Affine algebras at infinite distance limits in the Heterotic String

Veronica Collazuol

IPhT CEA/Saclay

Based on:

- `arXiv:2203.01341` with A. Herraez, M. Graña
- `arXiv:2210.13471` with A. Herraez, M. Graña and H. Parra De Freitas

String Phenomenology 2023

July 6, 2023

Motivation

SYMMETRY ENHANCEMENT ⟷ MASSIVE VECTORS BECOMING MASSLESS

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INFINITE DISTANCE POINTS ⟷ INFINITE TOWER OF STATES BECOMING MASSLESS (SDC)

[Ooguri, Vafa, '07]

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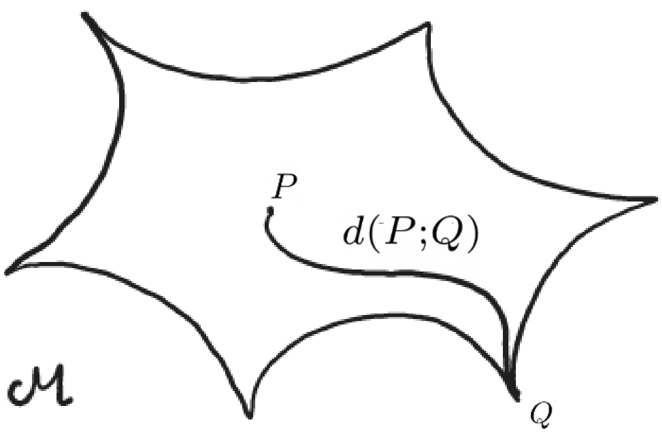
Swampland Distance Conjecture

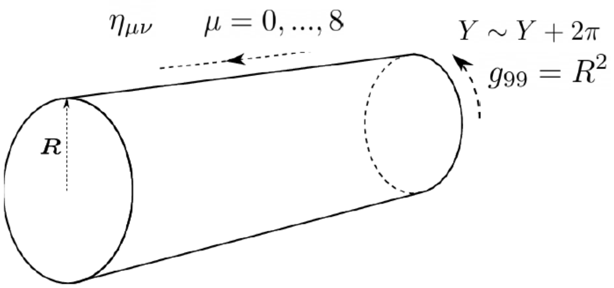


Moving from a point P in moduli space to another point Q an infinite distance away, there is a tower of states which becomes exponentially massless in Planck units

$$m(Q) \sim m(P)e^{-\alpha d(P;Q)}$$

Symmetry enhancements

Framework: Heterotic String on S^1



Moduli: radius R and Wilson line $A^I,\ I = 1, ..., 16$

Charges: $Z = (w, n, \pi^I)$

- w: winding
- n: momentum along S^1
- π^I: 16d momentum along the Heterotic torus

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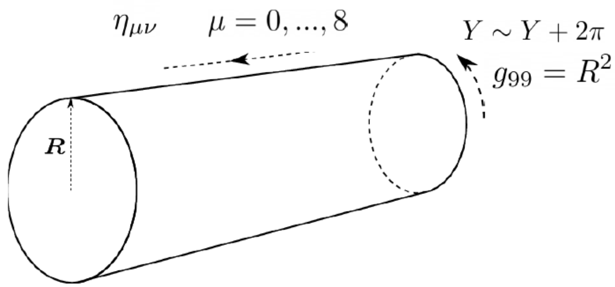

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momentum along S[1]

Charges: $Z = (w, n, \pi^I)$

winding

16d momentum along the Heterotic torus

The mass of a state depends on the point of moduli space

$$M = M(Z, R, A^I)$$

Symmetry enhancements

Massless vectors ⟷ space-time gauge bosons ⟷ worldsheet currents

$$M(Z, R, A^I) = 0 \longrightarrow \{J^a(z)\}$$

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- At special points: $G^r \times U(1)^{17-r}$ whose currents satisfy

$$J^a(z)J^b(w) \sim \frac{k^{ab}}{(z-w)^2} + \frac{i f_c^{ab} J^c(w)}{z-w}$$

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↓ zero modes

$$[J^a, J^b] = i f_c^{ab} J^c$$

classified in [Font, Fraiman, Graña, Nuñez, Parra de Freitas ‘18-'21]

Example: $E_8 \oplus E_8 \oplus U(1)$

For $A = 0$ and generic radius, the massless states are

- $J^I(z) = i\partial Y^I(z)$
- $J^9(z) = i\sqrt{2}\partial Y^9(z)$
- $J^\alpha(z) = e^{i\alpha^I Y^I(z)}$ $\qquad \alpha \in \Gamma_8 \times \Gamma_8$

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Their zero modes satisfy the algebra

$$[J^I, J^J] = [J^I, J^9] = [J^9, J^9] = 0$$

$$[J^I, J^\alpha] = \alpha^I J^\alpha$$

$$[J^\alpha, J^\beta] = \begin{cases} \epsilon(\alpha,\beta) J^{\alpha+\beta} & \alpha+\beta \text{ root}, \\ \alpha^I J^I & \alpha = -\beta, \\ 0 & \text{otherwise} \end{cases}$$

$E_9 \oplus E_9 / \sim$

For $A = 0$ and with $R \to \infty$, the massless states are

- $J^I_n(z) = i\partial Y^I(z) e^{inY^9(z)}$
- $J^9_n(z) = i\sqrt{2}\partial Y^9(z) e^{inY^9(z)}$ — Momentum towers
- $J^\alpha_n(z) = e^{i\alpha^I Y^I(z)} e^{inY^9(z)}$ — $\alpha \in \Gamma_8 \times \Gamma_8$

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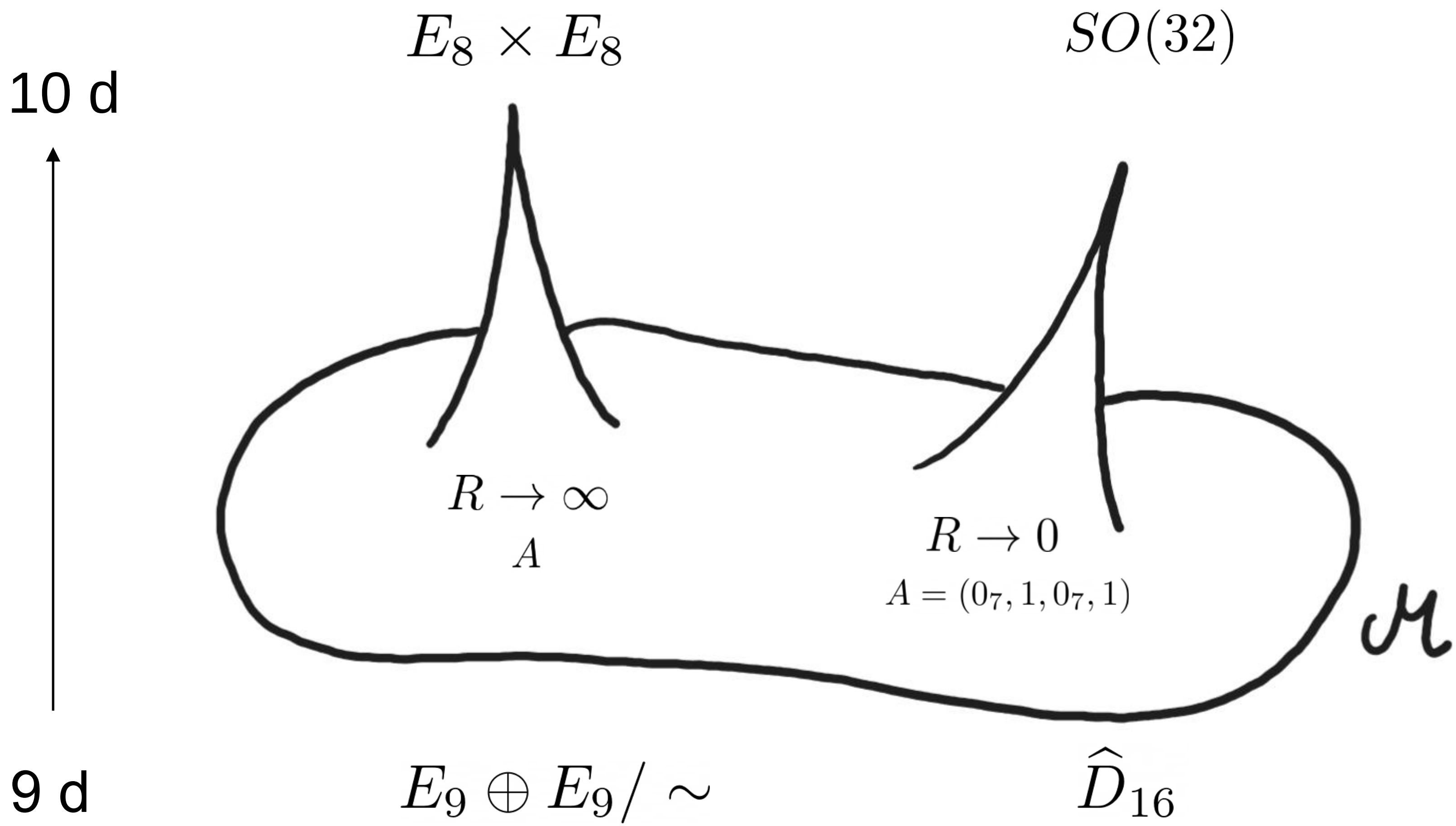
Central extension:

$$[\partial Y^9, \partial Y^9] = 0$$
$$[\partial Y^9, J^I_n] = 0$$
$$[\partial Y^9, J^\alpha_n] = 0$$

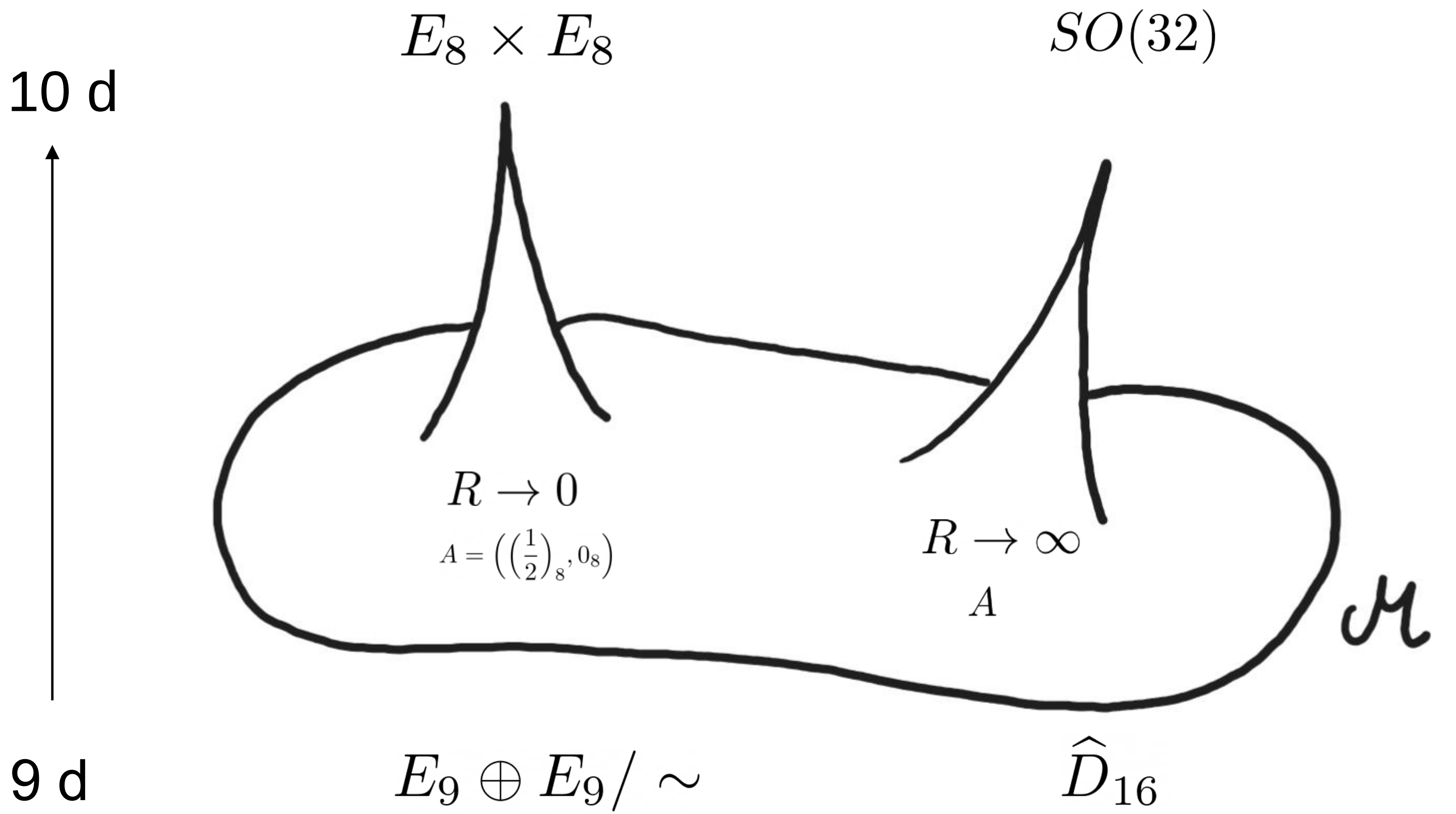
Decompactification limits 9d → 10d

$E_8 \times E_8$ theory



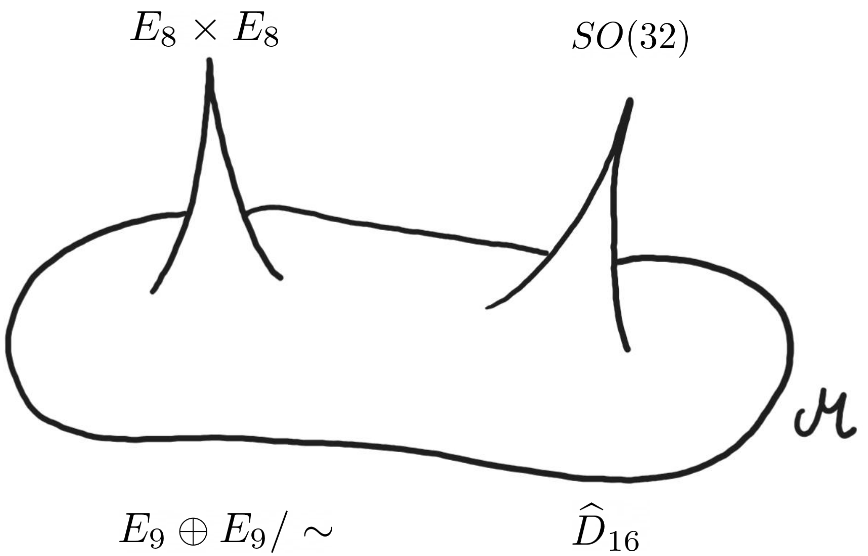
Decompactification limits 9d→10d

$SO(32)$ theory



General lesson

- In lower dimension we find the affine version of the higher dimensional algebra at infinite distance.



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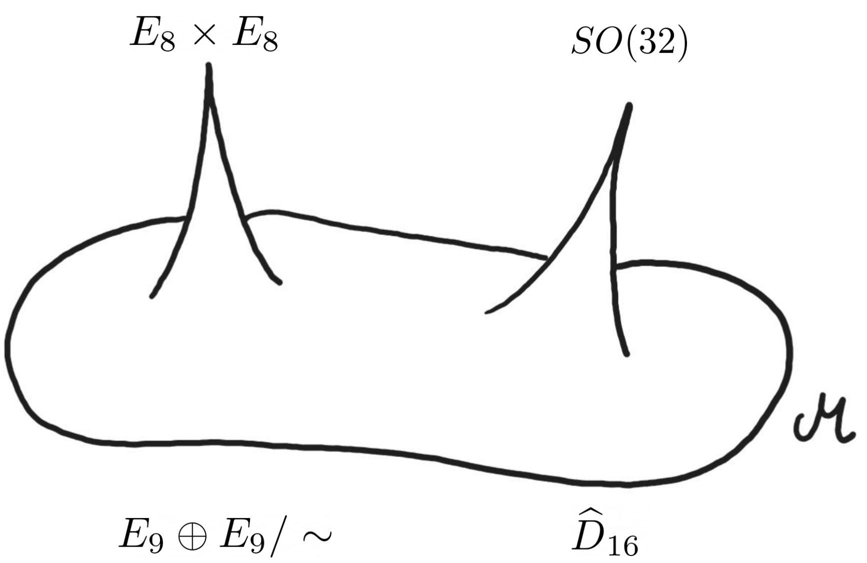

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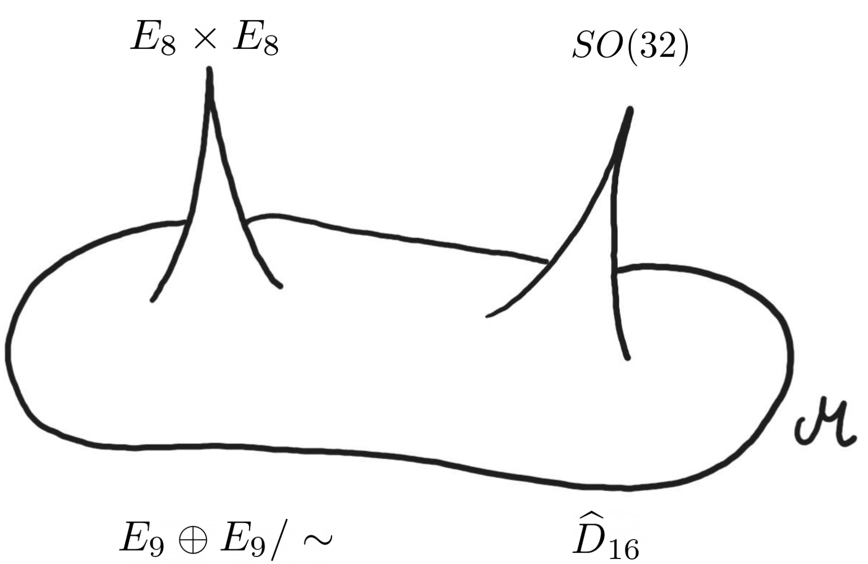

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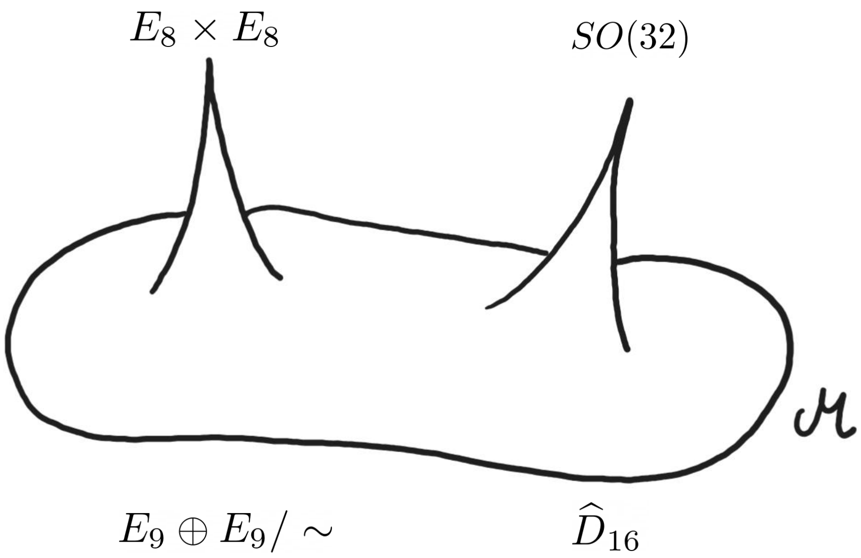

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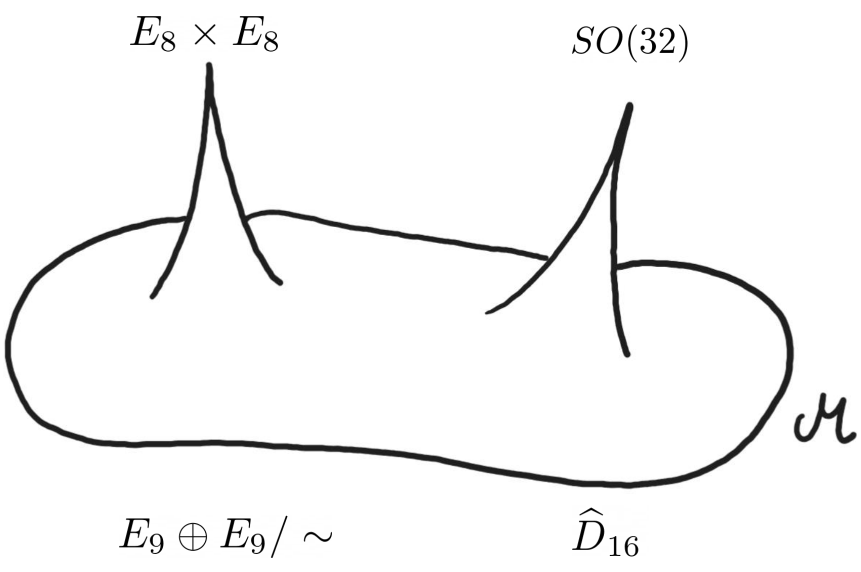

General decompactification limits

Framework: Heterotic String on T^d (e.g. d=2, dual to the case in [Lee, Weigand, '21] [Lee, Lerche, Weigand, '21])

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$\longrightarrow$ One central extension for each direction that is decompactified

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- Partial decompactification 8d⟶9d:

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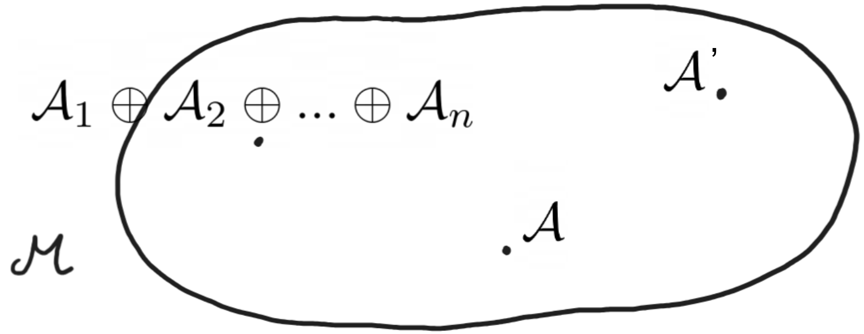

[Fraiman, Graña, Nuñez '18]

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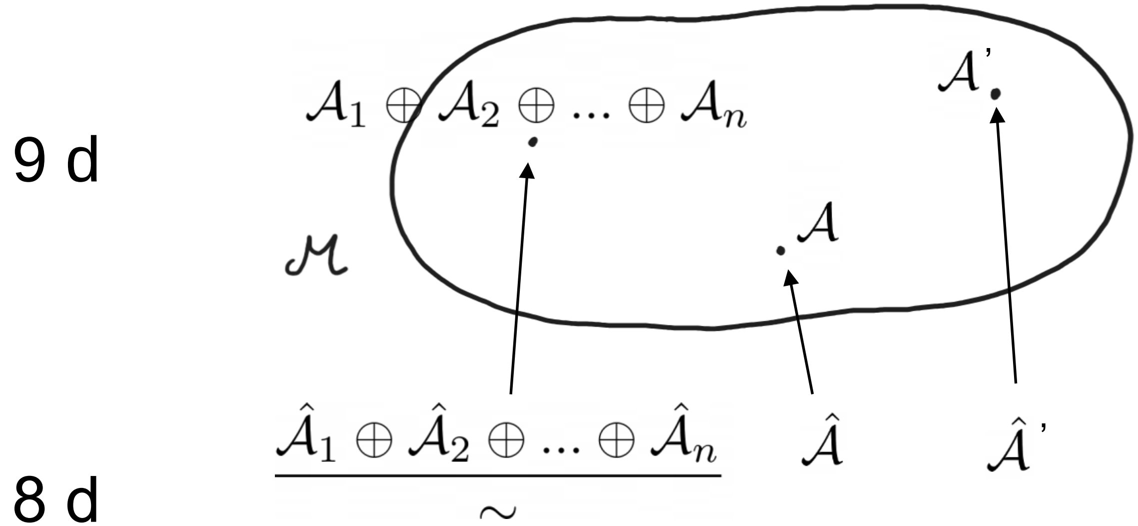

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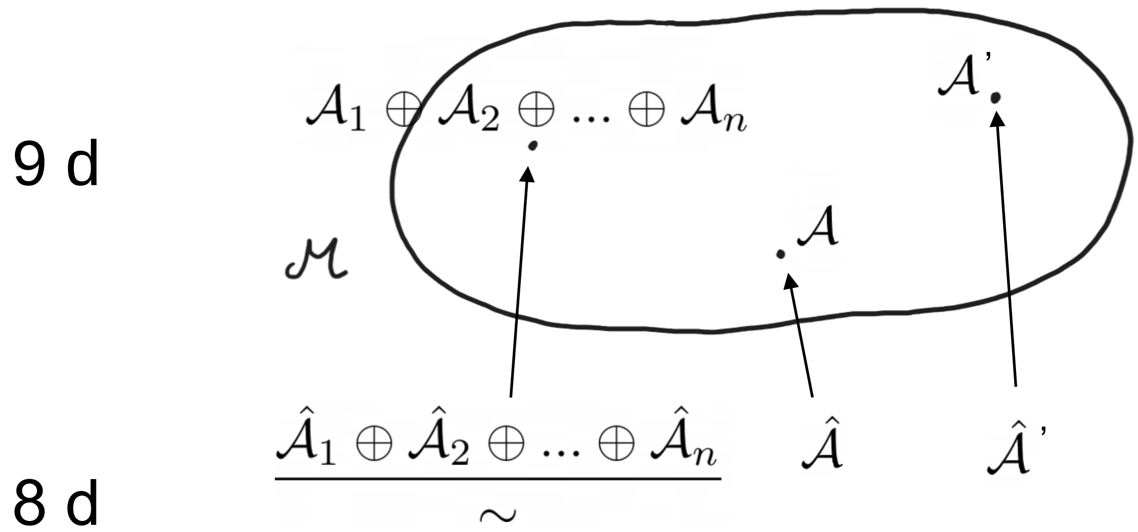

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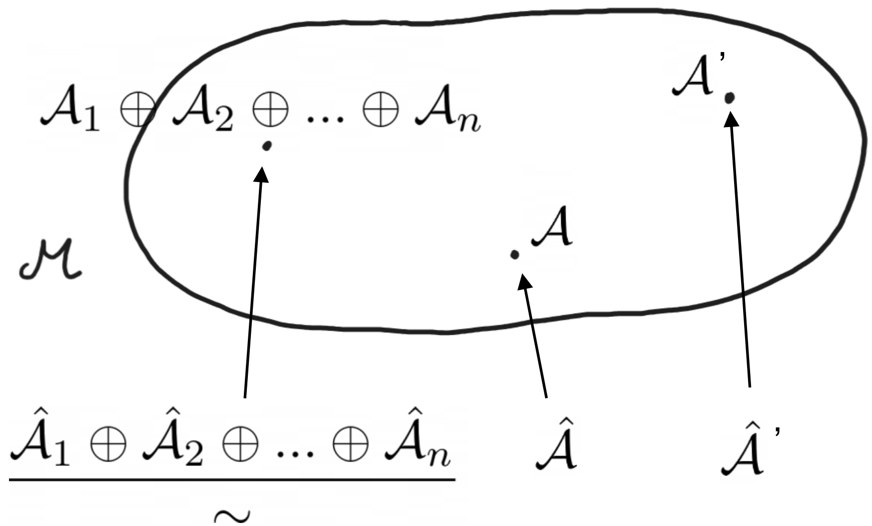

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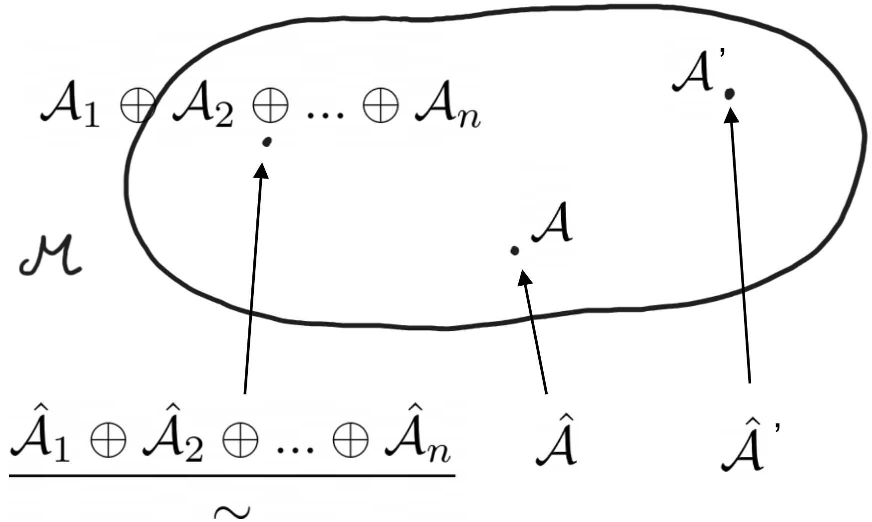

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[Fraiman, Graña, Nuñez '18]

  - We can find all the algebras in the higher dimensional theory
  - All the factors are made affine, also the U(1)'s
  - This generalizes to any d and any number of decompactified dimensions.

Conclusions

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- Match with the dual picture of F-theory on K3
  [Lee, Weigand, '21] [Lee, Lerche, Weigand, '21]

Thank you!