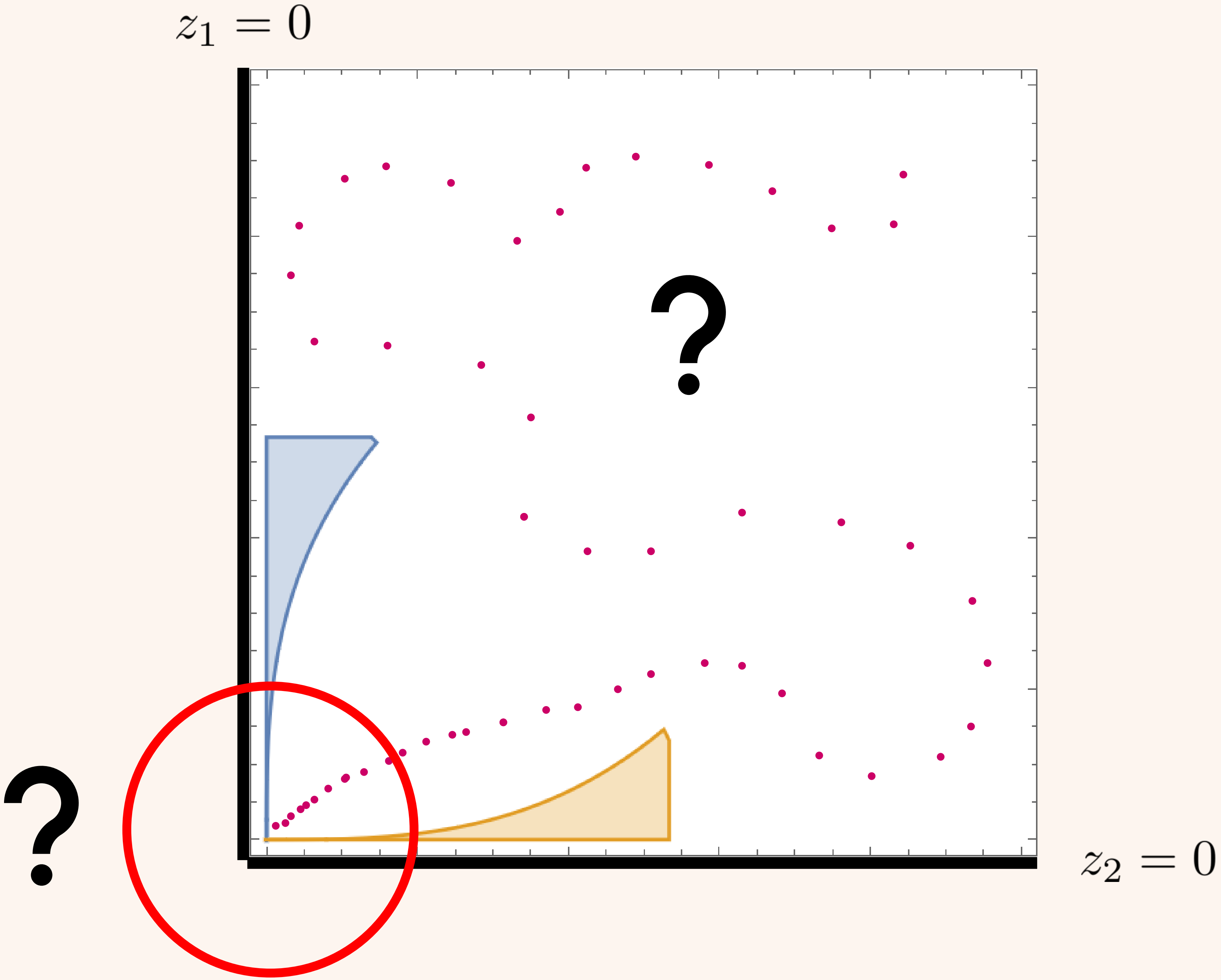
But! This is not good enough!

But! This is not good enough!



strict asymptotic regime: $y_1 \gg y_2 \gg \cdots \gg y_m$

$$y_i = \frac{1}{2\pi} \log \frac{1}{|z_i|}$$


$z_1 = 0$
?
?
$z_2 = 0$

To go beyond, necessary to include **all polynomial corrections.**

[Schmid; 1973] [Cattani, Kaplan, Schmid; 1986]

To go beyond, necessary to include **all polynomial corrections.**

[Schmid; 1973] [Cattani, Kaplan, Schmid; 1986]

For **one modulus** this has been done via a
`bulk reconstruction’ procedure

[Grimm; 2020] [Schnell] [Grimm, JM, van de Heisteeg; 2021]

To go beyond, necessary to include **all polynomial corrections.**

[Schmid; 1973] [Cattani, Kaplan, Schmid; 1986]

For **one modulus** this has been done via a `bulk reconstruction’ procedure

[Grimm; 2020] [Schnell] [Grimm, JM, van de Heisteeg; 2021]

We now have full control over these corrections for an **arbitrary number of moduli** and have a concrete algorithm to compute them in examples.

[Grimm, JM; to appear]

Idea: Perform 1-variable bulk reconstruction recursively

$$||G_4||^2 = ||h^{-1} G_4||_\infty^2 \,, \qquad h = h_m \cdots h_1$$

Idea: Perform 1-variable bulk reconstruction recursively

$$||G_4||^2 = ||h^{-1} G_4||^2_\infty \,, \qquad h = h_m \cdots h_1$$

In each step, get an infinite series

$$h_i = \left[1 + \sum_{k=0}^{\infty} g_{i,k} \left(\frac{y_i}{y_{i+1}} \right)^{-k} \right] \left(\frac{y_i}{y_{i+1}} \right)^{-\frac{1}{2} N^0_{(i)}}$$

sl(2) approximation — **Infinite tower of corrections** — **sl(2) grading operator**

Idea: Perform 1-variable bulk reconstruction recursively

$$||G_4||^2 = ||h^{-1}G_4||^2_\infty \,, \qquad h = h_m \cdots h_1$$

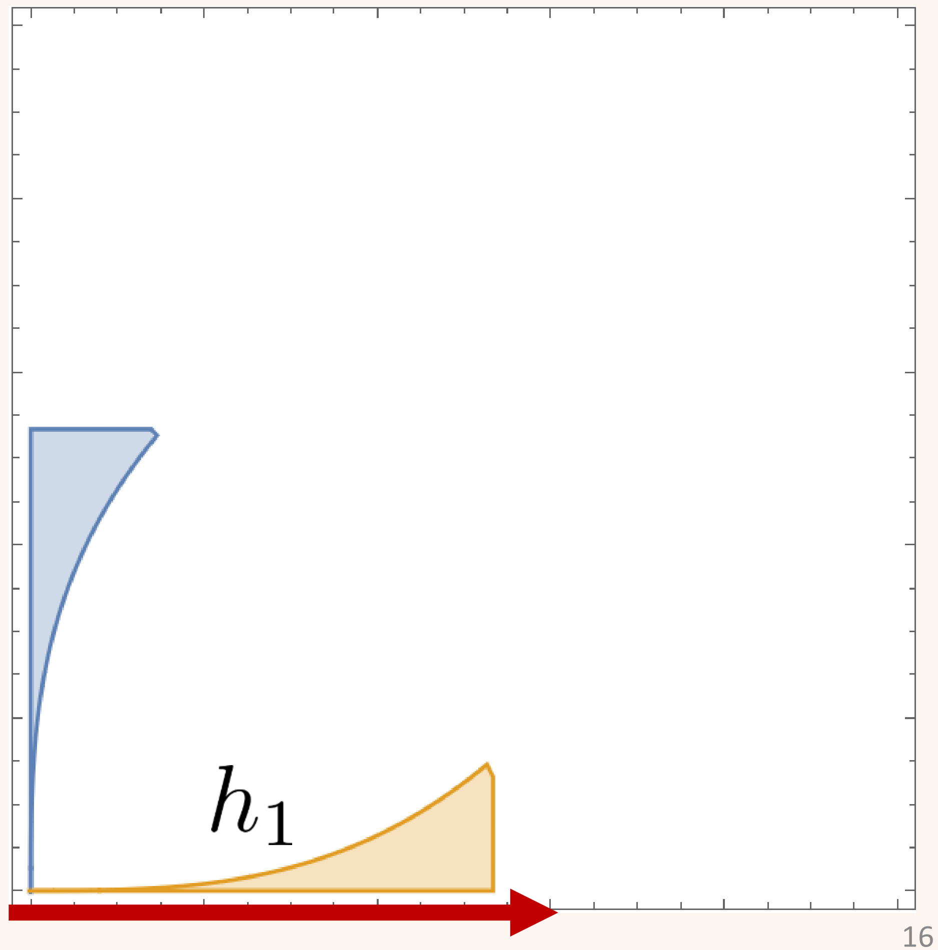
In each step, get an infinite series

$$h_i = \left[1 + \sum_{k=0}^{\infty} g_{i,k} \left(\frac{y_i}{y_{i+1}}\right)^{-k}\right] \left(\frac{y_i}{y_{i+1}}\right)^{-\frac{1}{2}N^0_{(i)}}$$

sl(2) approximation

Infinite tower of corrections

sl(2) grading operator



Idea: Perform 1-variable bulk reconstruction recursively

$$||G_4||^2 = ||h^{-1} G_4||^2_\infty \,, \qquad h = h_m \cdots h_1$$

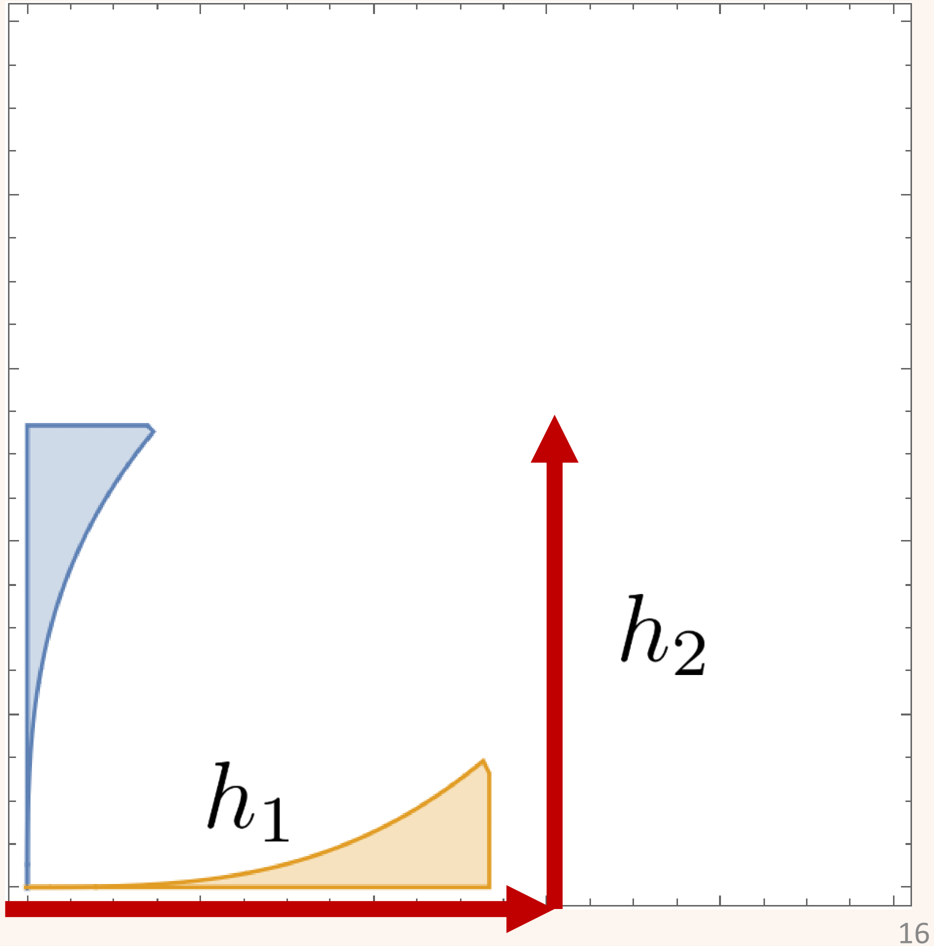
In each step, get an infinite series

$$h_i = \left[1 + \sum_{k=0}^{\infty} g_{i,k} \left(\frac{y_i}{y_{i+1}}\right)^{-k}\right] \left(\frac{y_i}{y_{i+1}}\right)^{-\frac{1}{2} N^0_{(i)}}$$

sl(2) approximation

Infinite tower of corrections

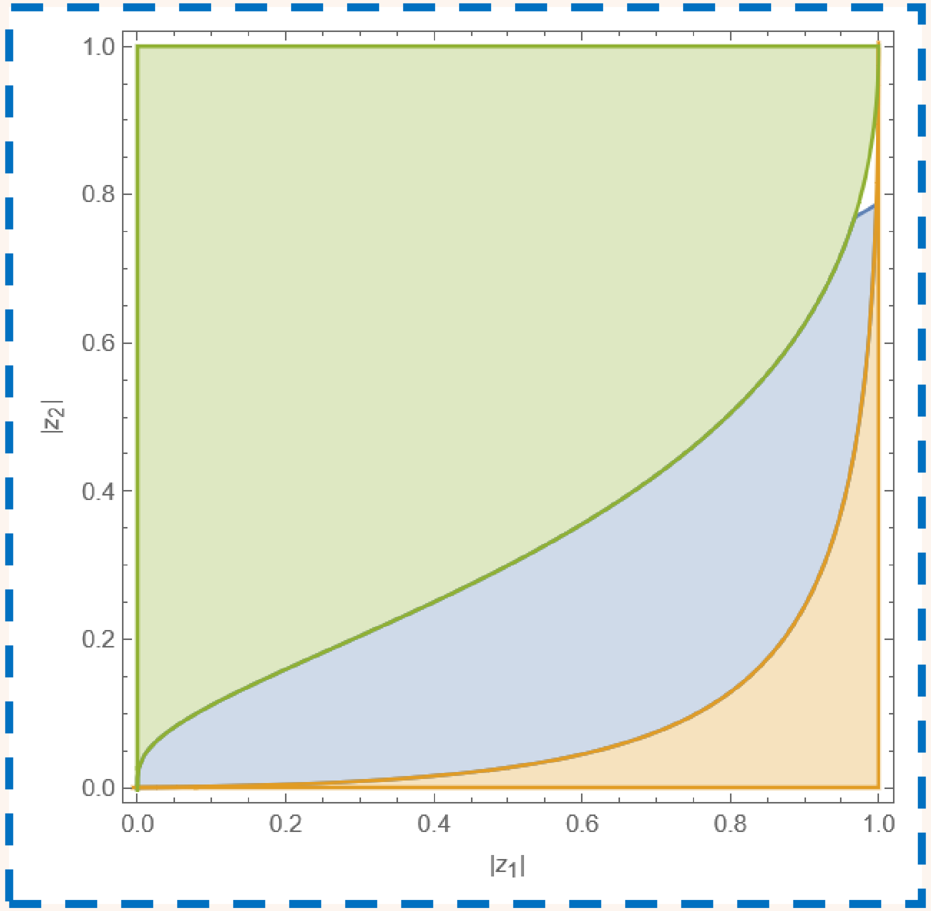
sl(2) grading operator



Highlight **two crucial features** of the finiteness proof

Highlight **two crucial features** of the finiteness proof

(1) **Refined Growth Sectors**



For each sl(2) weight, define subsectors

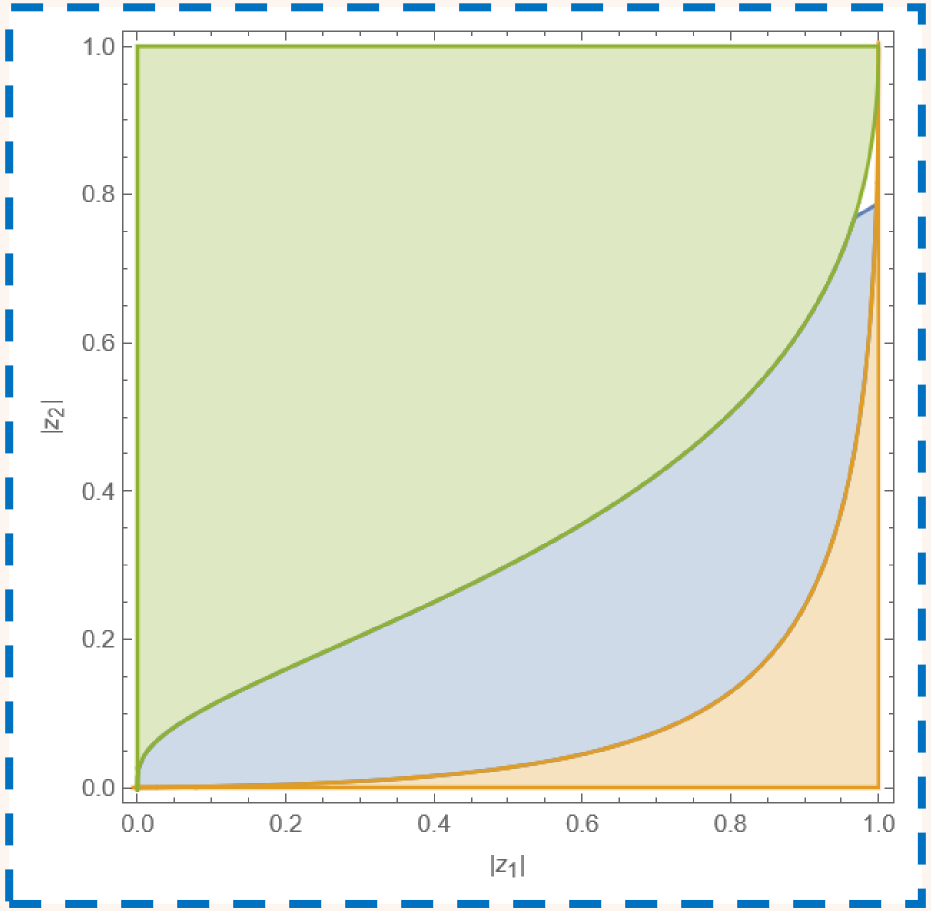
$$\left(\frac{y_1}{y_2}\right)^{\ell_1} \cdots y_m^{\ell_m} < K/\lambda\,, \quad \left(\frac{y_1}{y_2}\right)^{-\ell_1} \cdots y_m^{-\ell_m} < K/\lambda$$

together with their and consider all possible intersections.

$$\lambda \sim \text{smallest eigenvalue of } ||\cdot||_\infty$$

Highlight **two crucial features** of the finiteness proof

(1) **Refined Growth Sectors**



For each sl(2) weight, define subsectors

$$\left(\frac{y_1}{y_2}\right)^{\ell_1} \cdots y_m^{\ell_m} < K/\lambda\,, \quad \left(\frac{y_1}{y_2}\right)^{-\ell_1} \cdots y_m^{-\ell_m} < K/\lambda$$

together with their and consider all possible intersections.

$$\lambda \sim \text{smallest eigenvalue of } ||\cdot||_\infty$$

Tadpole grows with number of sl(2)-representations -> **tadpole conjecture?**

[Bena, Blåbäck, Graña, Lüst; 2020]
[Graña, Grimm, van de Heisteeg, Herraez, Plauschinn; 2022]

Highlight **two crucial features** of the finiteness proof

Highlight **two crucial features** of the finiteness proof

(2) **Scaling behaviour of corrections**

Necessary to use detailed properties of the expansion coefficients

$$g_{i,k}^{s^i} = 0\,, \quad \text{for } s_i^i \geq k\,, \qquad g_{i,k}^{s^i} \sim \prod_{j=1}^{i} \left(\frac{y_j}{y_{j+1}} \right)^{-s_j^i}$$

$$s^i = (s_1^i, \ldots, s_m^i)\,: \qquad s_j^i = \mathfrak{sl}(2)\text{-weight under ad}\, N_{(i)}^0$$

Highlight **two crucial features** of the finiteness proof

(2) **Scaling behaviour of corrections**

Necessary to use detailed properties of the expansion coefficients

$$g^{s^i}_{i,k} = 0\,, \quad \text{for } s^i_i \geq k\,, \qquad g^{s^i}_{i,k} \sim \prod_{j=1}^{i} \left(\frac{y_j}{y_{j+1}}\right)^{-s^i_j}$$

New!

$$s^i = (s^i_1, \ldots, s^i_m) : \qquad s^i_j = \mathfrak{sl}(2)\text{-weight under ad } N^0_{(i)}$$

Highlight **two crucial features** of the finiteness proof

(2) **Scaling behaviour of corrections**

Necessary to use detailed properties of the expansion coefficients

$$g_{i,k}^{s^i} = 0\,, \quad \text{for } s_i^i \geq k\,, \qquad g_{i,k}^{s^i} \sim \prod_{j=1}^{i} \left(\frac{y_j}{y_{j+1}}\right)^{-s_j^i}$$

New!

$$s^i = (s_1^i, \ldots, s_m^i) : \qquad s_j^i = \mathfrak{sl}(2)\text{-weight under ad } N_{(i)}^0$$

Possible to say more about general **Hodge inner products**, application to WGC?

[Bastian, Grimm, van de Heisteeg; 2020]

Thank you!