

# Kínation in String Theory

Based on [2212.10293](#) + works in progress

with Fien Apers, Joe Conlon, Ed Copeland, Martín Mosny

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See Also [2207.00567](#) J. Conlon, FR



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# Introduction

String cosmology

[Cicoli, Conlon, Maharana, Quevedo, Parameswaran, Zavala '23]

Desiderata:

inflation, QCD axion,  
quintessence, CC



Embed in ST



This Talk

What are typical dynamics induced by moduli potentials?

Steep potentials



Kination

"Asymptotic"  
cosmology

[Cicoli, Cuneillera, Padilla, Pedro '21]

[Calderon Infante, Ruiz, Valenzuela '22] [Apers, Conlon, Mosny, Revello '22]

[Shiu, Tonioni, Tran '23] x 2 [Cremonini, Gonzalo, Rajaguru, Tang, Wrase '23]

[Hebecker, Schreyer, Venken '23]

# What is kination?

Scalar rolling down a steep potential: 'kination' [Joyce '97]

$$\frac{1}{2}\dot{\phi}^2 \gg V(\phi) \qquad w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \simeq 1$$

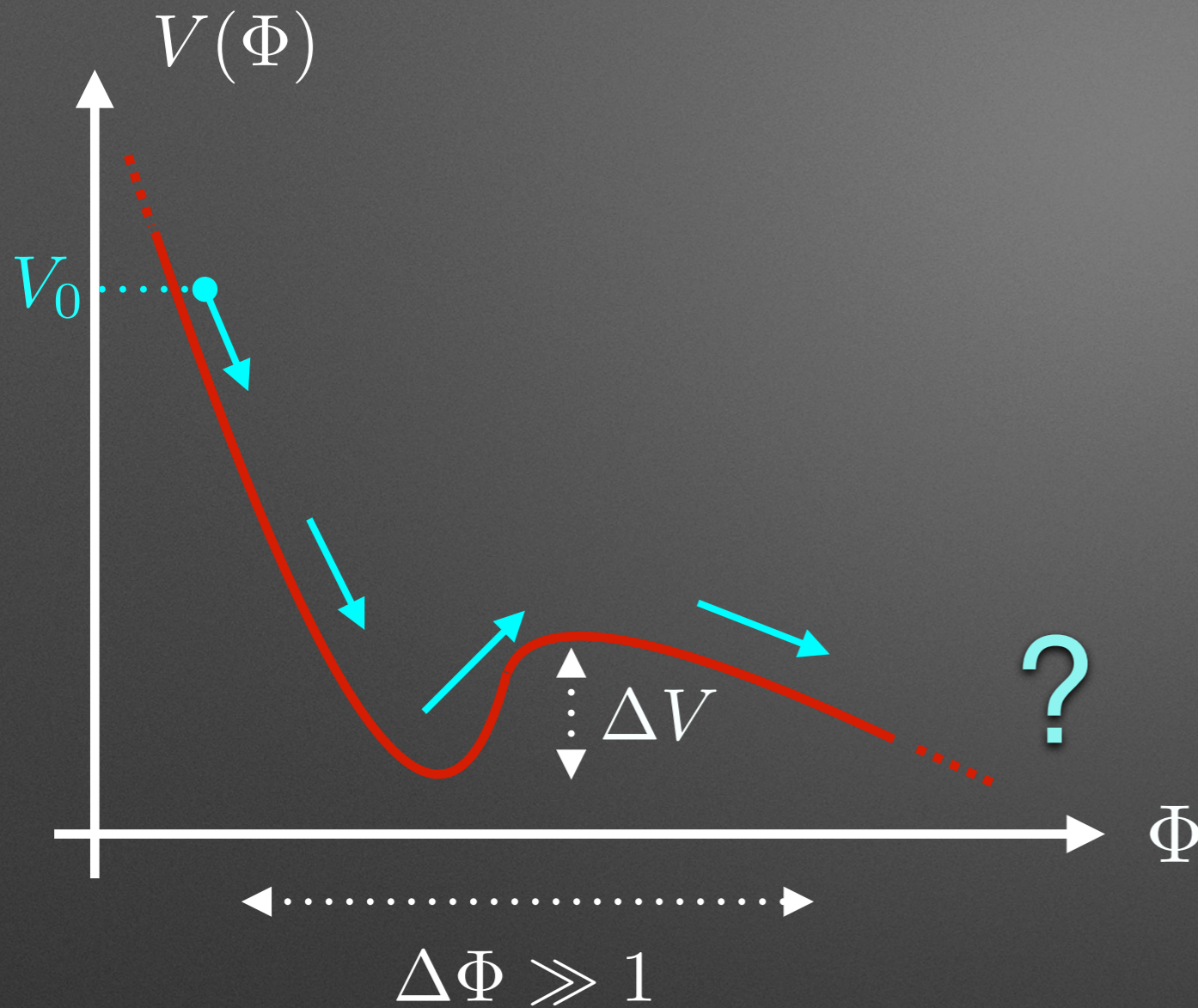
stiff fluid

$$\ddot{\phi} + 3H\dot{\phi} = - \cancel{\frac{\partial V}{\partial \phi}},$$

$$H^2 = \frac{1}{3M_P^2} \left( \cancel{V(\phi)} + \frac{\dot{\phi}^2}{2} \right)$$

$$\phi = \phi_0 + \sqrt{\frac{2}{3}} M_P \ln \left( \frac{t}{t_0} \right) \qquad a(t) \sim t^{1/3}$$

# Why care about Kination?



Potentials near boundaries of moduli space

$$V(\Phi) \sim e^{-\lambda_i \Phi_i}$$

[Grimm, Li, Valenzuela '19]+...

Specific example: LVS (GKP)

[Conlon, Quevedo, Suruliz '05]

[Balasubramanian, Berglund, Conlon, Quevedo '05]

$$\Phi = \sqrt{\frac{2}{3}} \ln \mathcal{V}$$

$V_0$  high after inflation as  $m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}$

# #1 Volume kination = Kasner

String to  
Einstein frame

$\mathcal{V}_{int}$

10d Kasner, solves vacuum EE

$$ds_E^2 = t^2 \left( -dt^2 + \sum_{i=1}^3 t^{-2/3} dx_i^2 + \sum_{j=1}^6 t^{2/3} dy_j^2 \right)$$



$$ds_{4d,E}^2 = -dt_E^2 + (2t_E)^{2/3} (dx_1^2 + dx_2^2 + dx_3^2)$$



$$\phi = \sqrt{\frac{2}{3}} \log \mathcal{V} \sim \sqrt{\frac{2}{3}} \log t \quad a(t) \sim t^{1/3} \quad \longrightarrow \quad \text{Kination}$$

Big crunch:  $ds^2 = -dt^2 + t^{-2/3} (dx_1^2 + dx_2^2 + dx_3^2)$

# A dynamical Swampland?

"Big Crunch" of the non-compact dimensions

4d picture

$$\left\{ \begin{array}{l} l_s \sim \frac{\sqrt{v}}{M_P} \quad \text{grows faster than } a(t) \\ \text{(Planck mass fixed in Ein. frame)} \end{array} \right.$$



Assumes knowledge of UV physics

No apparent issues  
with EFT approach

or "kinematic" Swampland constraints



Motivates a dynamical  
Swampland?

Constraints on the cosmological evolution

# What about the distance conjecture?

Distance conjecture:

[Ooguri, Vafa '06]

[Ooguri, Palti, Shiu, Vafa '19]

$$\Delta\phi \gg 1$$



towers of  
light states



Invalidate the EFT

"Static"

KK modes become light, but so do  $m_\phi, \Lambda_\phi$



Dynamics

KK modes above Hubble  $m_{KK}(t) \gg H(t)$



Cutoff is adiabatic  $\left| \frac{d\Lambda(t)_{KK}}{dt} \right| \ll \Lambda(t)_{KK}^2$



# #2 Trackers and axions

Reformulate as a dynamical system

See also [Shiu, Tonioni, Tran '23] x 2

$$x = \frac{\dot{s}}{\sqrt{6}Hs} \quad y = \frac{\dot{b}}{\sqrt{6}Hs} \quad z = \frac{1}{H} \sqrt{\frac{V(s)}{3}}$$

$$\begin{cases} \dot{x} = f_x(x, y, z) \\ \dot{y} = f_y(x, y, z) \\ \dot{z} = f_z(x, y, z) \end{cases}$$

$$T = s + ib$$

Matter/radiation + rolling scalar



Trackers

[Wetterich '88] [Ferreira, Joyce '98]  
[Copeland, Liddle, Wands '98]

What about **Axions**? [FR, WiP]

[Russo, Townsend '22]

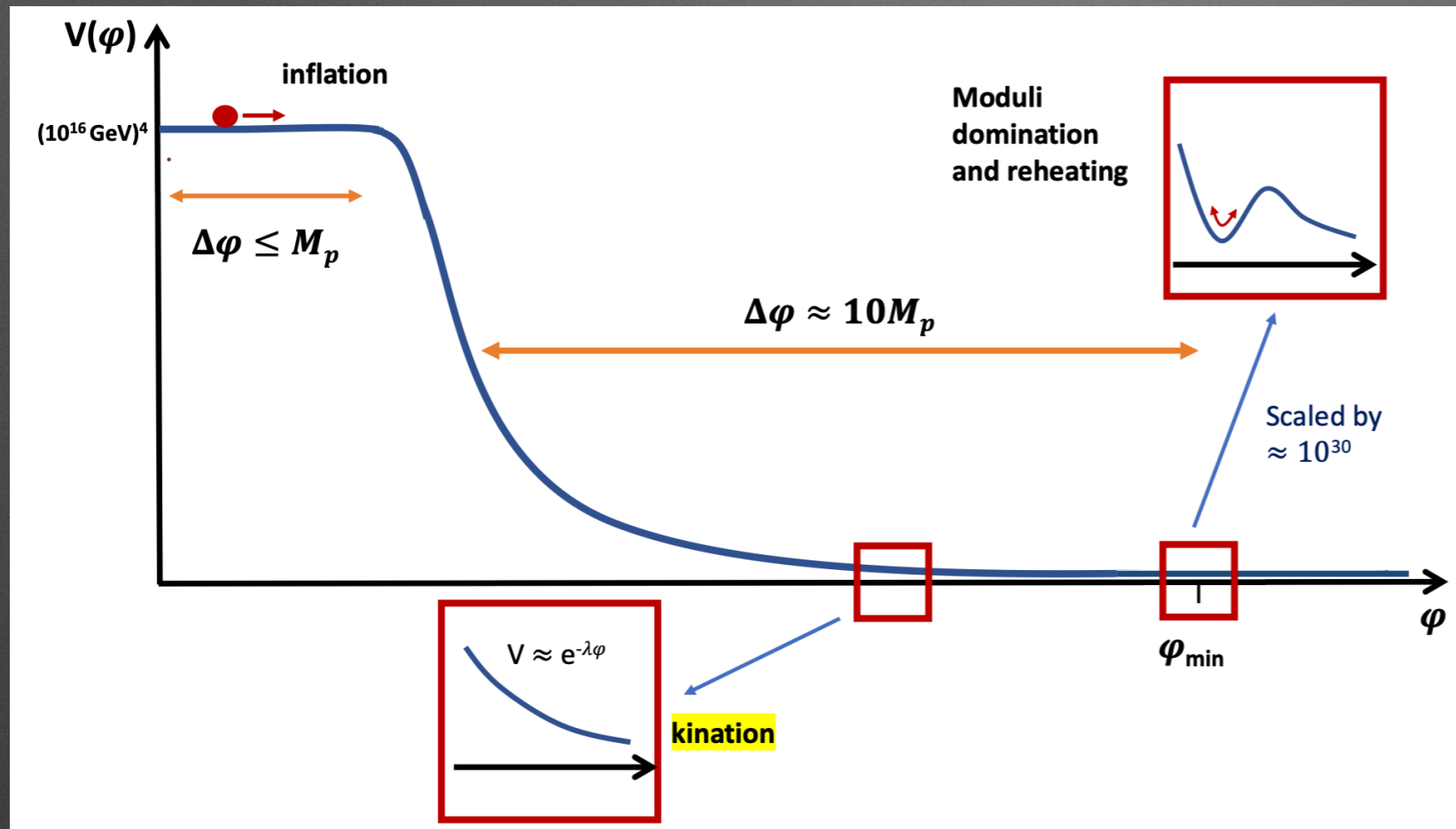
Universal coupling through kin term

$$\mathcal{L} \supset \frac{(\partial s)^2 + (\partial b)^2}{s^2} + \frac{V_0}{s^\lambda}$$

**Late time attractor**  $\dot{s}(t) \sim \dot{b}(t) \sim t^{2/\lambda-1}$  Fixed ratios  $x \sim y \sim z$



# #3 A non-standard picture



Scalar perturbations



small scale structure?

[Apers, Conlon, Copeland, FR WIP]

Figure taken from [Cicoli, Conlon, Maharana, Quevedo, Parameswaran, Zavala '23]

Other *signatures* of kination/tracker epochs?

Gravitational waves [Muia, Schachner, Quevedo, Villa '23]

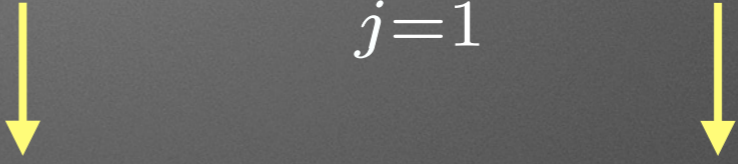
Axion kination [Co, Harigaya '20] [Co, Hall, Harigaya '20]  
[Gouttenoire, Servant, Simakachorn '21] x 2+...

Thank you for your attention!

# Kasner metric

Sourceless solution to Einstein Equations

$$ds^2 = -dt^2 + \sum_{i=1}^d t^{2p_i} dx_i^2 + \sum_{j=1}^{D-d-1} t^{2q_j} dy_j^2$$



"external"                      "internal"

One of the oldest solutions to GR! [Kasner 1921]

$$\sum_{i=1}^d p_i + \sum_{j=1}^{D-d-1} q_j = 1$$
$$\sum_{i=1}^d p_i^2 + \sum_{j=1}^{D-d-1} q_j^2 = 1$$

No isotropic expansion/contraction (unlike FLRW)

# Avoiding the catastrophe

- In a quantum universe: **radiation** will take over

$$\rho_{\text{kin}} \sim a^{-6} \quad \text{vs} \quad \rho_{\text{mat}} \sim a^{-3} \quad \rho_{\text{rad}} \sim a^{-4}$$

Tracker solutions!

See also [Conlon, FR '22]

**10d interpret:** Kasner is unstable to perturbations

$$ds^2 = ds_{\text{kasner}}^2 + \frac{2\epsilon t^{2/3}}{\sqrt{6}} \sum_{\substack{i=1,2,3 \\ j=4,\dots,9}} h_{ij}(t) dx_i dy_{j-3} \\ + 2\epsilon^2 t^{2/3} \sum_{\substack{i,j=1,2,3 \\ i < j}} h_{ij}^2(t) dx_i dx_j + \epsilon^2 t^{2/3} \sum_{i=1,2,3} h_{ii}^2(t) dx_i^2$$

Solves EE at first order if  $h'_{ij} = 1$

Exact solution if  $h_{ij} = 1$

**Radiation-like:**  $\rho \sim a^{-6} + \epsilon^2 a^{-4}$

# P.S. Avoiding the catastrophe

- In a quantum universe: radiation will take over

$$\rho_{\text{kin}} \sim a^{-6}$$

vs

$$\rho_{\text{mat}} \sim a^{-3}$$

$$\rho_{\text{rad}} \sim a^{-4}$$

Attractor solution with

$$\Omega_{\phi} \sim \Omega_{\gamma} \sim \mathcal{O}(1)$$

[Wetterich '88; Copeland, Liddle, Wands '98; Ferreira, Joyce '98]

[Conlon, Kallosh, Linde, Quevedo '08]

In realistic models (LVS)

localising the  
minimum



$$\Lambda_{\text{inf}} \gg \Lambda_{\text{EW}}$$

10d interpret:

Kasner is unstable to  
perturbations

[Conlon, FR '22]

[Apers, Conlon, Mosny, FR '22]

# The tracker solution

$$\left\{ \begin{array}{l} H^2 = \frac{1}{3M_P^2} \left( \rho_\gamma + \frac{1}{2} \dot{\Phi}^2 + V(\Phi) \right) \\ \dot{H} = -\frac{1}{2M_P^2} \left( \rho_\gamma + P_\gamma + \dot{\Phi}^2 \right) = -\frac{1}{2M_P^2} \left( \gamma \rho_\gamma + \dot{\Phi}^2 \right) \end{array} \right. \quad \text{Friedmann Equations}$$

$$+ \quad \dot{\rho}_\gamma = -3H (\rho_\gamma + P_\gamma) = -3H \gamma \rho_\gamma \quad \text{Energy conservation}$$

Attractor solution

$$V = V_0 e^{-\frac{\lambda \Phi}{M_P}}$$

$$\Omega_{\text{kin}} = \frac{3}{2} \frac{\gamma^2}{\lambda^2} \quad \Omega_{\text{pot}} = \frac{3(2-\gamma)\gamma}{2\lambda^2} \quad \Omega_\gamma = 1 - \Omega_{\text{kin}} - \Omega_{\text{pot}} = 1 - \frac{3\gamma}{\lambda^2}$$

[Wetterich '88; Copeland, Liddle, Wands '98; Ferreira, Joyce '98]