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Towards AdS Distances in String Theory

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Introduction: the AdS Distance Conjecture

- In string theory the vacuum energy ($V_{\text{min}} \equiv \Lambda$) is calculable.
- **AdS Conjecture**: Consider quantum gravity on AdS. There exists an infinite tower of states with mass scale M which, as $\Lambda \to 0$, behaves as $M \sim |\Lambda|^{\alpha}$, with α positive number. [Lüst, Palti, Vafa, 2019].
- Great interest in recent years: dark dimension scenario [Gonzalo, Obied, Montero, Valenzuela, Vafa, 2022], understanding scale separated vacua, e.g. [Shiu, Tonioni, Van Hemelryck, Van Riet 2023], etc...
- Do the AdS conjecture make sense? If yes, what is the physics behind?

Generalized distance and metric variations

- Compactifications parametrize deformations of the internal metric, RR fields, etc $\rightarrow$ **Distance Conjecture** [Ooguri, Vafa, 2006].
- External metric variations are particularly interesting:

$$ds_d^2 = e^{2\sigma} ds^2_{\mathrm{AdS}_d} \qquad\qquad \Lambda = -(d-1)(d-2)\, e^{-2\sigma}\,.$$

- We give to σ a dependence over $\mathrm{AdS}_d \longrightarrow S_d \supset -\int K_{\sigma\sigma}(\partial\sigma)^2$.
- Deformations in σ are equivalent to variations of Λ [Lüst, Palti, Vafa, 2019]

$$\Delta = -\frac{1}{2}\int_{\Lambda_i}^{\Lambda_f} \left(K_{(\sigma)} + K'\right)^{1/2} d\log\Lambda\,.$$

- **AdS Distance conjecture**: $(K_{(\sigma)} + K')^{1/2} \sim \mathcal{O}(1)$
  $m \sim e^{-\alpha\Delta} \sim |\Lambda|^{\alpha}$ [Lüst, Palti, Vafa, 2019].

Imaginary distance and conformal factor problem

- Let's see if we can develop this idea: we start with $ds_d^2 = e^{2\sigma} d\hat{s}^2_{\mathsf{AdS}_d}$.
- Very simple calculation: derive the EH action $S_d = \int d^d x \sqrt{-g}\, R_d$ with

$$R_d = e^{-2\,\sigma}\,\left(\hat{R}_d - (d-1)(d-2)\hat{g}_d^{mn}\partial_m\,\sigma\partial_n\,\sigma - 2(d-1)\,\hat{\nabla}_d^2\,\sigma\right)\,.$$

- Off-shell action for external volume variations:

$$S_d = \int d^d x \sqrt{-g_d}\,\left(e^{-2\sigma}\,\hat{R}_d - K_{\sigma\sigma}(\partial\,\sigma)^2\right)$$

with $K_{\sigma\sigma} = -(d-1)(d-2) \longleftarrow$ **negative definite metric!**

- This is the conformal factor problem in quantum gravity (big deal).

The inclusion of extra-dimensions: $\mathrm{AdS}_d \times Y_k$

External and internal metric variations $ds_D^2 = e^{2\sigma}\, d\hat{s}_d^2 + e^{2\tau}\, d\hat{s}_k^2$.

$$S_d \supset \int d^d x \sqrt{-g_E}\, \left[-K_{\sigma\sigma}\,(\partial\,\sigma)^2 - K_{\tau\tau}\,(\partial\,\tau)^2\right] \qquad \text{with} \qquad \tau = a\sigma$$

- Total metric over metric variations:

$$K_{\sigma\sigma}^{\text{tot}} = K_{\sigma\sigma} + K_{\tau\tau} = -(d-1)(d-2) + a^2 k^2 \left(\frac{d-1}{d-2} - \frac{k-1}{k}\right),$$

- Positivity is related to scale separation.
- $K_{\mathrm{AdS}_4\times S^7} = \frac{51}{2}, \qquad K_{\mathrm{AdS}_5\times S^5} = \frac{4}{3}, \qquad K_{\mathrm{AdS}_7\times S^4} = -\frac{114}{5}.$
- $K_{\mathrm{DGKT}} = -\frac{51}{128}$ ($a \neq 1$ + dilaton contribution).

Flux variations on $\mathrm{AdS}_7 \times S^4$ vacua

$$ds_{11}^2 = L^2 \left(ds^2_{\mathsf{AdS}_7} + 4\, ds^2_{S^4} \right) \text{ and } F_4 = -\tfrac{3}{8} L^3 \, \mathsf{vol}_{S^4} .$$

- Dual flux: $\hat{C}_6 = -L^6 z^{-6} \mathsf{vol}_{\mathbb{R}^{1,5}}$ with $F_7 = L^6 d\hat{C}_6 = 6\, L^6 \mathsf{vol}_{\mathsf{AdS}_7}$.
- Flux variations: $C_6 = e^{\alpha} \hat{C}_6$ with $\alpha_{\text{on-shell}} = 6 \log L$

$$F_7 = e^{\alpha} \hat{F}_7 + e^{\alpha} \, d\alpha \wedge \hat{C}_6 \qquad \longrightarrow \qquad F_4 = \star F_7$$

- Off-shell flux action:

$$S_F = \int d^{11}x \sqrt{-g}\, e^{2\alpha} \left(-\frac{1}{2} |\hat{F}_4|^2 - \frac{1}{2} (\partial_z \alpha)^2 + 6 z^{-1} g^{zz} \partial_z \alpha \right)$$

- We need “compensating" metric variations!

$$ds_{11}^2 = e^{2\sigma} d\hat{s}_7^2 + e^{2\tau} d\hat{s}_4^2 \qquad \text{with} \qquad d\hat{s}_7^2 = \frac{1}{z^2} \left(ds^2_{\mathbb{R}^{1,5}} + e^{2\sigma_1} dz^2 \right)$$

The action metric and on-shell conditions

- We derive $S = \int d^{11}x\sqrt{-g}(R - \frac{1}{2}|F_4|^2)$ and go to the Einstein frame

$$S_7 \supset \int \left(-K_{\sigma\sigma}\,(\partial_z\,\sigma)^2 - K_{\tau\tau}\,(\partial_z\,\tau)^2 - \frac{1}{2}\,e^{-12\sigma+2\alpha}(\partial_z\alpha)^2 \right.$$
$$\left. -12e^{-2\sigma-2\sigma_1}\,z\partial_z\,\sigma_1 + 6\,e^{-2\sigma-2\sigma_1-12\sigma+2\alpha}\,z\,\partial_z\alpha\right)$$

- On-shell conditions AdS$_7 \times S^4$: EOM: $\sigma + \sigma_1 = \tau$ and $\alpha = 6\sigma$
  Cancellation of linear terms: $\sigma_1 = \frac{\alpha}{2}$.

- Our final off-shell action ($\partial_z\sigma + \partial_z\sigma_1 = \partial_z\tau$, $\partial_z\alpha = 6\partial_z\sigma$, $\partial_z\sigma_1 = \frac{\partial_z\alpha}{2}$):

$$S_7 \supset \int d^7x\,\sqrt{-g_E}\left[-\frac{516}{5}\,(\partial_z\,\sigma)^2\right] \Longrightarrow K_{\mathrm{AdS}_7\times S^4} = \frac{516}{5}$$