Sharpening the Boundaries Between Flux Landscape and Swampland by Tadpole Charge

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based on **JHEP 12 (2021) 017** (arXiv: 2104.15030 [hep-th]).

Main purposes of the talk

- **Tadpole charge $N_{\rm flux}$ appears in the F-term moduli potential**
  - Details of period vector is not needed

- **Attractive properties of a certain background with $h^{1,1}=0$**
  - No Kähler moduli stabilization *ab initio*
  - A possible playground for testing the Swampland conjectures

- **Clarify the role of $N_{\rm flux}$ in the potential**
  - What happens to the potential minima with uncanceled $N_{\rm flux}$?

Compactifications and Moduli Fields

◆ Compactifications of Type IIB superstring theory

- 10-dimensional (10d) spacetime
  - Compactify 6d space → 4d effective field theory (EFT)

◆ Moduli fields

- Massless scalar fields from deformations of the internal 6d manifold
  - Complex-structure (cs) moduli ⋯ “shape”
  - Kähler moduli ⋯ “volume”
- Many moduli appear in the EFT

◆ Flux compactifications

- Non-trivial three-form fluxes ⋯ RR, NS-NS (NS) fluxes
- Generate a scalar potential for moduli fields
  - Obtain VEVs ⋯ **Moduli stabilization**

Formulation of the Moduli Stabilization

- Flux compactification → Scalar potential of moduli

  - **Scalar potential**

$$V = e^{K}\left[K^{i\bar{j}} D_i W \overline{D_{\bar{j}} W} - 3|W|^2\right]$$

$$K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$$
$$D_i = \partial_i + K_i$$

  - **Gukov-Vafa-Witten (GVW) type super potential**

S. Gukov, C. Vafa and E. Witten, Nucl.Phys.B **584** (2000) 69-108.

$$W_{\mathrm{GVW}} = \int_{\mathcal{X}} G_3 \wedge \Omega = \underbrace{\int F_3 \wedge \Omega}_{W_{\mathrm{RR}}} - S \underbrace{\int H_3 \wedge \Omega}_{W_{\mathrm{NS}}} \qquad (G_3 = F_3 - S H_3)$$

  - **Kähler potential**

$$K = \underline{-4} \log\left[-i(S - \overline{S})\right] - \log\left[-i \int_{\mathcal{X}} \Omega \wedge \overline{\Omega}\right]$$

specific property of the background (introduce later)
-1: usual case (no-scale type)

Tadpole Cancellation Condition

- The three-form fluxes are quantized:

$$\int_\Sigma F_3 = N_F \in \mathbb{Z}, \quad \int_\Sigma H_3 = N_H \in \mathbb{Z}.$$

  - These flux numbers discretize vacua

- **Tadpole cancellation condition**: consistency
  - Fluxes cannot be arbitrary numbers

$$S_{\rm IIB} = S_{\rm NS} + S_{\rm R} + S_{\rm CS} + S_{\rm local}$$

$$\supset -\frac{1}{8}\int \tilde{F}_5 \wedge \star\tilde{F}_5 + \frac{1}{8i{\rm ImS}}\int C_4 \wedge G_3 \wedge \bar{G}_3 + \frac{1}{2}\left(N_{\rm D3} - \frac{1}{2}N_{\rm O3}\right)\mu_3 \int_{\mathbb{R}^{1,3}} C_4,$$

$$\left(\tilde{F}_5 = dC_4 - \tfrac{1}{2}C_2 \wedge H_3 + \tfrac{1}{2}B_2 \wedge F_3\right)$$

$$\xrightarrow{\text{Bianchi id.}} \quad \underbrace{\int_{\mathcal{X}} H_3 \wedge F_3}_{N_{\rm flux}} - \underbrace{N_{\rm D3} + \frac{1}{2}N_{\rm O3}}_{\text{RR charges of D-branes and O-planes}} = 0$$

Problems and Difficulties

◆ Not all the moduli can be stabilized by the fluxes

- No-scale structure ⋯ Kähler moduli are flat directions
  - Non-perturbative effects

◆ No principle for flux choices

- Tadpole cancellation constrains; but still many degrees of freedom remain

◆ Huge Flux Landscape

F. Denef and M. R. Douglas, JHEP **0405**, 072 (2004), ⋯

- <u>Set of whole vacua in flux compactifications</u>
- Estimation: $\sim \mathcal{O}(10^{272000})$ vacua W. Taylor and Y. N. Wang, JHEP **12**, 164 (2015).
- SM-like models? Statistical approach?

M. R. Douglas, JHEP **0503**, 061 (2005), ⋯

◆ Difficulties in building Standard Model-like models

- Cosmological constant, CP violation, flavor and inflation etc.

$\boxed{\text{Understanding Landscape properties is still challenging!}}$

Swampland Conjectures

- Low energy EFT inconsistent with UV theory
  - In this talk, we call outside of the Landscape the **Swampland**.

- Many **Swampland conjectures** are proposed;

Today:

- de Sitter (dS) conjecture

  G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, arXiv:1806.08362 [hep-th].

  - Absence of stable dS vacua with all moduli stabilized

$$\mathrm{Min}(\nabla_i \nabla_j V) \leq -c \cdot V \qquad c\text{: } \mathcal{O}(1) \text{ positive constant}$$

- AdS/moduli separation conjecture

  F. Gautason, V. Van Hemelryck, and T. Van Riet, Fortsch. Phys.67, 1800091 (2019).

  - Limitation on the size of the Moduli mass and AdS radius

$$m_{\text{light}} R_{\text{AdS}} \leq c$$

m_{light}: lightest moduli mass, R_{AdS}: AdS size

c: $\mathcal{O}(1)$ positive constant

- AdS distance conjecture

  D. Lüst, E. Palti, and C. Vafa, Phys. Lett. B **797**, 134867 (2019).

  - Infinite tower of light KK states appear in the limit . $\Lambda \ll 1$

$$m_{\text{tower}} = c|\Lambda|^{\alpha}$$

m_{tower}: mass scale of the light states

Λ: cosmological constant

c, α: $\mathcal{O}(1)$ positive constant

Purposes of the Study

- **Understanding structure of the Landscape**
  - e.g., Classification by their cosmological constants

A. Strominger and E. Witten, Commun.Math.Phys. **101** (1985) 341.
P. Candelas, E. Derrick and L. Parkes, Nucl. Phys. B **407** (1993) 115.
O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, JHEP **0507** (2005) 066.
K. Becker, M. Becker and J. Walcher, Phys. Rev. D **76** (2007) 106002.
K. Becker, M. Becker, C. Vafa, and J. Walcher, Nucl.Phys.B **770** (2007) 1.
K. Becker, E. Gonzalo, J. Walcher, and T. Wrase, JHEP **12** (2022) 083.
J. Bardzell, E. Gonzalo, M. Rajaguru, D. Smith, T. Wrase, JHEP **06** (2022) 166.

- **Inspection of the Swampland conjectures**
  - A background with no Kähler moduli exists: **Mirror dual of** $T^6/(Z_3 \times Z_3)$
  - A possible playground for testing the Swampland conjectures

- **What does support the conjectures?** (if they hold)
  - Sharpen the boundary between Landscape/Swampland

A clue from general properties of V

◆ Sign of the scalar potential at minima

We factorize $V = e^K \left(K^{i\bar{j}} D_i W D_j \bar{W} - 3|W|^2 \right) \equiv e^K \tilde{V}$

$$\partial_i V = K_i V + e^K \partial_i \tilde{V} = 0 \longrightarrow V = -e^K \frac{\partial_i \tilde{V}}{K_i}$$

→ determined by the Kähler potential and $\partial_i \tilde{V}$ (at minima)

◆ Functional form of the scalar potential

- quadratic in the axio-dilaton S
- $K_{\mathrm{Im}S} < 0, K_{\mathrm{Re}S} = 0$

$\longrightarrow$ **Dependency on $\mathrm{Im}S$ will be universal**

N_{flux} appears in the scalar potential

- Expand $\tilde{V} = e^{-K} V$ with $\mathrm{Im}S$… We find that

$$\longrightarrow \boxed{\tilde{V} = \frac{1}{2}\partial^2_{\mathrm{ImS}}\tilde{V}(\mathrm{Im}S)^2 - e^{-K_{\mathrm{cs}}} N_{\mathrm{flux}} \mathrm{Im}S + C, \quad C \equiv \tilde{V}\Big|_{\mathrm{Im}S=0} \geq 0}$$

- **Coefficient of the linear term** is N_{flux}!
  - 10d consistency appears in the 4d potential in the nontrivial way
  - We observe this structure for both no-scale type and $h^{1,1} = 0$ case!
  - Some additional terms which does not destroy the structure are also OK.

- Implication of N_{flux} in resulting vacua
  - Restricted by the tadpole condition

- N_{flux} can be a messenger of the consistency

Implication of N_{flux} in resulting vacua

- Example: existence of a stable Minkowski vacuum
  - If exists, we find

$$\langle \mathrm{Im}S \rangle = \frac{N_{\text{flux}}}{e^{K_{\text{cs}}} \partial^2_{\mathrm{Im}S} \tilde{V}} \quad \text{with} \quad \partial^2_{\mathrm{Im}S} V > 0 \Leftrightarrow \partial^2_{\mathrm{Im}S} \tilde{V} > 0 \quad \text{(stability condition)}$$

→ Since $\mathrm{Im}S = g_s{}^{-1}$, $N_{\text{flux}} > 0$ is required.

(In the $h^{1,1} = 0$ case, SUSY does not require ISD flux*)

*K. Becker, M. Becker and J. Walcher, Phys. Rev. D **76** (2007) 106002.

- Stable dS vacuum? (w/ positive mass)
  - Again, N_{flux} is constrained (via a complicated relation).

→ **Tadpole cancellation is now linked to the existence of dS.**

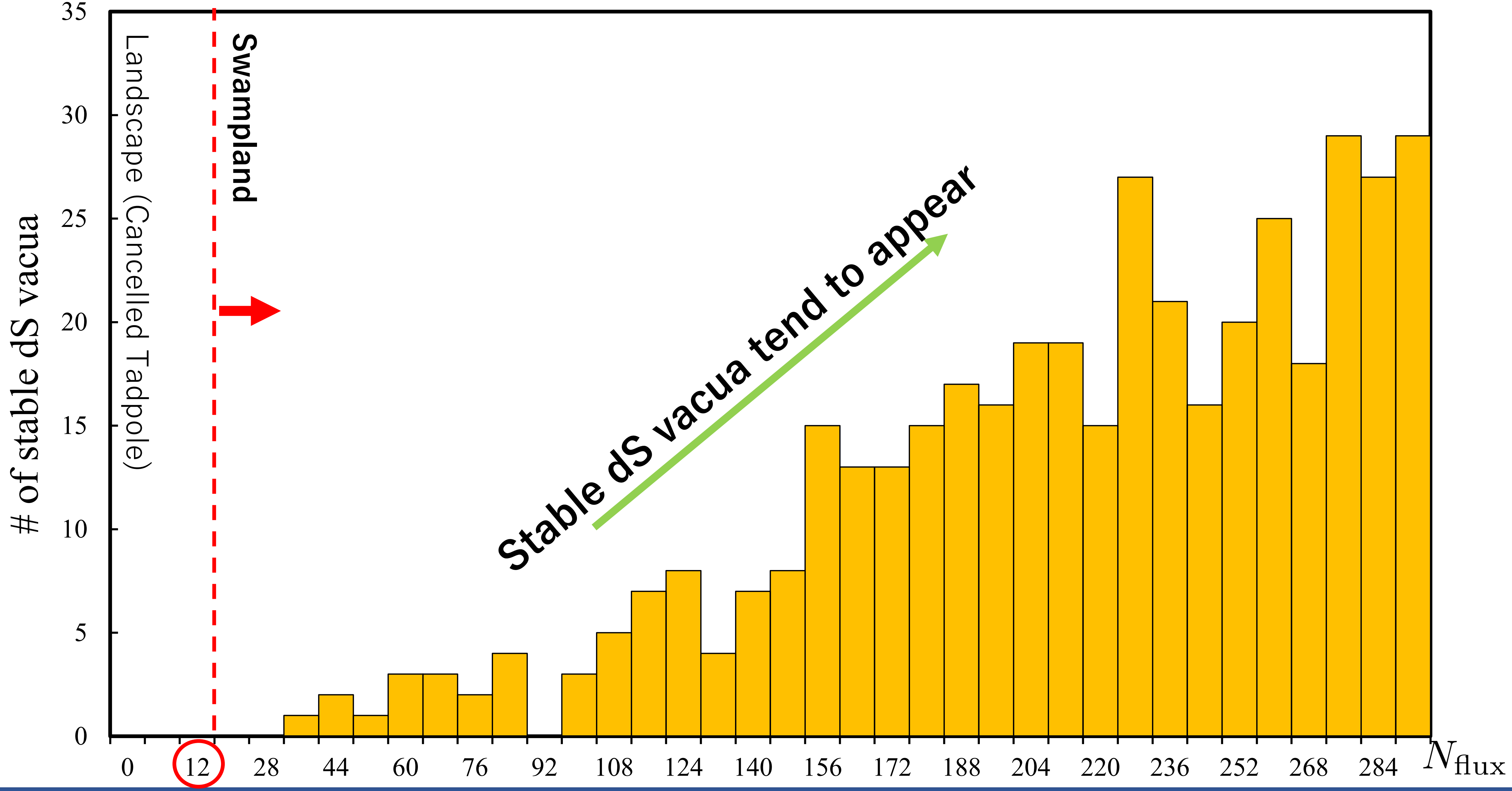
  - Numerical calculation in the $h^{1,1} = 0$ case would show the relation explicitly.
    - We consider only axio-dilaton and bulk moduli

Numerical search of stable dS vacua

- **Search configurations**
  - Fluxes: $-20 \leq \#(\text{flux}) \leq 20$ with $0 \leq N_{\text{flux}} \leq 300$
    - **Tadpole cancellation condition**: $N_{\text{flux}} \leq 12$
      - Vacua with $N_{\text{flux}} > 12$ fall into the Swampland, but we searched intentionally!
  - # of flux patterns: 7.9×10^9 and # of minima found: 6.7×10^8
- The dS conjecture holds – No dS vacua exist in the Landscape
  - **However, the dS vacua appear in the region $N_{\text{flux}} > 12$.**
  - The number of them **increases** as N_{flux} becomes larger.
  - Naively, we cannot find inconsistency by focusing only on V.

→ N_{flux} **characterizes the boundary between Landscape/Swampland in the 4d EFT**

Stable dS vacua are more likely to appear with larger N_{flux}



Summary and Conclusions

◆ $N_{\rm flux}$ is a parameter restricted by the 10d consistency.

In this talk, we pointed out;

- **Appearing in the 4d EFT in the nontrivial way**
- The Swampland conjectures are related to $N_{\rm flux}$.
  - $N_{\rm flux}$ as a **messenger**; resulting vacua notice its inconsistency via $N_{\rm flux}$.
  - **Landscape/Swampland boundary in the 4d EFT is controlled by** $N_{\rm flux}$.
- Implying the importance of tadpole condition in proving conjectures
  - Other applications of the emergence of $N_{\rm flux}$ in V ?

Thank you all for your attention!

More details in 2104.15030

Appendix

$N_{\rm flux}$ and AdS/moduli scale separation

- $N_{\rm flux}$ is also related to moduli masses and Λ;

  - Moduli mass matrix in SUSY AdS vacua (isotropic tori)

$$\frac{M^2_{\rm phys,AdS}}{\Lambda_{\rm AdS}} = \begin{pmatrix} \frac{2}{3} - \frac{19}{108}|x|^2 & \frac{2}{9}\bar{y} & -\frac{x}{2\sqrt{3}} - \frac{x\bar{y}}{9\sqrt{3}} & \frac{\bar{x}}{6\sqrt{3}} \\ \frac{2}{9}y & \frac{2}{3} - \frac{19}{108}|x|^2 & \frac{x}{6\sqrt{3}} & -\frac{x}{2\sqrt{3}} - \frac{\bar{x}y}{9\sqrt{3}} \\ -\frac{x}{2\sqrt{3}} - \frac{\bar{x}y}{9\sqrt{3}} & \frac{\bar{x}}{6\sqrt{3}} & -\frac{7}{3} - \frac{1}{36}|x|^2 & 1 \\ \frac{x}{6\sqrt{3}} & -\frac{\bar{x}}{2\sqrt{3}} - \frac{x\bar{y}}{9\sqrt{3}} & 1 & -\frac{7}{3} - \frac{1}{36}|x|^2 \end{pmatrix},$$

with $x \equiv (S-\bar{S})(\tau-\bar{\tau})\frac{D_\tau W_{\rm NS}}{W}, y \equiv (S-\bar{S})(\tau-\bar{\tau})\frac{\overline{D_\tau W_{\rm NS}}}{W}.$

  - $N_{\rm flux}$ in the SUSY AdS vacua

$$\frac{N_{\rm flux}}{\Lambda_{\rm AdS}} = \frac{8({\rm Im}S)^3}{3}\left(8 - \frac{|x|^2}{3}\right)$$

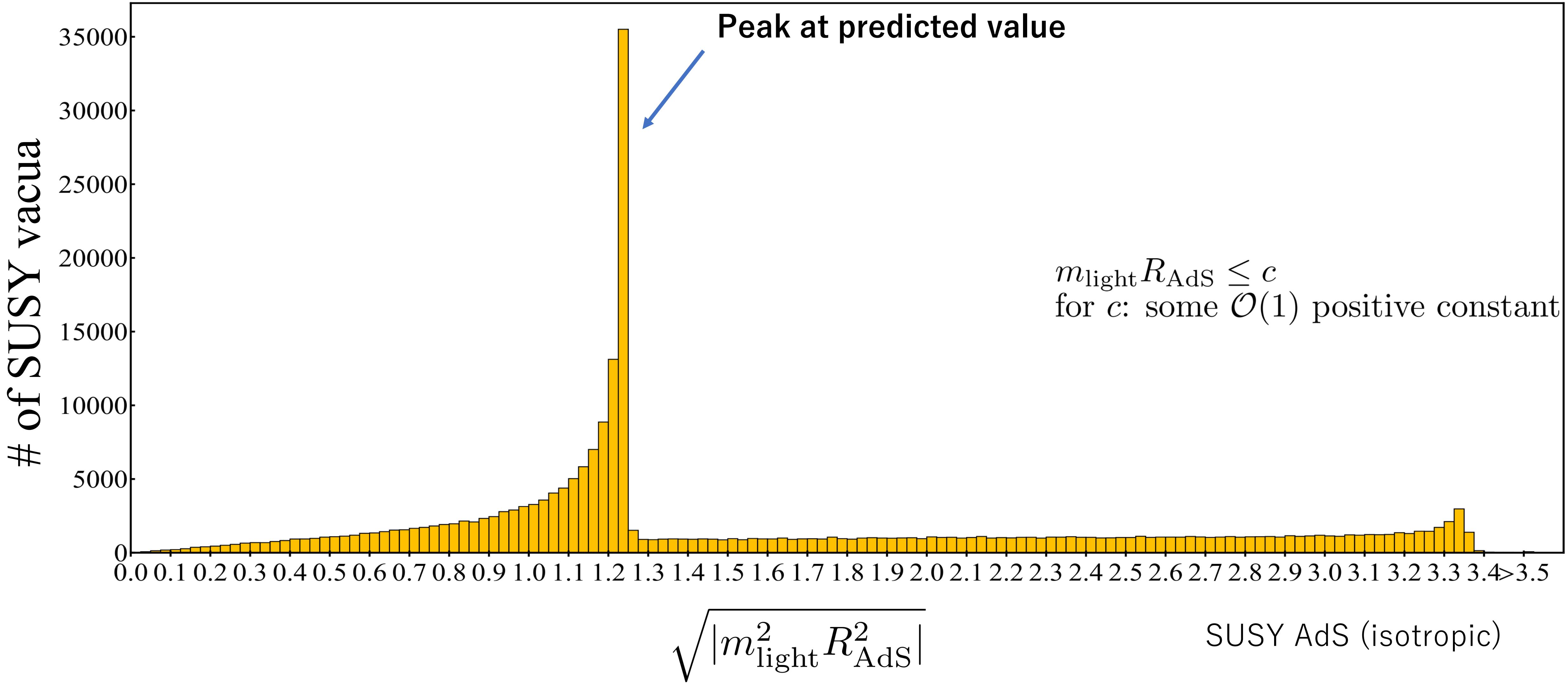
→ These are linked via x

  - Prediction

$$|m_{\rm light}R_{\rm AdS}| \simeq \frac{\sqrt{6}}{2} \simeq 1.22$$

with $8({\rm Im}S)^2\Lambda_{\rm AdS} \gg 9N_{\rm flux}, {\rm Arg}x = {\rm Arg}y$

N_{flux} supports the AdS/moduli scale separation conjecture



$m_{\mathrm{light}} R_{\mathrm{AdS}} \leq c$
for c: some $\mathcal{O}(1)$ positive constant

SUSY AdS (isotropic)