Cosmic Acceleration and Turns in the Swampland

Based on arXiv: 2306.17217

with Julian Freigang, Dieter Lüst, Marco Scalisi

Guoen Nian

Utrecht Univeristy

Outline

Introduction

The enough long-time cosmic acceleration requires slow-roll parameters

$$0 < \epsilon = -\frac{\dot{H}}{H^2} < 1, \; |\eta| = \left| \frac{\dot{\epsilon}}{H\epsilon} \right| < 1 \, .$$

The enough long-time cosmic acceleration requires slow-roll parameters

$$0 < \epsilon = -\frac{\dot{H}}{H^2} < 1, \; |\eta| = \left|\frac{\dot{\epsilon}}{H\epsilon}\right| < 1\,.$$

Meanwhile, the de Sitter conjecture:

$$|\nabla V| > cV, \; c \sim \mathcal{O}(1)\,,$$

for scalar potentials. From effective theory perspective,

$$\frac{|\nabla V|^2}{V^2} = 2\epsilon\left(\left(1+\frac{\eta}{2(3-\epsilon)}\right)^2 + \frac{\Omega^2}{H^2(3-\epsilon)^2}\right) \simeq 2\epsilon\left(1+\frac{\Omega^2}{9H^2}\right)\,.$$

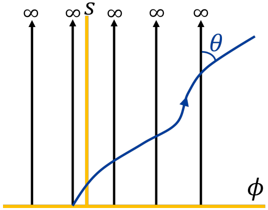
The acceleration parameter ϵ is generally small, so that we need the turning rate $\frac{\Omega}{H}$ to be large to fulfill the de Sitter conjecture in an accelerating universe.

Marco Scalisi's talk on Monday

A very common model from string effective theories is a hyperbolic plane with metric

$$\mathrm{d}\Delta^2 = \frac{n^2}{s^2}\left(\mathrm{d}s^2 + \mathrm{d}\phi^2\right) .$$

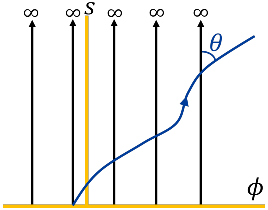
Under some scalar potential, the trajectory of the scalars will deviate from the geodesics by an angle θ, which is upper bounded due to the sharpened SDC. [Calderón-Infante, Uranga, Valenzuela, 20']



Marco Scalisi's talk on Monday

A very common model from string effective theories is a hyperbolic plane with metric

$$\mathrm{d}\Delta^2 = \frac{n^2}{s^2}\left(\mathrm{d}s^2 + \mathrm{d}\phi^2\right) .$$



Under some scalar potential, the trajectory of the scalars will deviate from the geodesics by an angle θ, which is upper bounded due to the sharpened SDC. [Calderón-Infante, Uranga, Valenzuela, 20']

Asymptotically, when time $t \to \infty$, θ is constant.

$$\frac{\Omega}{H} = \frac{|D_t \boldsymbol{T}|}{H} = \frac{|\sin\theta|\dot{\Phi}}{n}\frac{1}{H} = \frac{|\sin\theta|}{n}\sqrt{2\epsilon} \quad \Rightarrow \quad \boxed{\frac{\Omega}{H} \sim \mathcal{O}(\sqrt{\epsilon})},$$

with $\dot{\Phi}$ the speed of scalars along trajectory.

Generalization: product of two hyperbolic planes

The metric of the product of two hyperbolic planes:

$$\mathrm{d}\Delta^2 = \frac{n^2}{s^2}\left(\mathrm{d}s^2 + \mathrm{d}\phi^2\right) + \frac{m^2}{u^2}\left(\mathrm{d}u^2 + \mathrm{d}\psi^2\right) .$$

The trajectories are various and more complicated.

Generalization: product of two hyperbolic planes

The metric of the product of two hyperbolic planes:

$$\mathrm{d}\Delta^2 = \frac{n^2}{s^2}\left(\mathrm{d}s^2 + \mathrm{d}\phi^2\right) + \frac{m^2}{u^2}\left(\mathrm{d}u^2 + \mathrm{d}\psi^2\right) .$$

The trajectories are various and more complicated.

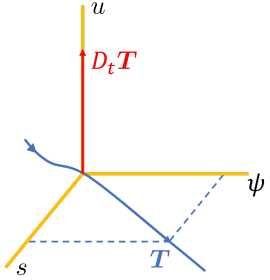
1) Saxion-another-axion trajectories

The trajectories can be thought as in the subspace with metric

$$\mathrm{d}\Delta^2 = \frac{n^2}{s^2}\mathrm{d}s^2 + \frac{m^2}{u^2}\mathrm{d}\psi^2 .$$

The turning rate is

$$\frac{\Omega}{H} = \frac{|D_t \boldsymbol{T}|}{H} = \frac{\sin^2\theta}{m}\sqrt{2\epsilon} \;\Rightarrow\; \boxed{\frac{\Omega}{H} \sim \mathcal{O}(\sqrt{\epsilon})} .$$



2) Saxion-saxion trajectories: $\mathrm{d}\Delta^2 = \frac{n^2}{s^2}\mathrm{d}s^2 + \frac{m^2}{u^2}\mathrm{d}u^2 \ \Rightarrow\ \Omega = 0\,.$ trivial

2) Saxion-saxion trajectories: $\mathrm{d}\Delta^2 = \frac{n^2}{s^2}\mathrm{d}s^2 + \frac{m^2}{u^2}\mathrm{d}u^2 \Rightarrow \Omega = 0$. trivial

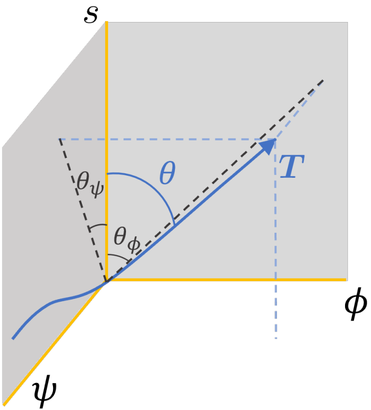
3) Saxion-axion-axion trajectories

$$\frac{\Omega^2}{H^2} = 2\epsilon \cos^4\theta \left(\frac{1}{n^2} \frac{\tan^2\theta_\phi}{\cos^2\theta_\phi} + \frac{1}{m^2} \tan^4\theta_\psi \right) ,$$

where $\tan^2\theta = \tan^2\theta_\phi + \tan^2\theta_\psi$.

It is θ upper bounded by the sharpened Distance Conjecture. Even if we take $\theta \to \frac{\pi}{2}$:

$$\max_{\theta\to\frac{\pi}{2}} \frac{\Omega^2}{H^2} = 2\epsilon \max\left\{ \frac{1}{m^2}, \frac{1}{n^2} \right\} \Rightarrow \boxed{\frac{\Omega}{H} \sim \mathcal{O}(\sqrt{\epsilon})} .$$



Further generalization: product of N hyperbolic planes

We can further generalize to the product of N hyperbolic planes, with metric

$$\mathrm{d}\Delta^2 = \sum_{i=1}^{N} \frac{n_i^2}{s_i^2} \left(\mathrm{d}s_i^2 + \mathrm{d}\phi_i^2\right) \,.$$

Any infinite distance limit in Calabi–Yau complex structure moduli space can have a metric of a product of hyperbolic planes asymptotically. [Grimm, Palti, Valenzuela, '18][Grimm, Li, Palti, '18][Calderón-Infante, Uranga, Valenzuela, 20']

Further generalization: product of N hyperbolic planes

We can further generalize to the product of N hyperbolic planes, with metric

$$\mathrm{d}\Delta^2 = \sum_{i=1}^{N} \frac{n_i^2}{s_i^2} \left(\mathrm{d}s_i^2 + \mathrm{d}\phi_i^2\right) \, .$$

Any infinite distance limit in Calabi–Yau complex structure moduli space can have a metric of a product of hyperbolic planes asymptotically. [Grimm, Palti, Valenzuela, '18][Grimm, Li, Palti, '18][Calderón-Infante, Uranga, Valenzuela, 20']

Consider the very general trajectories which deviate from some saxion s and move along all axion directions.

The turning rate is just the generalization of that of two hyperbolic planes:

$$\frac{\Omega^2}{H^2} = 2\epsilon \cos^4\theta \left(\frac{1}{n^2} \frac{\tan^2\theta_\phi}{\cos^2\theta_\phi} + \sum_{i=2}^{N} \frac{1}{n_i^2} \tan^4\theta_{\phi_i} \right) < 2\epsilon \max\left\{ \frac{1}{n^2}, \frac{1}{n_i^2} \right\} \quad \Rightarrow \quad \boxed{\frac{\Omega}{H} \sim \mathcal{O}(\sqrt{\epsilon})} .$$

Moving away from the boundary of moduli space

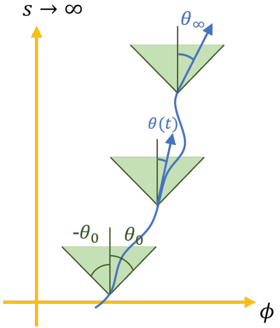
So far we assumed $\theta = \text{const}$ due to on the boundary of the moduli space.

Again consider the hyperbolic space with one saxion and one axion. Very simply, the non-constant deviation angle $\theta(t)$ contributes to the turning rate by

$$\Omega = \left| \frac{\sin\theta}{n} \dot{\Phi} - \dot{\theta} \right| .$$

Moving inside the moduli space slightly, the expression of $\dot{\theta}(s)$ can be derived by Taylor expanding $\theta(s)$ around $1/s = 0$. The upper bound of $\dot{\theta}(s)$ typically is

$$\left| \frac{\dot{\theta}(s)}{H} \right| \leq \frac{2\sqrt{2}}{n} \theta_0 \sqrt{\epsilon} \Rightarrow \boxed{\frac{\Omega}{H} \sim \mathcal{O}(\sqrt{\epsilon})}.$$



Conclusion

- There is always the turning rate $\frac{\Omega}{H} \sim \mathcal{O}(\sqrt{\epsilon})$ near the boundary of the field space;
- It is very challenging to fulfill the de Sitter conjecture under cosmic acceleration and near the boundary of moduli space. Large curvature or a large number of scalars approaching infinities is needed, but it is hard to achieve in string theory;
- The sound speed in the context of multi-field inflation $c_s = (1 + 4\Omega^2/M^2)^{-1/2}$ is close to unity.

Thank you!

The Taylor expansion of θ gives $\dot{\theta}$:

$$\theta(s) = \theta_\infty + \sum_{n>0} \frac{c_n}{s^n} \simeq \theta_\infty + \frac{c_k}{s^k} \text{ for first non-zero } c_k \Rightarrow \dot{\theta}(s) \simeq -k \frac{c_k}{s^k} \frac{\dot{s}}{s} \,.$$

With N hyperbolic planes, the turning rate

$$\Omega = \sqrt{\sum_{i=1}^{N} \left(\frac{\sin \theta_i}{n_i} \dot{\Phi}_i - \dot{\theta}_i \right)^2} \,.$$

The more hyperbolic planes, the larger the turning rate.