Remembrance of Things Past

Large Scales of Primordial Inflation at Late Times

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Overview

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The Idea

The question that we ask is

"To which extend the high scales of primordial inflation are transmitted to late times? "
→ Some sort of "fossilization" [1]

Why ?
→ Very important for re-summing the leading large logs of inflationary gravitons on a general cosmological background
→ Also, to reveal the fate of graviton loops at late times

Importance

We want to understand how graviton mode function, $u(t,k)$, changes on a general cosmological background which experiences inflation.

The 1-loop de Sitter result for plane wave gravitons [2] is

$$u(t,k) = u_0(t,k)\left\{1 + \frac{16GH^2}{3\pi}\ln^2(a) + O(G^2)\right\}. \qquad (1)$$

The Model

For this purpose, we pick the following model with 2 massless minimally coupled scalar fields, $A(x)$ and $B(x)$,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu A \partial_\nu A g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\left(1+\frac{1}{2}\lambda A\right)^2 \partial_\mu B \partial_\nu B g^{\mu\nu}\sqrt{-g} \ . \quad (2)$$

→ Mimicks the derivative couplings in the case of gravity!
→ Also, we know the large log structure of it [3, 4]

These fields are spectator to inflation!

The Cosmic History

The background geometry is

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x}$$
$$\Longrightarrow \qquad H(t) \equiv \frac{\dot{a}}{a} \quad , \quad \epsilon(t) \equiv -\frac{\dot{H}}{H^2} \ . \tag{3}$$

Aim is not to devise a new inflation or reheating mechanism. So, simply

Scalar-Driven Inflation
+
ΛCDM (Late times)

The composite slow-roll parameter is [5]

$$\epsilon(t) \equiv \frac{1}{2}\Big[1-\tanh(n-n_{\rm eq})\Big]\times\epsilon_{\varphi}(t)+\frac{1}{2}\Big[1+\tanh(n-n_{\rm eq})\Big]\times\epsilon_{\Lambda(t)} \ . \tag{4}$$

Here, n is number of e-foldings, $n \equiv \ln\left[\frac{a(t)}{a(t_i)}\right]$, and $n_{\rm eq} = 59.1$.

The Cosmic History

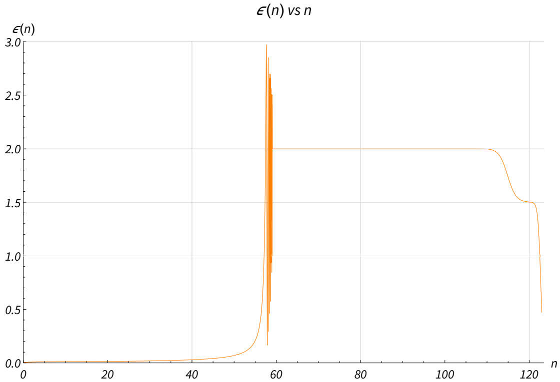

Figure: Evolution of the composite slow roll parameter (4) over cosmic history.

The Main Object of Interest

We are interested in time evolution of $\langle A \rangle$, VEV of the A field (2).

How?

$\rightarrow$ Starobinsky's Stochastic Formalism [6]:

- Find the effective potential, V_{eff}
- Introduce a stochastic random field
- Solve the Langevin equation

The Technique: $V_{\rm eff}$

In the presence of constant A field, expectation value of the variation of the action (2) yields the effective potential

$$\frac{\delta S[A,B]}{\delta A} = \partial_\mu\Big[\sqrt{-g}\, g^{\mu\nu}\partial_\nu A\Big] - \frac{\lambda}{2}\Big(1+\frac{\lambda}{2}A\Big)\partial_\mu B\partial_\nu B g^{\mu\nu}\sqrt{-g}$$
$$\longrightarrow \partial_\mu\Big[\sqrt{-g}\, g^{\mu\nu}\partial_\nu A\Big] - \frac{\lambda\sqrt{-g}\, g^{\mu\nu}\partial_\mu\partial'_\nu i\Delta(x;x')|_{x'=x}}{2(1+\frac{\lambda}{2}A)} = 0\ . \tag{5}$$

We have

$$V'_{\rm eff}(A_0) = \frac{\lambda}{2}\frac{g^{\mu\nu}\partial_\mu\partial'_\nu i\Delta(x;x')|_{x'=x}}{(1+\frac{\lambda}{2}A_0)}\ , \tag{6}$$

where we define $\mathcal{T}[a](t) \equiv g^{\mu\nu}\partial_\mu\partial'_\nu i\Delta(x;x')|_{x'=x}$.

The Technique: Langevin equation

The stochastic random field $\mathcal{A}(t,\vec{x})$ of this formalism obeys the Langevin equation,

$$-3H(t)\Big[\dot{\mathcal{A}}(t,\vec{x}) - \dot{\mathcal{A}}_0(t,\vec{x})\Big] = \frac{\frac{\lambda}{2}\mathcal{T}[a](t)}{1+\frac{\lambda}{2}\mathcal{A}(t,\vec{x})} \; , \tag{7}$$

where the stochastic "jitter" $\mathcal{A}_0(t,\vec{x})$ is the infrared-truncated, free field mode sum

$$\mathcal{A}_0(t,\vec{x}) \equiv \int_{a_i H_i}^{aH} \frac{d^3k}{(2\pi)^3} \sqrt{\frac{H^2(t_k)C(\epsilon(t_k))}{2k^3}} \times \left(e^{-i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + e^{i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right) \; , \tag{8}$$

where $a(t_k)H(t_k) \equiv k$, inflation begins at time t_i and $C(\epsilon) \equiv \frac{1}{\pi}\Gamma^2(\frac{1}{2}+\frac{1}{1-\epsilon})[2(1-\epsilon)]^{\frac{2}{1-\epsilon}}$.

The Technique: Main Equations I

Solving (7) with $\mathcal{A}(t_i, \vec{x}) = 0$ yields

$$\mathcal{A}(t,\vec{x}) = \frac{2}{\lambda}\left[\sqrt{1 - \frac{\lambda^2}{6}\int_{t_i}^{t} dt' \frac{\mathcal{T}[a](t')}{H(t')}} - 1\right] + \Big(\text{Stochastic acceleration}\Big)\,. \tag{9}$$

Approximate analytic expression for $\mathcal{T}[a](t)$ can be obtained through $\mathcal{V}[a](t) \equiv i\Delta(x:x)$ [7],

$$\mathcal{T}[a](t) = -\frac{1}{2}\left(\frac{d}{dt} + (D-1)H\right)\dot{\mathcal{V}}[a](t)\,. \tag{10}$$

The expressions for $\mathcal{V}[a](t)$ are obtained by summing ultraviolet$(a(t)H(t) < k)$ and infrared$(a(t)H(t) > k)$ modes during and after inflation. [5]

The Technique: Main Equations II

During inflation,

$$\mathcal{V}_{\rm div} = -\frac{1}{8}\left(\frac{D-2}{D-4}\right)\frac{[D-2\epsilon(t)]H^{D-2}(t)}{\Gamma\left(\frac{D-1}{2}\right)(4\pi)^{\frac{D-1}{2}}}\,, \tag{11}$$

$$\mathcal{V}_{\rm fin} = -\frac{H^2(t)}{8\pi^2} + \frac{1}{4\pi^2}\int_{t_i}^{t} dt' H^3\left(t'\right)\left[1-\epsilon\left(t'\right)\right] C\left(\epsilon\left(t'\right)\right)\,. \tag{12}$$

After inflation, t_e denoting the end of inflation,

$$\begin{aligned}\mathcal{V}_{\rm fin} = &\frac{(2-\epsilon)H^2}{8\pi^2}\ln\left(\frac{k_e}{aH}\right) - \frac{H^2}{8\pi^2}\left(\frac{k_e}{aH}\right)^2 \\ &+ \frac{1}{4\pi^2 a^2(t)}\int_{t_e}^{t} dt'\left[\epsilon\left(t'\right)-1\right] H\left(t'\right) a^2\left(t'\right) \\ &\qquad \times \cos^2\left[\int_{t_2(t_k)}^{t} dt''\frac{k}{a\left(t''\right)}\right] \times H^2\left(t_1\right) C\left(\epsilon\left(t_1\right)\right) \\ &+ \frac{1}{4\pi^2}\int_{t}^{t_2(t_i)} dt'\left[\epsilon\left(t'\right)-1\right] H\left(t'\right) \times H^2\left(t_1\right) C\left(\epsilon\left(t_1\right)\right)\,.\end{aligned} \tag{13}$$

The Technique: Main Equations III

Then, time evolution of $\langle A \rangle$ is given by

$$\left\langle \Omega \middle| \sqrt{8\pi G} A(x) \middle| \Omega \right\rangle = \frac{2\sqrt{8\pi G}}{\lambda} \left[\sqrt{1 - \frac{\lambda^2}{6} \int_{t_i}^{t} dt' \frac{\mathcal{T}[a](t')}{H(t')}} - 1 \right] + \Big(\text{Stochastic acceleration} \Big) \, . \tag{14}$$

The Technique: de Sitter Case

In the de Sitter case, we have [3, 4]

$$\mathcal{T}\Big[\text{de Sitter}\Big](t) = -\frac{H^D\Gamma(D)}{(4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})} \longrightarrow -\frac{3H^4}{8\pi^2} \qquad \Big(D=4\Big)\;, \tag{15}$$

and

$$\Big\langle \Omega \Big| A(x) \Big| \Omega \Big\rangle \longrightarrow \frac{2}{\lambda}\left[\sqrt{1+\frac{\lambda^2 H^2 \ln(a)}{16\pi^2}} - 1\right] + \Big(\text{Stochastic acceleration}\Big)\;. \tag{16}$$

The Technique: Renormalizaiton

We employ dimensional regularization with a suitable counter-term

$$\Delta\mathcal{V}[a](t) = \left(\frac{D-2}{8}\right)\frac{[D-2\epsilon(t)]H^2(t)}{\Gamma\left(\frac{D-1}{2}\right)(4\pi)^{\frac{D-1}{2}}}\left(\frac{\mu^{D-4}}{D-4}\right) . \tag{17}$$

We have, with the choice $\mu = e^{\alpha} \times H(t_e)$,

$$\lim_{D\to 4}\Big[\mathcal{V}_{\rm div}[a](t) - \Delta\mathcal{V}[a](t)\Big] = \frac{R(t)}{48\pi^2}\ln\left[\frac{\mu}{H(t)}\right] = \frac{R(t)}{48\pi^2}\ln\left[\frac{H(t_e)}{H(t)}\right] + \frac{\alpha R(t)}{48\pi^2} , \tag{18}$$

where $R(t) = (D-1)(D-2\epsilon)H^2(t)$.

$$\mathcal{V}_{\rm ren}(t) \equiv \mathcal{V}_{\rm fin}[a](t) + \frac{R(t)}{48\pi^2}\ln\left[\frac{H(t_e)}{H(t)}\right] \qquad \mathcal{V}_{\rm run}(t) \equiv \frac{\alpha R(t)}{48\pi^2} . \tag{19}$$

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The Technique: Time Evolution Equation

Expressing everything in terms of renormalized contributions, we have

$$\int_{t_i}^{t} dt' \frac{\mathcal{T}_{\rm ren}}{H(t')} = -\frac{1}{2}\int_0^n dn' \left[\frac{d}{dn'} + (3-\epsilon)\right] \frac{d\mathcal{V}_{\rm ren}}{dn'} \equiv -\frac{\mathcal{I}(n)}{16\pi G} \, . \quad (20)$$

The expectation value of the dimensionless field can be expressed in terms of $\mathcal{I}(n)$ as,

$$\left\langle \Omega \middle| \sqrt{8\pi G}\, A(x) \middle| \Omega \right\rangle = \frac{2\sqrt{8\pi G}}{\lambda} \left[\sqrt{1 + \frac{1}{12}\left(\frac{\lambda}{\sqrt{8\pi G}}\right)^2 \mathcal{I}(n)} - 1\right] + \Big(\text{Stochastic acceleration}\Big) \, . \quad (21)$$

Perturbative

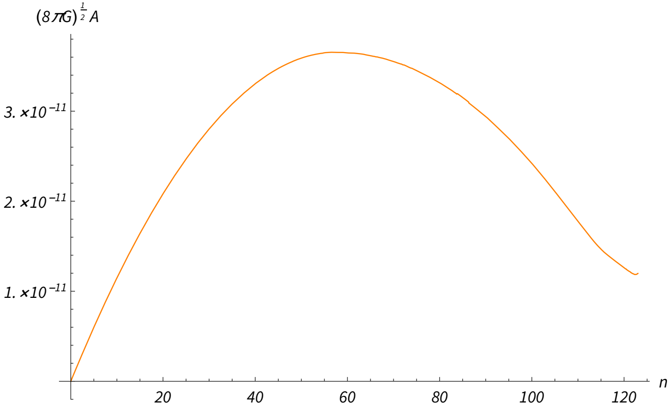

Figure: Evolution of the expectation value of $\sqrt{8\pi G}\,A$ for $\lambda = \frac{1}{10} \times \sqrt{8\pi G}$. [8]

Breakdown of Perturbation

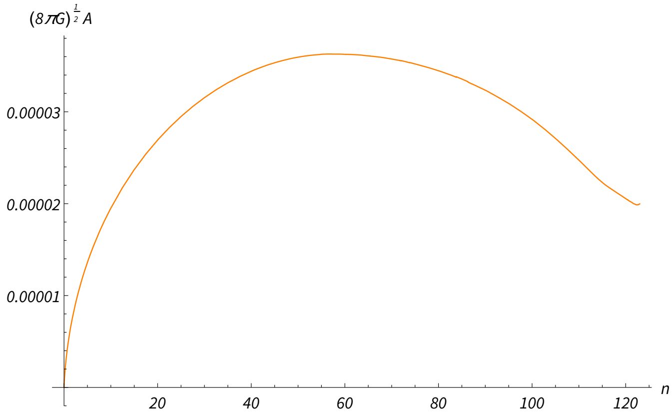

Figure: Evolution of the expectation value of $\sqrt{8\pi G}\, A$ for $\lambda = 10^6 \times \sqrt{8\pi G}$. [8]

Conclusion

On an arbitrary cosmological background (3) which has undergone primordial inflation

- **VEV of A fossilizes** ,i.e , large scales of inflation are transmitted to late times

An arbitrarily large amplitude can be built up provided inflation takes long enough. Yet, diminish in the amplitude during the post-inflationary phase is fixed.

- An arbitrarily large effect can be transmitted to late times

We can extend this technique for quantum gravity on general cosmological background which experiences primordial inflation.

Other Graphs I

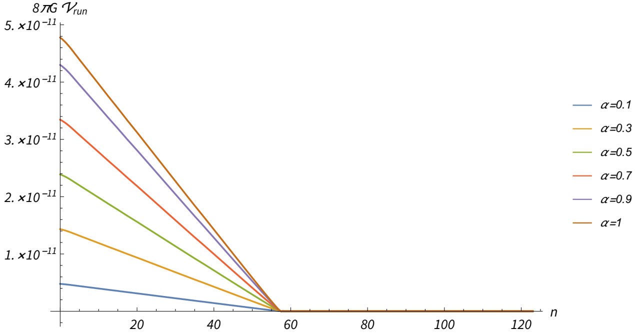

Figure: Evolution of ($8\pi G$ times) the running contribution (19) to the coincident propagator for different values of α. Note that $\mathcal{V}_{\rm run}$ vanishes exactly during radiation domination, and is too small to show up afterwards.

Other Graphs II

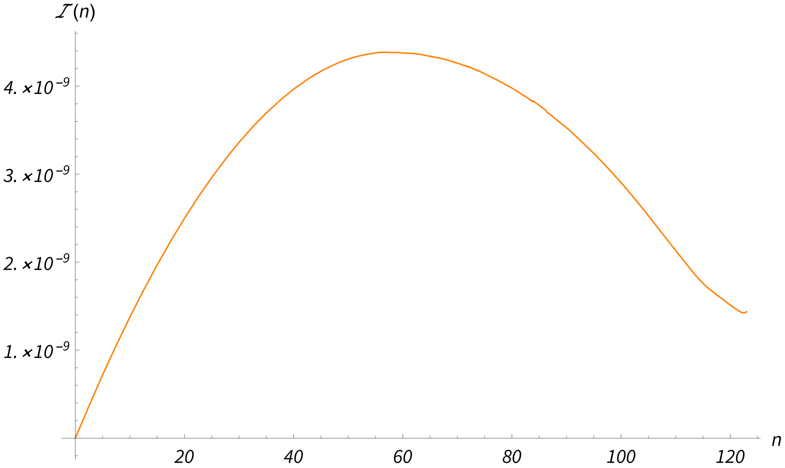

Figure: Evolution of $\mathcal{I}(n)$ from the beginning of primordial inflation to today.

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