Late-time accelerating cosmologies

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based on works in collaboration with G. Shiu and H.V. Tran

- *Accelerating universe at the end of time* [hep-th/2303.03418]
- *Late-time attractors and cosmic acceleration* [hep-th/2306.07327]

[+ see also G. Shiu's talk in the plenary session]

Accelerated cosmological expansion and string theory

- observations:

  present-day accelerated cosmic expansion,
  huge scale hierarchies

- string-theoretic considerations:

  all couplings are dynamical ($g = g(\phi)$)

  - Dine-Seiberg problem: hard to find weakly-coupled vacua
  - small coupling constants: possibly natural at field-space boundary

    [see G. Shiu's talk]

    [+ also A. Hebecker, D. Andriot, T. Wrase and M. Scalisi's talks]

main idea:

characterize scalar-field cosmological solutions
that asymptotically approach the moduli-space boundary

- field content:
  - canonically-normalized scalars ϕ^a, $a = 1, \dots, n$
    - e.g. string theory (minimal, unless stabilized): dilaton, radions
  - multi-exponential potential $V = \sum_{i=1}^{m} \Lambda_i \, \mathrm{e}^{-\kappa_d \gamma_{ia} \phi^a}$
    - e.g. string theory: non-trivial curvature, NSNS-fluxes, heterotic Yang-Mills fluxes, type-II RR-fluxes, type-II D-brane/O-plane sources and generic Casimir-energy terms

[see G. Shiu's talk]

- geometry: FLRW-metric $d\tilde{s}^2_{1,d-1} = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}l^2_{\mathbb{E}^{d-1}}$, with $H = \dfrac{\dot{a}}{a}$

  ϵ-parameter: $\epsilon = -\dfrac{\dot{H}}{H^2} = 1 + q$; accelerated expansion if $\epsilon < 1$

in this talk, we will see:

1. a *universal bound on late-time cosmic acceleration*
2. a class of theories with a *universal cosmological attractor* solution

1. Bounds on cosmic acceleration

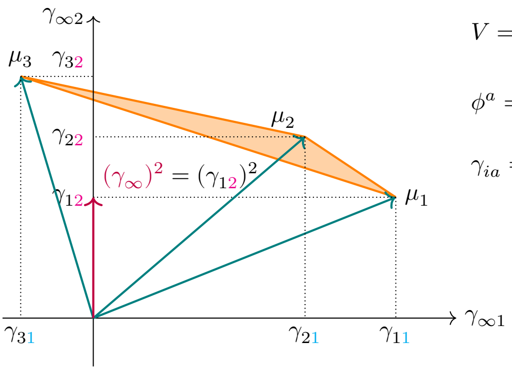
if all $\Lambda_i > 0$, let $\gamma_\infty^a = \begin{cases} \gamma^a, & \gamma^a = \min_i {\gamma_i}^a > 0 \\ 0, & \gamma^a \leq 0 \end{cases}$

then analytic late-time bounds

$$d - 1 \geq \epsilon \geq \frac{d-2}{4} \, (\gamma_\infty)^2$$

[see G. Shiu's talk]
[mathematical proofs in the papers]

example:



$$V = \sum_{i=1}^{3} \Lambda_i \, \mathrm{e}^{-\kappa_d \gamma_{ia} \phi^a}$$

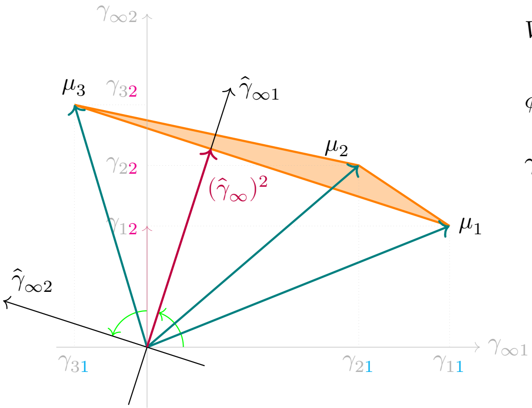
$$\phi^a = \phi^1, \phi^2$$

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{pmatrix}$$

- the bound can be maximized by a field-space basis rotation:

$$d-1 \geq \epsilon \geq \frac{d-2}{4}\,(\hat{\gamma}_\infty)^2$$

example:



$$V = \sum_{i=1}^{3} \Lambda_i \, \mathrm{e}^{-\kappa_d \gamma_{ia} \phi^a}$$

$$\phi^a = \phi^1, \phi^2$$

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{pmatrix}$$

for the canonical string-frame radion $\tilde{\sigma}$ and the canonical d-dim. dilaton $\tilde{\delta}$, any perturbative potential term has the Einstein-frame form $V = \Lambda\, \mathrm{e}^{\kappa_d[\gamma_{\tilde{\delta}}\tilde{\delta} - \gamma_{\tilde{\sigma}}\tilde{\sigma}]}$

- model-dependent $\tilde{\sigma}$-coupling
- for worldsheet Euler character χ_{E}, universal $\gamma_{\tilde{\delta}} = \dfrac{d}{\sqrt{d-2}} - \dfrac{\chi_{\mathrm{E}}}{2}\sqrt{d-2}$

general string-theoretic considerations:

- upper bound on $\gamma_{\tilde{\delta}}$: $\chi_{\mathrm{E}} \leq \chi_{\mathrm{E}}(\mathrm{S}^2) = 2$, so $\gamma_{\tilde{\delta}} \geq \dfrac{2}{\sqrt{d-2}}$
- lower bound on ϵ: $\epsilon \geq \dfrac{d-2}{4}\,(\gamma_\infty)^2 \geq \dfrac{d-2}{4}\,{\gamma_{\tilde{\delta}}}^2 \geq 1$

possible ways out:

- theory not at weak string coupling
- stabilized dilaton
- presence of negative-definite potential terms:
  bound takes a different form, less obvious but still restrictive!

[discussion of potentials with terms of both signs in the papers]

[more in upcoming work]

2. Scaling cosmologies

Scaling cosmologies: general characterization

- scaling cosmologies are solutions with constant positive ϵ
  - power-law scale factor: $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{\epsilon}}$

- complete analytic characterization, if $\operatorname{rank} \gamma_{ia} = m$ (where $M_{ij} = \gamma_{ia}\gamma_j{}^a$):
  - field-space trajectory $\phi_*^a(t) = \phi_0^a + \frac{2}{\kappa_d}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}\gamma_i{}^a (M^{-1})^{ij}\right] \ln\frac{t}{t_0}$
  - ϵ-parameter $\epsilon = \frac{d-2}{4}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}(M^{-1})^{ij}\right]^{-1}$

Collinucci, Nielsen, Van Riet [hep-th/0407047]

- notes:
  - no slow roll: $w = \frac{T_* - V_*}{T_* + V_*} = -1 + \frac{2\epsilon}{d-1}$, $\quad \ddot{\phi}_*^a \propto H\dot{\phi}_*^a \propto \frac{\partial V}{\partial \phi_{*a}} \propto \frac{1}{t^2}$
  - all scalar-potential terms decay identically: $V_i[\phi_*^a(t)] = V_i(t_0)\left(\frac{t_0}{t}\right)^2$

Scaling cosmologies: relevance

- late-time scale factor is bounded by power-law behaviors

  remember: $d-1 \geq \epsilon \geq \frac{d-2}{4}\left(\hat{\gamma}_{\infty}\right)^2$

- scaling solutions are perturbative late-time attractors

  see e.g. Hartong, Ploegh, Van Riet, Westra [gr-qc/0602077]

▶ new result:

we can analyically prove that

- if all terms in the potential are positive-definite, i.e. if $\Lambda_i > 0$
- if $\lambda^i = \sum_{j=1}^{m} (M^{-1})^{ij} \geq 0$ and $\sum_{i=1}^{m} \lambda^i > 0$ (i.e. no subdominant terms)

then scaling cosmologies are late-time attractors, irrespectively of initial conditions, and saturate the universal bound, i.e. they have $\epsilon = \frac{d-2}{4}\left(\hat{\gamma}_{\infty}\right)^2$

[mathematical proof in the papers]

field-space trajectory is a straight line

original field basis:

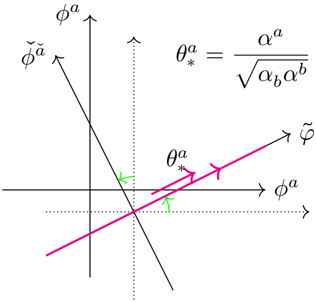
$$\phi^a_*(t) = \phi^a_\infty + \frac{\alpha^a}{\kappa_d} \ln \frac{t}{t_\infty}$$

rotated field basis:

$$\check{\phi}^{\check{a}}_*(t) = \check{\phi}^{\check{a}}_\infty$$

$$\tilde{\varphi}_*(t) = \tilde{\varphi}_\infty + \frac{1}{\kappa_d}\frac{2}{\gamma_*} \ln \frac{t}{t_\infty}$$

note: on-shell potential $\tilde{V}_* = \tilde{\Lambda}\, \mathrm{e}^{-\kappa_d \gamma_* \tilde{\varphi}_*}$



- exact relationship between scalar-potential slope and ϵ-parameter:

$$-\frac{1}{V}\, \theta^a_* \frac{\partial V}{\kappa_d\, \partial \phi^a}(\phi_*) = \frac{1}{\kappa_d V} \sqrt{\frac{\partial V}{\partial \phi^a} \frac{\partial V}{\partial \phi_a}}(\phi_*) = \frac{2\sqrt{\epsilon}}{\sqrt{d-2}} = \gamma_*$$

to compare with Obied, Ooguri, Spodyneiko, Vafa [hep-th/1806.08362]
Ooguri, Palti, Shiu, Vafa [hep-th/1810.05506]

false for generic solution $\overline{\phi}^a$, unless $\eta, \Omega = 0$ ($\eta = -\dot{\epsilon}/(\epsilon H)$), Ω: non-geodesity):

$$\overline{\gamma}(\epsilon, \eta) = -\frac{1}{V} \frac{\dot{\phi}^a}{\sqrt{\dot{\phi}_b \dot{\phi}^b}} \frac{\partial V}{\kappa_d\, \partial \phi^a}(\overline{\phi}) \leq \frac{1}{\kappa_d V} \sqrt{\frac{\partial V}{\partial \phi_a} \frac{\partial V}{\partial \phi^a}}(\overline{\phi}) = \gamma(\epsilon, \eta, \Omega)$$

Achúcarro, Palma [hep-th/1807.04390]
see also Andriot, Horer, Tringas [hep-th/2212.04517]

Scaling cosmologies: non-slow-roll gradient flow

- in a scaling cosmology (no slow roll), we have $\phi^a_*(t) = \phi^a_\infty + \frac{\alpha^a}{\kappa_d} \ln \frac{t}{t_\infty}$, so
  - $\ddot{\phi}^a_*(t) = -\frac{1}{\kappa_d t^2}\, \alpha^a$
  - $(d-1)H\dot{\phi}^a_*(t) = \frac{1}{\kappa_d t^2}\, \frac{d-1}{\epsilon}\, \alpha^a$

  we can plug these into the scalar-field eq. to find the trajectory, i.e.

$$\ddot{\phi}^a_* + (d-1)H\dot{\phi}^a_* = \left[1 - \frac{\epsilon}{d-1}\right](d-1)H\dot{\phi}^a_* = -\frac{\partial V}{\partial \phi_a}(\phi_*)$$

  as the proportionality factor between $\dot{\phi}^a_*$ and $\partial V / \partial \phi_{*a}$ is universal, this happens to give a non-slow-roll gradient-flow trajectory (note: irrespectively of ϵ)

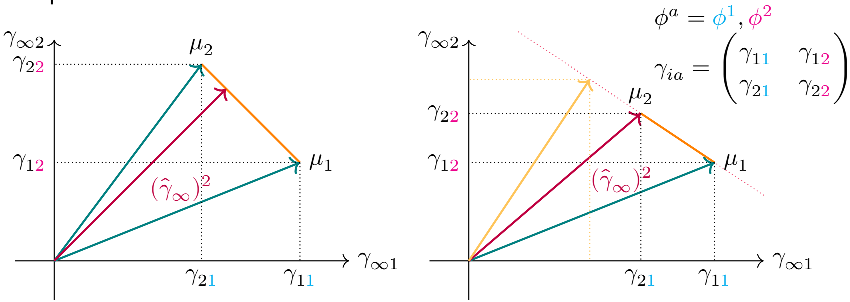
- in the slow-roll regime, one approximates

$$\ddot{\phi}^a_{\text{sr}} + (d-1)H\dot{\phi}^a_{\text{sr}} \overset{\frac{|\ddot{\phi}^a_{\text{sr}}|}{H|\dot{\phi}^a_{\text{sr}}|} \ll 1}{\simeq} (d-1)H\dot{\phi}^a_{\text{sr}} = -\frac{\partial V}{\partial \phi_a}(\phi_{\text{sr}})$$

  conclusion:

  although the field-space trajectory is accidentally the same,
  the physics (i.e. the time dependence of fields) is completely different

examples:



if distance vector from the origin to the convex-hull coupling hyperplane intersects convex hull itself too, we analytically find the late-time ϵ-parameter

$$\epsilon = \frac{d-2}{4}\,(\hat{\gamma}_\infty)^2 = \frac{d-2}{4}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}(M^{-1})^{ij}\right]^{-1}$$

else, we speculate that the potential is truncated, leaving $\epsilon = \frac{d-2}{4}\,(\hat{\gamma}_\infty)^2$

[more in upcoming work]

3. Conclusions

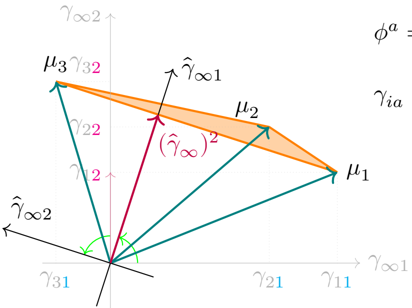
Recap

- analytic model-independent results, and handle on swampland conjectures:
  - universal:
    - bound for cosmic acceleration
      1-field 1-potential case: see also Rudelius [hep-th/2208.08989]
    - string-theoretic dilaton obstruction to acceleration and ways out
      compatible with strong de Sitter conjecture in Rudelius [hep-th/2101.11617]
    - general tension for slow roll
  - scaling cosmologies:
    - proof of convergence, irrespectively of initial conditions
    - relationship between acceleration and scalar-potential derivatives
      to compare with Obied, Ooguri, Spodyneiko, Vafa [hep-th/1806.08362]
      Ooguri, Palti, Shiu, Vafa [hep-th/1810.05506]
    - convex-hull criterion to diagnose cosmic acceleration
- other swampland conjectures:
  - precise handle on attempts for string-theoretic accelerated expansion
    Calderón-Infante, Ruiz, Valenzuela [hep-th/2209.11821]
    Cremonini, Gonzalo, Rajaguru, Tang, Wrase [hep-th/2306.15714]
  - importance of dynamical considerations in the swampland program
    Apers, Conlon, Mosny, Revello [hep-th/2212.10293]
  - asymptotic acceleration implies higher-dimensional de Sitter spacetime
    Hebecker, Schreyer, Venken [hep-th/2306.17213]

Thank you!

4. Backup material

- the bound can be maximized by a field-space basis rotation



$$\phi^a = \phi^1, \phi^2$$

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{pmatrix}$$

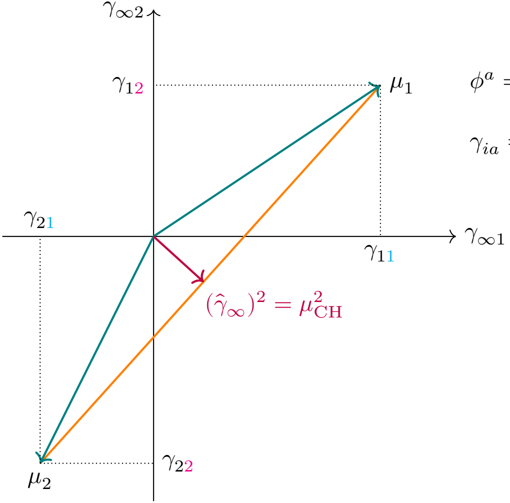
- in the optimal basis:

$$V = \left[\sum_{\sigma=1}^{\hat{m}} \Lambda_\sigma \, \mathrm{e}^{-\kappa_d \hat{\gamma}_{\sigma\check{a}} \hat{\phi}^{\check{a}}}\right] \mathrm{e}^{-\kappa_d \hat{\gamma}_\infty \hat{\varphi}} + \sum_{\iota=\hat{m}+1}^{m} \Lambda_\iota \, \mathrm{e}^{-\kappa_d \hat{\gamma}_{\iota\hat{\varphi}} \hat{\varphi} - \kappa_d \hat{\gamma}_{\iota\check{a}} \hat{\phi}^{\check{a}}}$$

  - the 1-field 1-term potential $\hat{V}_\infty = \hat{\Lambda}_\infty \, \mathrm{e}^{-\kappa_d \hat{\gamma}_\infty \hat{\varphi}}$ would give $\epsilon = \dfrac{d-2}{4} \, \hat{\gamma}_\infty^2$
  - the presence of other fields creates further steepness

observation:

even if $(\gamma_\infty)^2 = 0$, scaling cosmologies automatically saturate bound

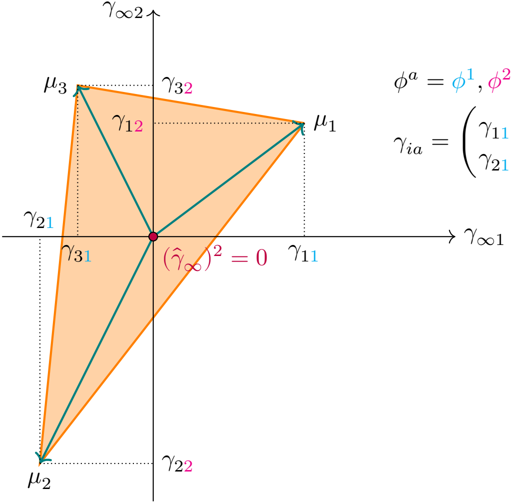


$$\phi^a = \phi^1, \phi^2$$

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}$$

observation:

there are plenty of possibilities for late-time acceleration!



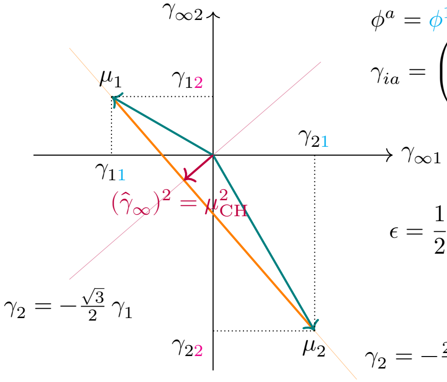
$$\phi^a = \phi^1, \phi^2$$

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}$$

4-dimensional potential: $V = \Lambda_1 \, e^{\kappa_4 \sqrt{2}\, \tilde{\phi}^1 - \kappa_4 \sqrt{\frac{2}{3}}\, \tilde{\phi}^2} + \Lambda_2 \, e^{-\kappa_4 \sqrt{2}\, \tilde{\phi}^1 + \kappa_4 \sqrt{6}\, \tilde{\phi}^2}$

Calderón-Infante, Ruiz, Valenzuela [hep-th/2209.11821]

- convex-hull hyperplane: $\gamma_2 = -\frac{2\sqrt{3}}{3}\,\gamma_1 - \frac{\sqrt{6}}{3}$
- orthogonal line: $\gamma_2 = \frac{\sqrt{3}}{2}\,\gamma_1$, intersection at $(\gamma_1, \gamma_2) = \left(-\frac{2\sqrt{2}}{7}, -\frac{\sqrt{6}}{7}\right)$



$$\phi^a = \phi^1, \phi^2$$

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} = \begin{pmatrix} -\sqrt{2} & \sqrt{\frac{2}{3}} \\ \sqrt{2} & -\sqrt{6} \end{pmatrix}$$

$$\epsilon = \frac{1}{2}\left[\left(-\frac{2\sqrt{2}}{7}\right)^2 + \left(-\frac{\sqrt{6}}{7}\right)^2\right] = \frac{1}{7}$$

- exact relationship between scalar-potential slope and ϵ-parameter:

$$-\frac{1}{V}\,\theta^a_*\,\frac{\partial V}{\kappa_d\,\partial\phi^a}(\phi_*) = \frac{1}{\kappa_d V}\sqrt{\frac{\partial V}{\partial\phi^a}\frac{\partial V}{\partial\phi_a}}(\phi_*) = \frac{2\sqrt{\epsilon}}{\sqrt{d-2}} = \gamma_*$$

to compare with Obied, Ooguri, Spodyneiko, Vafa [hep-th/1806.08362]
Ooguri, Palti, Shiu, Vafa [hep-th/1810.05506]

- the potential directional derivative and gradient norm are not necessarily related to ϵ for non-scaling solutions

  a generic solution $\overline{\phi}^a(t)$, gives ($\eta = -\dot{\epsilon}/(\epsilon H)$, Ω: non-geodesity factor)

$$\overline{\gamma} = -\frac{1}{V}\frac{\dot{\phi}^a}{\sqrt{\dot{\phi}_b\dot{\phi}^b}}\frac{\partial V}{\kappa_d\,\partial\phi^a}(\overline{\phi}) = \frac{2\sqrt{\epsilon}}{\sqrt{d-2}}\left[1-\frac{\eta}{(d-1)-\epsilon}\right]$$

$$\gamma = \frac{1}{\kappa_d V}\sqrt{\frac{\partial V}{\partial\phi_a}\frac{\partial V}{\partial\phi^a}}(\overline{\phi}) = \sqrt{\overline{\gamma}^2 + \frac{4\epsilon}{d-2}\frac{1}{[(d-1)-\epsilon]^2}\frac{\Omega^2}{H^2}}$$

Achúcarro, Palma [hep-th/1807.04390]