Conserved Currents at Infinite Distance in the Conformal Manifold

José Calderón Infante

Based on 2305.05693 with Florent Baume

String Phenomenology 2023, IBS Korea, 06/07/2023

The Swampland Distance Conjecture

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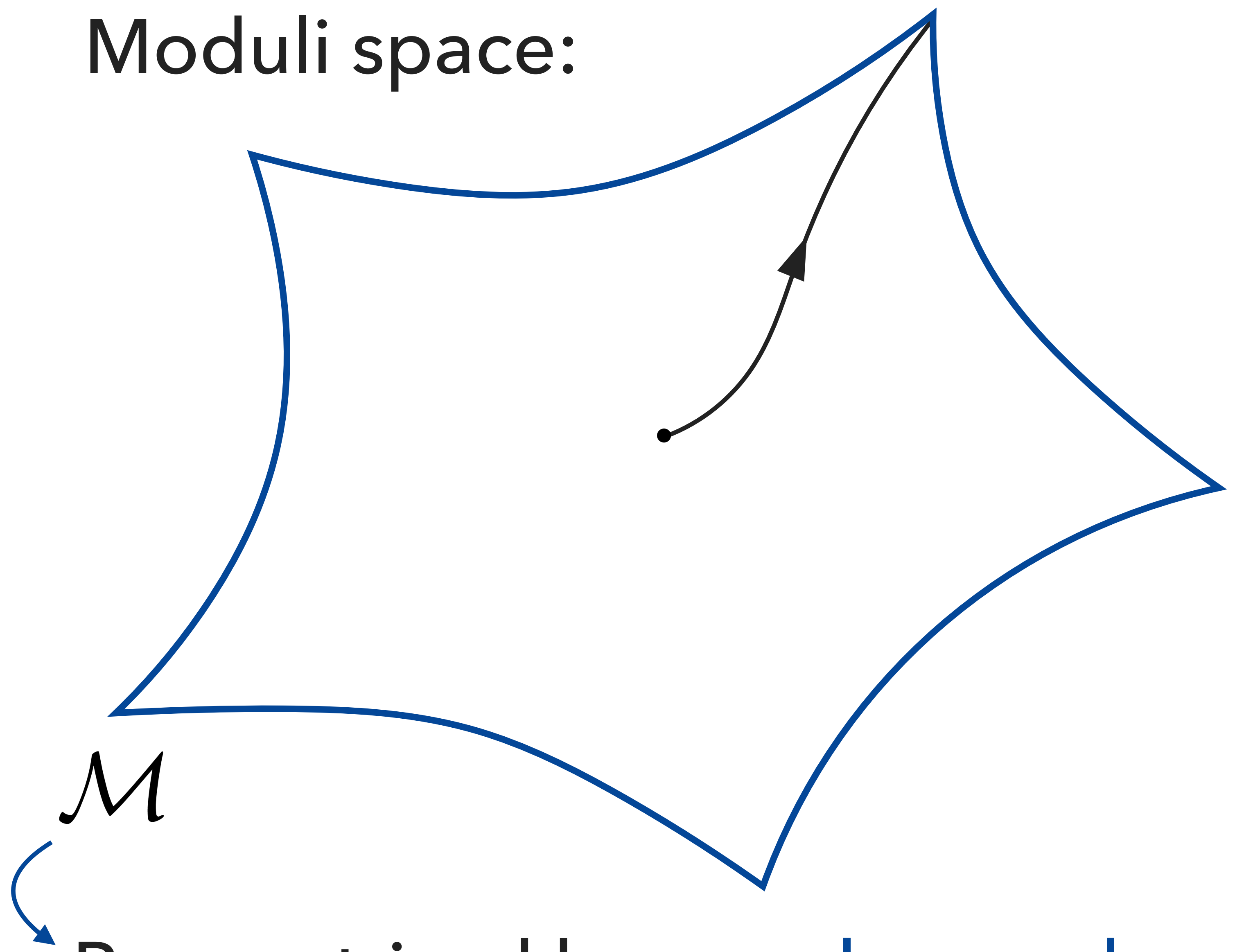

Parametrized by massless scalars

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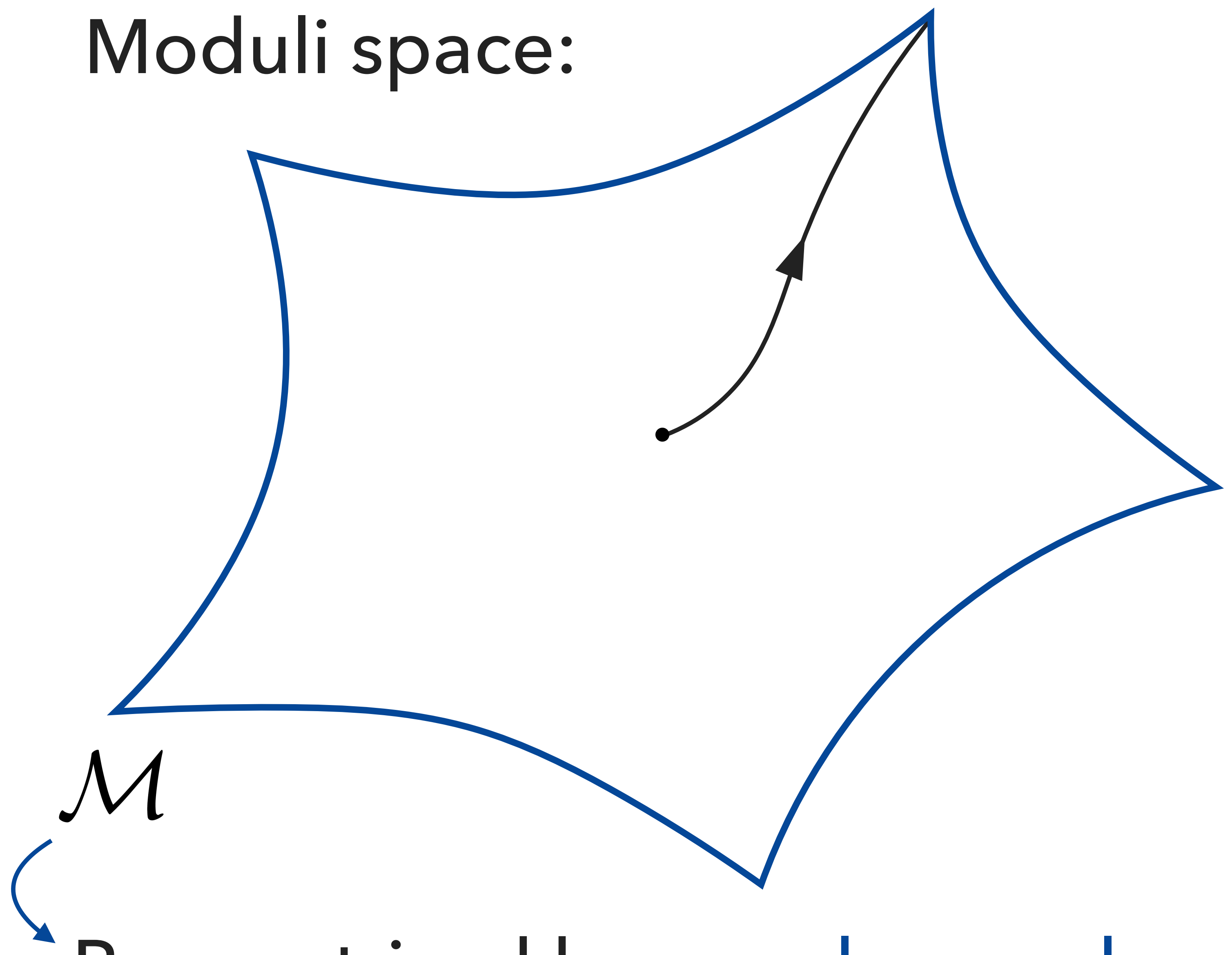

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There is an infinite tower of states becoming light at infinite-distance points in moduli space:

$$M_{tower} \sim e^{-\alpha_t D_\phi} \text{ as } D_\phi \to \infty \quad (M_{Pl} = 1)$$

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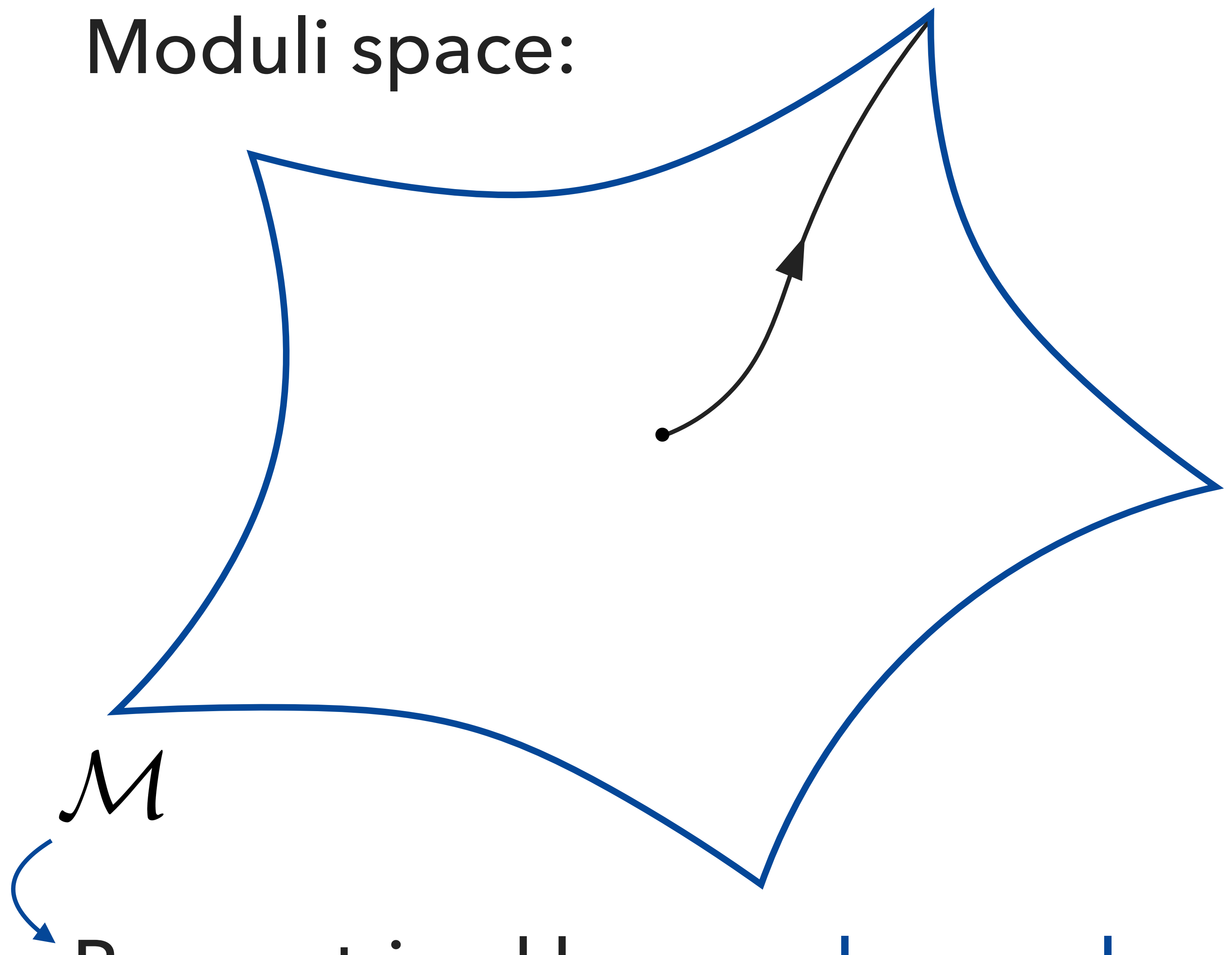

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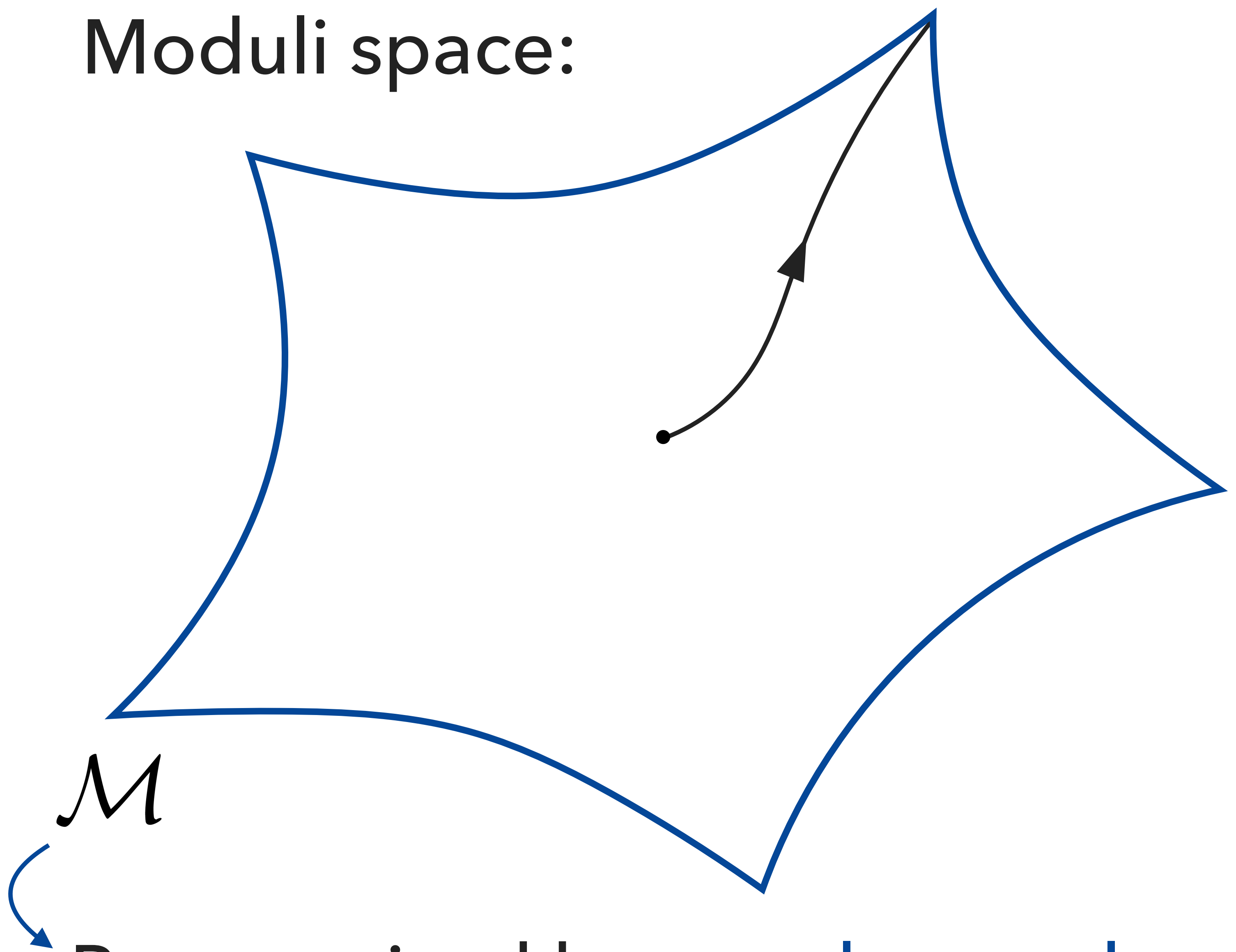

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  Lightest tower: [Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]

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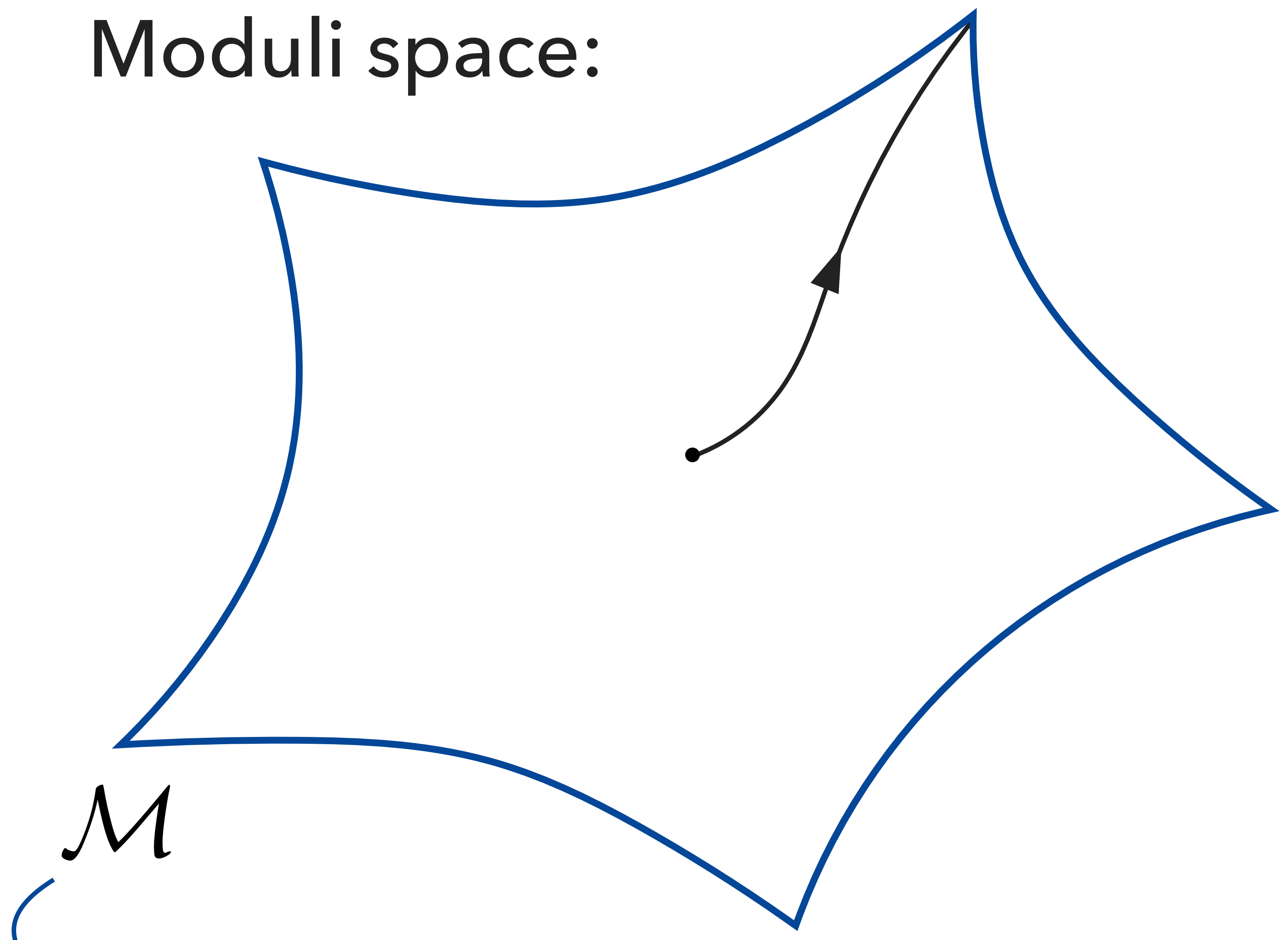

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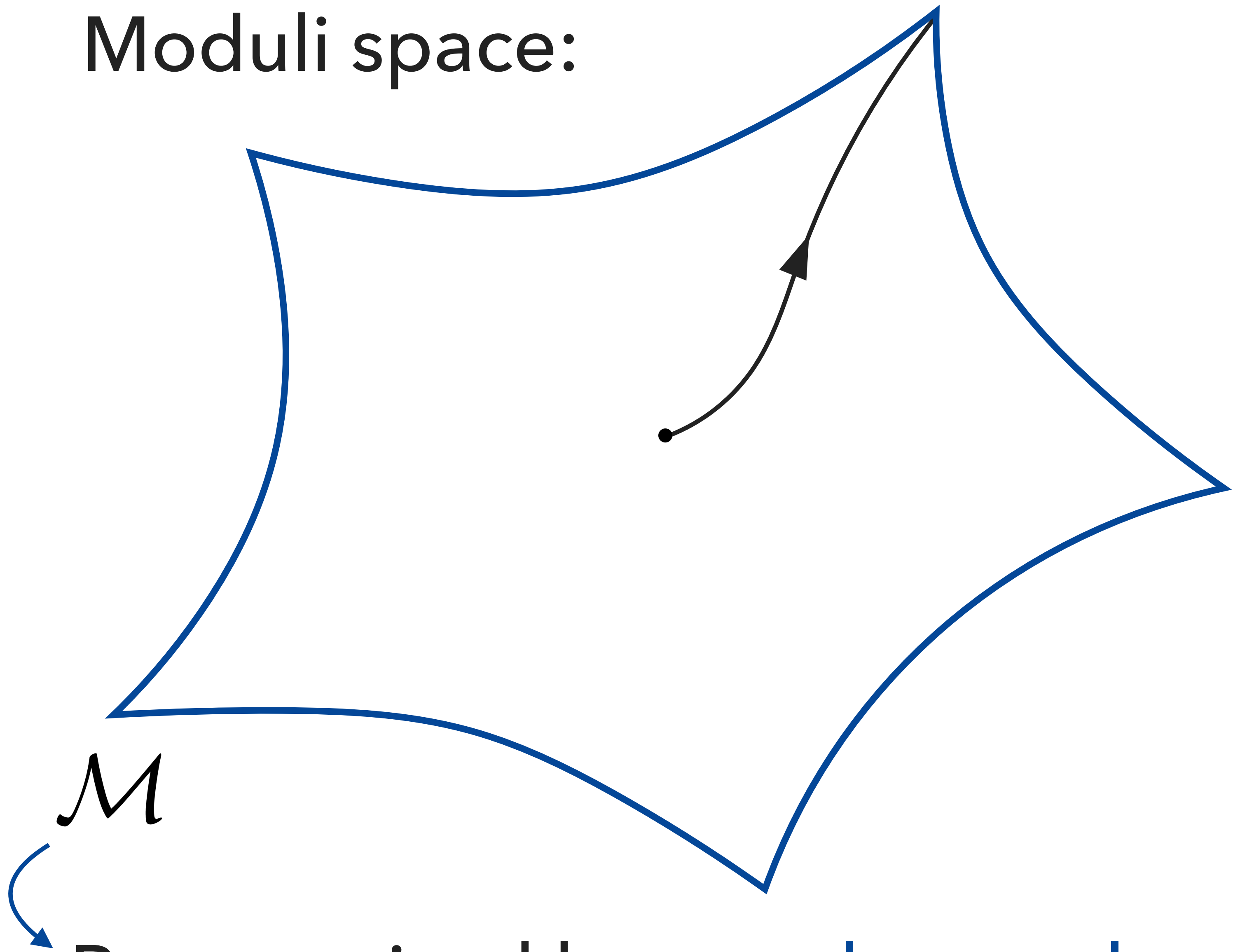

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Lots of top-down evidence!

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  [Grimm, Palti, Valenzuela '18] [Lee, Lerche, Weigand '18-'19]

  + many many more!

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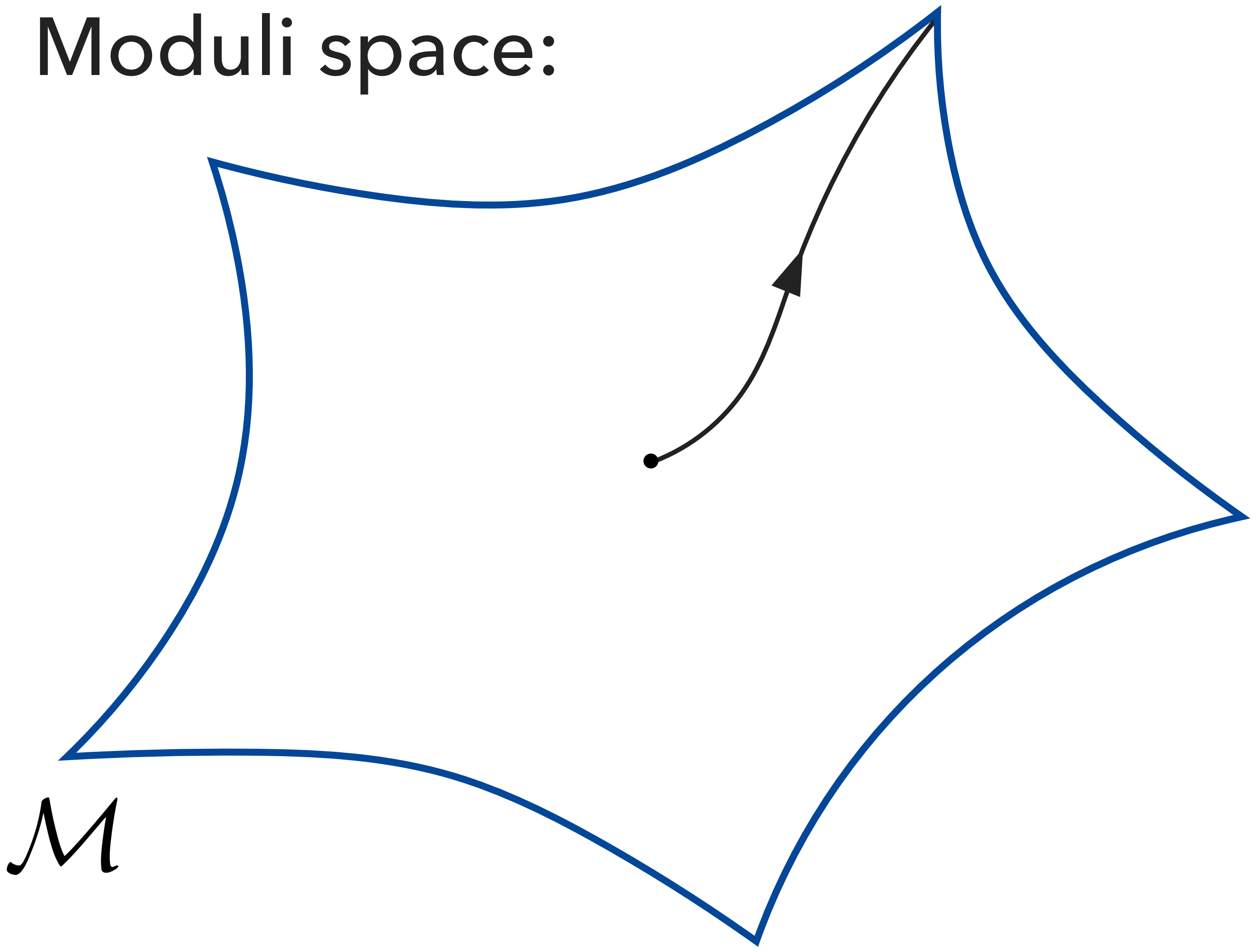

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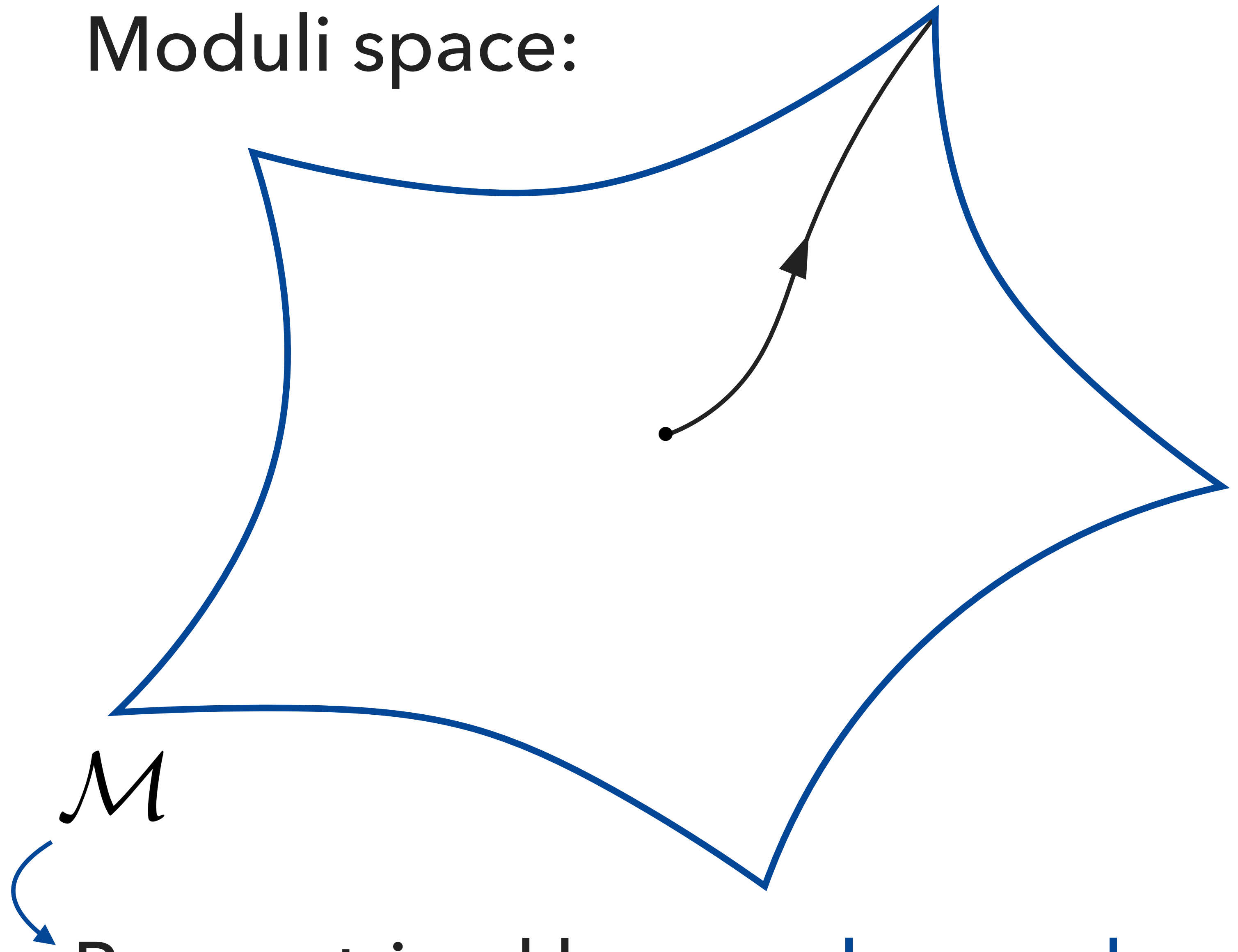

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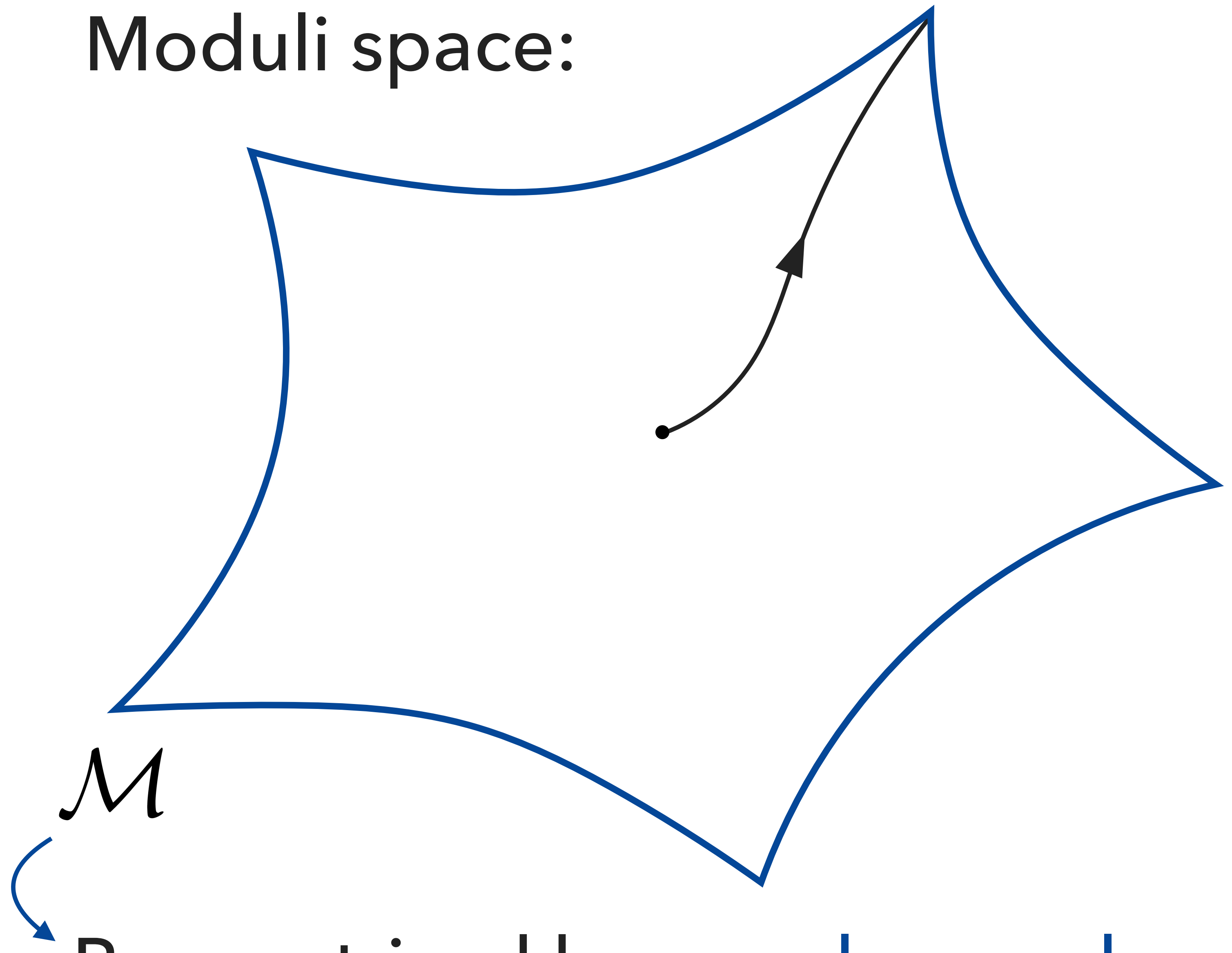

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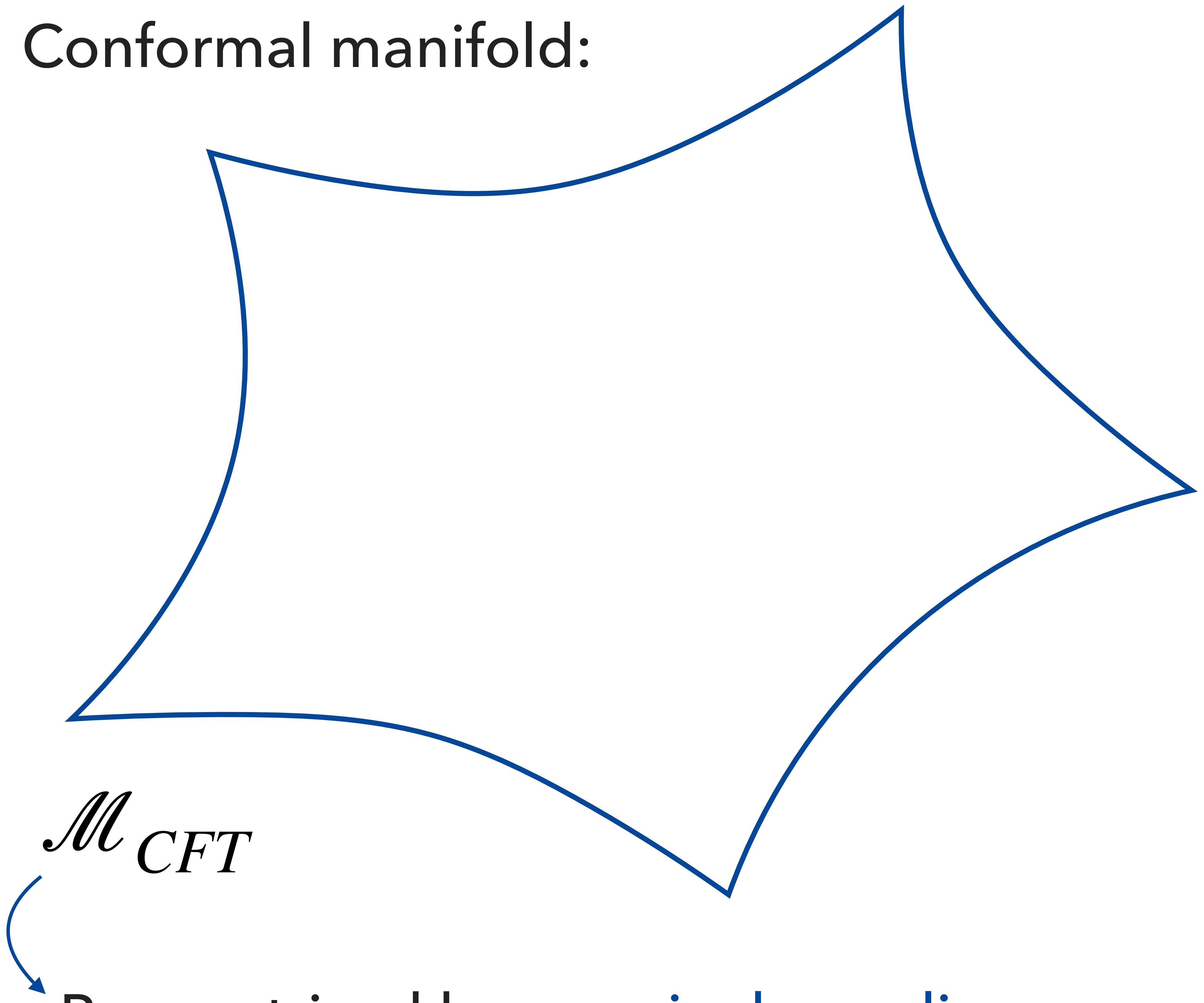

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Local CFT: Posses energy-momentum tensor

→ Dynamical gravity in the bulk!

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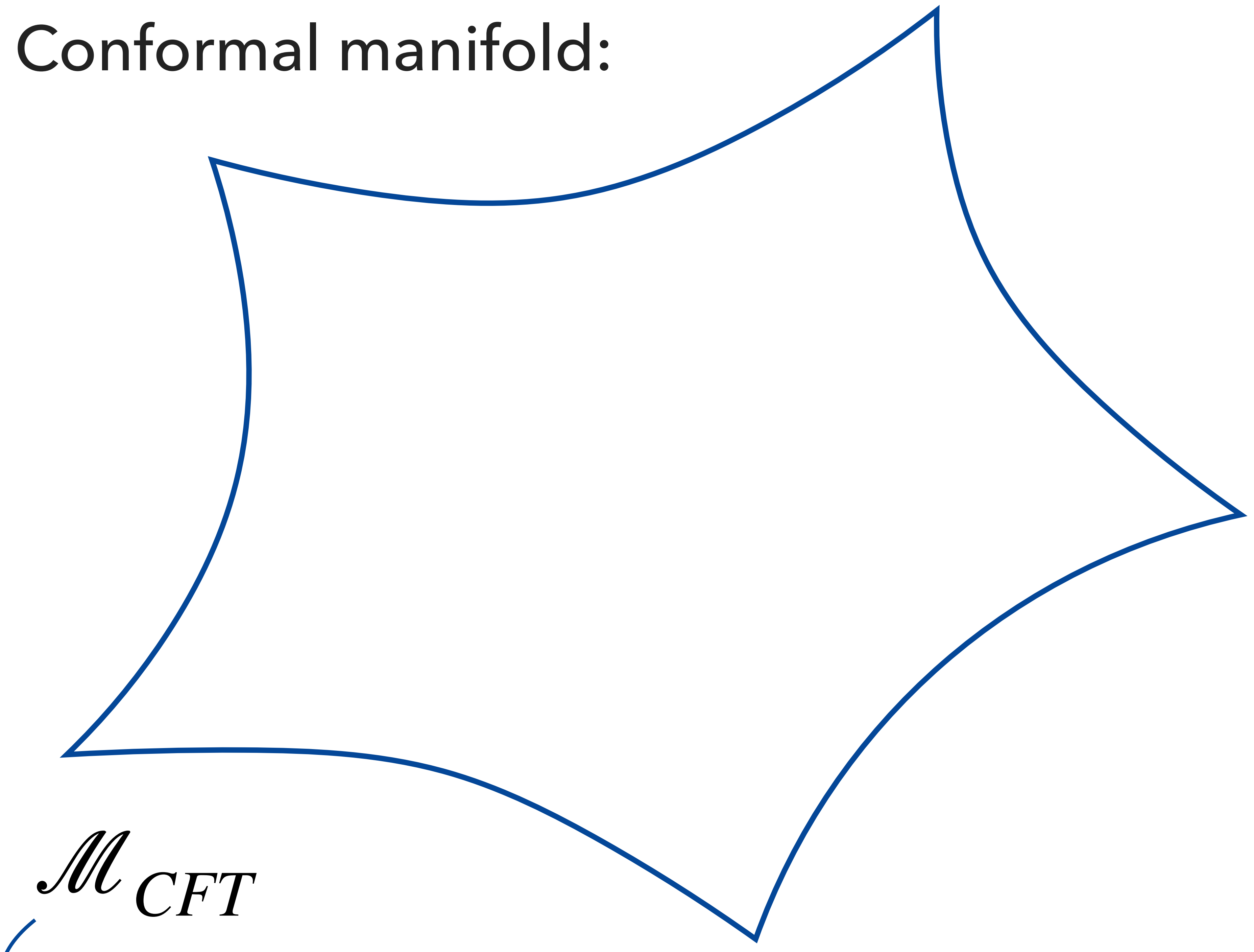

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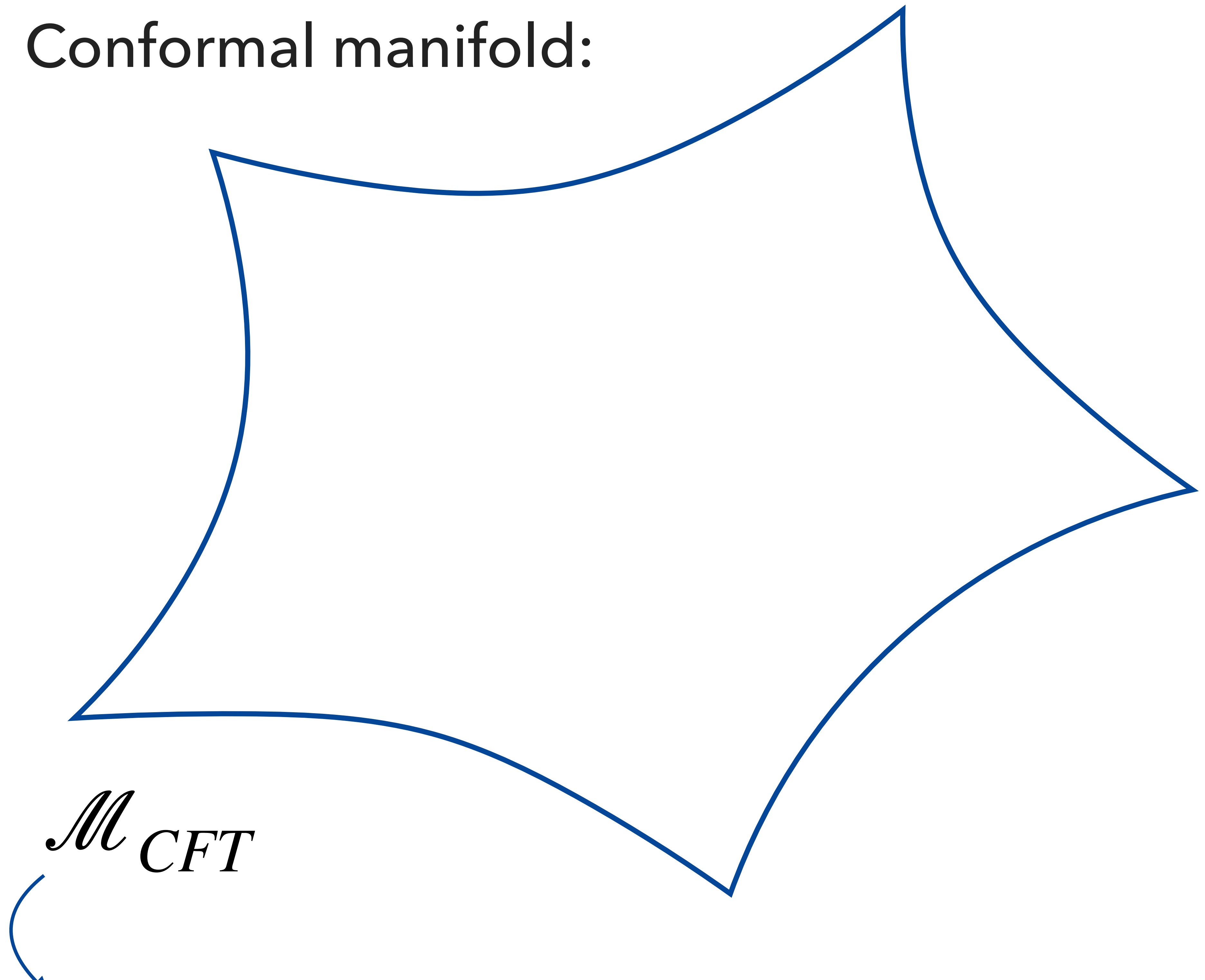

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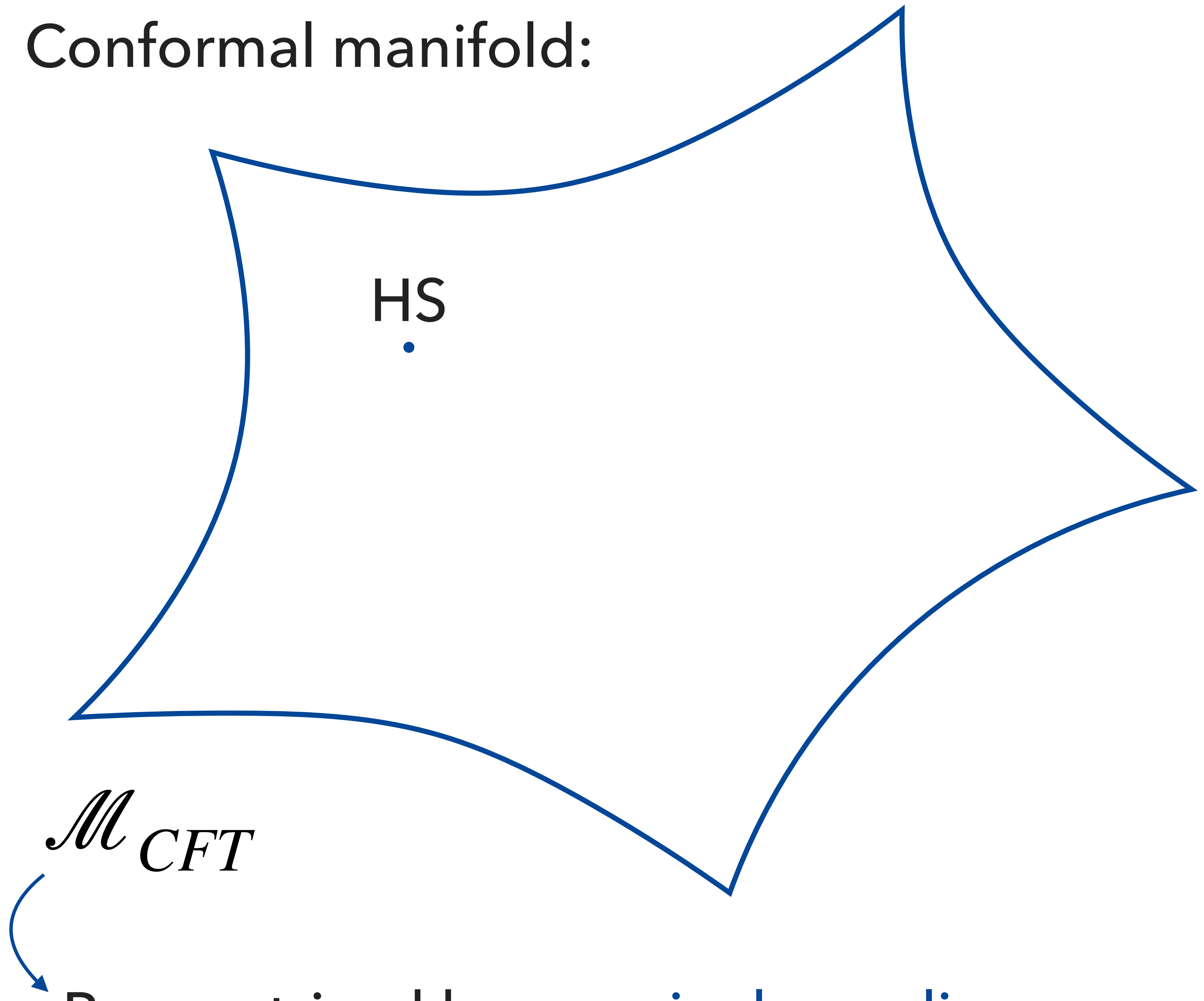

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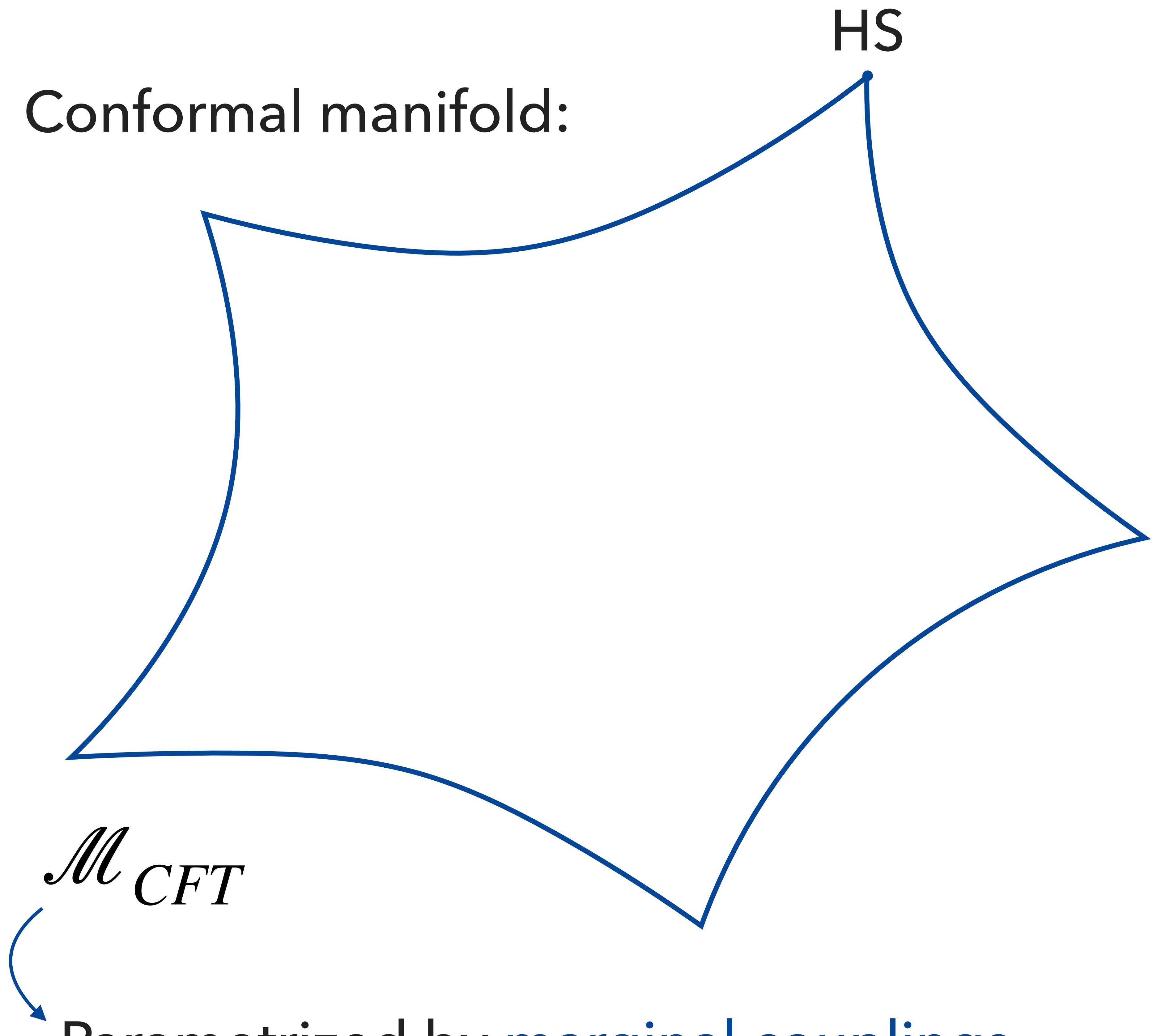

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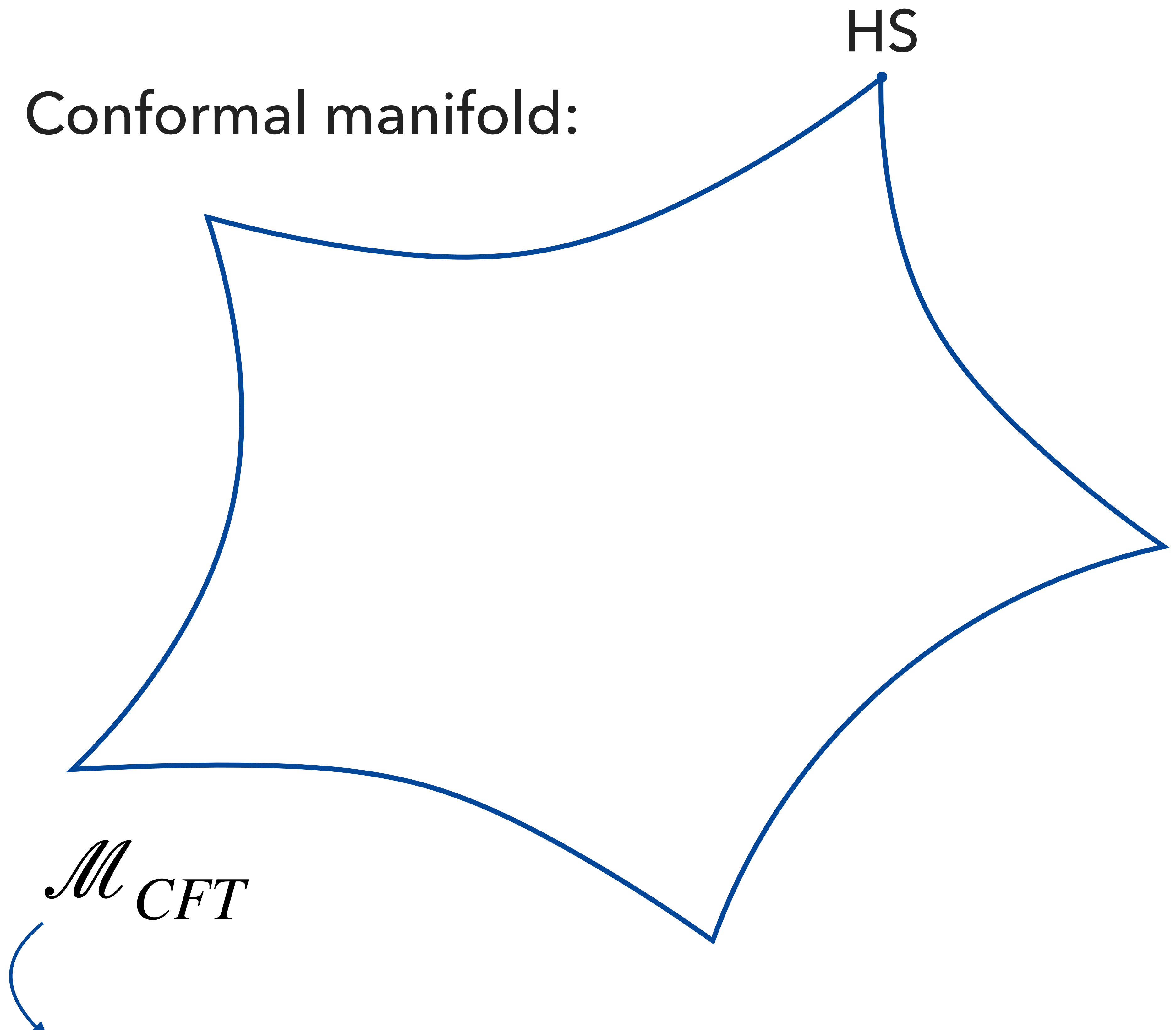

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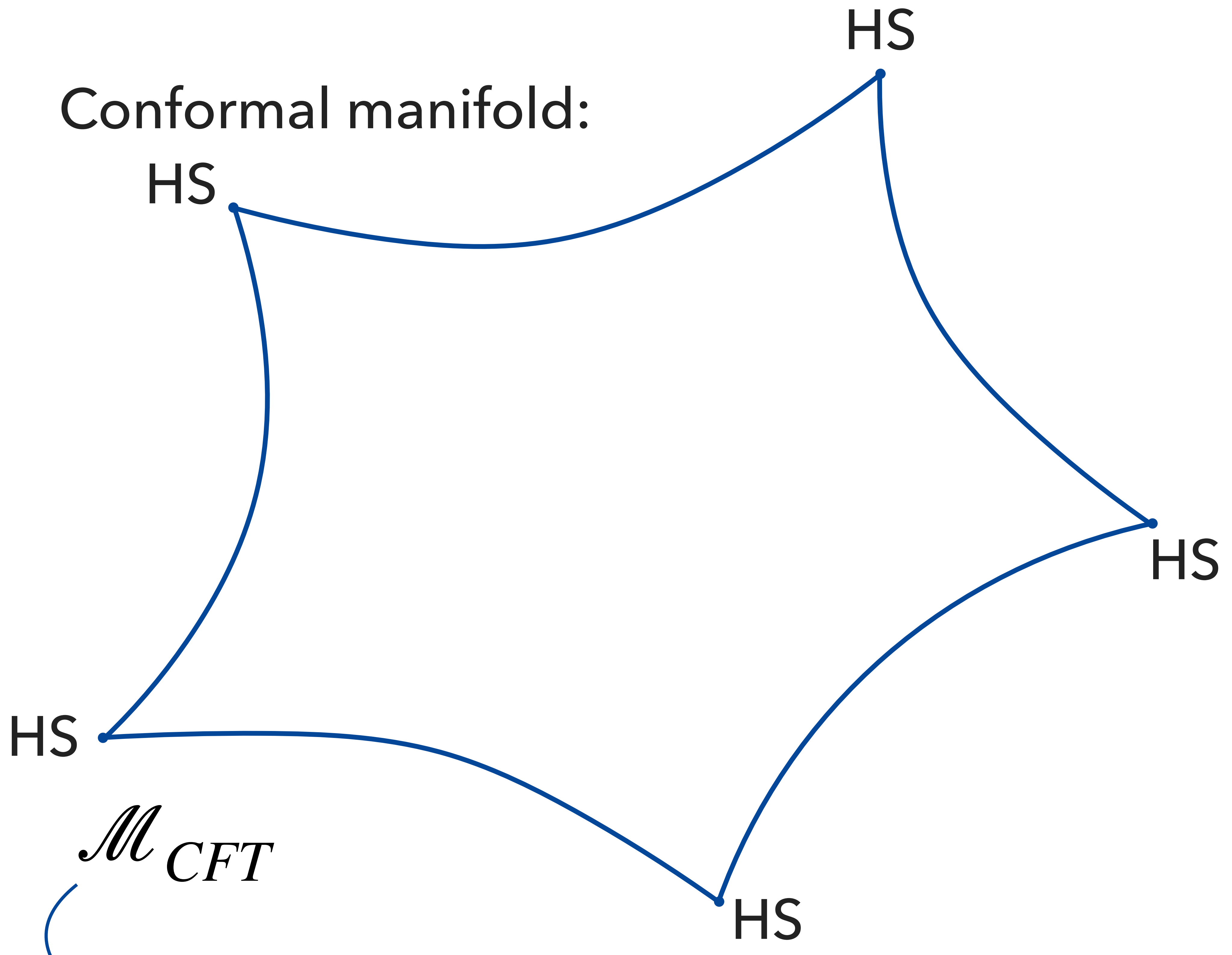

$\mathcal{M}_{CFT}$

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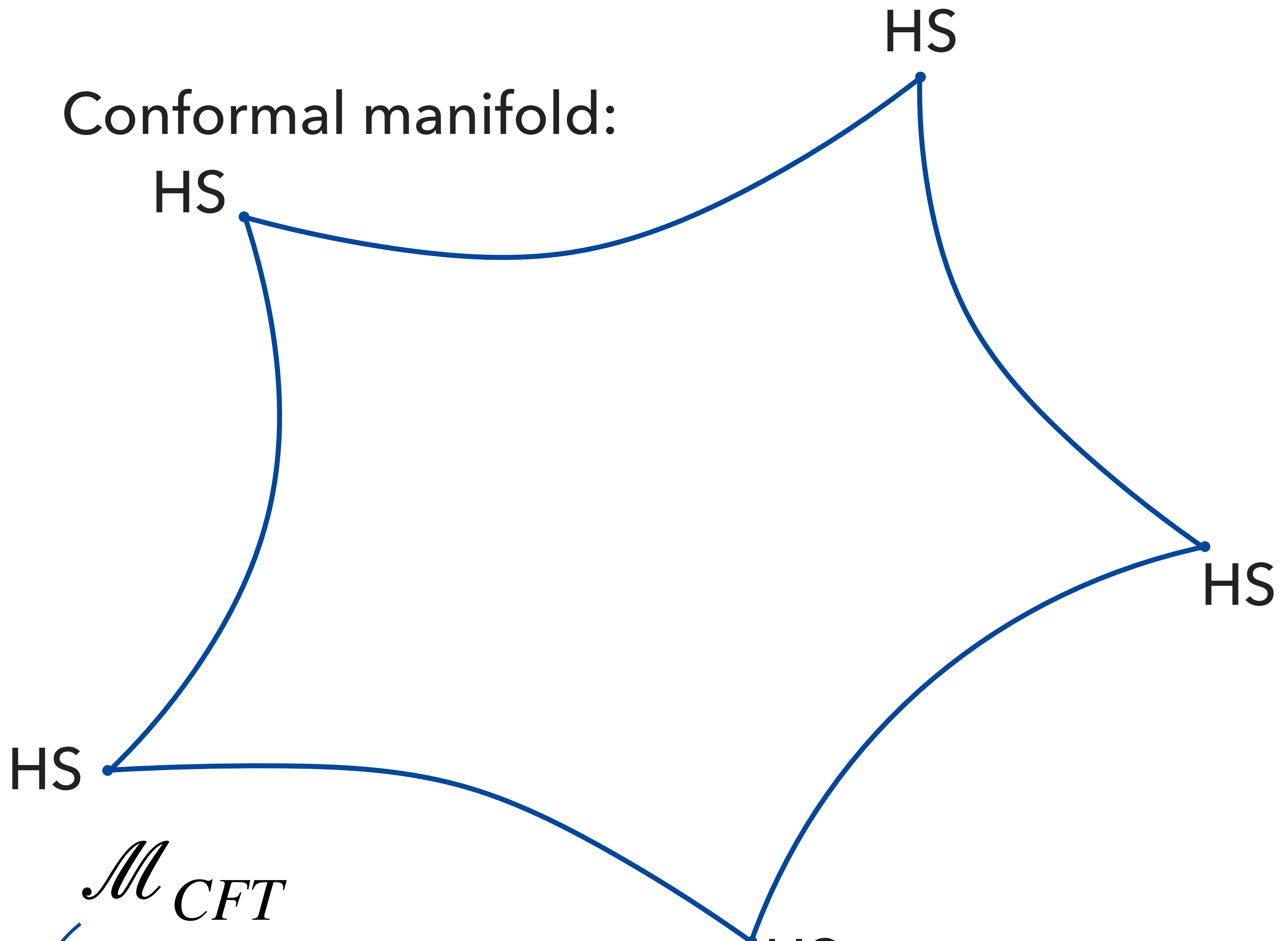

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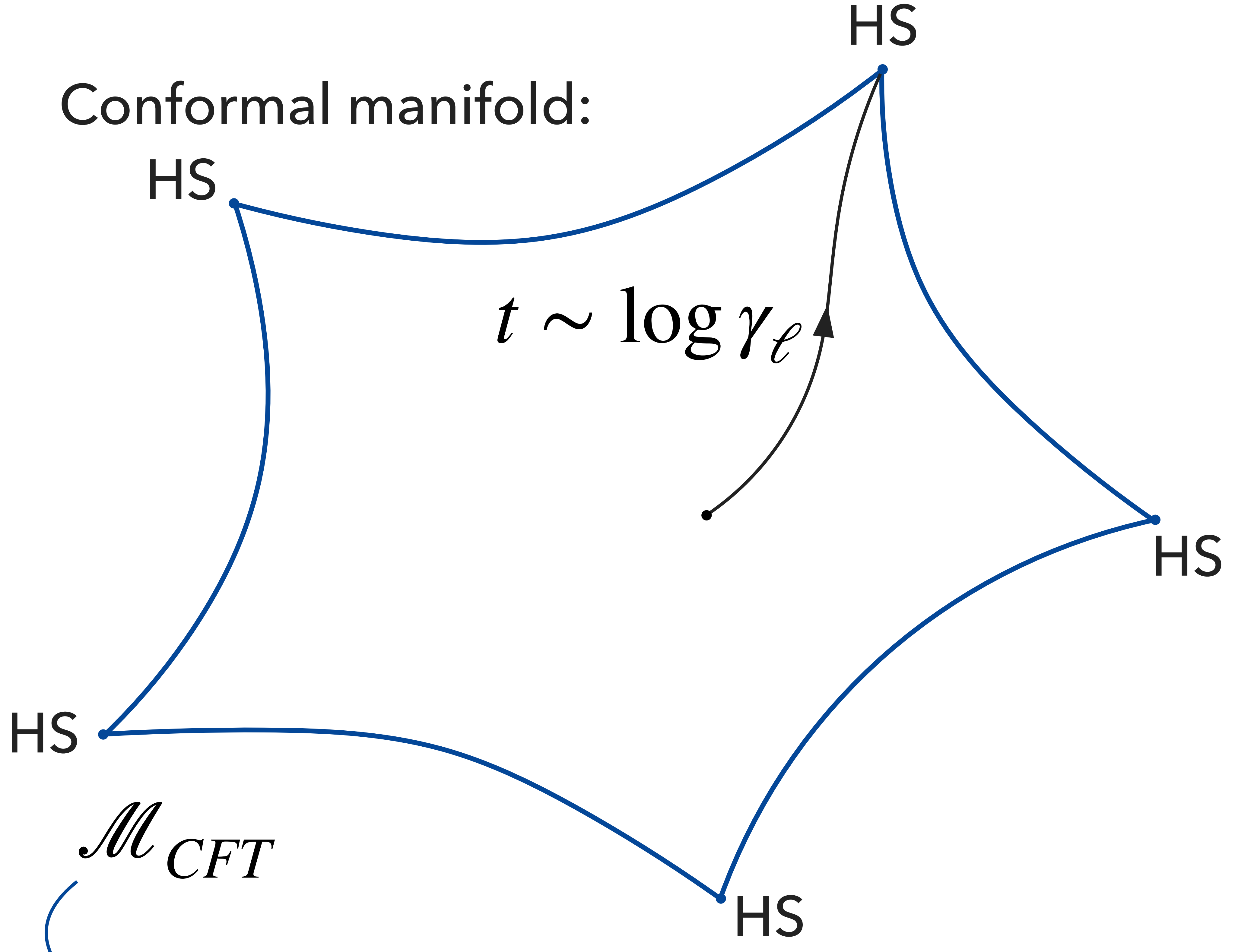

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Towards a CFT Distance Theorem

Can we prove this using CFT techniques?

[Perlmutter, Rastelli, Vafa, Valenzuela '20]

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! No extra assumption, e.g., no supersymmetry

+ existence of energy-momentum is crucial!

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Conformal perturbation theory

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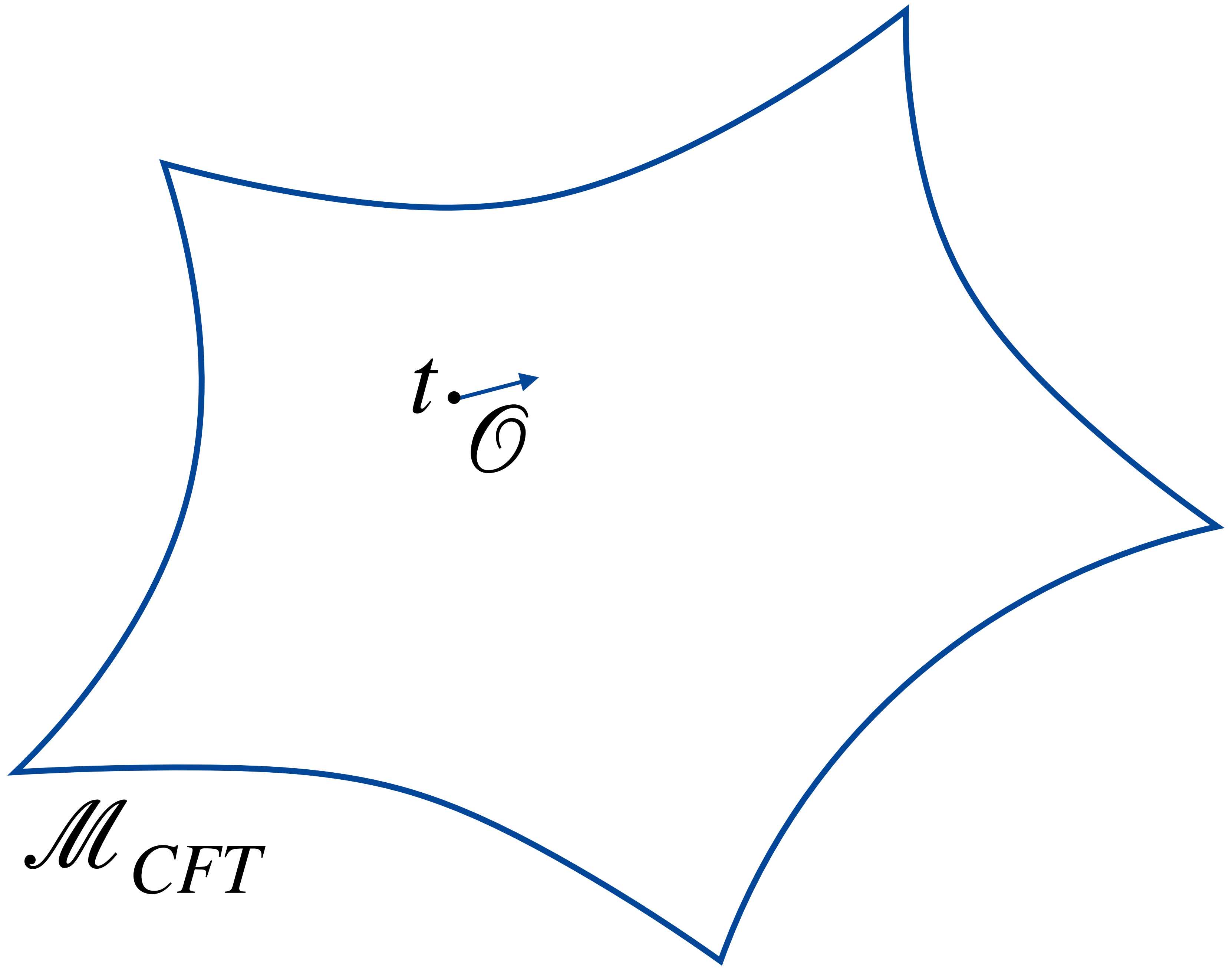

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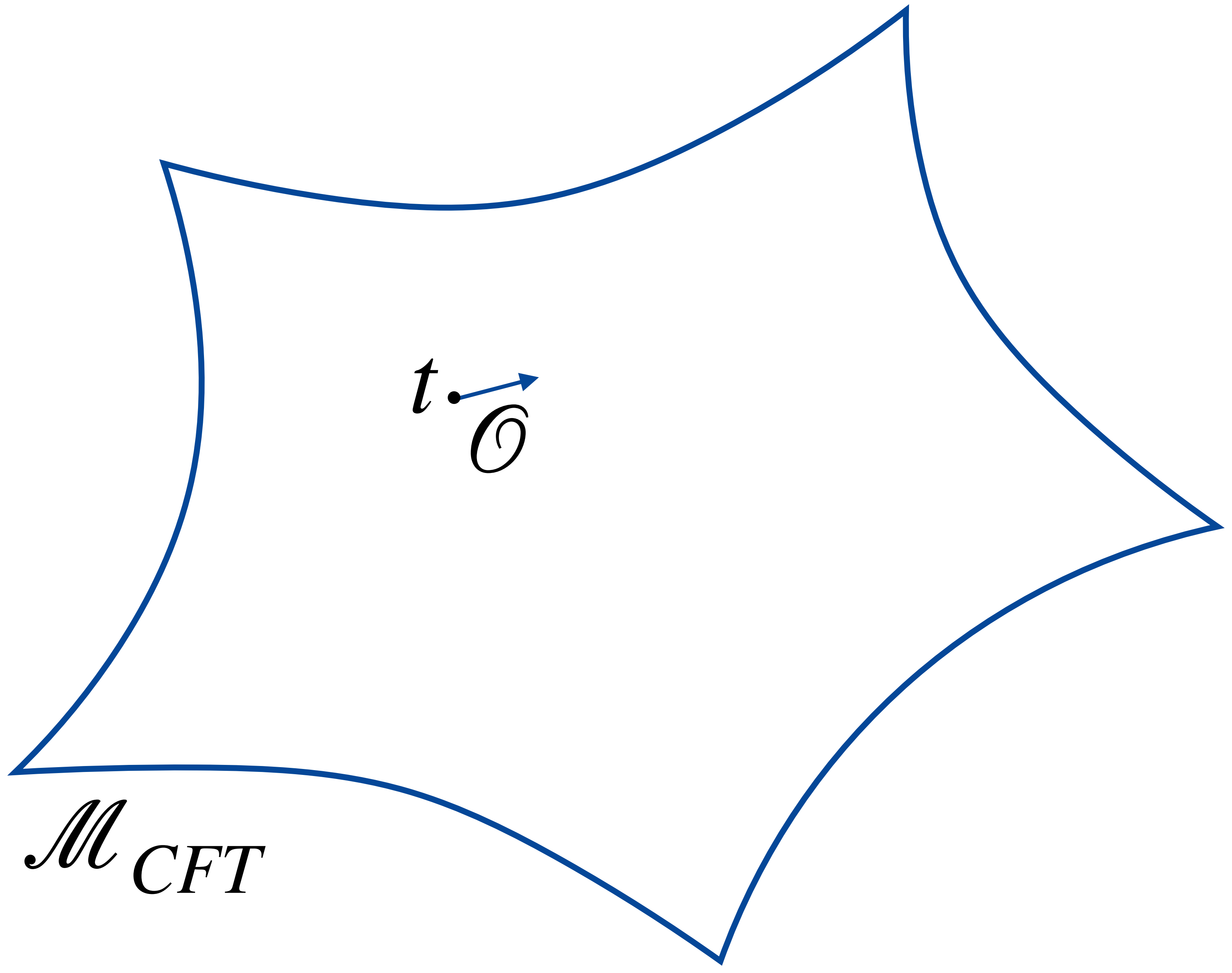

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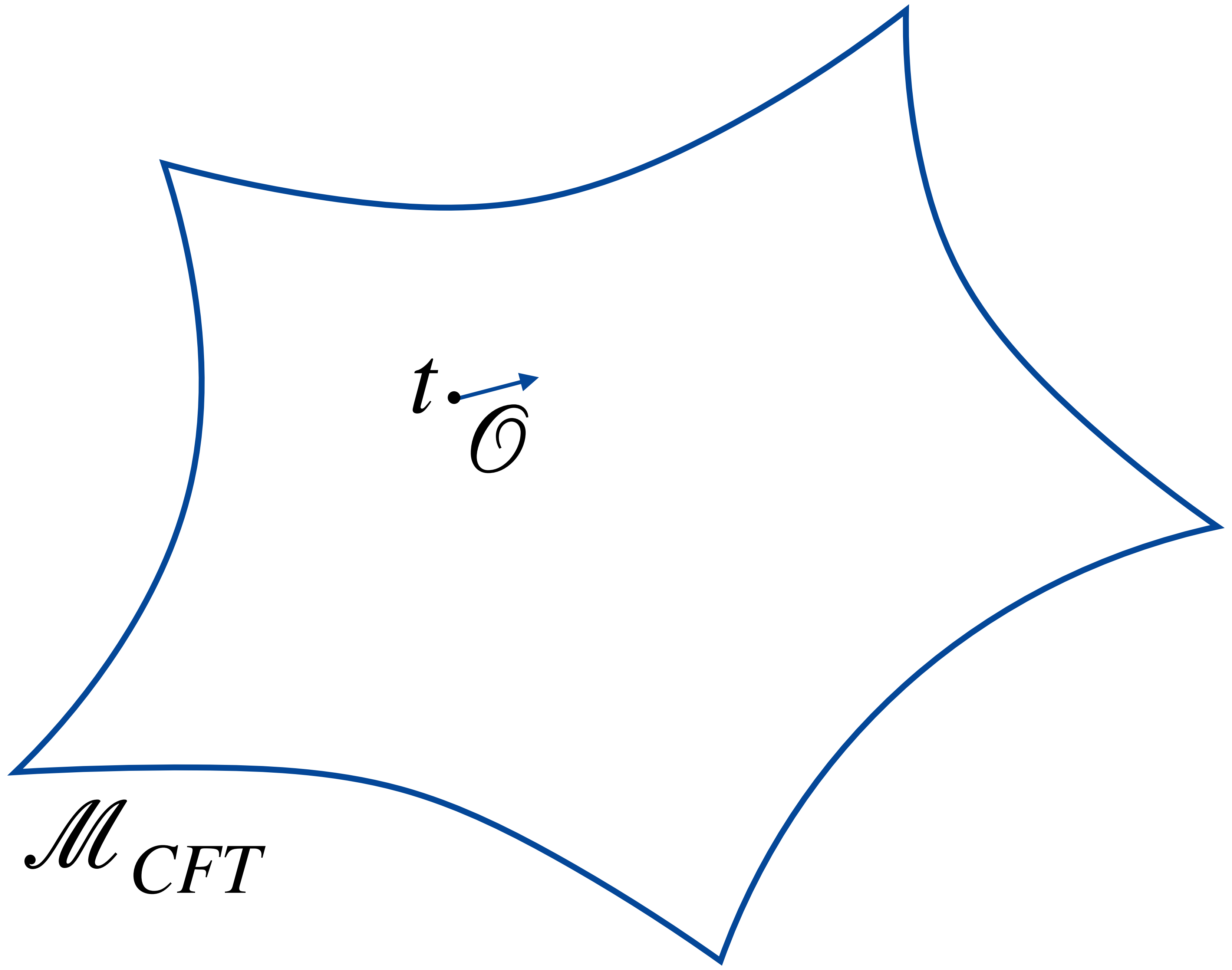

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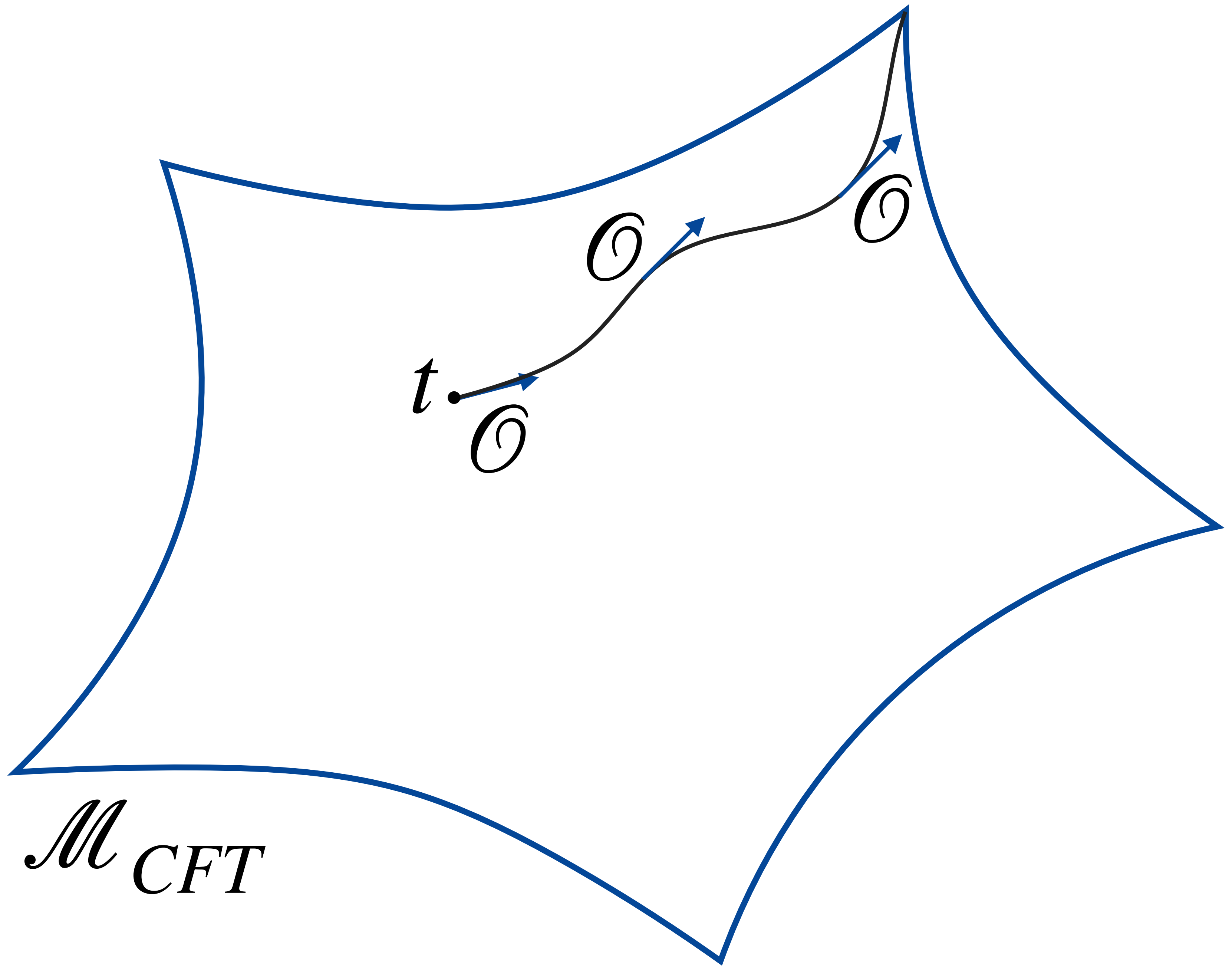

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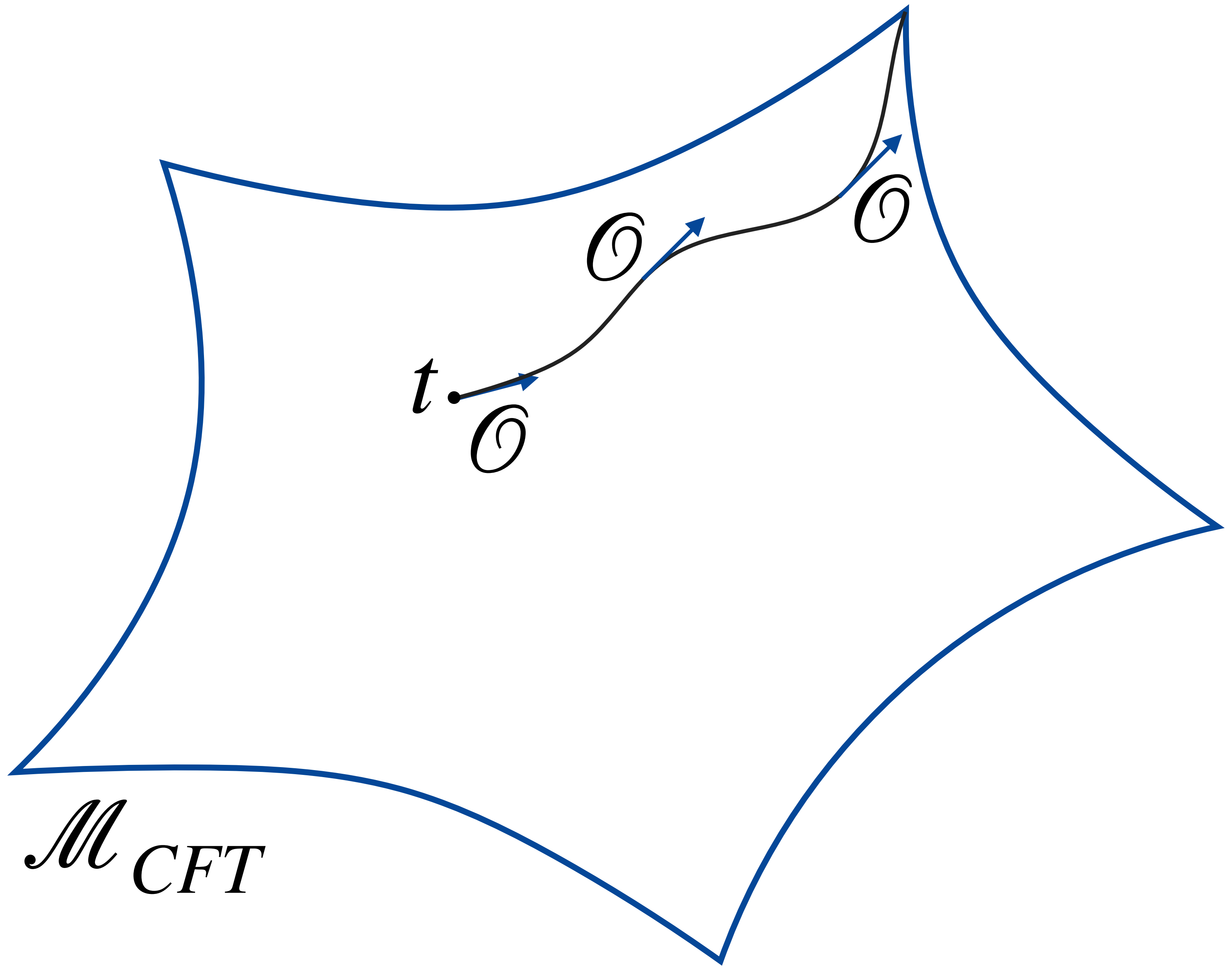
Trajectory!



Sketch of the Proof

Conformal perturbation theory

$$\delta \left\langle J_\ell J_\ell \right\rangle_t = \delta t \int \left\langle J_\ell J_\ell \mathcal{O} \right\rangle_t$$

$$\downarrow$$

$$\delta\gamma_\ell = -\, C_{JJ\mathcal{O}}(t)\, \delta t$$

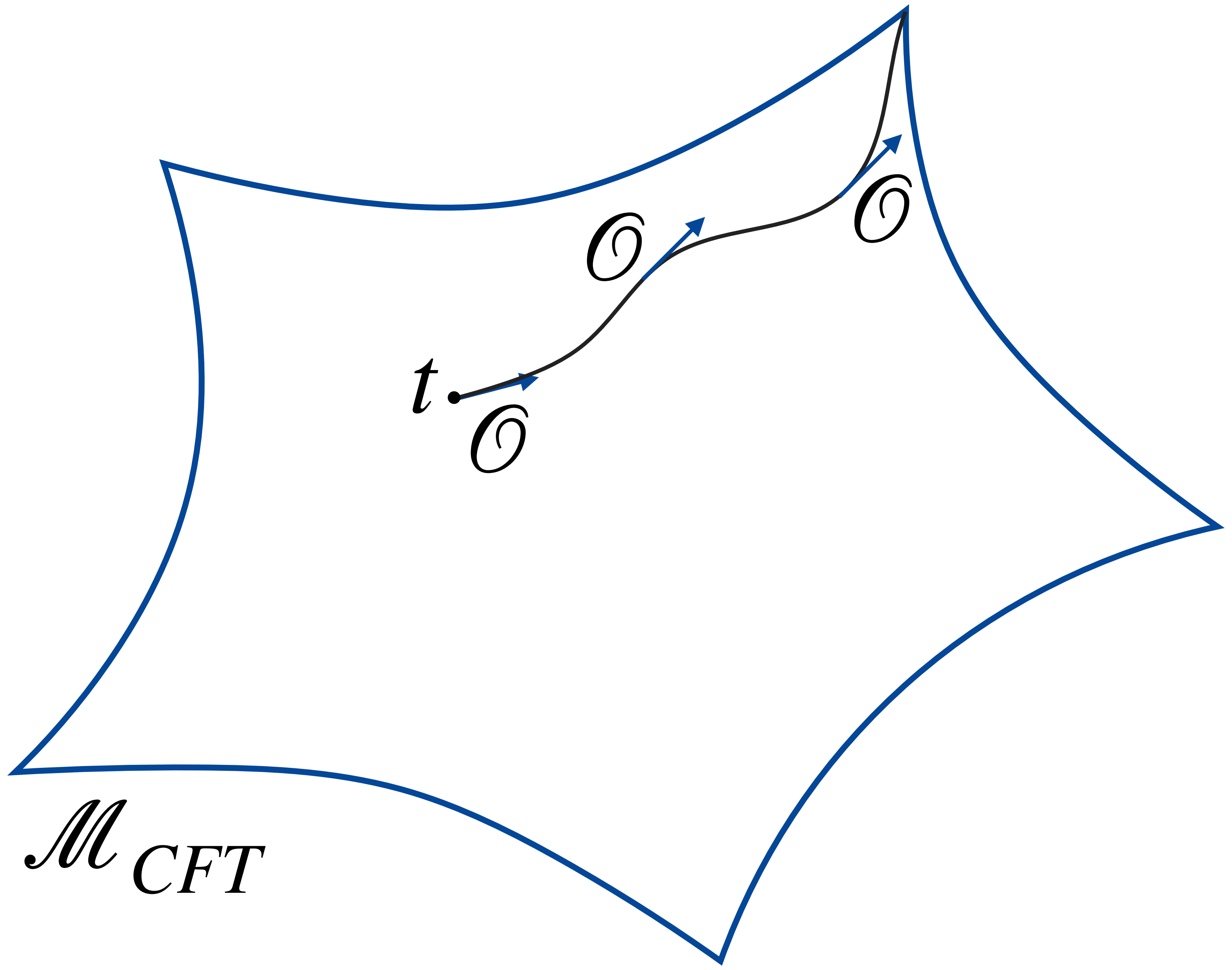
$$\downarrow$$

$$\boxed{\frac{d\gamma_\ell}{dt} = -\, C_{JJ\mathcal{O}}}$$

Zamolodchikov distance!

(given $\langle \mathcal{O}\mathcal{O} \rangle \sim 1$)

Trajectory!



Sketch of the Proof

(Weakly-broken) HS symmetry

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$$\partial \cdot J_\ell = g\, K_{\ell-1}$$

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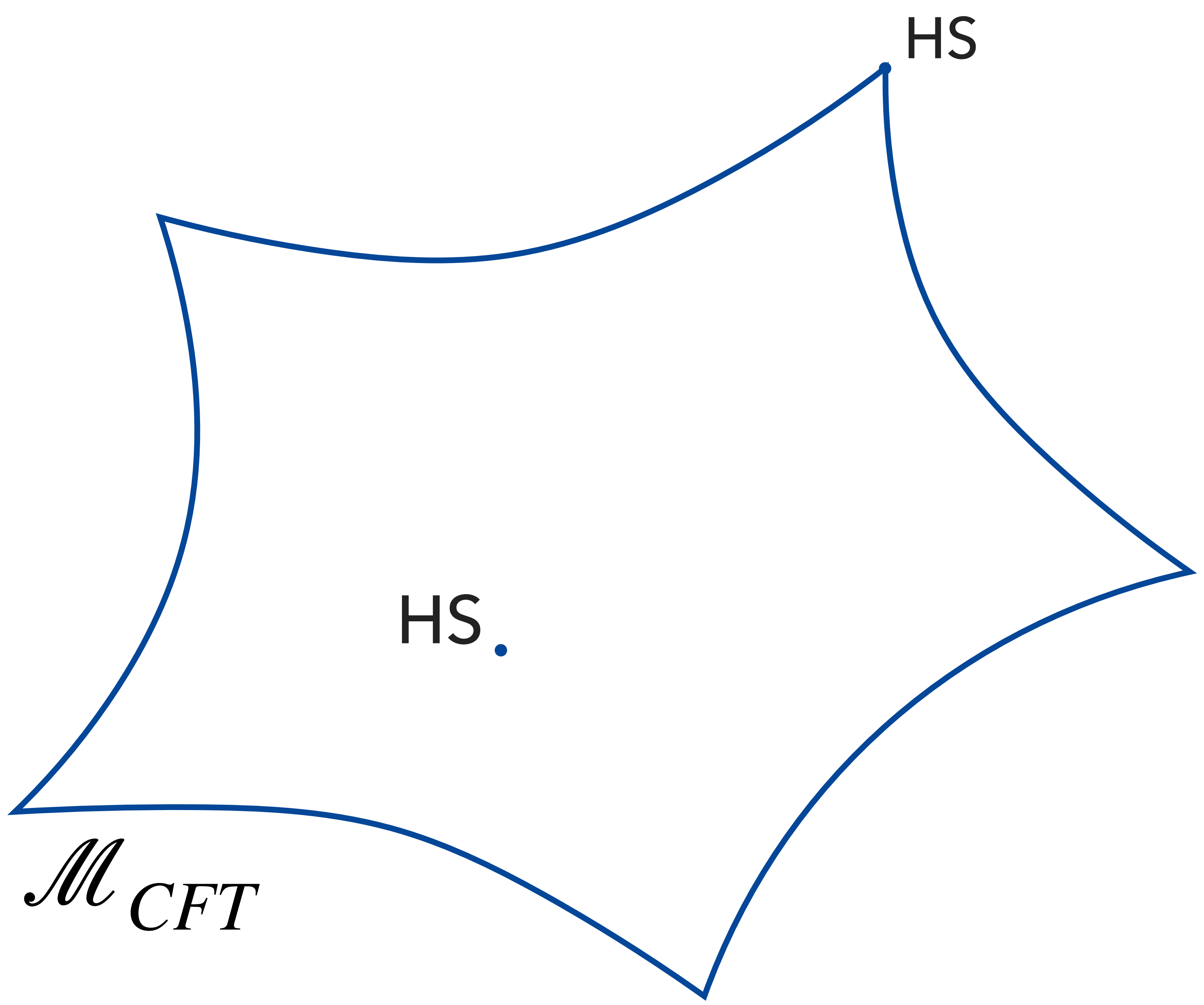

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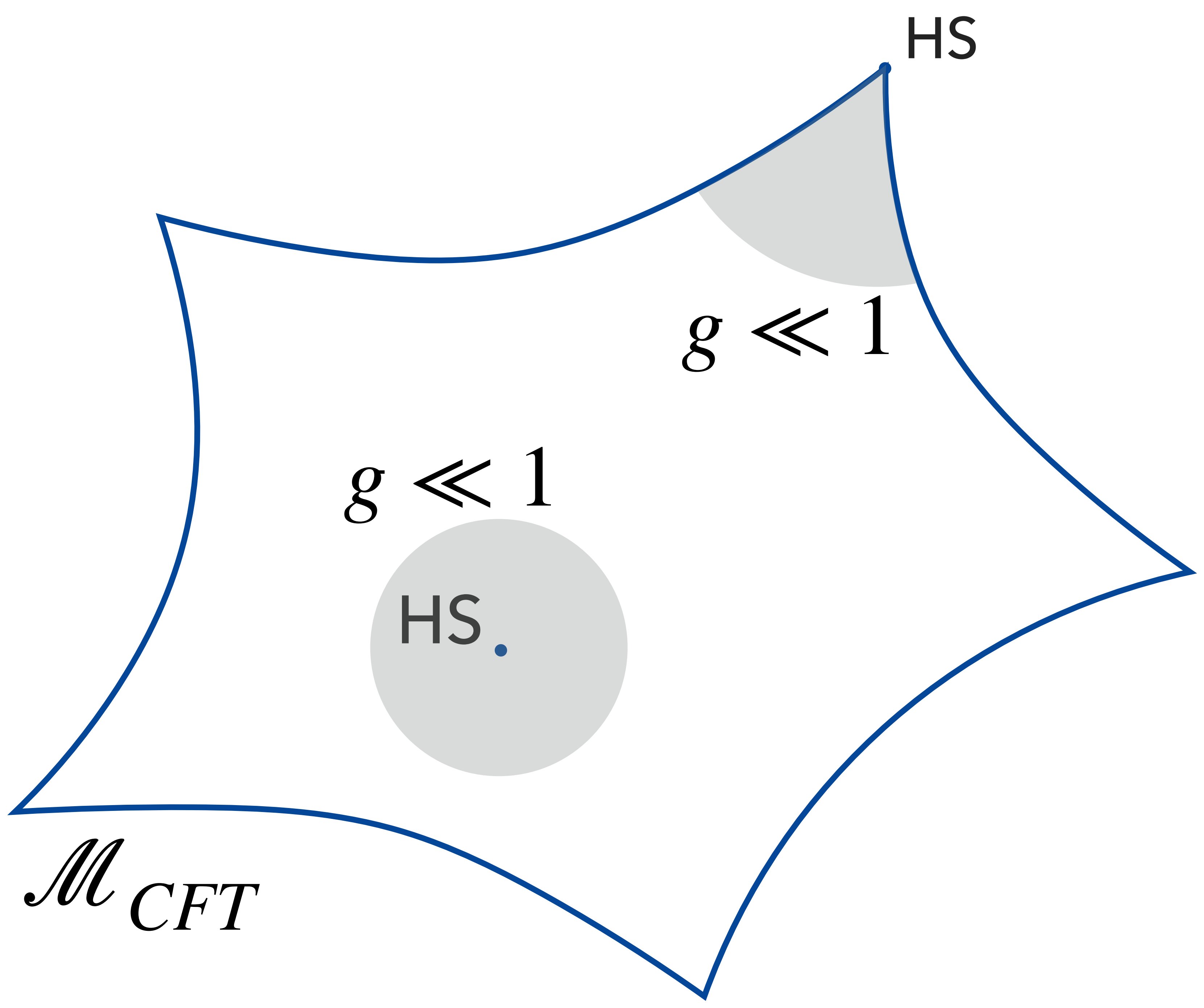

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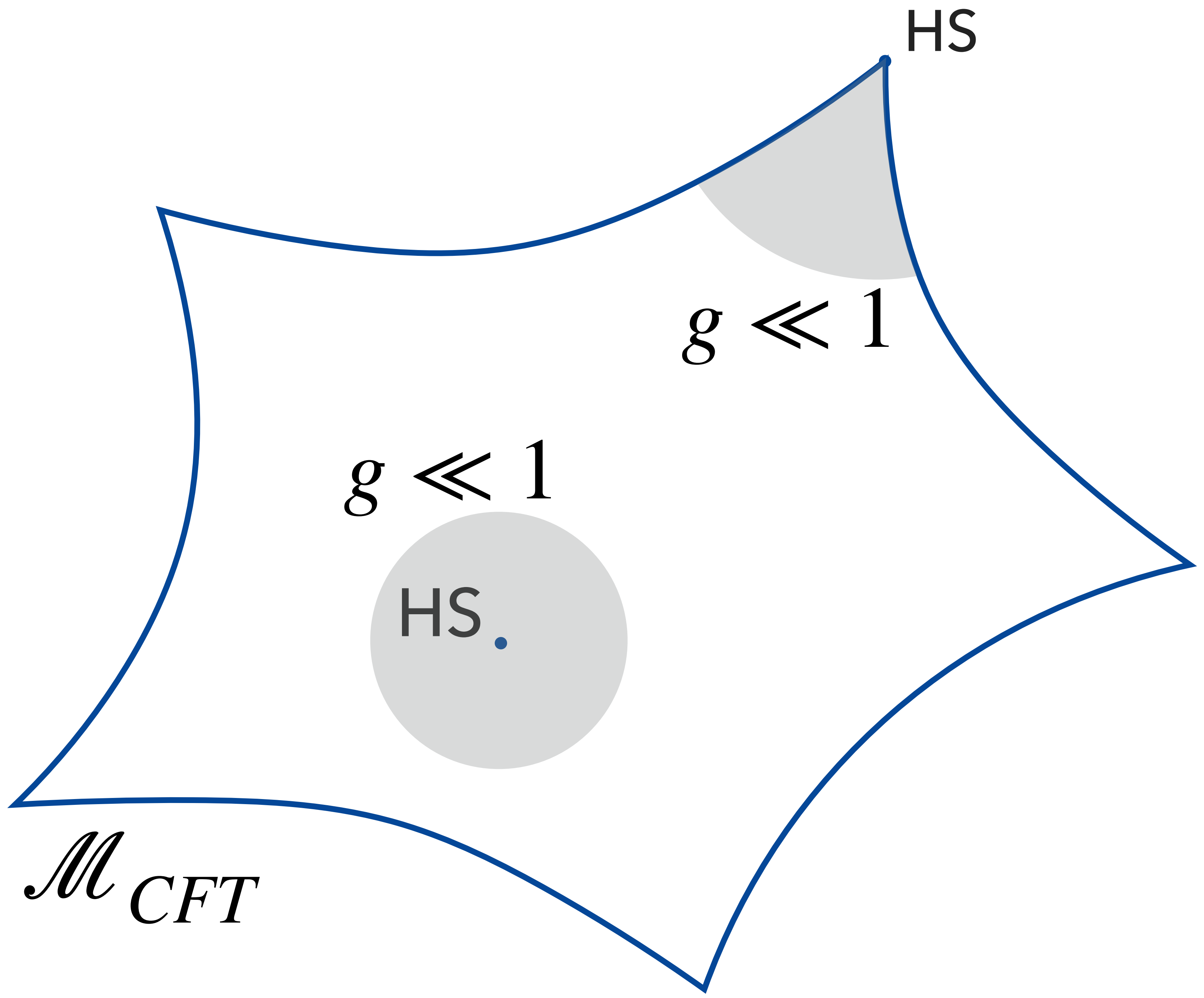


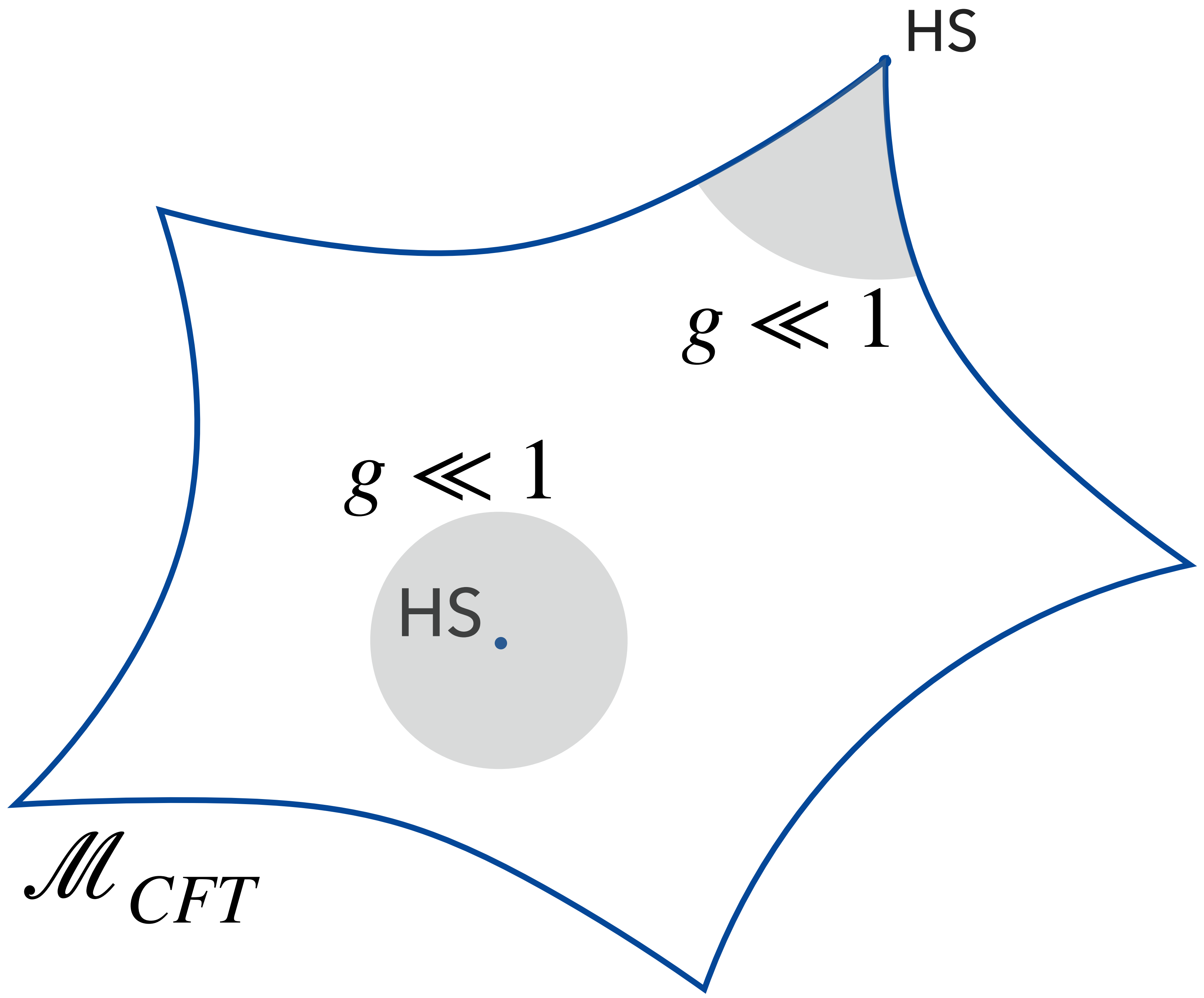
(Weakly-broken) HS symmetry

$$\partial \cdot J_\ell = g\, K_{\ell-1}$$

$$\downarrow$$

$$C_{JJ\mathcal{O}} \simeq C^{HS}_{JJ\mathcal{O}} + C^{HS}_{JK\mathcal{O}}\, g + C^{HS}_{KK\mathcal{O}}\, g^2 + \cdots$$

Sketch of the Proof



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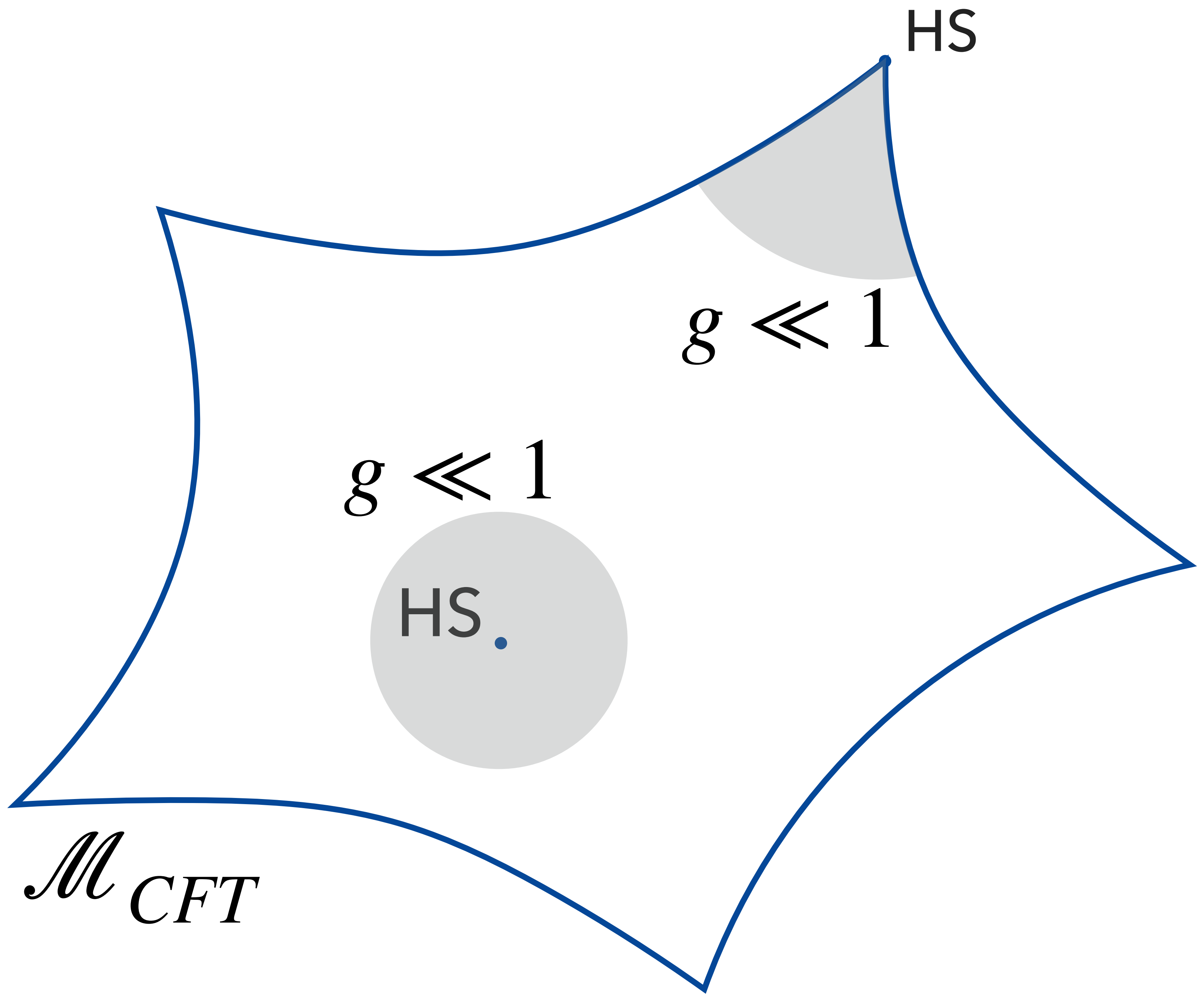
Essentially, as $g \to 0$:

$$\partial \cdot \langle J_\ell J_\ell \mathcal{O} \rangle = g\, \langle J_\ell K_{\ell-1} \mathcal{O} \rangle$$

Sketch of the Proof



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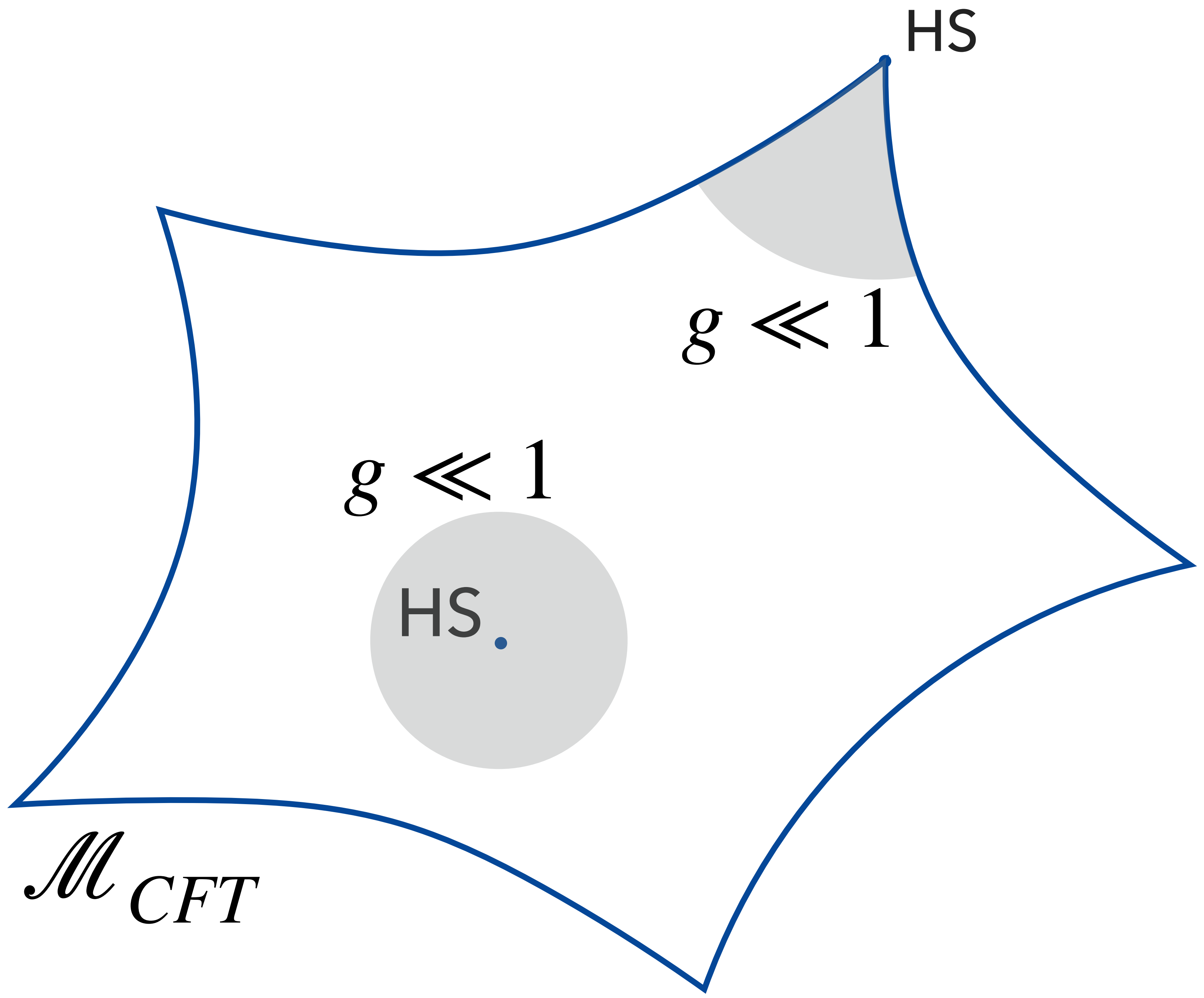

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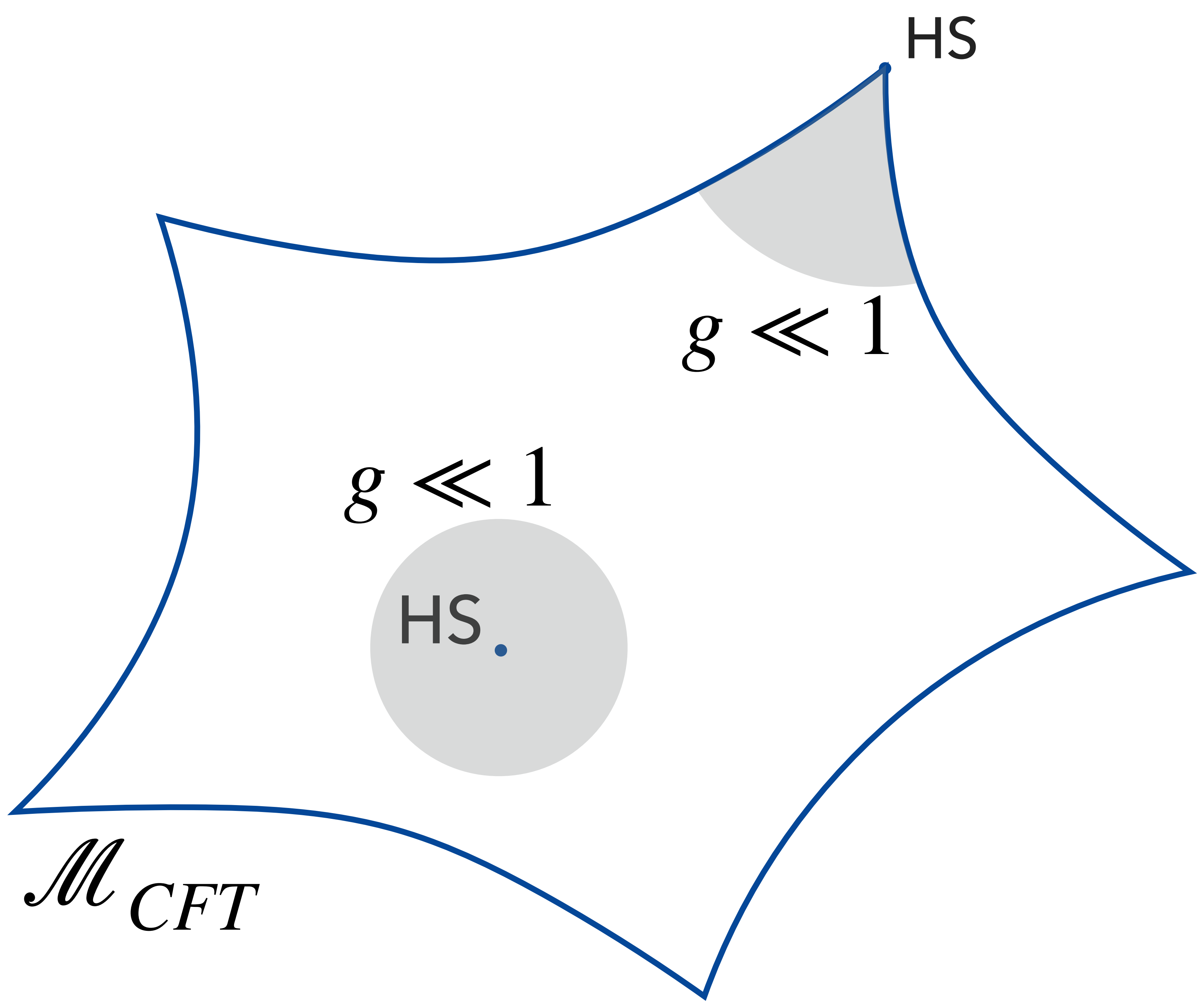
A bit more complicated than this...

→ More details in [Baume, JC '23]

Sketch of the Proof



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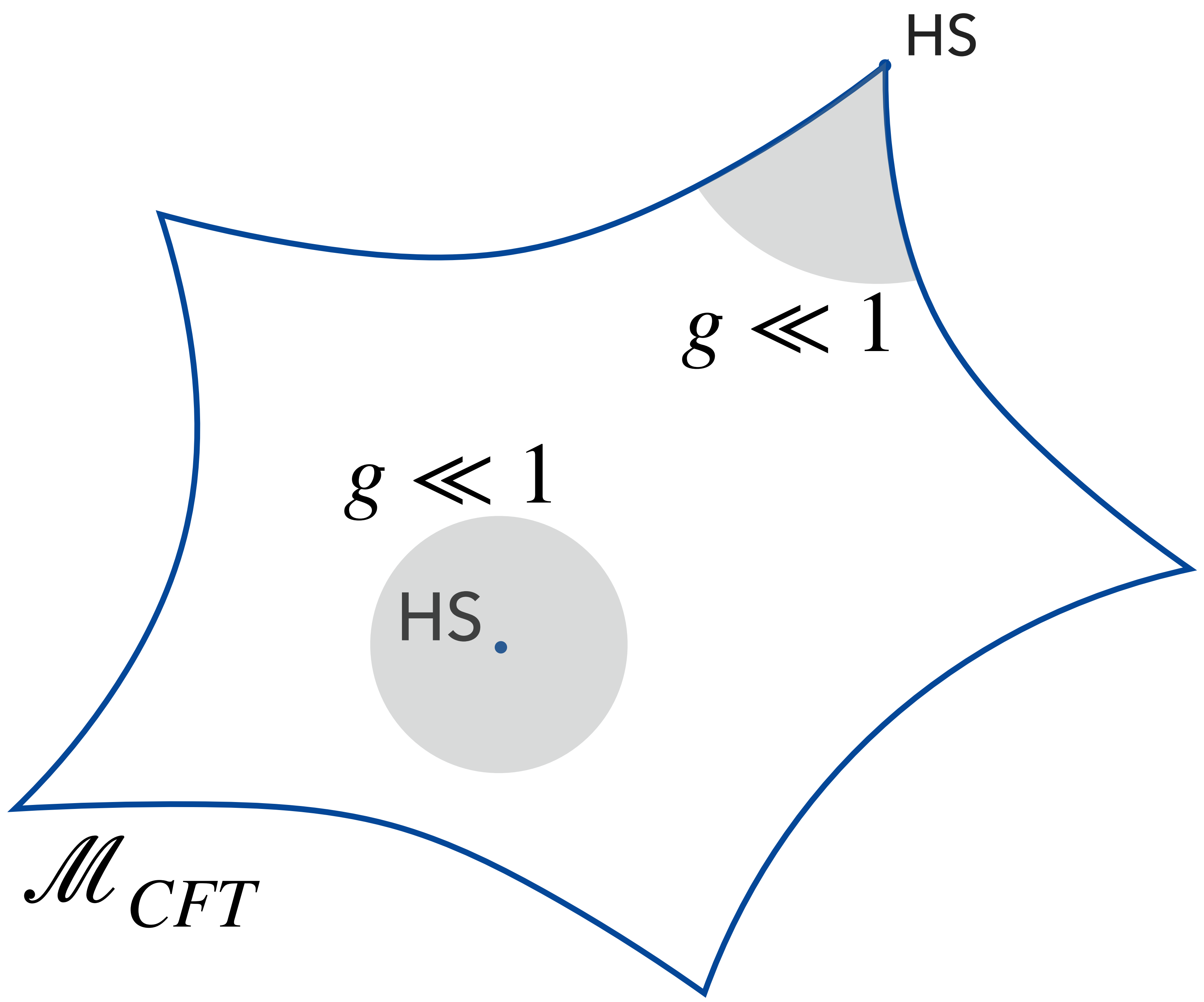

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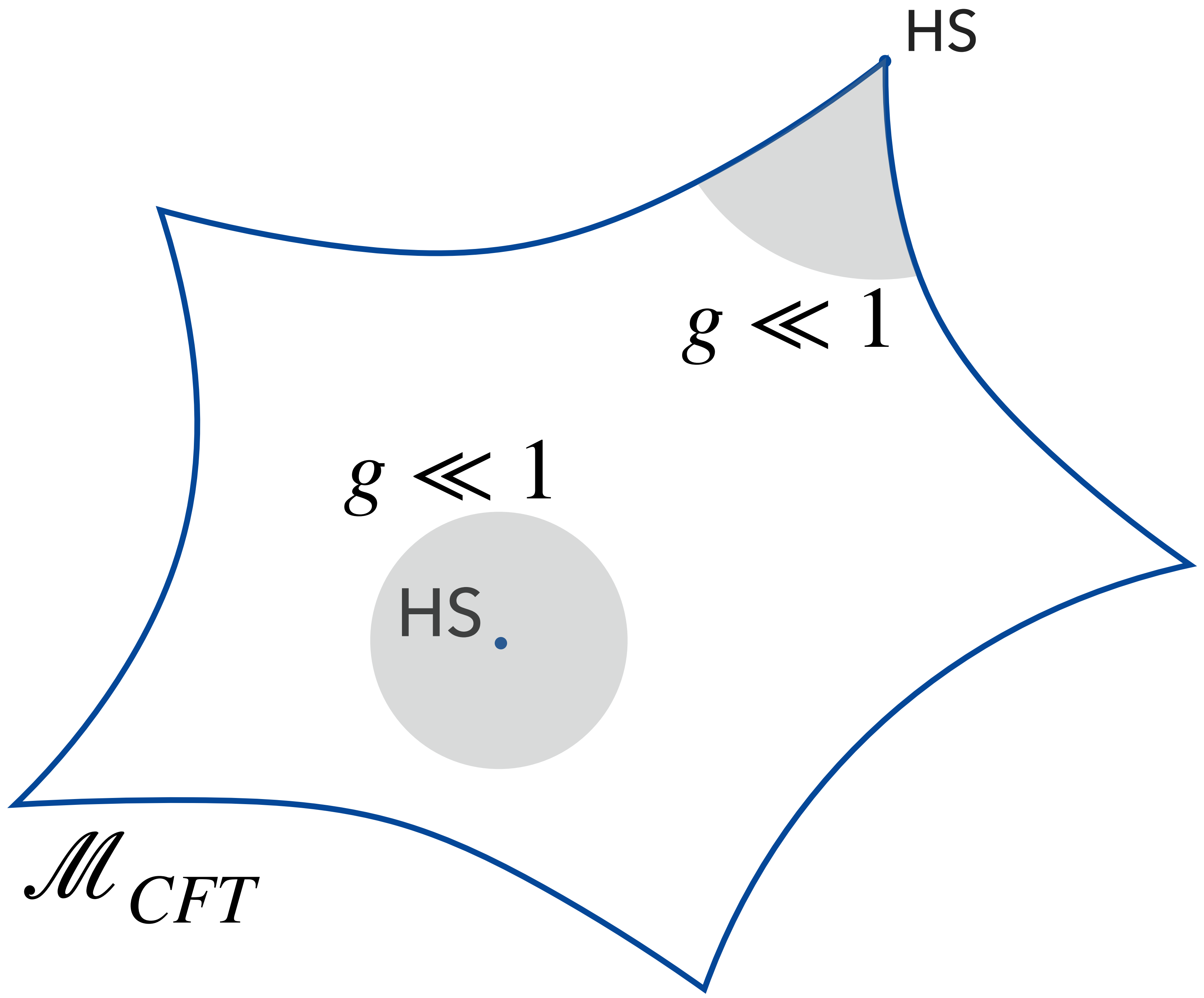
HS symmetry constraints

Sketch of the Proof



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HS symmetry constraints

$$\Big\downarrow\ \gamma_\ell \sim g^2$$

$$\boxed{C_{JJ\mathcal{O}} \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0}$$

Sketch of the Proof

Conformal perturbation theory

$$\delta \left\langle J_\ell J_\ell \right\rangle_t = \delta t \int \left\langle J_\ell J_\ell \mathcal{O} \right\rangle_t$$

$$\downarrow$$

$$\delta\gamma_\ell = -\, C_{JJ\mathcal{O}}(t)\, \delta t$$

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$$\boxed{\frac{d\gamma_\ell}{dt} = -\, C_{JJ\mathcal{O}}}$$

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➡ $\boxed{\gamma_\ell \to 0 \text{ as } t \to \infty \quad \forall\, J_\ell, \mathcal{O} : \text{All HS points are at infinite distance}}$ ✓

Towards a CFT Distance Theorem

CFT Distance Conjecture

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point ⟶ Infinite distance

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point ⟶ Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\, C_{JJ\mathcal{O}}$$

⊕

$$C_{JJ\mathcal{O}} \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0$$

(Weakly-broken) HS symmetry

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point ⟶ Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\,C_{JJ\mathcal{O}}$$

$$\oplus$$

$C_{JJ\mathcal{O}} \lesssim \gamma_\ell$ as $\gamma_\ell \to 0$

(Weakly-broken) HS symmetry

II. Infinite distance ⟶ HS point

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point → Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\, C_{JJ\mathcal{O}}$$

⊕

$$C_{JJ\mathcal{O}} \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0$$

(Weakly-broken) HS symmetry

II. Infinite distance → HS point

No HS symmetry

↓ ?

$$\exists \mathcal{O} : \; C_{JJ\mathcal{O}} \neq 0$$

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point ⟶ Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = - C_{JJ\mathcal{O}}$$

⊕

$$C_{JJ\mathcal{O}} \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0$$

(Weakly-broken) HS symmetry

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CFT Distance Criterion ↓ Sufficient but not necessary

Finite distance

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point ⟶ Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\, C_{JJ\mathcal{O}}$$

⊕

$C_{JJ\mathcal{O}} \lesssim \gamma_\ell$ as $\gamma_\ell \to 0$

(Weakly-broken) HS symmetry

II. Infinite distance ⟶ HS point

No HS symmetry

↓ ?

$$\exists \mathcal{O} :\ C_{JJ\mathcal{O}} \neq 0$$

CFT Distance Criterion ↓ Sufficient but not necessary

Finite distance

III. $\gamma_\ell \sim e^{-\alpha_\ell\, t}$

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point → Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\,C_{JJ\mathcal{O}}$$

⊕

$C_{JJ\mathcal{O}} \lesssim \gamma_\ell$ as $\gamma_\ell \to 0$

(Weakly-broken) HS symmetry

II. Infinite distance → HS point

No HS symmetry

↓ ?

$$\exists \mathcal{O} :\ C_{JJ\mathcal{O}} \neq 0$$

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Finite distance

III. $\gamma_\ell \sim e^{-\alpha_\ell\, t}$

$\exists \mathcal{O} :\ C^{HS}_{KK\mathcal{O}} \neq 0$?

Towards a CFT Distance Theorem

CFT Distance Conjecture

I. HS point ⟶ Infinite distance ✓

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\,C_{JJ\mathcal{O}}$$

⊕

$C_{JJ\mathcal{O}} \lesssim \gamma_\ell$ as $\gamma_\ell \to 0$

(Weakly-broken) HS symmetry

II. Infinite distance ⟶ HS point

No HS symmetry

↓ ?

$\exists \mathcal{O} : C_{JJ\mathcal{O}} \neq 0$

CFT Distance Criterion ↓ Sufficient but not necessary

Finite distance

III. $\gamma_\ell \sim e^{-\alpha_\ell t}$

$\exists \mathcal{O} : C^{HS}_{KK\mathcal{O}} \neq 0$?

↓

$\frac{d\gamma_\ell}{dt} \simeq -\,C^{HS}_{KK\mathcal{O}}\gamma_\ell$ as $\gamma_\ell \to 0$

Conformal perturbation theory

⊕

(Weakly-broken) HS symmetry

Towards a CFT Distance Theorem

CFT Distance Conjecture

✓

I. HS point ⟶ Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\,C_{JJ\mathcal{O}}$$

⊕

$$C_{JJ\mathcal{O}} \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0$$

(Weakly-broken) HS symmetry

II. Infinite distance ⟶ HS point

No HS symmetry

↓ ?

$$\exists \mathcal{O} :\; C_{JJ\mathcal{O}} \neq 0$$

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$$\exists \mathcal{O} :\; C^{HS}_{KK\mathcal{O}} \neq 0$$?

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Conformal perturbation theory

⊕

(Weakly-broken) HS symmetry

↳ **Thank you for your attention!**

Backup slides

Conformal Perturbation Theory for HS operators

- Deform the theory with an operator: $\mathscr{L}_{t+\delta t} = \mathscr{L}_t + \delta t\, \mathcal{O}$

$$\rightarrow \quad \delta \left\langle J_\ell J_\ell \right\rangle = \delta t \int d^d y \left\langle J_\ell J_\ell \mathcal{O}(y) \right\rangle_{reg} + \mathcal{O}(\delta t^2)$$

- $\mathcal{O}$ marginal operator ⟶ Theory remains conformal

$$\rightarrow \quad \delta \left\langle J_\ell J_\ell \right\rangle = - \frac{H_{12}^{\ell}}{|x|^{2\Delta_\ell - 2\ell}} 2 \delta\Delta_\ell \log|x|$$

Shift in conformal dimension linked to log divergences!

- Conformal structures (see e.g. [Costa, Penedones, Poland, Rychkov '11])

$$\rightarrow \quad \left\langle J_\ell J_\ell \mathcal{O} \right\rangle = \sum_{n=0}^{\ell} C^n_{JJ\mathcal{O}} \Theta_n$$

$\ell + 1$ conformal structures

Values of w_n:
[Sen, Tachikawa '18]

$$C_{JJ\mathcal{O}} = \sum_{n=0}^{\ell} w_n C^n_{JJ\mathcal{O}}$$

$$\delta\Delta_\ell = - \delta t\, C_{JJ\mathcal{O}} + \mathcal{O}(\delta t^2) \rightarrow \boxed{\frac{d\Delta_\ell}{dt} = - C_{JJ\mathcal{O}}}$$

Valid everywhere in the conformal manifold

Distances from Conformal Perturbation Theory

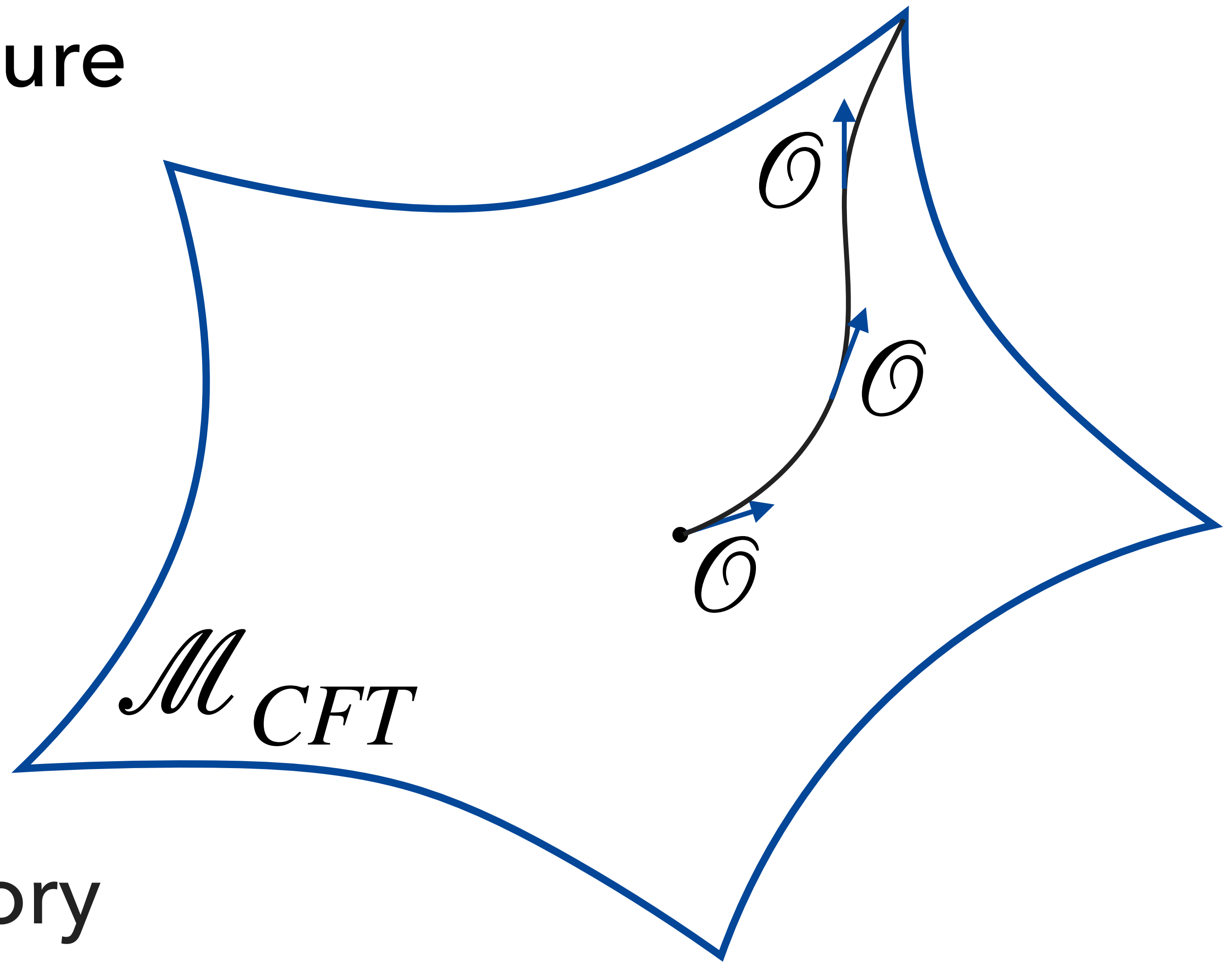
$$\boxed{\frac{d\Delta_\ell}{dt} = -\,C_{JJ\mathcal{O}}} \xrightarrow{C_{JJ\mathcal{O}}} \boxed{\Delta_\ell(t)}$$

CFT Distance Conjecture

Two important questions:

Not necessarily a geodesic !

- What is the meaning of t ?
  1. **1.** $\mathcal{O}$ defines a direction in $\mathcal{M}_{CFT}$ ⟶ $\mathcal{O}(t)$ defines a trajectory
  2. **2.** $t + \delta t$ represents how we move ⟶ t is a parameter of the trajectory
  3. **3.** Take $\langle \mathcal{O}\mathcal{O} \rangle \sim 1 \longrightarrow ds^2 \sim \langle \mathcal{O}\mathcal{O} \rangle\, dt^2 \sim dt^2 \longrightarrow$ t is the distance along the trajectory!



- How do we learn about $C_{JJ\mathcal{O}}$?

Usually: CPT for $\langle JJ\mathcal{O} \rangle$ ⟶ Complicated equations involving all spectrum of operators and conformal blocks (See e.g. [Behan '18])

Here: Something different! In particular, weakly-broken HS symmetry

A CFT Distance Criterion

$$\boxed{\frac{d\Delta_\ell}{dt} = -\,C_{JJ\mathcal{O}}} \xrightarrow{C_{JJ\mathcal{O}}} \boxed{\Delta_\ell(t)}$$

CFT Distance Conjecture

Consider a point with $\Delta_\ell = \Delta_\ell^*$ and $\Delta_\ell(t)$ invertible in a neighbourhood: $C_{JJ\mathcal{O}}(t) \longrightarrow \boxed{C_{JJ\mathcal{O}}(\Delta_\ell)}$

Take parametrization: $C_{JJ\mathcal{O}} \propto \left(\Delta_\ell - \Delta_\ell^*\right)^{a+1} \qquad a \in \mathbb{R}$

$$\Rightarrow \; t \propto \left(\Delta_\ell - \Delta_\ell^*\right)^{-a}$$

Finite distance $\longleftrightarrow$ $a < 0$

Infinite distance $\longleftrightarrow$ $a \geq 0$

CFT Distance criterion:

Finite distance $\longleftrightarrow$ $\exists\,\mathcal{O}$ (marginal operator) such that $a < 0$

Infinite distance $\longleftrightarrow$ $a \geq 0 \quad \forall\,\mathcal{O}$ (marginal operator)

Notice! Any point with $C_{JJ\mathcal{O}} \neq 0$ any automatically at finite distance!

JJO Correlator at HS Points

HS point → Ward identities

$$\boxed{\partial \cdot J_\ell = 0 + \text{(contact terms)}}$$

At generic points!

- $\left\langle \partial J_\ell \, J_\ell \, \mathcal{O} \right\rangle = 0$ → Recursive relation among all $C^n_{JJ\mathcal{O}}$ → $\boxed{C^n_{JJ\mathcal{O}} = v^n \, C^\ell_{JJ\mathcal{O}}}$

→ $C^{HS}_{JJ\mathcal{O}} = C^\ell_{JJ\mathcal{O}} \boxed{\sum_{n=0}^{\ell} w_n v^n}$ $= 0$? No :(...Need to work a bit harder

- Integrated Ward identity!

Twist conservation: $\Delta_s - s = d - 2$

1. Define conserved charges: $Q_\ell = \int_S J_\ell$ → $[Q_\ell, J_\ell] \sim \sum_s J_s$

2. Use: $\int_S \langle J_\ell J_\ell \mathcal{O} \rangle \propto \left\langle [Q_\ell, J_\ell] \; \mathcal{O} \right\rangle$

JJO Correlator at HS Points

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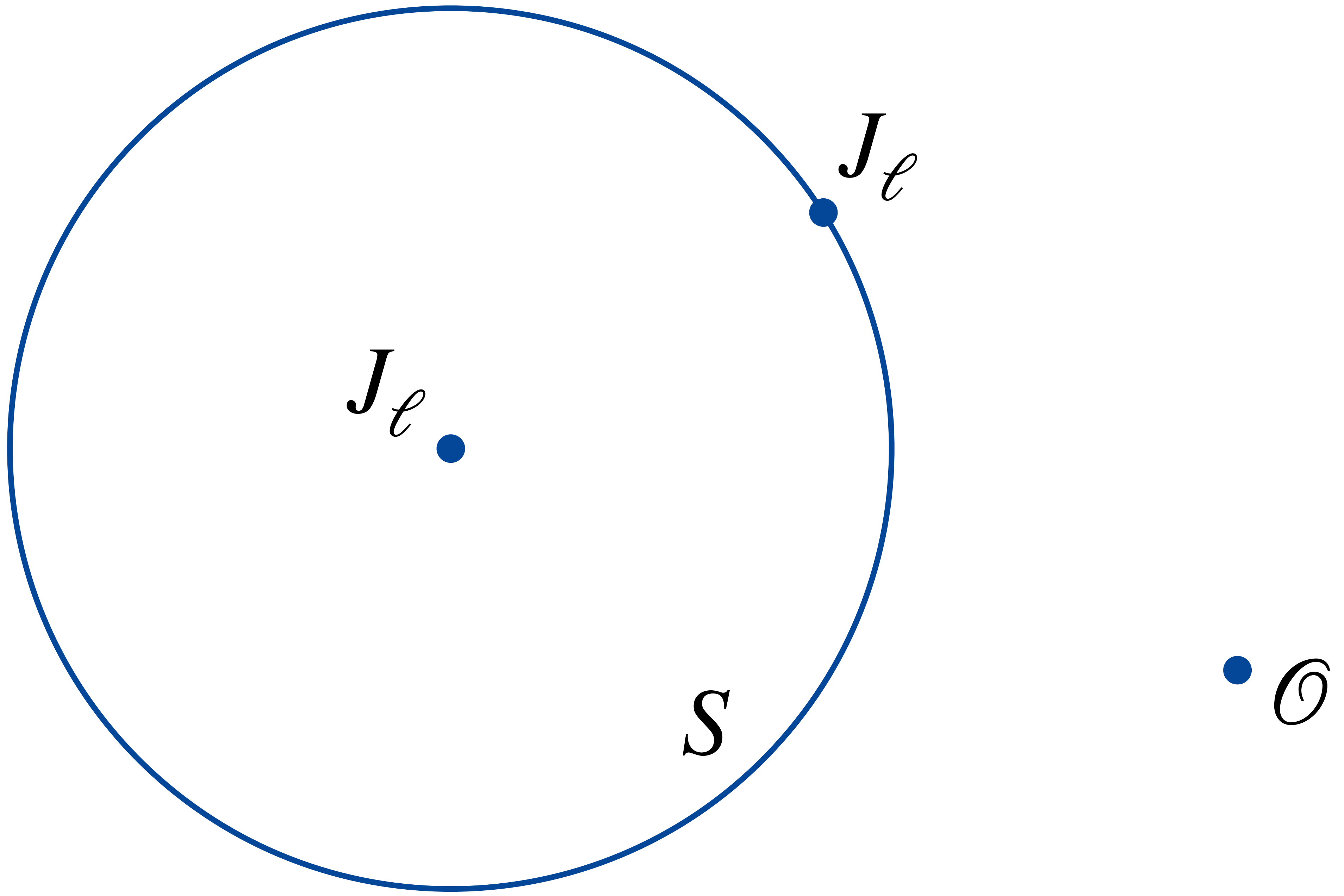
Surface integral



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HS point → Ward identities

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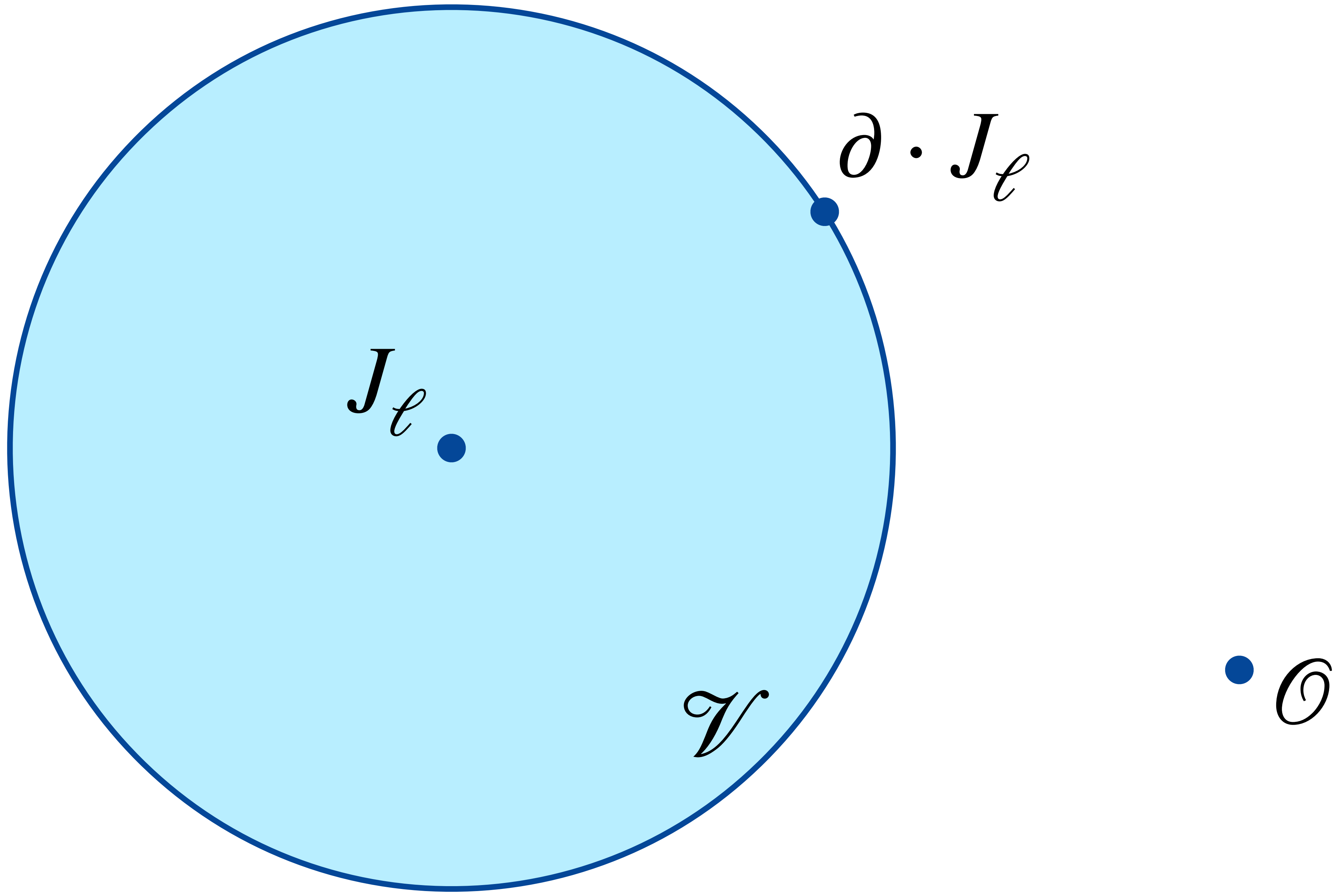
Surface integral + Stokes theorem



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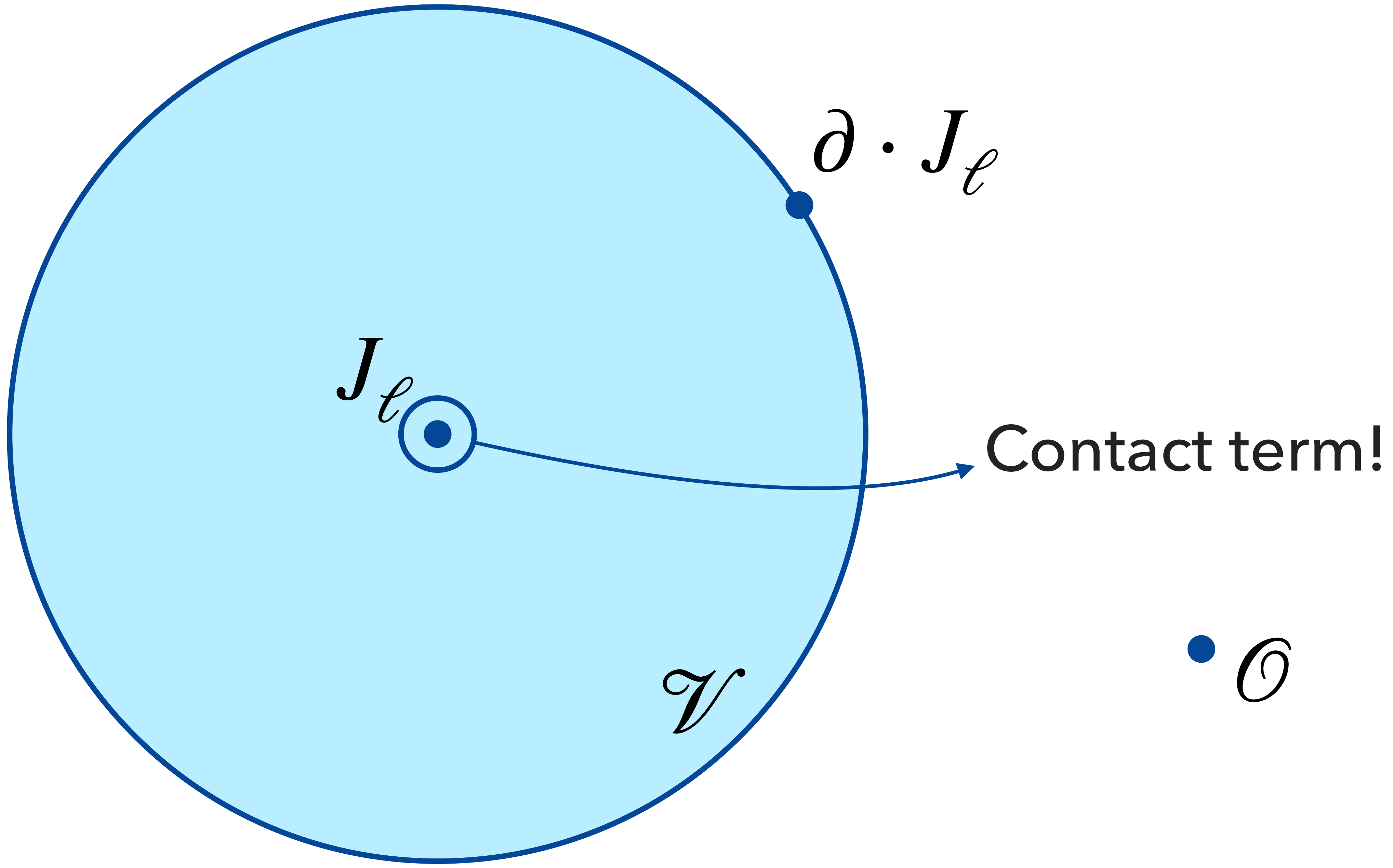
Surface integral + Stokes theorem + Ward identity



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At generic points!

- $\left\langle \partial J_\ell \, J_\ell \, \mathcal{O} \right\rangle = 0 \rightarrow$ Recursive relation among all $C^n_{JJ\mathcal{O}}$ $\rightarrow$ $\boxed{C^n_{JJ\mathcal{O}} = v^n \, C^\ell_{JJ\mathcal{O}}}$

$$\rightarrow C^{HS}_{JJ\mathcal{O}} = C^\ell_{JJ\mathcal{O}} \boxed{\sum_{n=0}^{\ell} w_n v^n} = 0 \ ?$$

No :(…Need to work a bit harder

- Integrated Ward identity!

Twist conservation: $\Delta_s - s = d - 2$

1. Define conserved charges: $Q_\ell = \int_S J_\ell \rightarrow [Q_\ell, J_\ell] \sim \sum_s J_s$

$$\boxed{\left\langle J_\ell J_\ell \mathcal{O} \right\rangle = 0 \quad \forall J_\ell, \mathcal{O}}$$

$$\uparrow$$

2. Use: $\int_S \left\langle J_\ell \, J_\ell \, \mathcal{O} \right\rangle \propto \left\langle [Q_\ell, J_\ell] \ \mathcal{O} \right\rangle \rightarrow k \, C^\ell_{JJ\mathcal{O}} \propto \sum_\ell \left\langle J_\ell \, \mathcal{O} \right\rangle = 0 \rightarrow C^\ell_{JJ\mathcal{O}} = 0$

Non-vanishing!

JKO Correlator at HS Points

HS point → Ward identities

$$\boxed{\partial \cdot J_\ell = 0 + \text{(contact terms)}}$$

At generic points!

- $\left\langle \partial J_\ell \, K_{\ell-1} \, \mathcal{O} \right\rangle = 0 \rightarrow$ Recursive relation among all $C^n_{JK\mathcal{O}}$ $\rightarrow$ $\boxed{C^n_{JK\mathcal{O}} = v^n \, C^\ell_{JK\mathcal{O}}}$

$\langle J_\ell J_\ell \mathcal{O} \rangle$: ℓ equations for $\ell + 1$ variables

$\langle J_\ell K_{\ell-1} \mathcal{O} \rangle$: ℓ equations for ℓ variables

Enough? No :(…Need to work a bit harder

- Integrated Ward identity!

Twist conservation: $\Delta_s - s = d$

1. Define conserved charges: $Q_\ell = \int_S J_\ell \rightarrow [Q_\ell, K_{\ell-1}] \sim \sum_s K_s$

A priori could not vanish!

2. Use: $\int_S \langle K_{\ell-1} \, J_\ell \, \mathcal{O} \rangle \propto \left\langle [Q_\ell, K_{\ell-1}] \; \mathcal{O} \right\rangle \rightarrow k \, C^{\ell-1}_{JK\mathcal{O}} \propto \sum_\ell \langle K_\ell \, \mathcal{O} \rangle = \langle K_0 \, \mathcal{O} \rangle$

Non-vanishing!

…Need to work a bit harder

JKO Correlator at HS Points

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At generic points!

- $\left\langle \partial J_\ell \, K_{\ell-1} \, \mathcal{O} \right\rangle = 0$ → Recursive relation among all $C^n_{JK\mathcal{O}}$ → $\boxed{C^n_{JK\mathcal{O}} = v^n \, C^\ell_{JK\mathcal{O}}}$

  $\left\langle J_\ell J_\ell \mathcal{O} \right\rangle$: ℓ equations for $\ell + 1$ variables

  $\left\langle J_\ell K_{\ell-1} \mathcal{O} \right\rangle$: ℓ equations for ℓ variables

  Enough? No :(…Need to work a bit harder

- Charge conservation identity! [Maldacena, Zhiboedov '11]

  Involves presence of energy-momentum tensor → Dynamical gravity in the bulk !

  $$\left\langle [Q_\ell, T_2 K_{\ell-1} \mathcal{O}] \right\rangle = 0 \rightarrow \beta_\ell \left\langle J_\ell K_{\ell-1} \mathcal{O} \right\rangle + \cdots = 0 \quad \forall x_i$$

  For all positions of operators!

  Non-vanishing! [Maldacena, Zhiboedov '11]

Goal achieved for $\ell = 4$!

Goal: Show that only consistent solution is such that this contribution vanishes → $\boxed{\left\langle J_\ell K_{\ell-1} \mathcal{O} \right\rangle = 0 \quad \forall J_\ell, K_{\ell-1}, \mathcal{O}}$

Enough! Any HS point contains J_4