Small Black Hole EXPLOSIONS

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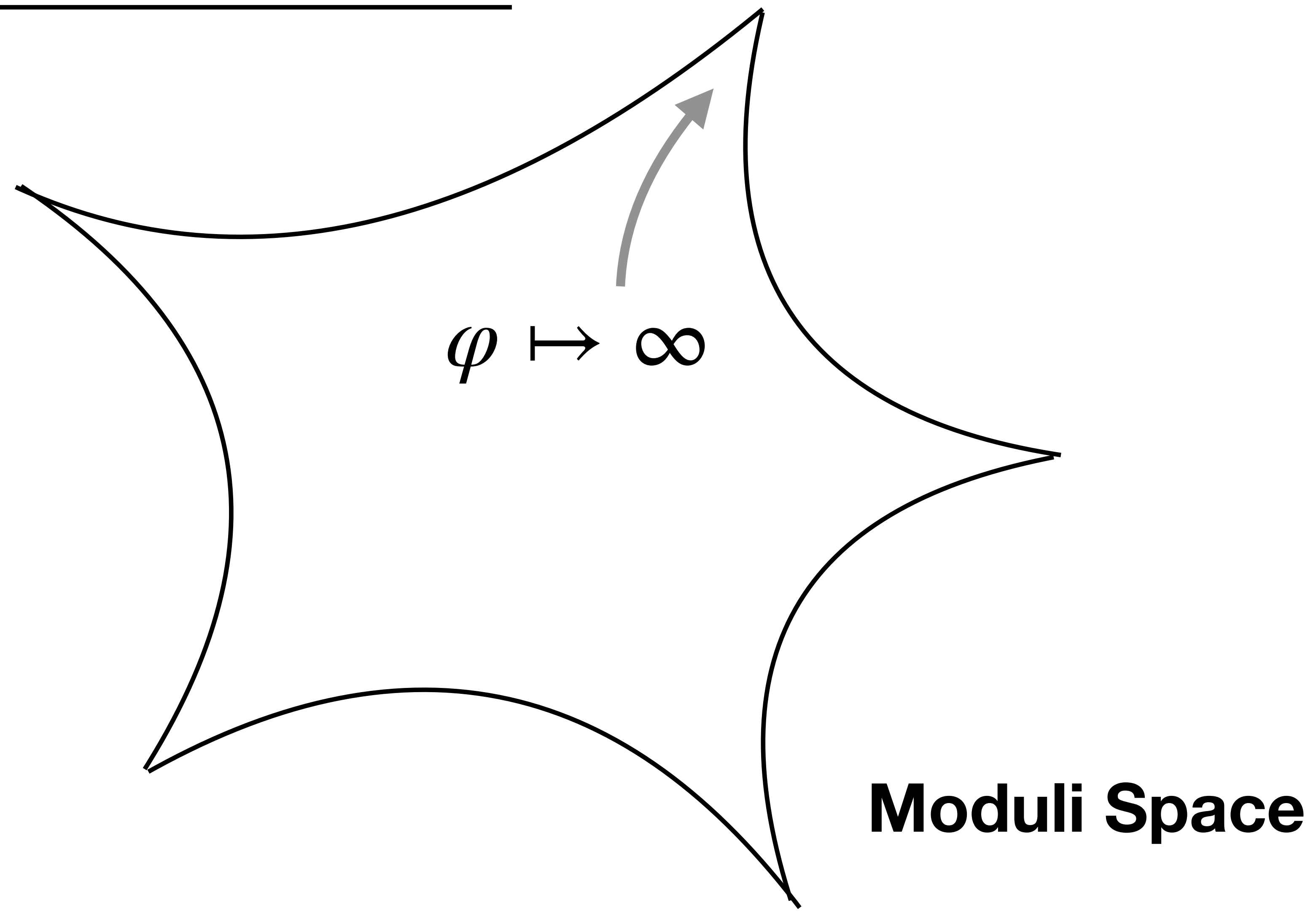
Based on **2303.15903** with **J.Huertas** and **A.M.Uranga**

The motivations

- The explorations of **Infinite Distance Limits** in Moduli space

  Research Method: Spacetime dependent solutions of EFTs with some scalars running to infinity



- It provides realisations of these infinite distance limits through **Realistic Configurations**.

- It allows a direct study of non-trivial **scalar potentials** growing at infinity and potentially obstructing the possibility of a dynamical exploration of these regimes.

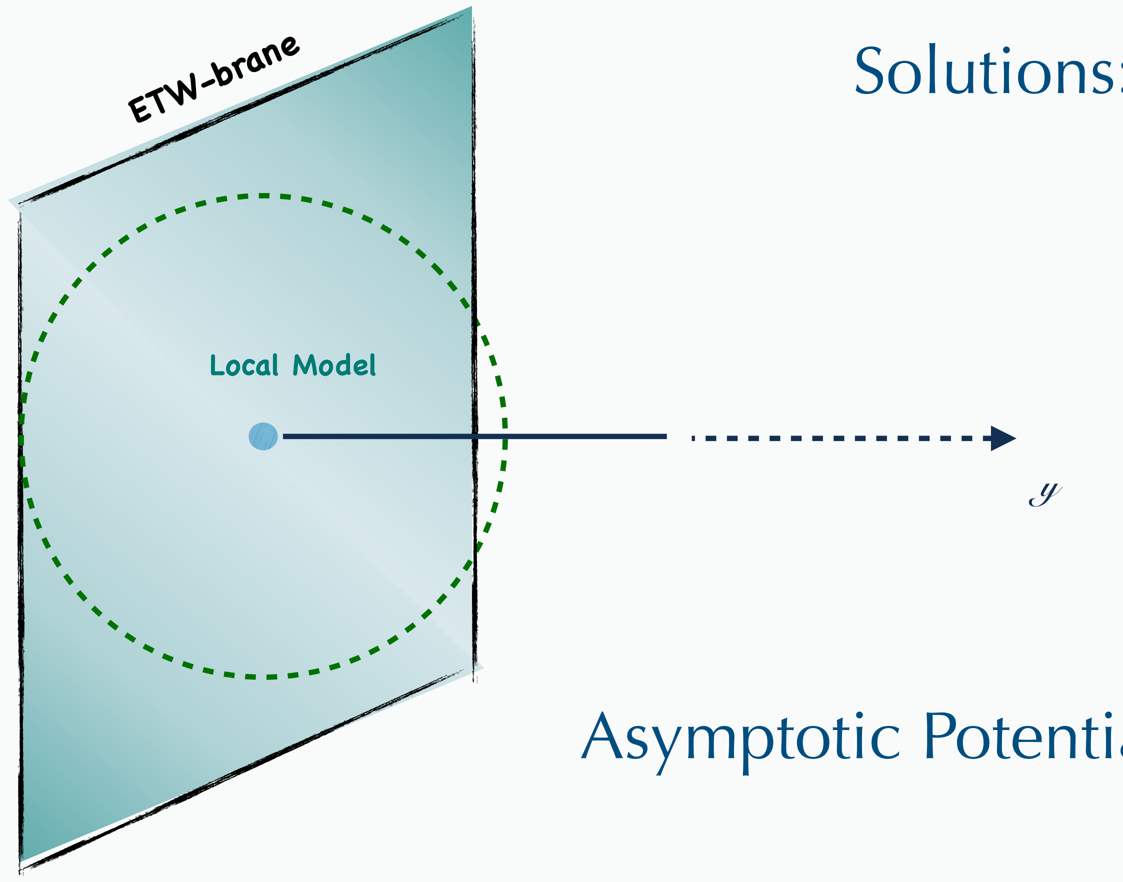
MAIN GOAL: To determine which type of potential realizes such obstructions.

Dynamical Cobordisms

[Buratti,Calderon,Delgado,Uranga 2021] [R.A., Calderon-Infante, Delgado,Huertas,Uranga 2022] [Blumenhagen,Cribiori,Kneissl,Makridou 2022]

- They are running solutions along one spacetime coordinate;
- They show a Ricci singularity at finite distance Δ in spacetime;
- The scalars of the theory go to infinity when we approach to the singularity.

Singularity source: **End of The World-brane**



Solutions:

$$ds^2 = e^{-2\sigma(y)} ds^2_{d-1} + dy^2$$
$$\sigma(y) \simeq \pm \frac{4}{(d-2)\delta^2} \log y$$
$$\varphi(y) \simeq -\frac{2}{\delta} \log y$$

Asymptotic Potential:

$$V(\varphi) = -c a(\varphi) e^{\delta\varphi}$$

Scaling Relations:

$$\mathcal{D} \simeq -\frac{2}{\delta} \log \Delta \qquad |R| \simeq e^{\delta \mathcal{D}}$$

Top perspective

Defect in the complete theory which trivialize the Cobordism Group as predicted by the Cobordism Conjecture:

$$\Omega_d^{QG} = [0]$$

Bottom perspective

Small Black Holes

[Hamada, Montero, Vafa, Valenzuela 2021]
[R.A., J.Huertas, A.Uranga 2023]

They are **static**, **spherically symmetric** and **electrically charged** solutions of the following 4d action:

$$S = \int d^4x\sqrt{-g}\,[R - 2\,|d\phi|^2 + \frac{1}{2g^2}\,|F|^2]$$

with an exponential **gauge coupling** function:

$$g = e^{-\alpha\phi}, \quad \phi \mapsto \infty$$

Their mathematical form is the following:

$$ds^2 = -\,e^{2U(\tau)}dt^2 + e^{-2U(\tau)}\left(\frac{d\tau^2}{\tau^4} + \frac{1}{\tau^2}d\Omega_2^2\right) \qquad \phi = \frac{1}{1+\alpha^2}\phi_0 + \frac{\alpha}{1+\alpha^2}\log\left(-q\tau + e^{\alpha\phi_0}\right)$$

$$e^{-2U} = e^{-\frac{2\alpha}{1+\alpha^2}\phi_0}\left(-q\tau + e^{\alpha\phi_0}\right)^{\frac{2}{1+\alpha^2}}$$

where $\quad q^2 = 2(1+\alpha^2)Q^2\,.$

The scalar is driven to $\phi \mapsto +\infty$ due to the **attractor mechanism**.

2d perspective

Let's consider the 4d action:

$$S_{4d} = \int d^4x\sqrt{-g_4}\{R_4 - 2g_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} - \frac{1}{2}Im\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\Sigma\mu\nu} - \frac{1}{2}Im\mathcal{N}^{\Lambda\Sigma}G_{\Lambda\mu\nu}G^{\mu\nu}_\Sigma - 2V\}$$

using the compactification ansatz:

$$ds_4^2 = ds_2^2 + e^{2\sigma}d\Omega_2^2$$

$$F^\Lambda = \sqrt{2}p^\Lambda \sin\theta d\theta \wedge d\varphi \qquad G_\Lambda = \sqrt{2}q_\Lambda \sin\theta d\theta \wedge d\varphi$$

we get the reduced 2d action:

$$S_{2d} = \int d^2x\sqrt{-g_2}e^{2\sigma}\{R_2 + 2(\partial\sigma)^2 - 2g_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + 2e^{-2\sigma} - 2e^{-4\sigma}V_{BH} - 2V\}$$

where the black hole potential is:

$$V_{BH} = -\frac{1}{2}\left(p^\Lambda Im\mathcal{N}_{\Lambda\Sigma}p^\Sigma + q_\Lambda Im\mathcal{N}^{\Lambda\Sigma}q_\Sigma\right)$$

(attractor mechanism)

The Effective Potential

Let's assume to be in the following regime:

- AdS 2d metric: $ds_2^2 = v_1\left(-r^2dt^2 + \frac{dr^2}{r^2}\right)$
- S^2 size: $e^{2\sigma} = v_2$
- Constant moduli.

Let's define the **Entropy Functional**: [Sen,2005]

$$\mathcal{E}(v_1, v_2, p, q) = -4\pi\sqrt{-g_2}\mathcal{L}_2 = 8\pi v_2 - 8\pi v_1 + 8\pi V_{BH}\frac{v_1}{v_2} + 8\pi V v_1 v_2$$

The conditions to get an AdS vacua are:

(i) $\frac{\partial \mathcal{E}}{\partial v_1} = 0 \qquad V v_2^2 - v_2 + V_{BH} = 0 \longrightarrow v_2 = \frac{1-\sqrt{1-4VV_{BH}}}{2V} \equiv V_{eff}$

(ii) $\frac{\partial \mathcal{E}}{\partial v_2} = 0 \qquad 1 - V_{BH}\frac{v_1}{v_2^2} + V v_1 = 0 \longrightarrow v_1 = \frac{v_2}{\sqrt{1-4VV_{BH}}}$

(iii) $\left.\frac{\partial \mathcal{E}}{\partial z^i}\right|_{z_H^i} = 0 \qquad \frac{v_1}{v_2}\left.\frac{\partial V_{BH}}{\partial z^i}\right|_{z_H^i} + v_1 v_2\left.\frac{\partial V}{\partial z^i}\right|_{z_H^i} = 0$

Effective Potential

[Bellucci, Ferrara, Marrani, Yeranyan, 2008]

Classes of 4d Potentials

Let's assume an exponential growing approximation in the regime $\phi \mapsto +\infty$:

$$V = V_0 e^{\delta\phi} \qquad \text{with} \qquad V_0 > 0, \quad \delta > 0$$

[R.A., Calderon, Delgado, Huertas, Uranga 2023]

The Effective Potential becomes:

$$V_{eff} = \frac{1 - \sqrt{1 - 8V_0 Q^2 e^{(\delta-2\alpha)\phi}}}{2V_0 e^{\delta\phi}}$$

- $\delta < 2\alpha$ **Subcritical** case : the Small black hole behavior is preserved $V_{eff} \sim V_{BH}$
- $\delta = 2\alpha$ **Critical** case : Small black hole solutions are still present $V_{eff} = \frac{1 - \sqrt{1 - 8V_0 Q^2}}{2V_0} e^{-2\alpha\phi}$
- $\delta > 2\alpha$ **Supercritical** case : Small black hole **EXPLOSION**
  - → Puffed-up finite size Black Hole
  - → Finite size Black Hole runway to larger and larger sizes

2d Dynamical Cobordism

- Critical/ Subcritical Regime

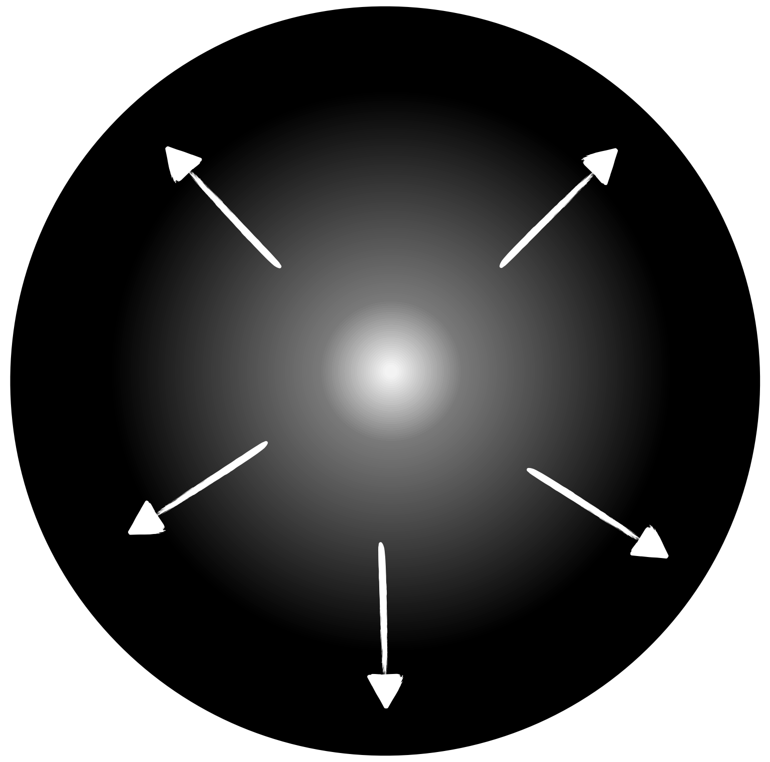
The 2d metric takes the form: $ds_2^2 = -e^{2U}dt^2 + \underbrace{e^{-2U}dr^2}_{d\tilde{r}^2}$ where $U = -\frac{1}{1+\alpha^2}\log(A/r)$

The associated scaling relations are:

$$\Delta \simeq \int e^{-U}dr \propto r^{\frac{\alpha^2}{1+\alpha^2}}, \qquad |R| \simeq \Delta^{-2}, \qquad D \simeq -\frac{\sqrt{1-\alpha^2}}{\alpha^2}\log\Delta, \qquad \delta = \frac{2\alpha^2}{\sqrt{1-\alpha^2}}$$

- Supercritical Regime: Small black hole **EXPLOSION**

The scalar potential is too large, it stops the running of the scalars towards infinity and it obstructs the realization of a Dynamical Cobordism.
Instead to have an ETW brane, the 2d running configuration ends up in an AdS vacuum corresponding to the near horizon geometry of the finite size black hole.

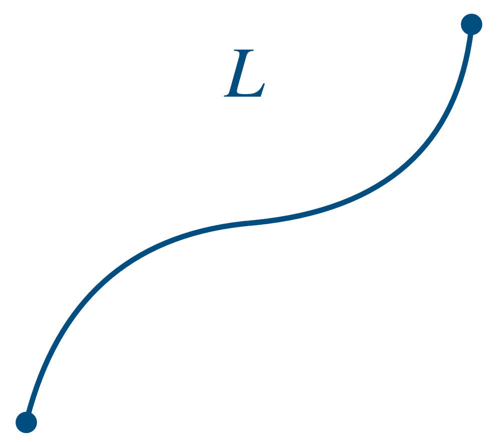


x AdS Geometry

Freed-Witten Condition

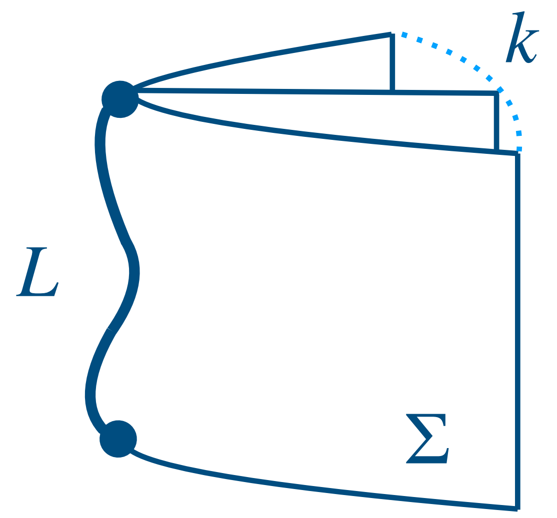
- The gauging procedure changes the allowed observable operators present in the theory: [Banks, Seiberg 2011]

Ungauged theory: $S \propto \int d^4x \left(|dV_1|^2 + |H_3|^2 \right)$



$= e^{i \int_L V_1}$

Gauged theory: $S \propto \int d^4x \left(|dV_1 - kB_2|^2 + |H_3|^2 \right)$



$= e^{i \int_L V_1 + k \int_\Sigma B_2}$

- Considering 4d theory arising from flux compactification of String Theory,

Magnetic monopoles $\longrightarrow$ $\Pi_{D3} = \sum_\Lambda (q_\Lambda a_\Lambda + q^\Lambda b^\Lambda)$ Gauging $\longrightarrow$ $\Pi_{D5} = \sum_\Lambda (g_\Lambda a_\Lambda + g^\Lambda b^\Lambda)$

The lack of gauge invariance is a realization of the Freed-Witten condition:

$$\int_{\Pi_{D3}} F_3 = q_\Lambda g^\Lambda - q^\Lambda g_\Lambda = -\langle \mathcal{G}, \mathcal{Q} \rangle = k$$

If $k = 0$ the background is consistent;

If $k \in Z_*$ we have to **add k strings** ending on the D3-brane to source the extra k-units of flux.

Intersection number: $\langle \mathcal{G}, \mathcal{Q} \rangle = \Pi_{D3} \cdot \Pi_{D5}$

Explosions from Topological obstructions

Let's impose the **Freed-Witten condition** to the stringy background associated to 4d $\mathcal{N} = 2$ gauged supergravities:

- Regular BHs [Cacciatori, Klemm 2010] [Dall'Agata, Gnecchi 2011]

In the literature many regular Black Hole solutions of these 4d theories are valid under the condition:

$$\langle \mathcal{G}, \mathcal{Q} \rangle = -k \in Z_*$$

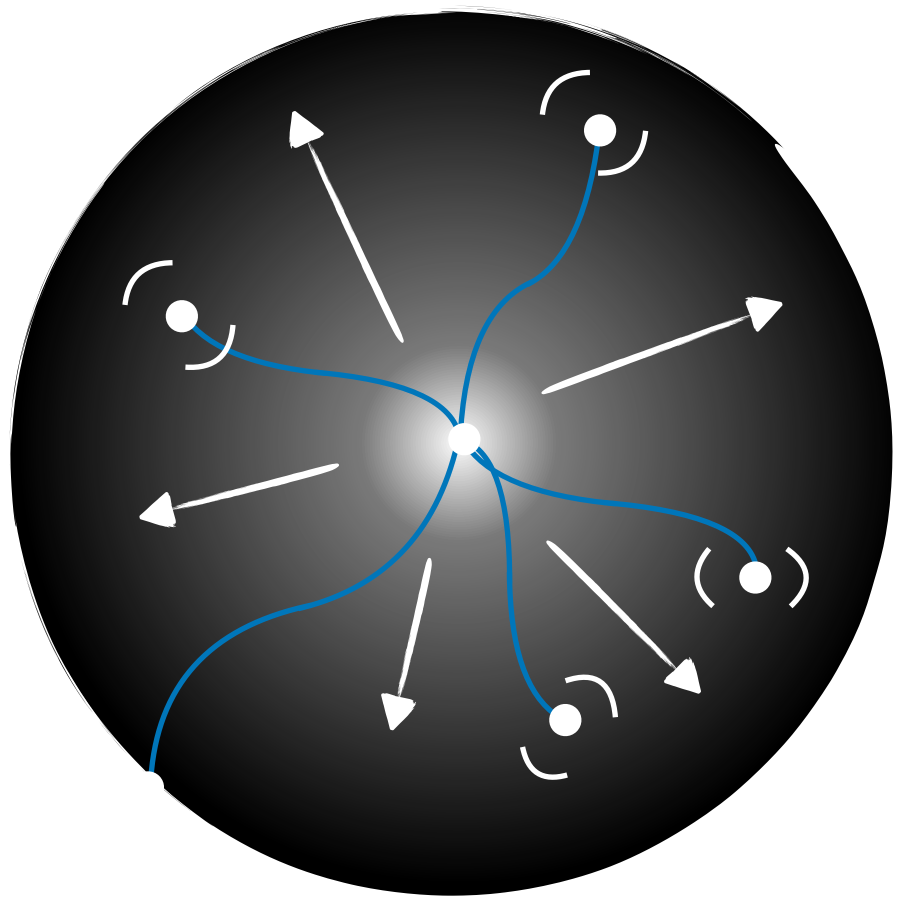
Such solutions are not complete, but they require the introduction of k strings.

- Small BHs

If $\langle \mathcal{G}, \mathcal{Q} \rangle = -k$ we have to add k strings attached to the Small Black Hole.

The 2d action takes the following additional contribute:

$$2kT_0 \int d^2x \sqrt{-g_2}$$



The effective potential becomes:

$$V_{eff} = \frac{(1-kT_0) - \sqrt{(1-kT_0)^2 - 4V_{BH}V}}{2V} \longrightarrow \textbf{EXPLOSION}$$

Cobordism Distance Conjecture

In a consistent theory of QG, every infinite field distance limit can be realized as a Dynamical Cobordism running into an ETW-brane

[Buratti, Calderon, Delgado, Uranga 2021]

Is the set of small black holes enough rich to explore all possible infinite limits?

In the absence of potentials the Completeness Conjecture assures us that for each weak gauge coupling limit there exists a small black hole solution capable to explore it.

What happens when I add a scalar potential to the action?

Small Black Hole Criterion

In a consistent theory of QG with scalar-dependent Abelian gauge couplings, the allowed 4d potentials are constrained by the requirement that they allow for the existence of a set of small black holes whose scalars are still able to explore any infinite distance limit of the theory (i.e. they remain subcritical or critical).

We test the criterion on 4d $\mathcal{N} = 2$ gauged supergravity theories with scalar potential given by:

$$V = g^{i\bar{j}} D_i \mathscr{L} \overline{D_{\bar{j}} \mathscr{L}} - 3 |\mathscr{L}|^2$$

It is satisfied ✓

- In the limit where all the moduli go to infinity (Large Moduli Limit) a necessary condition for the criterion to be satisfied is:

$$g^{i\bar{j}} \partial_i \mathscr{K} \partial_{\bar{j}} \mathscr{K} = 3$$

Summary

- We studied the possible obstructions for the realization of a Dynamical Cobordism due to the introduction of a non-trivial potential for the scalars;

- We identified a bound condition on the growing of the potential, up to that the Dynamical Cobordism is stopped;

- We showed the presence of a topological obstruction for small black hole solutions of $\mathcal{N} = 2$ supergravities

- We formulated, in the Small Black Hole Criterion, the requirement of capabrility of small black hole solutions to continue to explore infinite distance limits despite the presence of the two obstruction mechanisms.

Thanks for your attention!