Cosmological gravitational particle production of massive spin-2 particles

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Hassan Rosen Bigravity

Two interacting metrics (spin-2 dofs)

$$S=\int \mathrm{d}^4x\Big[\underbrace{\frac{M_g^2}{2}\sqrt{-g}\,R[g]+\frac{M_f^2}{2}\sqrt{-f}\,R[f]}_{\text{Einstein-Hilbert}}\quad\underbrace{-m^2M_*^2\sqrt{-g}\,V(\mathbb{X};\beta_n)}_{\text{interaction between metrics}}$$

$$\underbrace{+\sqrt{-g}\,\mathcal{L}_g(g,\phi_g)+\sqrt{-f}\,\mathcal{L}_f(f,\phi_f)}_{\text{minimal theory}}\quad\underbrace{+\sqrt{-g_\star}\,\mathcal{L}_\star(g_\star,\phi_\star)}_{\text{nonminimal theory}}\Big]\,.$$

Matter sector: the inflatons

Bigravity: mass eigen modes

- Expand two metrics around same background
- Mass eigen modes can be identified: one massless, one massive

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_g} h_{\mu\nu}\,, \qquad f_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_f} k_{\mu\nu}$$

$$\frac{u_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_f} + \frac{k_{\mu\nu}}{M_g}\,, \qquad \frac{v_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_g} - \frac{k_{\mu\nu}}{M_f}$$

Massless spin-2 (GW)

Massive spin-2

Bigravity: spin-2 dofs

- Massless spin-2: 2 dofs (2 tensor modes, +/x)
- Massive spin-2: 2s+1=5 dofs (2 tensor & 2 vector & 1 scalar modes)
- Massless / massive modes decouple at quadratic order

$$S = \int \mathrm{d}^4x \left[\sqrt{-\bar{g}}\, \bar{\mathcal{L}}(\bar{g}, \bar{\phi}) + \sqrt{-\bar{g}}\, \mathcal{L}^{(2)}_{\text{massless}} + \sqrt{-\bar{g}}\, \mathcal{L}^{(2)}_{\text{massive}} + \text{interactions} \right]$$

$$\frac{u_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_f} + \frac{k_{\mu\nu}}{M_g}, \qquad \frac{v_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_g} - \frac{k_{\mu\nu}}{M_f}$$

The “Minimal” theory

- Each inflaton couples to one metric.
- The “massive” and “massless” sector decouples at quadratic order.

	Original field variables	Mass eigenstates
Metric	$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_g} h_{\mu\nu}\,, \quad f_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_f} k_{\mu\nu}$	$\frac{u_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_f} + \frac{k_{\mu\nu}}{M_g}\,, \quad \frac{v_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_g} - \frac{k_{\mu\nu}}{M_f}$
Inflaton	$\phi_g = \bar{\phi}_g + \varphi_g\,, \quad \phi_f = \bar{\phi}_f + \varphi_f$	$\frac{\varphi_u}{M_*} = \frac{\varphi_g}{M_f} + \frac{\varphi_f}{M_g}\,, \quad \frac{\varphi_v}{M_*} = \frac{\varphi_g}{M_g} - \frac{\varphi_f}{M_f}$
Lagrangian	$+\sqrt{-g}\,\mathcal{L}_g(g,\phi_g) + \sqrt{-f}\,\mathcal{L}_f(f,\phi_f)$	$\sqrt{-\bar{g}}\left[\mathcal{L}^{(2)}_{\text{massless}}(u_{\mu\nu},\varphi_u) + \mathcal{L}^{(2)}_{\text{massive}}(v_{\mu\nu},\varphi_v)\right]$

Massive spin-2 Lagrangian ("Minimal" theory)

- Massive spin-2 action: the same as GW action plus a Fierz-Pauli mass term
- Couples with inflaton perturbations.

$$\mathcal{L}^{(2)}_{vv} = -\frac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu}\nabla_\nu v + \frac{1}{2}\nabla_\mu v\nabla^\mu v$$
$$+ \left(\bar{R}_{\mu\nu} - M_P^{-2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(v^{\mu\lambda}v_\lambda{}^\nu - \frac{1}{2}v^{\mu\nu}v\right)$$
$$- \frac{1}{2}m^2\left(v^{\mu\nu}v_{\mu\nu} - v^2\right)$$

Identical to GW action

Fierz-Pauli mass term

$$\mathcal{L}^{(2)}_{v\,\varphi_v} = M_P^{-1}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_v + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_v\right)\left(v^{\mu\nu} - \frac{1}{2}\bar{g}^{\mu\nu}v\right) - V'(\bar{\phi})\varphi_v v\right]$$

$$\mathcal{L}^{(2)}_{\varphi_v\varphi_v} = -\frac{1}{2}\nabla_\mu\varphi_v\nabla^\mu\varphi_v - \frac{1}{2}V''(\bar{\phi})\varphi_v^2$$

Spin-2 couples with inflaton perturbations

Scalar-Vector-Tensor decomposition

- Decompose the metric perturbation into spatial scalars, vectors and tensor.
- Degrees of freedom: 2 for tensor, 2 for vector, 1 for scalar.
- SVT sectors decouple at quadratic order.

$$v_{00} = a^2 E, \quad v_{0i} = a^2(\partial_i F + G_i), \quad v_{ij} = a^2(\delta_{ij} A + \partial_i \partial_j B + \partial_i C_j + \partial_j C_i + D_{ij})$$

⬇

$$\mathcal{L}^{(2)}_{\text{massive}} = \mathcal{L}^{(2)}_{\text{tensor}}(D_{ij}, \cdots) + \mathcal{L}^{(2)}_{\text{vector}}(G_i, C_i, \cdots) + \mathcal{L}^{(2)}_{\text{scalar}}(A, B, E, F, \cdots)$$

SVT decomposition (“Minimal” theory)

- Spin-2 dofs: 2 tensor & 2 vector & 1 scalar (3-tensors, not 4-tensors)
- In the scalar sector, the spin-2 dof (B) is mixed with inflaton perturbation

$$\mathcal{L}^{(2)}_{\mathrm{tensor},\mathbf{k}} = \frac{1}{2}a^2\left[\tilde{D}'_{ij}\tilde{D}'_{ij} - (k^2 + a^2m^2)\tilde{D}_{ij}\tilde{D}_{ij}\right]$$

SVT decomposition (“Minimal” theory)

- Spin-2 dofs: 2 tensor & 2 vector & 1 scalar (3-tensors, not 4-tensors)
- In the scalar sector, the spin-2 dof (B) is mixed with inflaton perturbation

$$\mathcal{L}^{(2)}_{\mathrm{tensor},\mathbf{k}} = \frac{1}{2}a^2\left[\tilde{D}'_{ij}\tilde{D}'_{ij} - (k^2 + a^2m^2)\tilde{D}_{ij}\tilde{D}_{ij}\right]$$

$$\mathcal{L}^{(2)}_{\mathrm{vector},\mathbf{k}} = \frac{a^4k^2m^2}{k^2+a^2m^2}|\tilde{C}'_i|^2 - a^4k^2m^2|\tilde{C}_i|^2$$

SVT decomposition (“Minimal” theory)

- Spin-2 dofs: 2 tensor & 2 vector & 1 scalar (3-tensors, not 4-tensors)
- In the scalar sector, the spin-2 dof (B) is mixed with inflaton perturbation

$$\mathcal{L}^{(2)}_{\mathrm{tensor},\mathbf{k}} = \frac{1}{2}a^2\left[\tilde{D}'_{ij}\tilde{D}'_{ij} - (k^2 + a^2m^2)\tilde{D}_{ij}\tilde{D}_{ij}\right]$$

$$\mathcal{L}^{(2)}_{\mathrm{vector},\mathbf{k}} = \frac{a^4k^2m^2}{k^2 + a^2m^2}|\tilde{C}'_i|^2 - a^4k^2m^2|\tilde{C}_i|^2$$

$$\mathcal{L}^{(2)}_{\mathrm{scalar},\mathbf{k}} = K_\varphi\,|\tilde{\hat{\varphi}}'_v|^2 - M_\varphi\,|\tilde{\hat{\varphi}}_v|^2 + K_B\,|\tilde{B}'|^2 - M_B\,|\tilde{B}|^2 + L_2\,\tilde{\hat{\varphi}}^{*\prime}_v\tilde{B}' + L_1\,\tilde{\hat{\varphi}}^*_v\tilde{B}' - L_0\,\tilde{\hat{\varphi}}^*_v\tilde{B}$$

Scalar mode coupled with inflaton

$$K_\varphi = \frac{a^2}{2}\frac{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{9}{4}a^4m^2\left(m^2 - m_H^2\right)H^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17a}$$

$$M_\varphi = \frac{a^2}{2}\frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2 + c_0}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17b}$$

$$\begin{aligned}
c_{10} &= H^4 \\
c_8 &= \tfrac{1}{2}a^2H^2\Big[\left(12m^2H^2 + 8H^4 - 14H^2m_H^2 - m_H^4\right) + 4\frac{HV'(\bar\phi)\bar\phi'}{aM_P^2} + 2H^2V''(\bar\phi)\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(36m^4H^2 + 72m^2H^4 - 82m^2H^2m_H^2 - 64H^4m_H^2 \\
&\qquad - 7m^2m_H^4 + 40H^2m_H^4 + 8m_H^6\big) \\
&\qquad + 8\left(3m^2 - 4m_H^2\right)\frac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 16\left(m^2 - m_H^2\right)H^2V''(\bar\phi)\Big] \\
c_4 &= \tfrac{3}{8}a^6\Big[4H^2\big(9m^6H^2 + 36m^4H^4 + 16m^2H^6 - 30m^4H^2m_H^2 - 76m^2H^4m_H^2 \\
&\qquad - 3m^4m_H^4 + 31m^2H^2m_H^4 + 24H^4m_H^4 + 6m^2m_H^6 - 6H^2m_H^6 - 3m_H^8\big) \\
&\qquad - 4m^2H^2\left(H^2 - m_H^2\right)\frac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + \left(36m^4H^2 + 8m^2H^4 - 94m^2H^2m_H^2 + m^2m_H^4 + 48H^2m_H^4\right)\frac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \left(36m^4H^2 - 58m^2H^2m_H^2 - m^2m_H^4 + 24H^2m_H^4\right)H^2V''(\bar\phi)\Big] \\
c_2 &= \tfrac{9}{32}a^8m^2\Big[H^2\big(18m^6H^2 + 120m^4H^4 + 128m^2H^6 - 78m^4H^2m_H^2 - 384m^2H^4m_H^2 \\
&\qquad - 9m^4m_H^4 + 132m^2H^2m_H^4 + 128H^4m_H^4 + 23m^2m_H^6 - 32H^2m_H^6 - 16m_H^8\big) \\
&\qquad - 8H^2\left(2m^2H^2 - 2m^2m_H^2 + m_H^4\right)\frac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + 4\left(6m^4H^2 - 22m^2H^2m_H^2 + m^2m_H^4 + 14H^2m_H^4\right)\frac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 4\left(m^2 - m_H^2\right)\left(12m^2H^2 - 10H^2m_H^2 - m_H^4\right)H^2V''(\bar\phi)\Big] \\
c_0 &= \tfrac{27}{32}a^{10}m^4\Big[-2H^2\left(2m^2H^2 - 2m^2m_H^2 + m_H^4\right)\frac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad - m^2\left(2H^2 - m_H^2\right)\left(4H^2 + m_H^2\right)\frac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \left(m^2 - m_H^2\right)\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)H^2V''(\bar\phi)\Big]
\end{aligned}$$

$$K_B = \frac{a^6m^2}{8}\frac{\left(8m^2H^2 - 6H^2m_H^2 - m^2m_H^2\right)k^4}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17c}$$

$$M_B = \frac{a^6m^2}{8}\frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17d}$$

$$\begin{aligned}
c_{10} &= H^2\left(8m^2H^2 - 8H^4 - 2H^2m_H^2 - m^2m_H^2\right) \\
c_8 &= a^2H^2\Big[\big(30m^4H^2 + 32m^2H^4 - 96H^6 - 3m^4m_H^2 - 56m^2H^2m_H^2 \\
&\qquad + 48H^4m_H^2 + 5m^2m_H^4 + 6H^2m_H^4\big) \\
&\qquad + \left(4m^2 - 24H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_6 &= \tfrac{3}{8}a^4m^2\Big[\big(96m^4H^4 + 144m^2H^6 - 6m^4H^2m_H^2 - 252m^2H^4m_H^2 - 192H^6m_H^2 \\
&\qquad + 8m^2H^2m_H^4 + 200H^4m_H^4 - 10H^2m_H^6 - m^2m_H^6\big) \\
&\qquad + \left(8m^2m_H^2 - 16H^2m_H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_4 &= \tfrac{3}{8}a^6m^4\Big[\big(36m^4H^4 - 48m^2H^6 + 64H^8 - 12m^2H^4m_H^2 - 32H^6m_H^2 \\
&\qquad - 12m^2H^2m_H^4 + 4H^4m_H^4 + 12H^2m_H^6 - 3m^2m_H^6 + 2m_H^8\big) \\
&\qquad - \left(24m^2H^2 - 16H^4 - 12m^2m_H^2 - 8H^2m_H^2 + 8m_H^4\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big]
\end{aligned}$$

$$L_2 = \frac{a^3m^2\bar\phi'}{2M_PH}\frac{H^2k^4 + \frac{3}{2}a^2\left(m^2 - m_H^2\right)H^2k^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17e}$$

$$L_1 = -\frac{a^4m^2\bar\phi'}{M_P}\frac{\left(H^2 - \frac{1}{4}m_H^2 - \frac{1}{2}\frac{aHV'(\bar\phi)}{\phi'}\right)k^4 - \frac{3}{2}a^2\left(m^2 - m_H^2\right)\left(H^2 + \frac{1}{4}m_H^2 + \frac{1}{2}\frac{aHV'(\bar\phi)}{\phi'}\right)k^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17f}$$

$$L_0 = \frac{a^3m^2\bar\phi'}{2M_PH}\frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17g}$$

$$\begin{aligned}
c_{10} &= H^4 \\
c_8 &= \tfrac{1}{2}a^2H^4\Big[\left(9m^2 + 12H^2 - 13m_H^2\right) - 4\tfrac{aHV'(\bar\phi)}{\phi'}\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(18m^4H^2 + 32m^2H^4 + 64H^6 - 48m^2H^2m_H^2 - 64H^4m_H^2 \\
&\qquad + m^2m_H^4 + 28H^2m_H^4\big) \\
&\qquad + 8\left(-4m^2H^2 + 4H^4 + m^2m_H^2\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_4 &= \tfrac{3}{16}a^6m^2H^2\Big[\big(18m^4H^2 - 24m^2H^4 + 256H^6 - 54m^2H^2m_H^2 - 160H^4m_H^2 \\
&\qquad + 9m^2m_H^4 + 60H^2m_H^4 - 7m_H^6\big) \\
&\qquad + 4\left(-30m^2H^2 + 32H^4 + 12m^2m_H^2 + 4H^2m_H^2 - 7m_H^4\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_2 &= \tfrac{9}{16}a^8m^4H^2\left(2H^2 - m_H^2\right)\Big[-\left(4H^2 + m_H^2\right)\left(3m^2 - 4H^2 - m_H^2\right) \\
&\qquad + 4\left(-3m^2 + 2H^2 + 2m_H^2\right)\tfrac{aHV'(\bar\phi)}{\phi'}\Big]
\end{aligned}$$

Scalar sector detail ("Minimal" theory)

- Summary of transformations in the scalar sector

$$\mathcal{L}^{(2)}_{\text{scalar}}(A, B, E, F, \varphi_v)$$

$$\xrightarrow{\text{eliminate constraints}} \mathcal{L}^{(2)}_{\text{scalar}}(B, \hat{\varphi}_v)$$

$$\xrightarrow{\text{eliminate kinetic mixing}} \mathcal{L}^{(2)}_{\text{scalar}}(\mathcal{B}, \Pi)$$

$$\mathcal{L}^{(2)}_{\text{scalar},\mathbf{k}} = K_\Pi\,|\tilde{\Pi}'|^2 - M_\Pi\,|\tilde{\Pi}|^2 + K_\mathcal{B}\,|\tilde{\mathcal{B}}'|^2 - M_\mathcal{B}\,|\tilde{\mathcal{B}}|^2 + \lambda_1\,\tilde{\Pi}^*\tilde{\mathcal{B}}' - \lambda_0\,\tilde{\Pi}^*\tilde{\mathcal{B}}$$

$$K_\Pi = K_\varphi = \frac{a^2}{2}\,\frac{H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{9}{4}a^4m^2(m^2 - m_H^2)H^2}{H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)}$$

$$K_\mathcal{B} = \frac{4K_\varphi K_B - L_2^2}{4k^4K_\varphi} = \frac{3a^6m^2(m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4m^2(m^2 - m_H^2)}\,,$$

Summary

- Superheavy (> inflationary scale) spin-2 particles can be gravitationally produced during inflation.
- In Hassan Rosen bigravity with “minimal” matter coupling, production in the scalar sector dominates.
- Aside: We find a generalized Higuchi bound for the massive spin-2 in FRW.

$$m^2 > m_H^2(\eta) = 2H^2\left[1 - \epsilon\right]$$

Extra slides

Generalized Higuchi bound (“Minimal” theory)

- The theory has a lower bound on mass m, below which the scalar sector constains a ghost with wrong sign kinetic term
- Higuchi: derived the bound for de-Sitter (the Higuchi bound)
- This work: derived the bound for FRW
- Implies an inflationary scale lower bound for spin-2 mass!

$$m^2 > m_H^2(\eta) = 2H^2\left[1-\epsilon\right] \quad \text{where} \quad \epsilon \equiv -H'/(aH^2)$$

$$K_{\mathcal{B}} = \frac{4K_\varphi K_B - L_2^2}{4k^4 K_\varphi} = \frac{3a^6 m^2 (m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4 m^2 (m^2 - m_H^2)}$$

A. Higuchi (1987), *Forbidden mass range for spin-2 field theory in de Sitter spacetime*

Scalar sector detail (“Minimal” theory)

- The kinetic mixing in the scalar sector can be eliminated by a change of variable

$$\mathcal{L}^{(2)}_{\mathrm{scalar},\mathbf{k}} = K_\varphi\,|\tilde{\hat{\varphi}}_v'|^2 - M_\varphi\,|\tilde{\hat{\varphi}}_v|^2 + K_B\,|\tilde{B}'|^2 - M_B\,|\tilde{B}|^2 + L_2\,\tilde{\hat{\varphi}}_v^{*\prime}\tilde{B}' + L_1\,\tilde{\hat{\varphi}}_v^{*}\tilde{B}' - L_0\,\tilde{\hat{\varphi}}_v^{*}\tilde{B}$$

$$\mathcal{L}^{(2)}_{\mathrm{scalar},\mathbf{k}} = K_\Pi\,|\tilde{\Pi}'|^2 - M_\Pi\,|\tilde{\Pi}|^2 + K_{\mathcal{B}}\,|\tilde{\mathcal{B}}'|^2 - M_{\mathcal{B}}\,|\tilde{\mathcal{B}}|^2 + \lambda_1\,\tilde{\Pi}^*\tilde{\mathcal{B}}' - \lambda_0\,\tilde{\Pi}^*\tilde{\mathcal{B}}$$

$$\tilde{\hat{\varphi}}_v = \tilde{\Pi} + \kappa(\eta)\,\tilde{\mathcal{B}} \qquad \text{and} \qquad \tilde{B} = k^{-2}\tilde{\mathcal{B}}$$

$$\kappa(\eta) = -\frac{L_2}{2k^2K_\varphi}$$

A change of variable decouples the two at early/late times

Gravitational particle production

Cosmological expansion → Time-dependent Lagrangian → Time-dependent effective frequency → Particle production

$$\mathrm{d}s^2 = a(\eta)^2(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2) \quad \Rightarrow \quad \mathcal{L} = a^2\left[\frac{1}{2}(\partial_\eta\phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}a^2m^2\phi^2\right]$$

$$|\beta_k|^2 = \frac{\omega_k\left|a\tilde{\phi}_k\right|^2}{2} + \frac{\left|\partial_\eta(a\tilde{\phi}_k)\right|^2}{2\omega_k} - \frac{1}{2} \quad \Leftarrow \quad [\partial_\eta^2 - \nabla^2 + a^2(\eta)m_{\mathrm{eff}}^2(\eta)](a\phi) = 0$$

$$n_k(\eta) = a(\eta)^{-3}\frac{k^3}{2\pi^2}|\beta_k|^2$$

particle production!

Birrell & Davies (1982), Parker & Toms (2009), L H Ford (2021), ...

The "Nonminimal" theory

- The inflaton couples the "effective" metric.
- The "massive" sector only contain the massive spin-2 field.

	Original field variables	Mass eigenstates
Metric	$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_g} h_{\mu\nu}\,, \quad f_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2}{M_f} k_{\mu\nu}$	$\frac{u_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_f} + \frac{k_{\mu\nu}}{M_g}\,, \quad \frac{v_{\mu\nu}}{M_*} = \frac{h_{\mu\nu}}{M_g} - \frac{k_{\mu\nu}}{M_f}$
Inflaton	$\phi_\star = \bar{\phi}_\star + \varphi_\star$	
Lagrangian	$+\sqrt{-g_\star}\,\mathcal{L}_\star(g_\star, \phi_\star)$	$\sqrt{-\bar{g}}\left[\mathcal{L}^{(2)}_{\text{massless}}(u_{\mu\nu}, \varphi_\star) + \mathcal{L}^{(2)}_{\text{massive}}(v_{\mu\nu})\right]$

$$(g_\star)_{\mu\nu} = \frac{a^2}{(a+b)^2} g_{\mu\nu} + \frac{ab}{(a+b)^2}\left(g_{\mu\lambda}\left(\sqrt{g^{-1}f}\right)^{\lambda}{}_{\nu} + \left(\sqrt{g^{-1}f}\right)_{\mu}{}^{\lambda} g_{\lambda\nu}\right) + \frac{b^2}{(a+b)^2} f_{\mu\nu}$$

Massive spin-2 Lagrangian (“Nonminimal” theory)

- Massive spin-2 action: contains a Fierz-Pauli mass term
- Contains terms that depend on ϕ potential. Gives “wrong” de-Sitter limit.

$$\mathcal{L}^{(2)}_{vv} = -\frac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu}\nabla_\nu v + \frac{1}{2}\nabla_\mu v \nabla^\mu v$$
$$+ \left(\bar{R}_{\mu\nu} + \frac{1}{2}M_P^{-2}(\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi} + \bar{g}_{\mu\nu}\bar{\mathcal{L}}(\bar{g},\bar{\phi}))\right)v^{\mu\lambda}v_\lambda{}^\nu$$
$$- \frac{1}{2}\left(\bar{R}_{\mu\nu} + M_P^{-2}(\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi} + \bar{g}_{\mu\nu}\bar{\mathcal{L}}(\bar{g},\bar{\phi}))\right)v^{\mu\nu}v$$
$$- \frac{1}{2}m^2(v_{\mu\nu}v^{\mu\nu} - v^2)$$

Distinct from GW action

Fierz-Pauli mass term

SVT decomposition ("Nonminimal" theory)

- Spin-2 dofs: 2 tensor & 2 vector & 1 scalar
- The scalar sector now contains only one d.o.f.

$$\mathcal{L}^{(2)}_{\text{tensor},\mathbf{k}} = \frac{1}{2}a^2\left[\tilde{D}'_{ij}\tilde{D}'_{ij} - \left(k^2 + a^2\mu_2^2\right)\tilde{D}_{ij}\tilde{D}_{ij}\right]$$

$$\mathcal{L}^{(2)}_{\text{vector},\mathbf{k}} = \frac{a^4k^2\mu_1^2}{k^2 + a^2\mu_1^2}|\tilde{C}'_i|^2 - a^4k^2\mu_2^2|\tilde{C}_i|^2$$

$$\mathcal{L}^{(2)}_{\text{scalar},\mathbf{k}} = K_B\,|\tilde{B}'|^2 - M_B\,|\tilde{B}|^2$$

$$\mu_1^2 = m^2 - \Lambda + 3H^2 - a^{-1}H'$$

$$\mu_2^2 = m^2 - \Lambda + 3H^2 + 2a^{-1}H'$$

SVT

- Spin-2 dof
- In the scal

$$\mathcal{L}^{(2)}_{\text{tensor},\mathbf{k}} = \frac{1}{2}$$

$$\mathcal{L}^{(2)}_{\text{vector},\mathbf{k}} = \frac{}{k}$$

$$\mathcal{L}^{(2)}_{\text{scalar},\mathbf{k}} = K$$

$$\mu_1^2 = m^2$$

$$\mu_2^2 = m^2$$

$$K_B = \frac{a^4}{P}\left[c_6 k^6 + c_4 k^4\right] \tag{3.46a}$$

$$c_6 = -4\dot{H}^2$$

$$c_4 = 3a^2\left(m^2+H^2\right)\left(m^2+3H^2-\dot{H}\right)\left(m^2+3H^2+2\dot{H}\right)$$

$$M_B = \frac{a^6}{P^2}\left[c_{10}k^{10} + c_8 k^8 + c_6 k^6 + c_4 k^4\right] \tag{3.46b}$$

$$\begin{aligned} c_{10} = {} & 12\left(m^2+H^2\right)\left(m^2+3H^2\right)^3 + 16\left(m^2+3H^2\right)^2\left(6m^2+7H^2\right)\dot{H} \\ & + 4\left(m^2+3H^2\right)\left(63m^2+71H^2\right)\dot{H}^2 + 8\left(25m^2+27H^2\right)\dot{H}^3 - 48\dot{H}^4 \\ & - 32H\left(m^2+3H^2\right)\dot{H}\ddot{H} - 48H\dot{H}^2\ddot{H} \end{aligned}$$

$$\begin{aligned} c_8 = {} & 12a^2\left(m^2+3H^2+2\dot{H}\right) \times \Big[2\left(m^2+H^2\right)\left(m^2+3H^2\right)^2\left(2m^2+5H^2\right) \\ & + \left(m^2+3H^2\right)\left(19m^4+64m^2H^2+49H^4\right)\dot{H} + 2\left(7m^4+20m^2H^2+17H^4\right)\dot{H}^2 \\ & - \left(23m^2+25H^2\right)\dot{H}^3 + 2\dot{H}^4 - 2H\left(m^2+H^2\right)\left(m^2+3H^2\right)\ddot{H} \\ & - 4H\left(m^2+H^2\right)\dot{H}\ddot{H}\Big] \end{aligned}$$

$$\begin{aligned} c_6 = {} & 9a^4\left(m^2+H^2\right)\left(m^2+3H^2-\dot{H}\right)\left(m^2+3H^2+2\dot{H}\right)^2 \\ & \times \left[7\left(m^2+H^2\right)\left(m^2+3H^2\right) + 17\left(m^2+H^2\right)\dot{H} - 8\dot{H}^2\right] \end{aligned}$$

$$c_4 = 27a^6\left(m^2+H^2\right)^2\left(m^2+3H^2-\dot{H}\right)^2\left(m^2+3H^2+2\dot{H}\right)^3$$

where $\dot{H} \equiv a^{-1}H'$, $\ddot{H} \equiv a^{-2}H'' - a^{-1}HH'$, and

$$\begin{aligned} P = {} & 4\left[m^2+3H^2+3\dot{H}\right]k^4 \\ & + 12a^2\left[\left(m^2+H^2\right)\left(m^2+3H^2\right) + 2\left(m^2+H^2\right)\dot{H} - \dot{H}^2\right]k^2 \\ & + 9a^4\left(m^2+H^2\right)\left(m^2+3H^2-\dot{H}\right)\left(m^2+3H^2+2\dot{H}\right) . \end{aligned} \tag{3.47}$$

Gradient instability ("Nonminimal" theory, vector sector)

- If the squared effective sound speed (c_s^2) is negative at any time, the mode function blows up at high-k due to a period of exponential growth.
- This scenario usually happens around end of inflation, when Hubble-prime is large enough.

$$\tilde{\chi}_r'' \approx -c_s^2 k^2 \tilde{\chi}_r \quad \Rightarrow \quad \tilde{\chi}_r \propto \exp\left[\pm \int^{\eta} \mathrm{d}\eta'\, k\sqrt{-c_s^2}\right]$$

$$c_s^2 \propto m^2 - \Lambda + 3H^2 + 2a^{-1}H'$$

Ghost instability ("Nonminimal" theory, scalar sector)

- The kinetic term can momentarily vanish for small enough mass.
- Effectively sets a UV cutoff p_max for physical momentum:

$$p_{\max}(\eta) = \sqrt{\frac{3}{4}} H \frac{\sqrt{m^2/H^2+1}\,\sqrt{m^2/H^2+3+\epsilon}\,\sqrt{m^2/H^2+3-2\epsilon}}{|\epsilon|}$$

$$
\begin{aligned}
\mathcal{L}_{v\ \mathrm{decay}} = M_{\mathrm{P}}^{-2}\Big[&2v_{\mu\nu}u^{\mu\nu}(\nabla_\lambda\varphi_v)(\nabla^\lambda\bar{\phi}) - 4v_{\mu\nu}u^{\nu}{}_{\lambda}(\nabla^\mu\bar{\phi})(\nabla^\lambda\varphi_v) \\
&+ 2vu_{\mu\nu}(\nabla^\mu\bar{\phi})(\nabla^\nu\varphi_v) - 4v_{\mu\nu}u^{\nu}{}_{\lambda}(\nabla^\lambda\bar{\phi})(\nabla^\mu\varphi_v) + 2v_{\mu\nu}u^{\mu\nu}\varphi_v V'(\bar{\phi})\Big] \\
+ M_{\mathrm{P}}^{-1}\Big[&2v_{\mu\nu}(\nabla^\mu\varphi_u)(\nabla^\nu\varphi_v) - v(\nabla_\mu\varphi_u)(\nabla^\mu\varphi_v) - v\varphi_u\varphi_v V''(\bar{\phi})\Big] \\
+ (&v\varphi_v\varphi_v\ \text{terms})\,,
\end{aligned}
$$

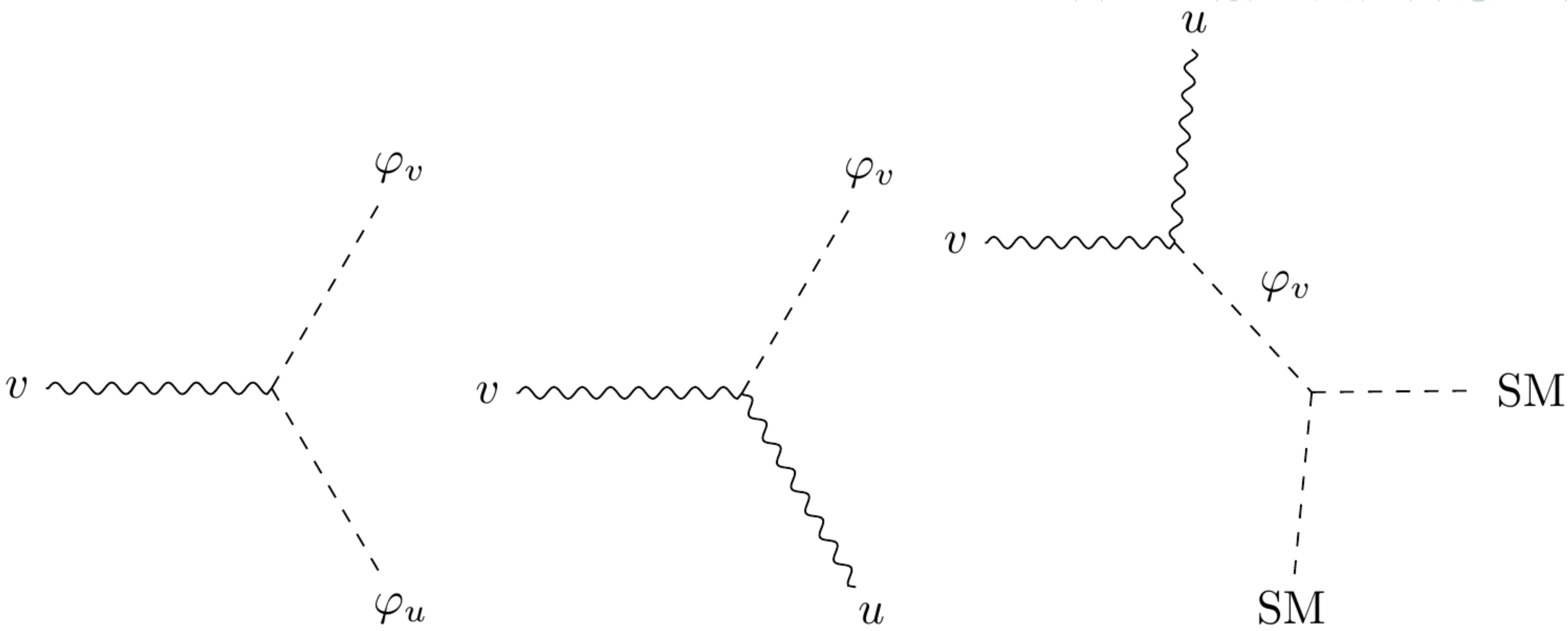

Figure 9. Direct and indirect decay of $v_{\mu\nu}$ in the minimally-coupled theory.

$$\Gamma \sim \left(\frac{m^2}{M_{\rm P}}\right)^2 \frac{1}{m} \sim \frac{m^3}{M_{\rm P}^2}$$

$$V(\mathbb{X};\beta_n) \equiv \sum_{n=0}^{4} \beta_n S_n(\mathbb{X})\,, \quad S_n(\mathbb{X}) \equiv \mathbb{X}^{\mu_1}{}_{[\mu_1} \ldots \mathbb{X}^{\mu_n}{}_{\mu_n]}\,, \quad \mathbb{X}^{\mu}{}_{\sigma}\mathbb{X}^{\sigma}{}_{\nu} \equiv g^{\mu\lambda} f_{\lambda\nu}$$