Perturbations in $O(D, D)$ cosmology

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based on

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Motivation

- In general relativity, the metric $g_{\mu\nu}$ is the only gravitational field.
- Dark matter, dark energy, tensions within ΛCDM (H_0 tension, lensing amplitude...) $\Rightarrow$ need to move beyond GR cosmology?
- In string theory, the closed-string massless sector includes:
  - the metric, $g_{\mu\nu}$;
  - an antisymmetric 2-form potential, $B_{\mu\nu}$;
  - the dilaton, ϕ.
- Furthermore, these fields map into each other under T-duality.

Natural 'stringy' extension of general relativity:

Consider $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ as the fundamental gravitational multiplet.

$O(D, D)$ symmetry

Low-energy limit of string theory $\supset$ bosonic (NS-NS) supergravity,

$$S = \int \mathrm{d}^D x \sqrt{-g} e^{-2\phi} \left(R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} \right) .$$

In addition to diffeo.s and B-field gauge symmetry, action is invariant under T-duality. For arbitrary backgrounds of $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$, generalizes to the Buscher transformation, e.g. $x^\mu = (x^a, x^3)$: (Buscher, 1987)

$$g_{ab} \mapsto g_{ab} - \frac{g_{3a}g_{3b} - B_{3a}B_{3b}}{g_{33}} , \qquad g_{3a} \mapsto \frac{B_{3a}}{g_{33}} , \quad g_{33} \mapsto \frac{1}{g_{33}} ,$$

$$g^{33} \mapsto g_{33} - B_{3a} g^{ab} B_{b3} , \qquad g^{3a} \mapsto -B_{3b} g^{ba} , \quad g^{ab} \mapsto g^{ab} ,$$

$$B_{ab} \mapsto B_{ab} - \frac{g_{3a}B_{3b} - B_{3a}g_{3b}}{g_{33}} , \qquad B_{3a} \mapsto \frac{g_{3a}}{g_{33}} , \quad \phi \mapsto \phi - \frac{1}{2} \ln g_{33} .$$

Looks highly non-trivial! Note that this typically changes the geometry. The full implied group of transformations is $O(D, D)$.

Overview of double field theory

We can make the $O(D, D)$ symmetry manifest using the formalism of double field theory (DFT). (Siegel; 1993) (Hull, Zwiebach; 2009)

- In DFT we describe D-dimensional physics using $D + D$ coordinates, $x^A = (\tilde{x}_\mu, x^\nu)$, $A = 1, \ldots, 2D$.
- Doubled vector indices are raised and lowered using the $\mathbf{O}(D, D)$-invariant $\mathcal{J}$:

$$\mathcal{J}_{AB} = \begin{pmatrix} 0 & \mathbf{1}_D \\ \mathbf{1}_D & 0 \end{pmatrix} = \mathcal{J}^{AB} \, .$$

- DFT also enjoys doubled diffeomorphism symmetry, which unifies ordinary diffeomorphisms and B-field gauge transformations.

Double trouble?

What does it mean to ‘double’ the number of spacetime dimensions?

- Closure of doubled diffeomorphisms $\Rightarrow$ section condition,

$$\partial_A \partial^A = 2\, \partial_\mu \tilde{\partial}^\mu = 0\,.$$

- A natural choice is $\tilde{\partial}^\mu = 0$ (all fields indep. of $\tilde{x}_\mu$). The theory is not truly ‘doubled’: it is just a repackaging of D-dimensional physics.
- c.f. the level-matching condition for closed-string massless modes is $p_\mu \tilde{p}^\mu = 0$, where $\tilde{p}^\mu$ is the T-dual momentum. Translating this to position basis, we recover the section condition.

Ingredients of double field theory

The basic fields of double field theory are:

- the DFT dilaton d, providing a scalar density e^{-2d} of unit weight;
- an $\mathbf{O}(D,D)$, symmetric DFT metric $\mathcal{H}_{AB}$, satisfying

$$\mathcal{H}_{AB} = \mathcal{H}_{BA}\,, \qquad \mathcal{H}_A{}^C \mathcal{H}_B{}^D \mathcal{J}_{CD} = \mathcal{J}_{AB}\,.$$

On Riemannian backgrounds (i.e. usual spacetime), with the section choice $\tilde{\partial}^\mu = 0$: $\{d, \mathcal{H}_{AB}\} \to \{g_{\mu\nu}, B_{\mu\nu}, \phi\}$, such that

$$e^{-2d} = e^{-2\phi}\sqrt{-g}\,; \qquad \mathcal{H}_{AB} = \begin{pmatrix} g^{\mu\nu} & -g^{\mu\sigma}B_{\sigma\nu} \\ B_{\mu\rho}g^{\rho\nu} & g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu} \end{pmatrix}\,.$$

Buscher transformations are simply linear $O(D,D)$ rotations of $\mathcal{H}$.

The DFT Ricci scalar gives the ‘stringy’ gravitational Lagrangian,

$$\mathcal{R} = R + 4\Box\phi - 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\,.$$

DFT coupled to matter

We can extend $O(D,D)$ covariance to interactions with matter $\{\Upsilon_a\}$,

$$S = \int_\Sigma e^{-2d}\left[\frac{1}{16\pi G}\mathcal{R} + L_\mathrm{m}(\Upsilon_a)\right],$$

where L_m is an $\mathbf{O}(D,D)$-covariant Lagrangian for the additional matter fields $\{\Upsilon_a\}$, and the integral is taken over a D-dimensional section Σ.

Varying this action yields a DFT generalization of Einstein's equations, (SA, Cho, Park (2018))

$$G_{AB} = 8\pi G T_{AB},$$

where the DFT energy-momentum tensor T_{AB} is conserved on-shell.

Note: $O(D,D)$ covariance $\Rightarrow$ volume element must always be e^{-2d}.

Riemannian spacetime backgrounds

EDFEs give closed-string equations of motion plus source terms,

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu(\partial_\nu \phi) - \tfrac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} &= 8\pi G K_{(\mu\nu)} \ ; \\ \nabla^\rho \left(e^{-2\phi} H_{\rho\mu\nu} \right) &= 16\pi G e^{-2\phi} K_{[\mu\nu]} \ ; \\ R + 4\Box\phi - 4\partial_\mu \phi \partial^\mu \phi - \tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} &= 8\pi G T_{(0)} \ , \end{aligned}$$

where $K_{\mu\nu}$ and $T_{(0)}$ source $\mathcal{H}_{AB}$ and d, respectively.

Many common fields, particles, etc. admit a DFT embedding, from which we can derive their DFT energy-momentum tensors explicitly. Generally, $K_{\mu\nu}$ gives the kinetic part and $T_{(0)}$ the trace contribution.

Note: asymmetric $K_{\mu\nu}$ possible (e.g. fermions, strings) $\rightarrow$ source for H, which gives a topological obstruction to dualizing H to an axion.

Interpretation of $K_{\mu\nu}$ and $T_{(0)}$

Can arrange DFT energy-momentum tensor into terms sourcing

$$\begin{aligned} \delta g_{\mu\nu} : &\qquad T_{\mu\nu} \equiv e^{-2\phi} \left(K_{(\mu\nu)} - \frac{1}{2} g_{\mu\nu} T_{(0)} \right) , \\ \delta B_{\mu\nu} : &\qquad \Theta_{\mu\nu} \equiv e^{-2\phi} K_{[\mu\nu]} , \\ \delta\phi : &\qquad \sigma \equiv e^{-2\phi} T_{(0)} . \end{aligned}$$

Here $T_{\mu\nu}$ is the usual energy-momentum tensor in GR, while $\Theta_{\mu\nu}$ and σ represent sources for $B_{\mu\nu}$ and ϕ, respectively.

On-shell conservation of DFT energy-momentum tensor gives

$$\nabla^{\mu} T_{\mu\nu} + \frac{1}{2} H_{\nu}{}^{\mu\lambda} \Theta_{\mu\lambda} - \nabla_{\nu}\phi\, \sigma = 0 , \qquad \nabla^{\mu} \Theta_{\mu\nu} = 0 .$$

If we convert to Einstein frame via $g_{\mu\nu} \equiv e^{2\phi} \tilde{g}_{\mu\nu}$, components given by

$$\widetilde{T}_{\mu\nu} \equiv e^{2\phi} T_{\mu\nu} , \qquad \widetilde{\Theta}_{\mu\nu} \equiv \Theta_{\mu\nu} , \qquad \tilde{\sigma} \equiv e^{4\phi} (\sigma + T^{\mu}{}_{\mu}) .$$

Cosmology in DFT

Now consider $D = 4$ homogeneous and isotropic solutions in DFT.

Solving for these isometries in DFT yields the gravitational ansatz

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right] ,$$

$$B = \frac{hr^2}{\sqrt{1-kr^2}}\cos\vartheta\,\mathrm{d}r\wedge\mathrm{d}\varphi , \qquad \phi = \phi(t) .$$

This gives $H = \mathrm{d}B = h\,\mathrm{Vol}_3$ which is homogeneous and isotropic.

For matter, the DFT energy-momentum tensor is constrained as

$$K^{\mu}{}_{\nu} = \mathrm{diag}(K^t{}_t(t), K^r{}_r(t), \dots, K^r{}_r(t)) , \qquad T_{(0)} = T_{(0)}(t) .$$

Note: energy density and pressure,

$$\rho := \left(-K^t{}_t + \frac{1}{2}T_{(0)}\right)e^{-2\phi}, \qquad p := \left(K^r{}_r - \frac{1}{2}T_{(0)}\right)e^{-2\phi} .$$

O(D, D)-complete Friedmann equations

- Choose e.g. cosmic gauge, $N(t) = 1$ $H \equiv \dot{a}/a$. From the EDFEs we obtain **O**(D, D)-complete Friedmann equations (OFEs),

  (SA, Cho, Franzmann, Mukohyama, Park; 2019)

$$\frac{8\pi G}{3}\rho e^{2\phi} + \frac{h^2}{12a^6} = H^2 - 2\dot{\phi}H + \frac{2}{3}\dot{\phi}^2 + \frac{k}{a^2}\,,$$

$$\frac{4\pi G}{3}(\rho + 3p)e^{2\phi} + \frac{h^2}{6a^6} = -H^2 - \dot{H} + \dot{\phi}H - \frac{2}{3}\dot{\phi}^2 + \ddot{\phi}\,,$$

$$\frac{4\pi G}{3}(2\rho - \sigma)\,e^{2\phi} = -H^2 - \dot{H} + \frac{2}{3}\ddot{\phi}\,.$$

- DFT e-m conservation yields one non-trivial conservation law,

$$\dot{\rho} + 3H(\rho + p) + \dot{\phi}\sigma = 0\,.$$

Properties of the OFEs

The OFEs have various interesting properties.

- 3 OFEs + 1 conservation law ⇒ 3 independent equations (c.f. GR cosmology, which has 2 Friedmann eq.s + 1 conservation law).
- If $\dot{\phi} = \ddot{\phi} = 0$, $h = 0$ ⇒ GR cosmology; $\sigma \equiv \rho - 3p$ ('critical line').
- It is useful to define two equation-of-state parameters,

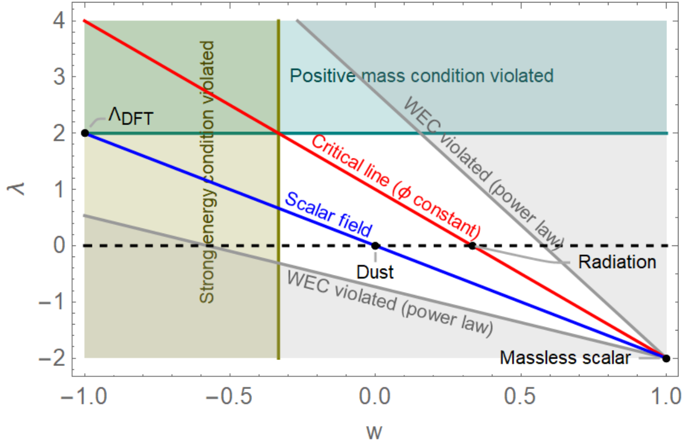
$$w := \frac{p}{\rho}\,; \qquad \lambda := \frac{\sigma}{\rho}\,.$$

- w is the usual parameter corresponding to pressure, while λ is the ratio of the dilaton source to the energy density.
- For constant w and λ ("generalized perfect fluid"), we can solve:

$$\rho = \rho_0 \frac{e^{-\lambda\phi}}{a^{3(1+w)}}\,.$$

Cosmological solutions

Identify various regions and types of matter in the (w, λ)-plane.



Also, pure DFT vacuum: $\rho = 0$ (Copeland, Lahiri, Wands; 1994)

Note: Naive supergravity case is $\lambda = 0 \Rightarrow$ radiation critical, dust is not.

Solution: radiation with H-flux and freezing dilaton

E.g. there is an analytic solution for radiation ($w = 1/3$, $\lambda = 0$), with non-vanishing H-flux, in which the dilaton is frozen at late times.

- Einstein-frame scale factor,

$$\tilde{a}^2 = \frac{\tau(C_1 + \Omega_{\rm rad} H_0^2 \tau)}{1 + k\tau^2}\,; \qquad \tau = \begin{cases} \tan(\eta - \eta_0) & \text{for} \quad k = 1\,, \\ \eta - \eta_0 & \text{for} \quad k = 0\,, \\ \tanh(\eta - \eta_0) & \text{for} \quad k = -1\,. \end{cases}$$

- The dilaton profile is (c.f. $h = 0$: Copeland, Lahiri, Wands; 1994)

$$e^{2\phi} = \left(\frac{C_1\tau}{\tau_* \left(C_1 + \Omega_{\rm rad} H_0^2 \tau\right)}\right)^{\pm\sqrt{3}} + \frac{1}{12}\frac{h^2}{C_1^2}\left(\frac{C_1\tau}{\tau_* \left(C_1 + \Omega_{\rm rad} H_0^2 \tau\right)}\right)^{\mp\sqrt{3}}\,,$$

  which converges to a constant as $\eta \to \infty$ (for $k \in \{0, -1\}$).
- For nonzero h, the string-frame scale factor $a = \tilde{a}e^{\phi}$ has a minimum $\Rightarrow$ purely classical bouncing cosmology.

Cosmological perturbations

In conformal gauge ($N = a$), write the perturbed metric as

$$g_{\mu\nu} = a^2 \begin{pmatrix} -(1+2A) & B_j \\ B_i & \delta_{ij} + h_{ij} \end{pmatrix} .$$

Under a scalar-vector-tensor (SVT) decomposition, these separate as

$$B_i \equiv \hat{B}_i + \partial_i B \,, \qquad h_{ij} \equiv 2C\delta_{ij} + 2(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2)E + 2\partial_{(i}\hat{E}_{j)} + \hat{E}_{ij}$$

($\partial^i \hat{B}_i = 0$, $\partial^i \hat{E}_i = 0$, $\partial^i \hat{E}_{ij} = 0$ and $\hat{E}^i{}_i = 0$). For the B-field, expand

$$\delta B_{(2)} \equiv f_i \, \mathrm{d}x^i \wedge \mathrm{d}\eta + \frac{1}{2} m_{ij} \, \mathrm{d}x^i \wedge \mathrm{d}x^j \,,$$

where $f_i = \partial_i f + \hat{f}_i$, $m_{ij} = \partial_i \hat{m}_j - \partial_j \hat{m}_i + \sqrt{g_3}\epsilon_{ijk}\partial^k m$ ($\partial^i \hat{f}_i = \partial_i \hat{m}^i = 0$).

Energy-momentum tensors: $T_{\mu\nu}$ has the standard SVT decomposition, while the non-trivial components of $\Theta_{\mu\nu}$ decompose as

$$\mathcal{J}_i = \partial_i \mathcal{J} + \hat{\mathcal{J}}_i \,, \qquad \mathcal{I}^i = \partial^i \mathcal{I} + \hat{\mathcal{I}}^i \,.$$

Scalar perturbations

E.g. scalar perturbations around DFT vacuum. Work in Einstein frame; spatially flat gauge, $\tilde{C} = \tilde{E} = 0$; expand in Fourier modes, $\partial_i \to ik_i$:

$$0 \simeq \left[\partial_0^2 + 2\tilde{\mathcal{H}}\partial_0 + k^2\right]\tilde{A}\,, \qquad \partial_0\tilde{A} \simeq k^2\tilde{B}\,,$$

$$-4h\left((\tilde{\mathcal{H}} + \partial_0\bar{\phi})B + \delta\phi\right) \simeq \left[\partial_0^2 - \left(2\tilde{\mathcal{H}} + 4\partial_0\bar{\phi}\right)\partial_0 + k^2\right]m\,,$$

$$-\frac{e^{-4\bar{\phi}}h}{2\tilde{a}^4}k^2 m \simeq \left[\partial_0^2 + 2\tilde{\mathcal{H}}\partial_0 + k^2 + \frac{2e^{-4\bar{\phi}}h^2}{\tilde{a}^4}\right]\delta\phi\,,$$

along with two constraint equations relating $\tilde{A}$, $\delta\phi$ and m.

For $h = 0$, reduces to damped oscillator equations. (e.g. Mueller; 1990)

Extra h-dependent terms mix components. (SA, Mukohyama; in progress)

Summary

- Double field theory coupled to matter gives a ‘stringy’ modified gravity; **O**(D, D) symmetry constrains the allowed interactions.
- The energy-momentum tensor has additional components corresponding to sources for $B_{\mu\nu}$ and ϕ.
- On homogeneous and isotropic backgrounds, DFT coupled covariantly to matter gives **O**(D, D) Friedmann equations.
- Various analytic solutions, including a bouncing solution with radiation, non-vanishing H-flux, and frozen dilaton at late times.
- Scalar perturbations mix in the presence of background H-flux. Non-trivial implications for structure formation?
- Future directions: H-flux baryogenesis, redshift due to dilaton-photon coupling, CMB observables. . .