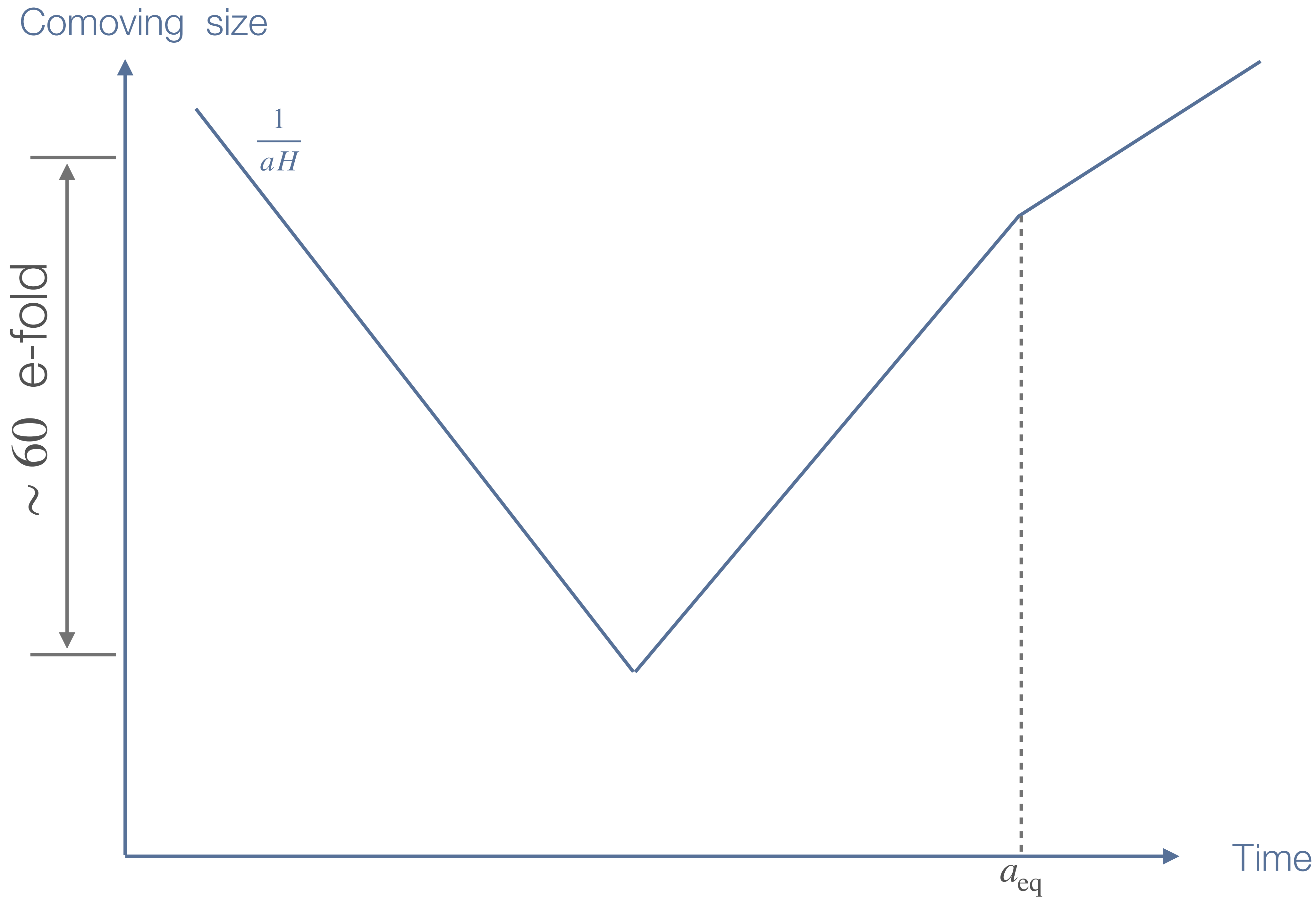
Gravitational wave signals from the inflationary era

LianTao Wang
Univ. of Chicago

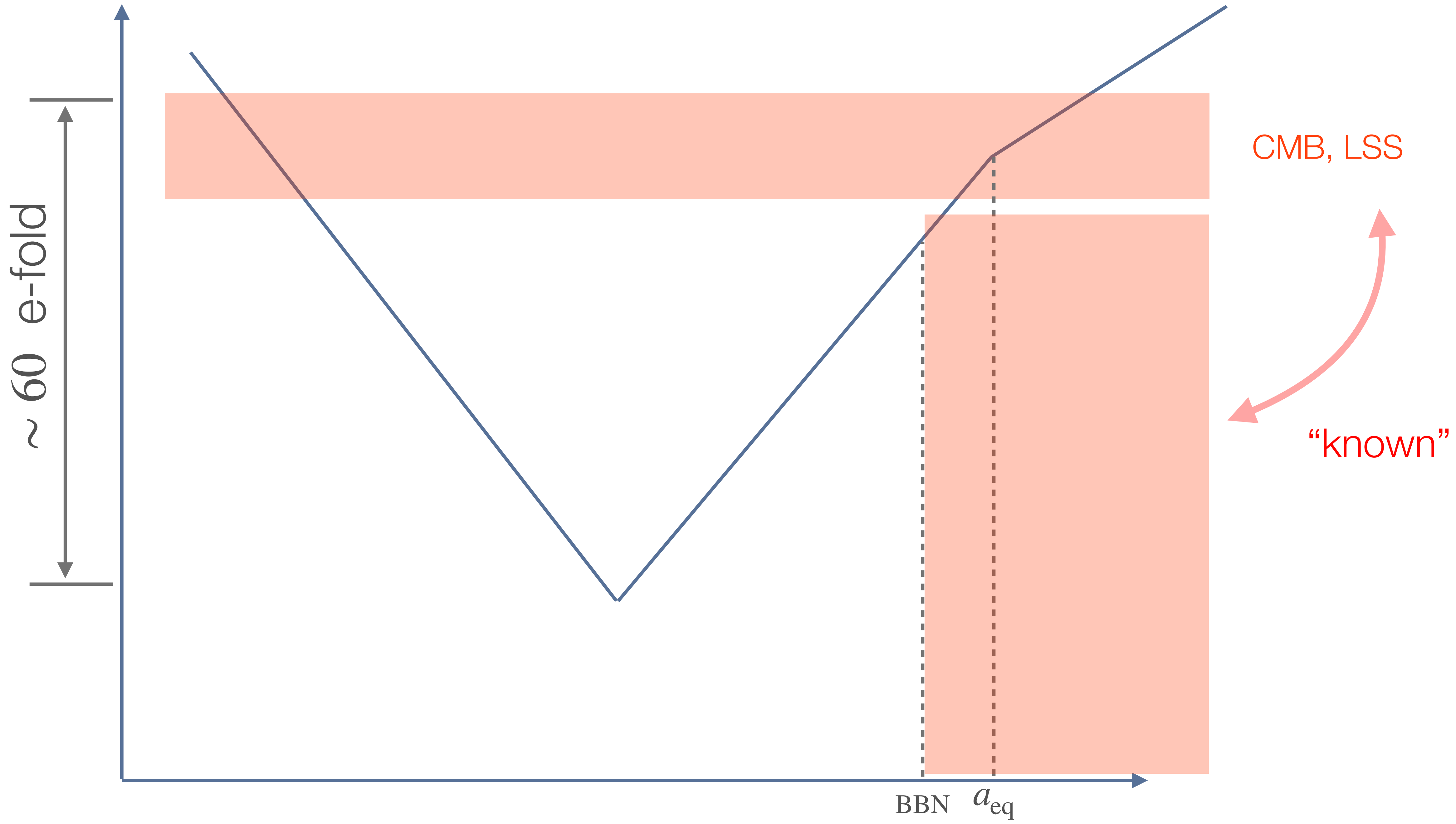
Work in collaboration with Haipeng An, Chen Yang, KunFeng Lyu, Boye Su, Siyi Zhou, Hanwen Tai 2009.12381, 2201.05171, and 2308.00070
with Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, 2307.12048

Workshop “The early universe, a window to new physics”. U. Florida. Oct 20, 2023

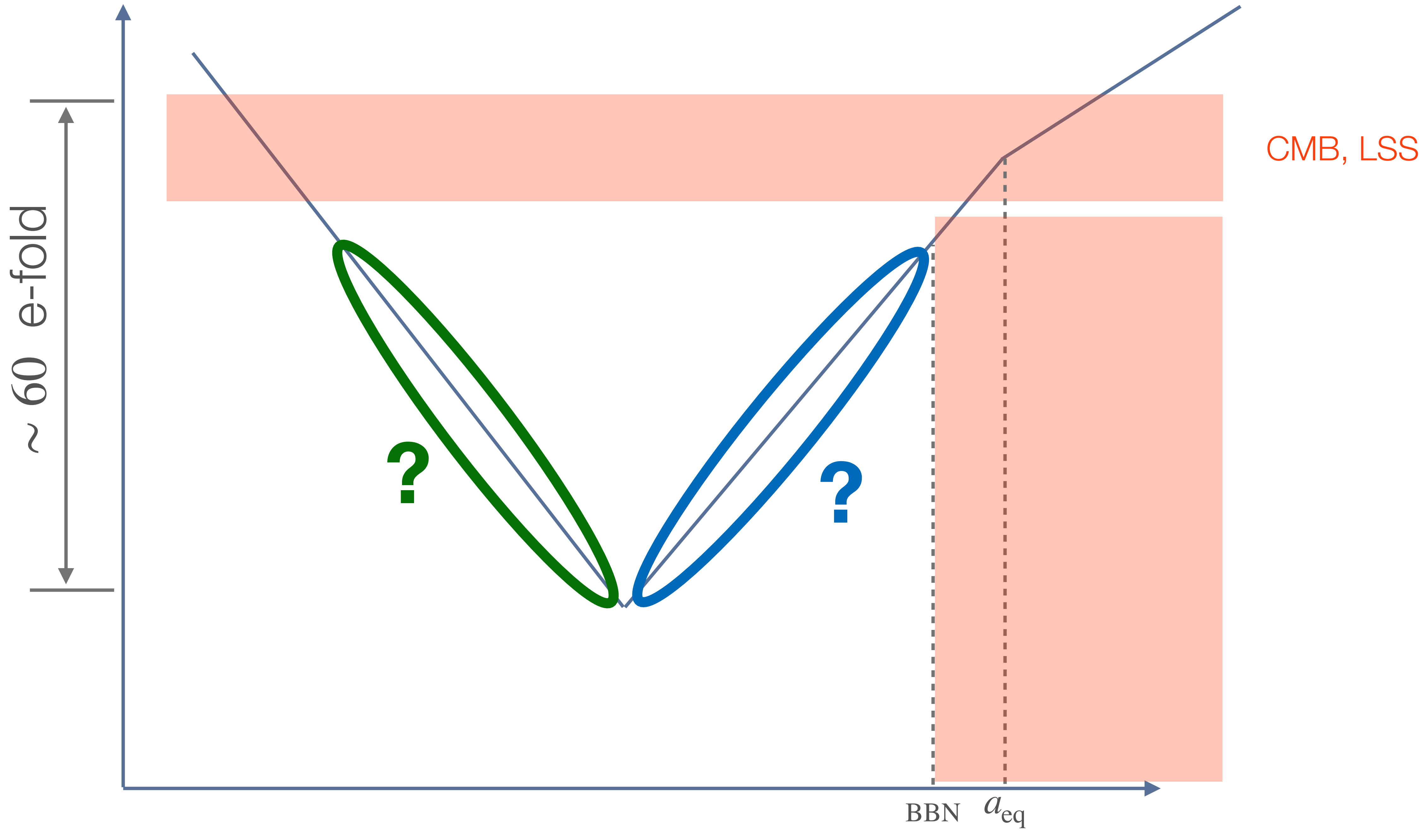
Early universe



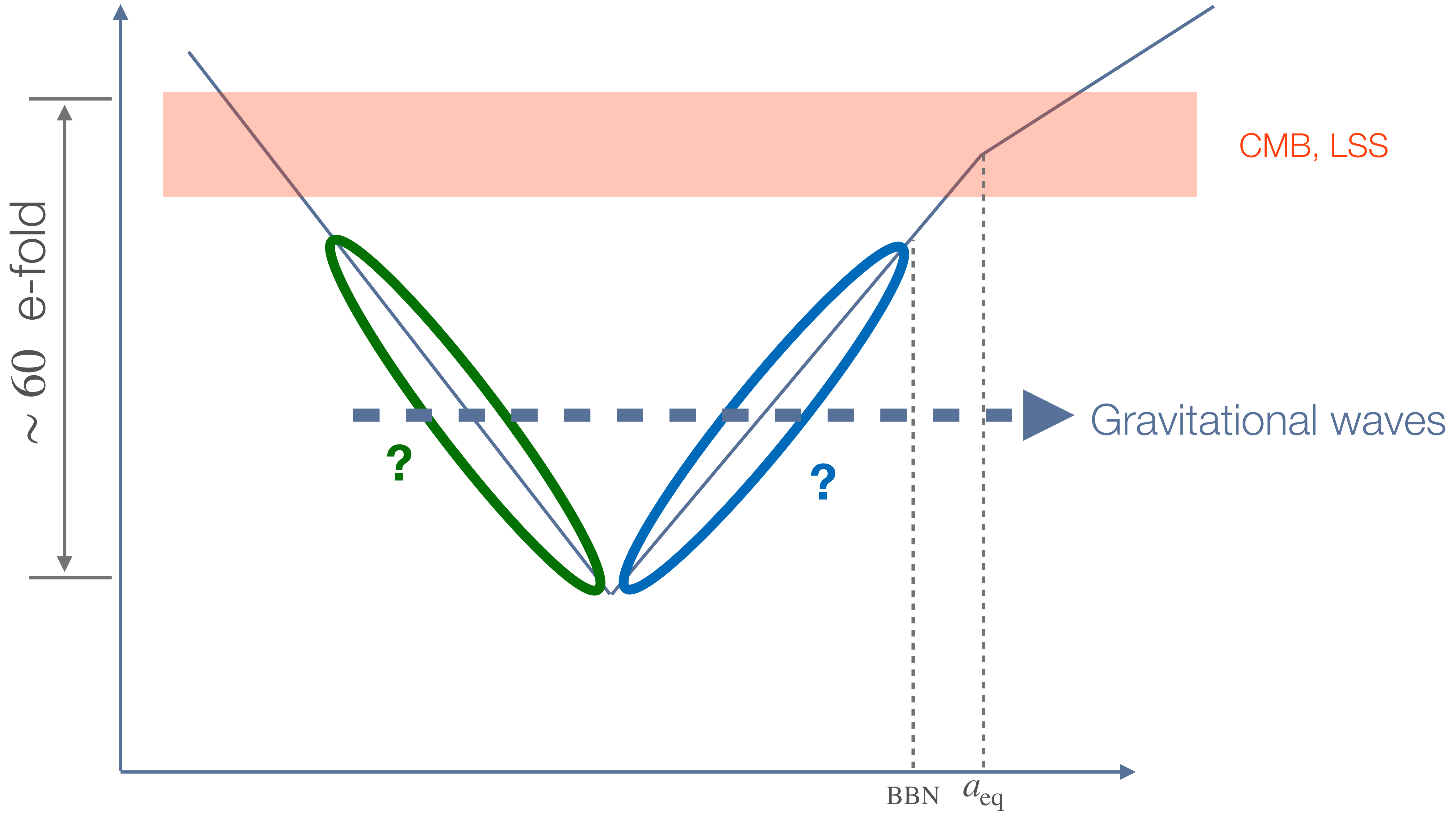
Early universe



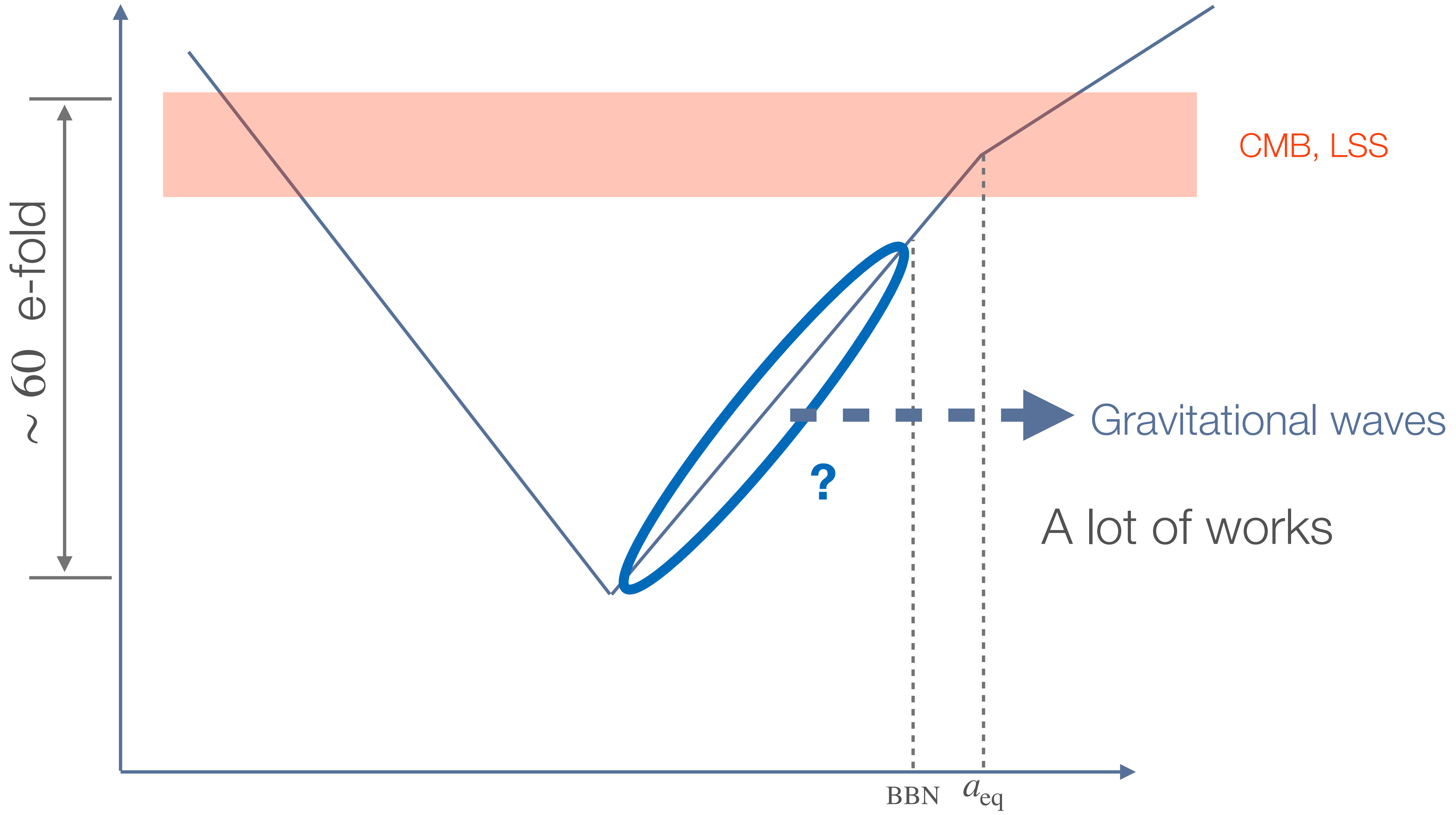
Early universe



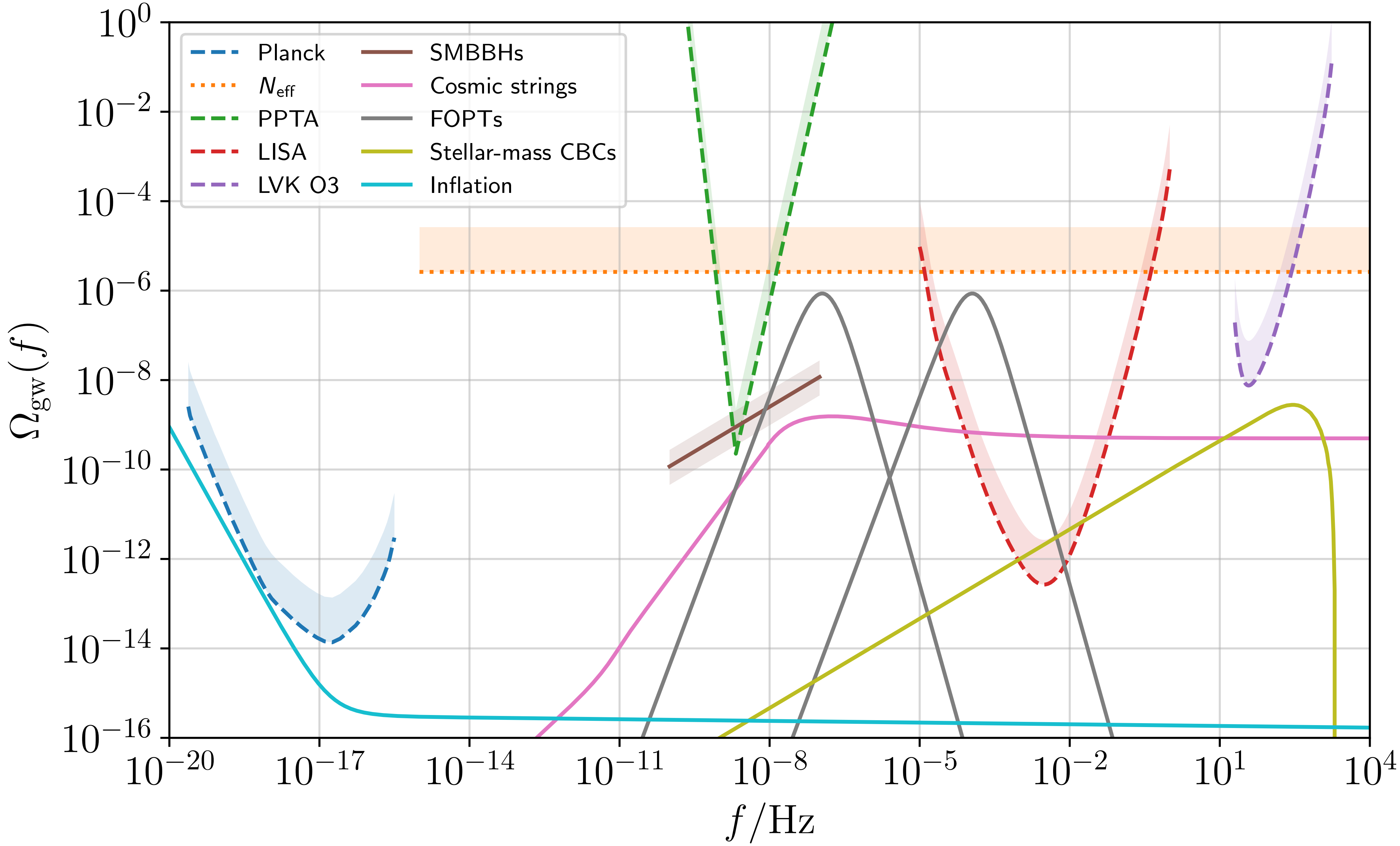
Early universe



Early universe

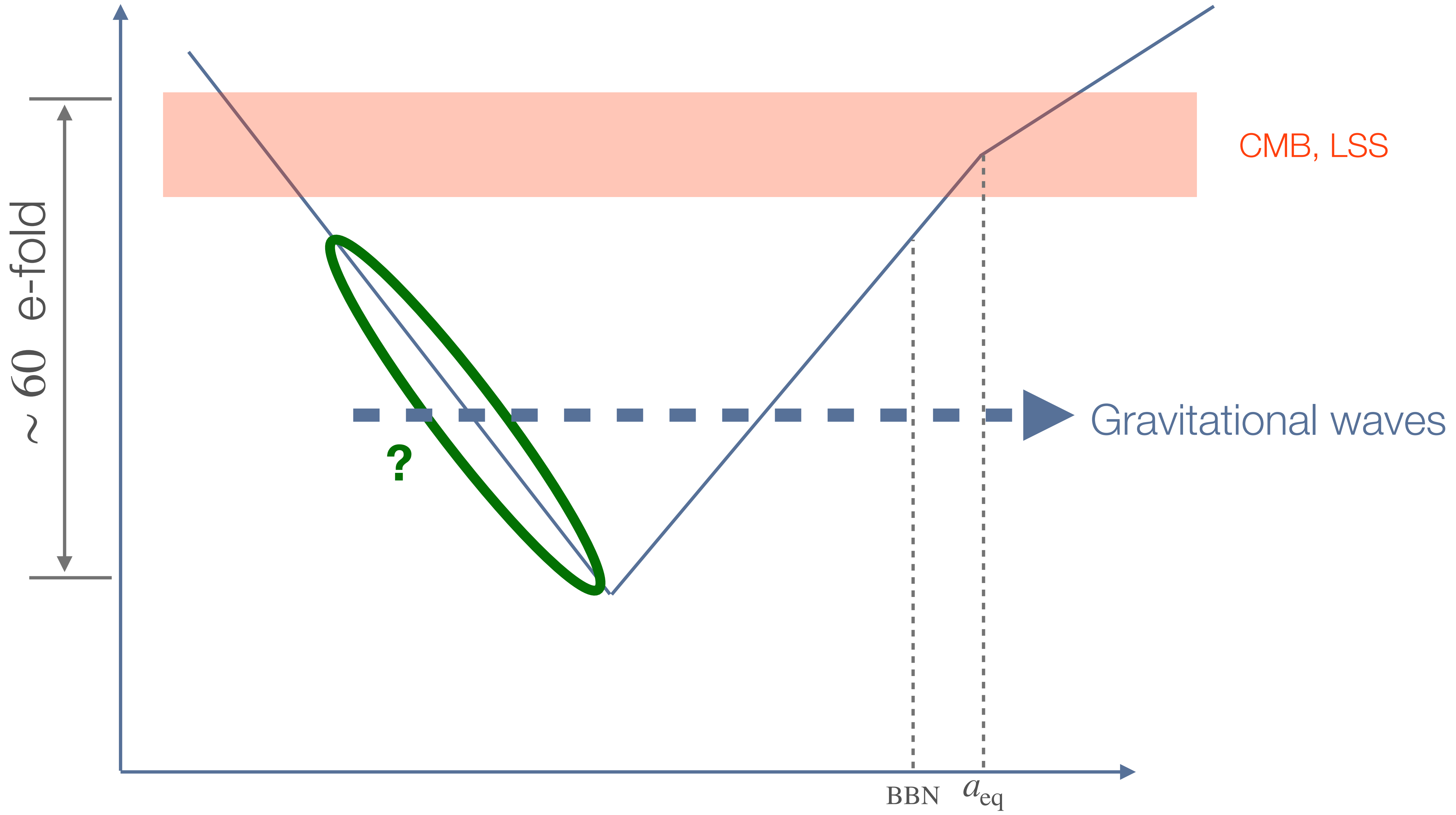


Review by Renzini et al, 2202.00178

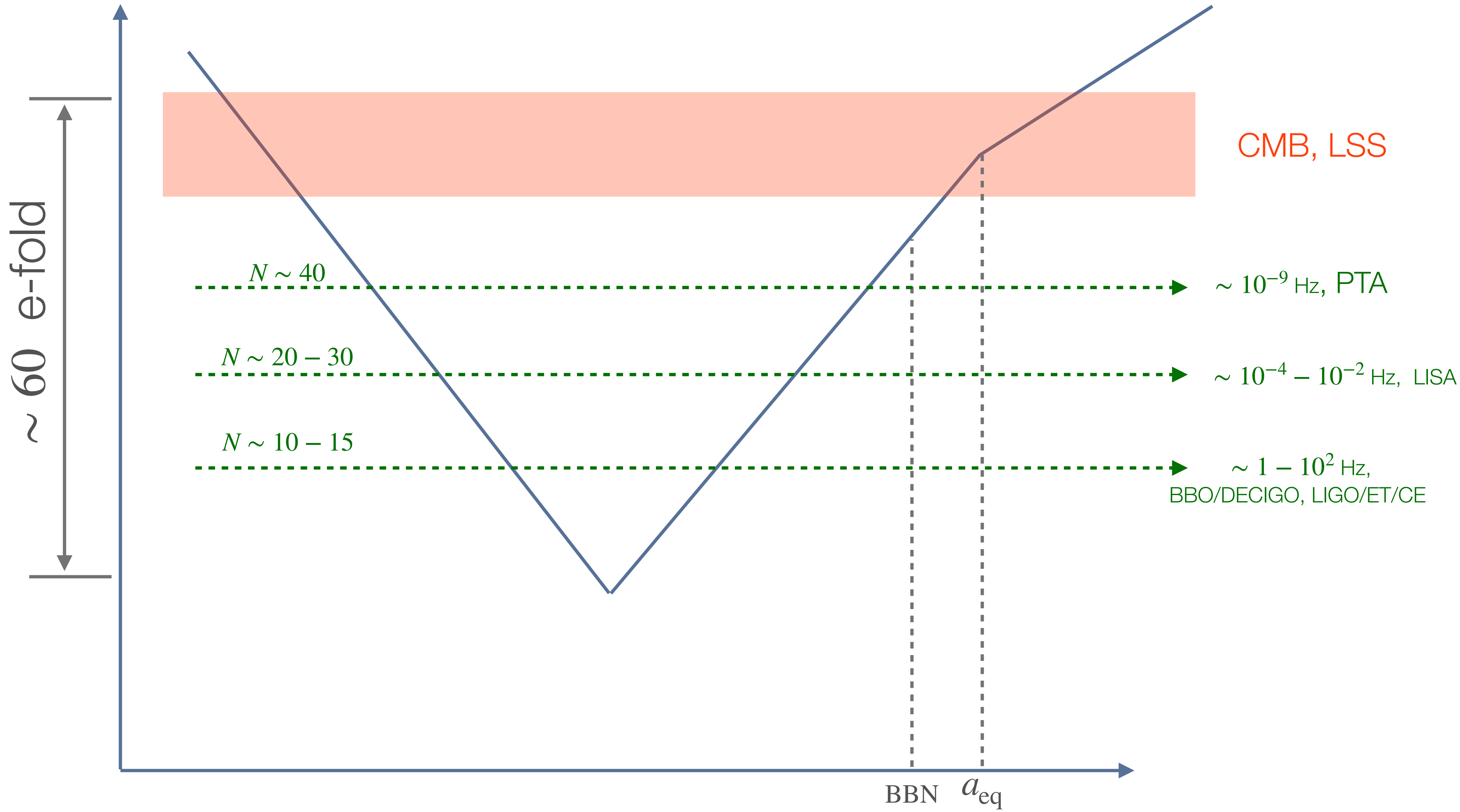


Typically, need something quite dramatic.

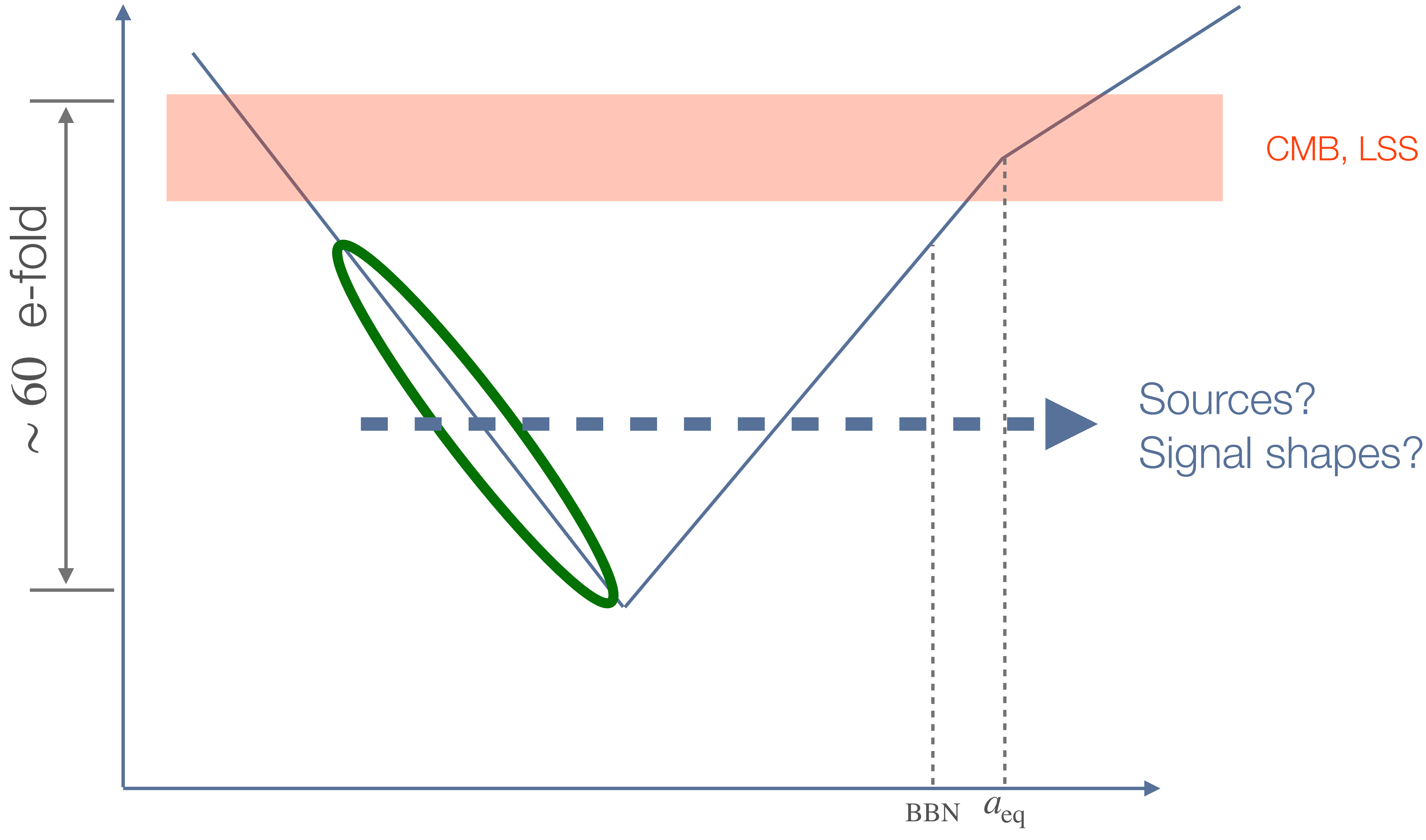
Early universe



Early universe



Early universe



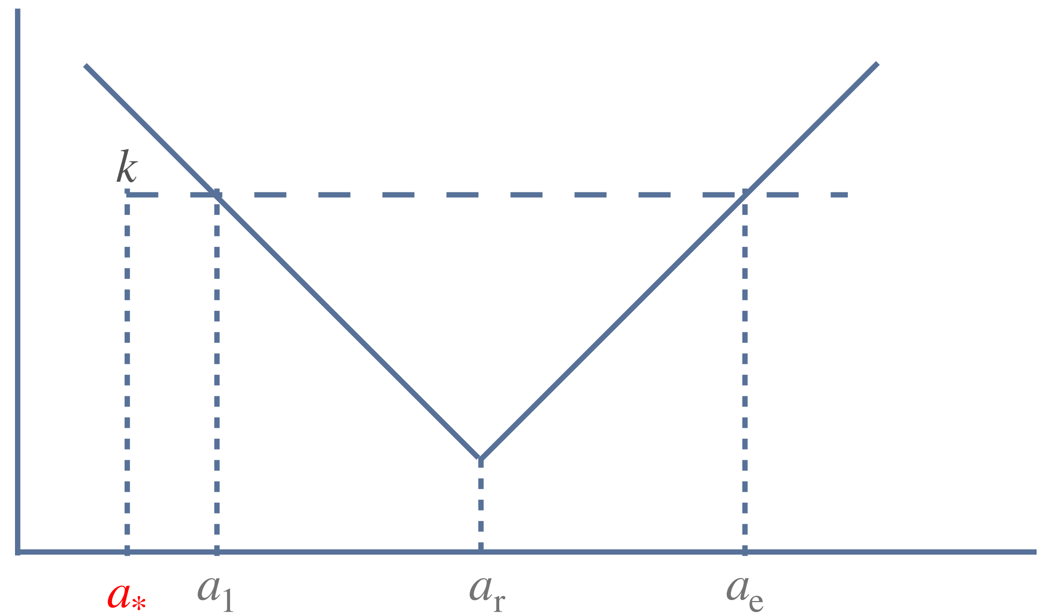
Rest of this talk

An overview, covering several recent developments

See talks by Arushi Bodas, Ramond Co, and
Josh Foster for related topics

Primary GW

Signal strength, dilution by inflation



1. GW mode k generated, at a_*, with strength $\rho^0_{\mathrm{GW}}(k)$

2. Mode evolve to horizon exit at a_1, diluted by a factor of

$$\left(\frac{a_*}{a_1}\right)^4 = \left(\frac{a_* H_I}{k}\right)^4$$

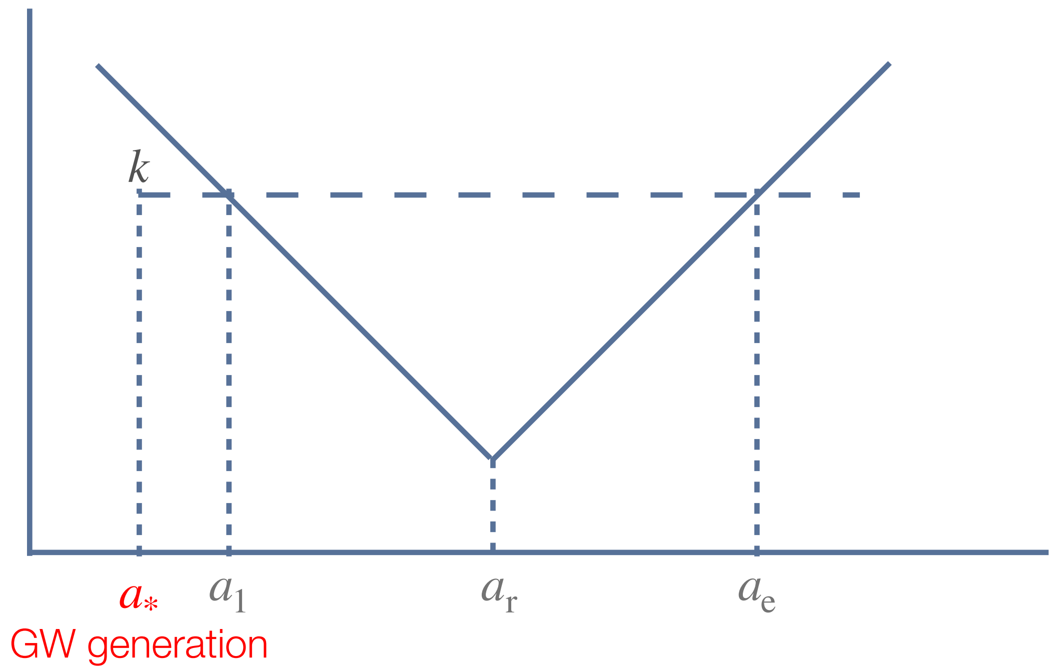
3. Mode evolves outside the horizon, from a_1 to a_e, diluted by a factor of

$$\left(\frac{a_1}{a_\mathrm{e}}\right)^2 = \left(\frac{a_\mathrm{r}}{a_\mathrm{e}}\right)^4$$

Same as dilution of radiation from a_r

4. Final GW strength $\Omega_{\mathrm{GW}} \propto \Omega_R \left(\frac{a_* H_I}{k}\right)^4$

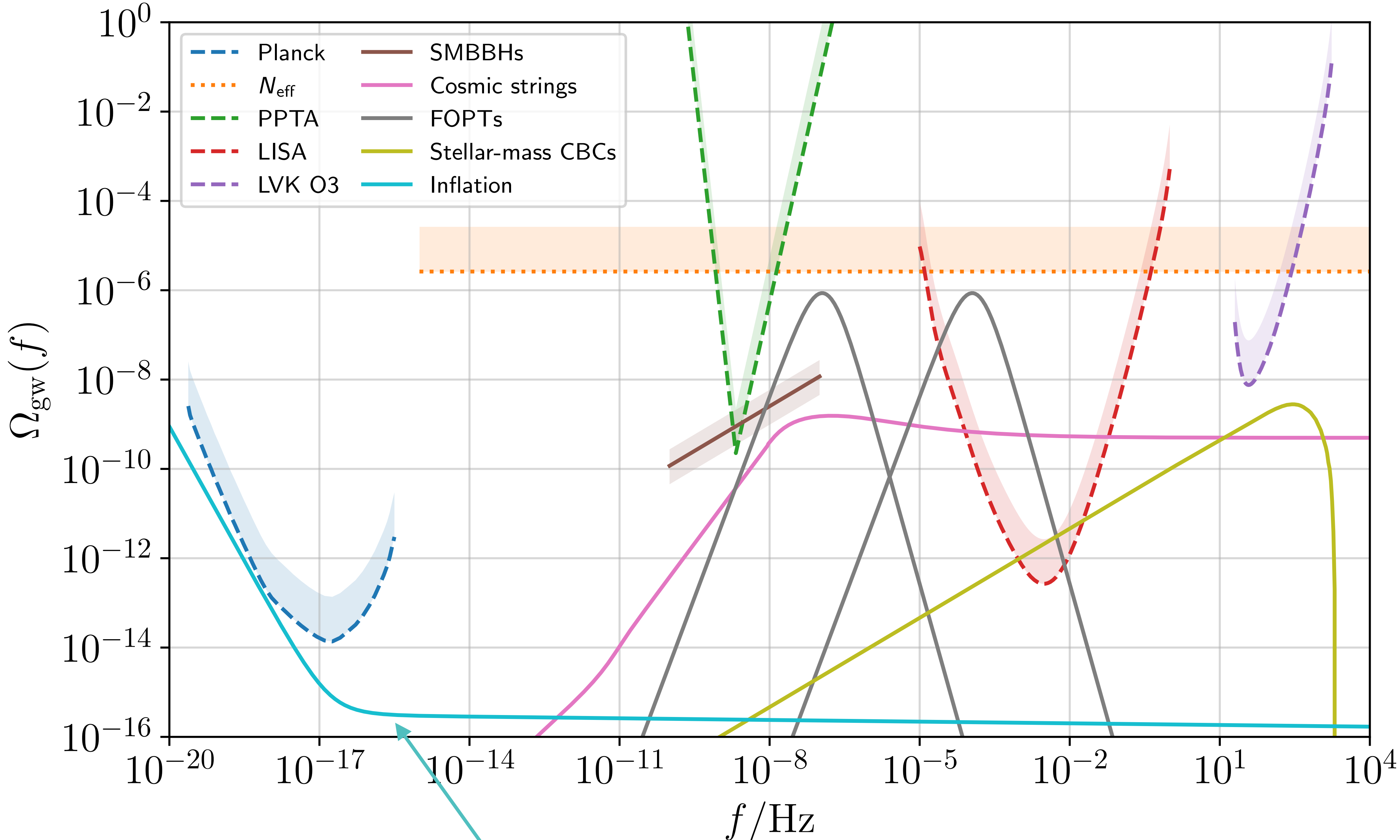
Signal strength, dilution by inflation



$$\Omega_{GW} \propto \Omega_R \left(\frac{a_* H_I}{k}\right)^4$$

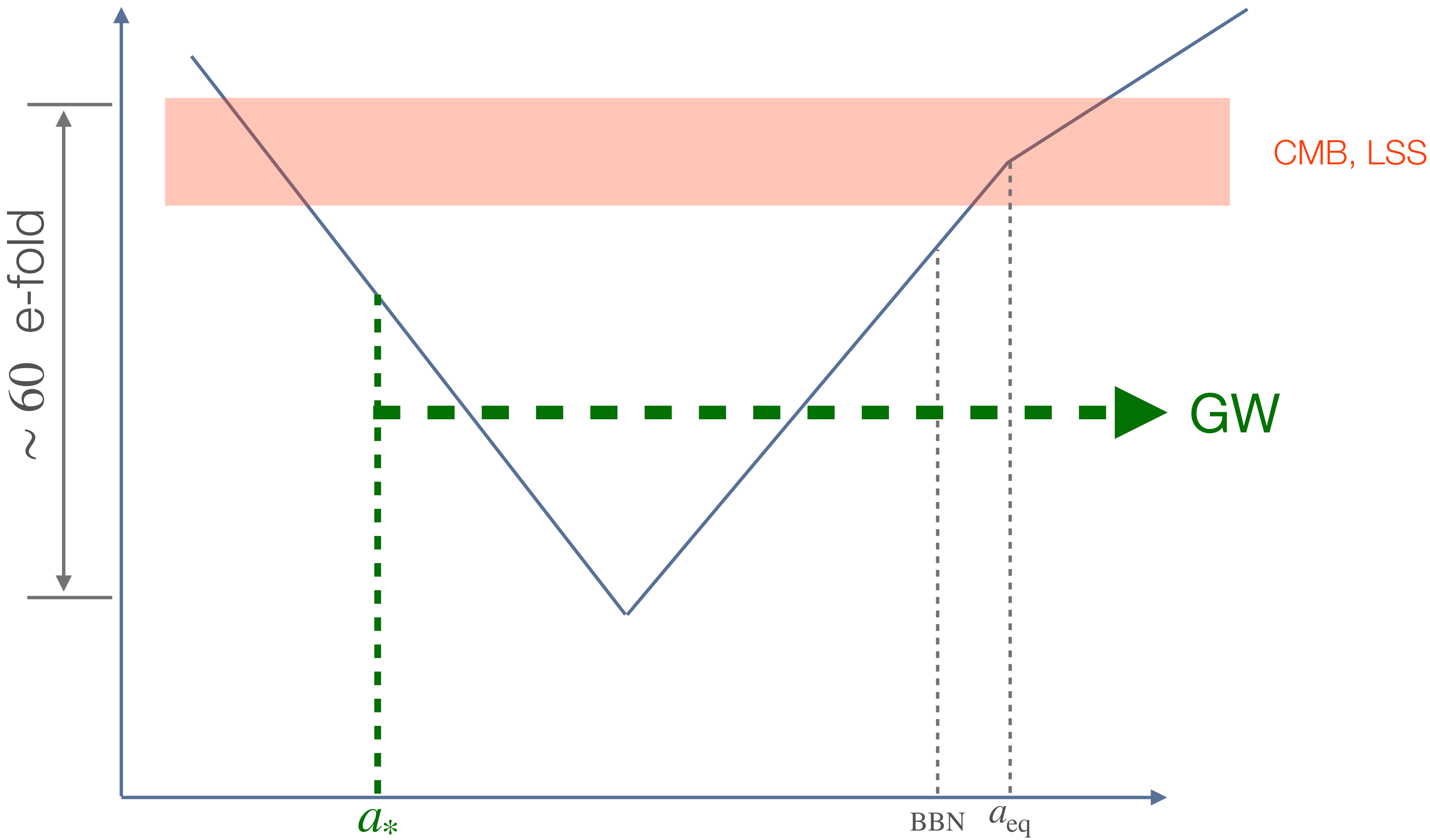
No significant additional dilution as long as we are looking at GW modes not too deep inside the horizon during its generation

Review by Renzini et al, 2202.00178



Directly generated by inflation

Early universe



Something dramatic generates GW
Is this plausible?

Inflation: a trigger for new dynamics

- Inflaton can travel a large distance in field space.
  - Can trigger dramatic changes in spectator sectors which couple to the inflaton.

The excursion of the inflaton

$$\Delta\phi \sim N_{\text{efold}}\sqrt{\epsilon}M_{\text{Planck}}$$

Large excursion of the inflaton field plausible, even if we restrict ourselves to the case where $\boldsymbol{\Delta\phi} < M_{\text{Planck}}$

This is the case even for a small part of inflation with $N_{\text{efold}} \approx O(\text{a few})$

Large excursion of inflaton as trigger

Inflaton often has an (approximate) shift symmetry

$$\phi \to \phi + c$$

However, it is expected to be broken at high scales by operators

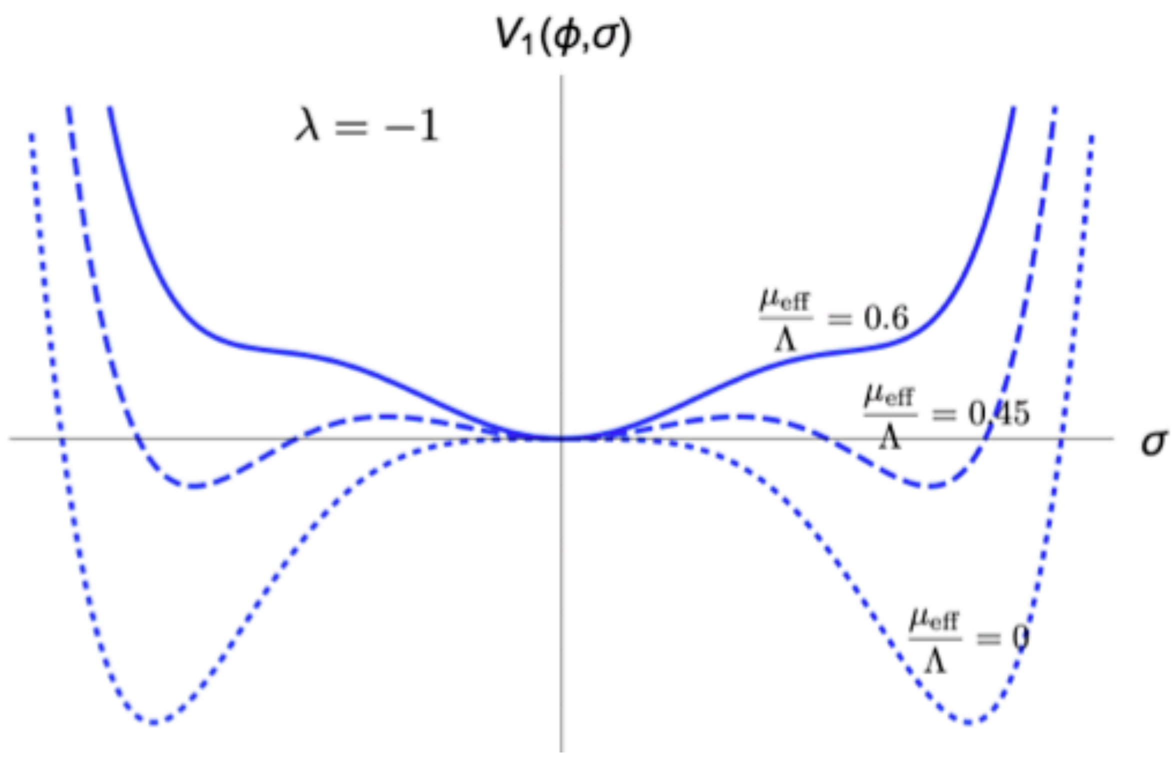
$$\frac{1}{M^d}\phi^n \mathcal{O}_{\text{spectator}} \qquad \text{e.g. } M = M_{\text{Planck}}$$

With large excursions of the inflaton, $\Delta\phi = \mathcal{O}(M_{\text{Planck}})$, we expect this can trigger large changes in the spectator sector.

For example: 1st order PT

$$V(\phi,\sigma) = -\frac{1}{2}\mu_{\rm eff}^2\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6 + V_{\rm inf}(\phi), \quad \mu_{\rm eff}^2 = -\left(m_\sigma^2 - c^2\phi^2\right)$$

$$c^2 \sim \frac{m_\sigma^2}{M^2} \ll 1$$

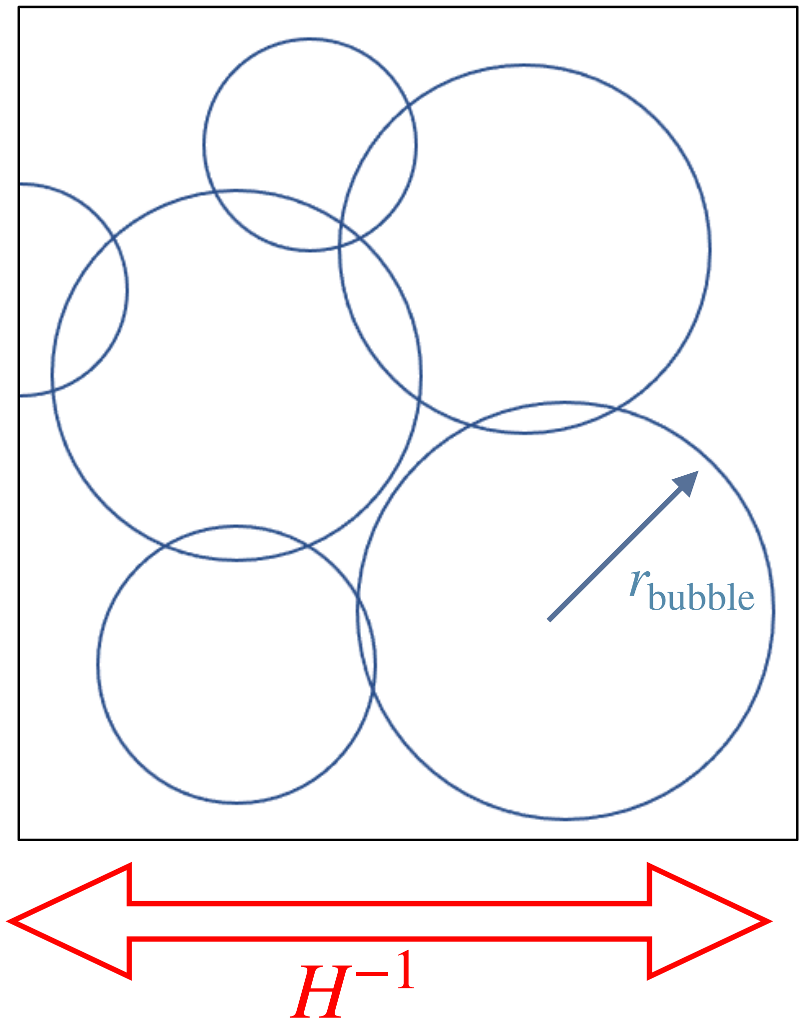


Rolling inflaton → (1st order) phase transition in the spectator sector

1st order phase transition

Phase transition is 1st order, and spectator sector does not dominate energy density:

$$S_4(t) \simeq S(t_*) + \beta(t_* - t) + \ldots \qquad \beta^4 \ll m_\sigma^4 \ll 3M_{\rm Pl}^2 H^2$$

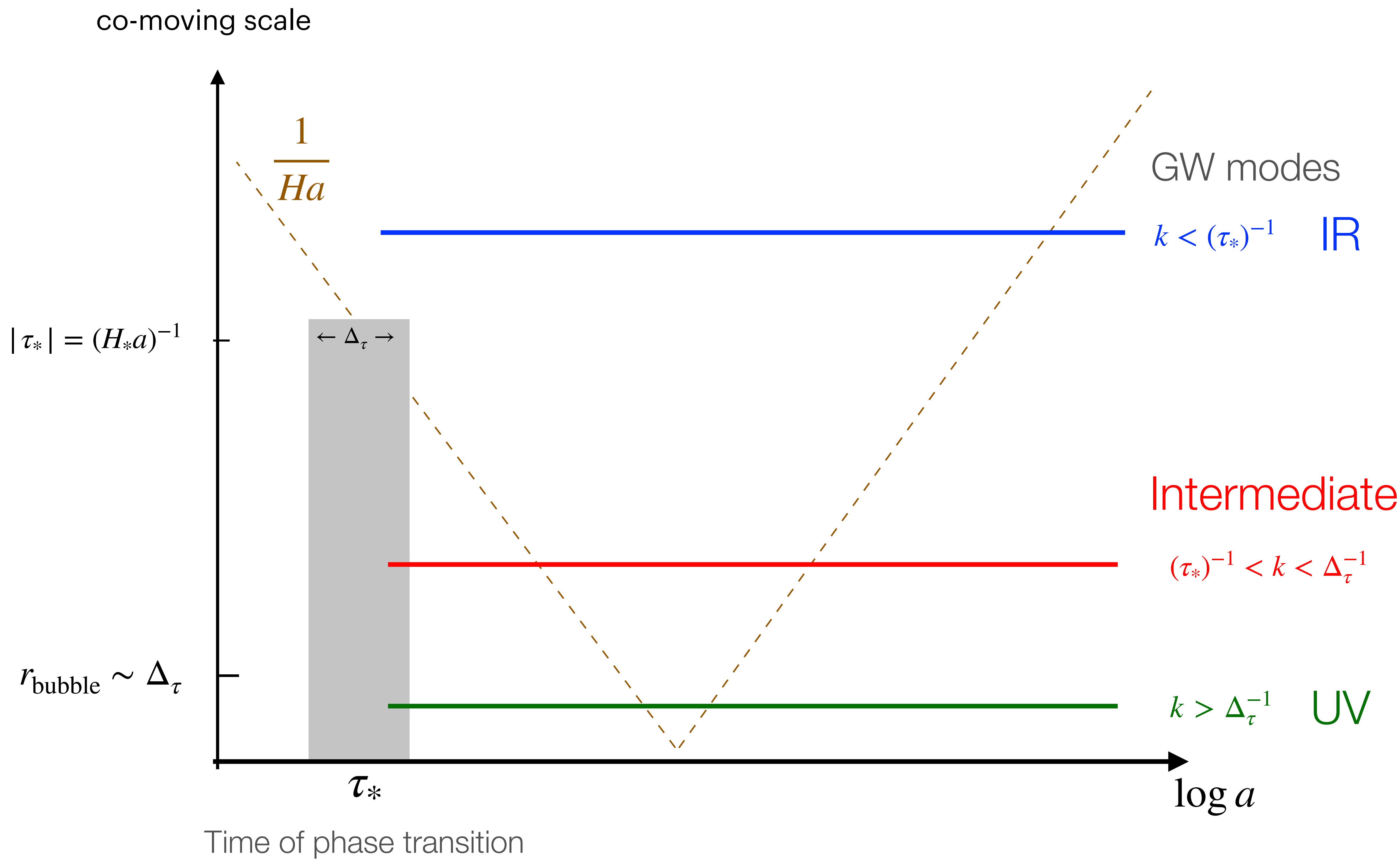


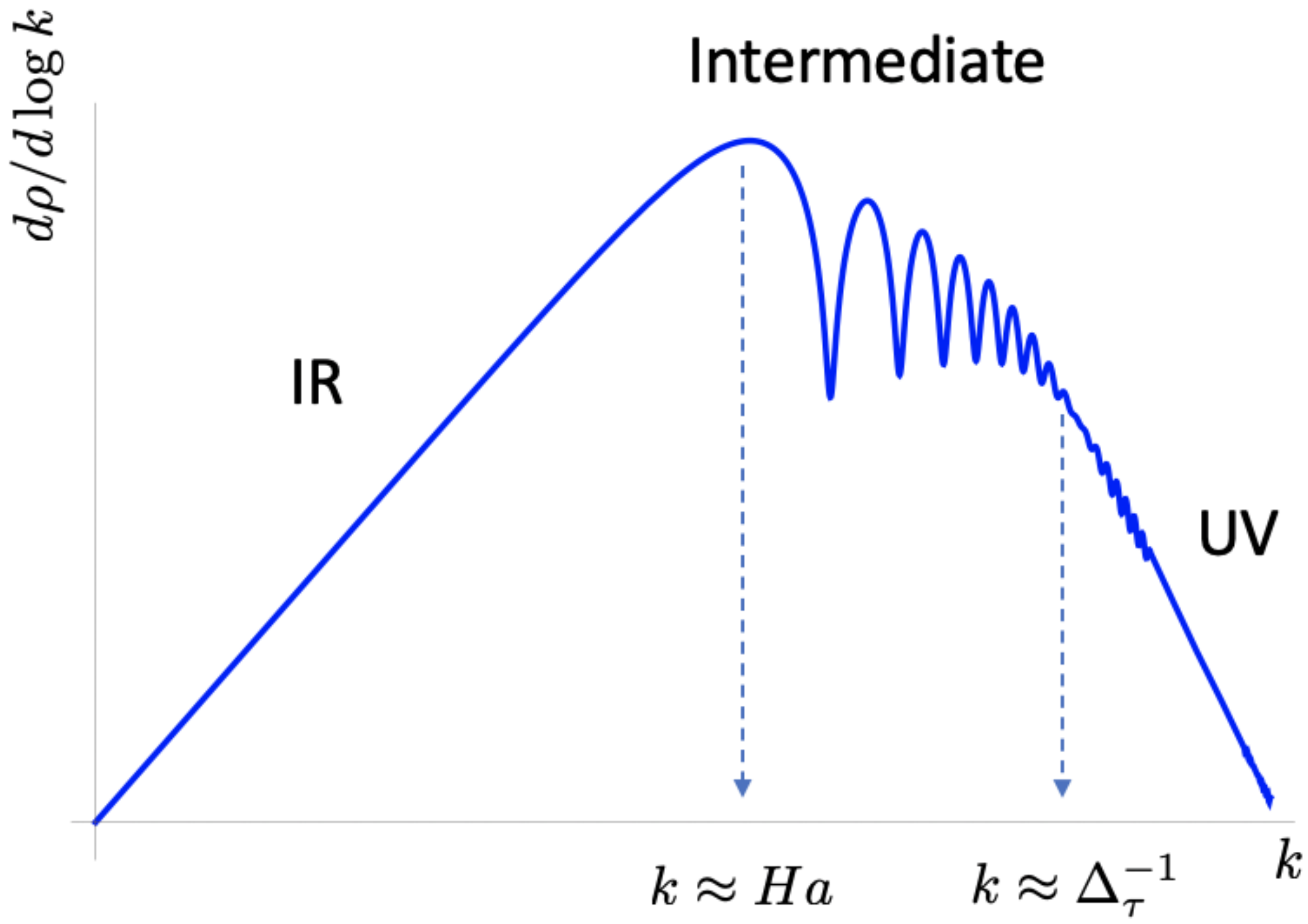
$$\beta^{-1} \sim r_{\rm bubble} \ll H^{-1}$$

$$t_{\rm bubble\ collision} \sim r_{\rm bubble} \ll H^{-1}$$

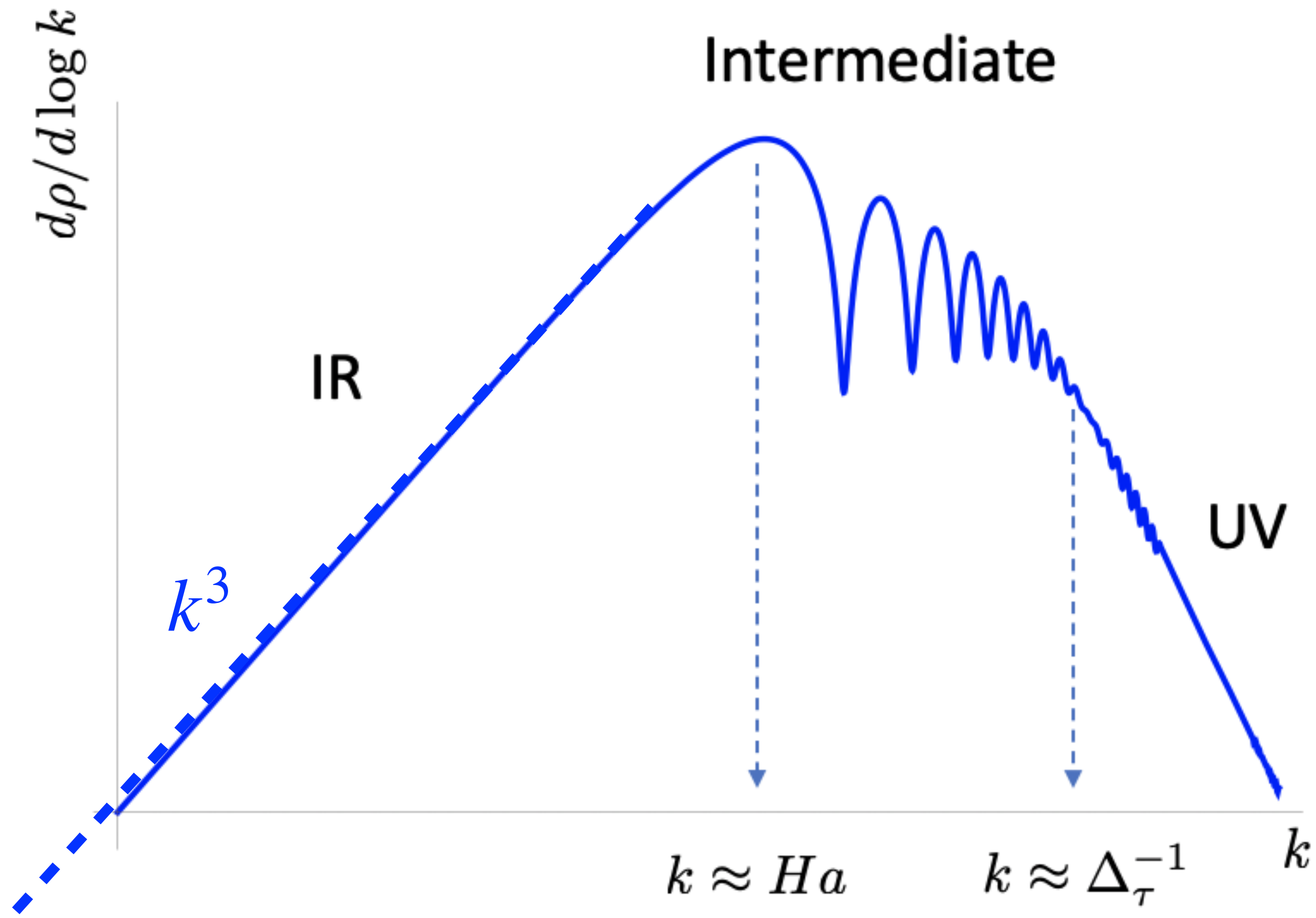
An instantaneous source of GW.

GW in three regimes



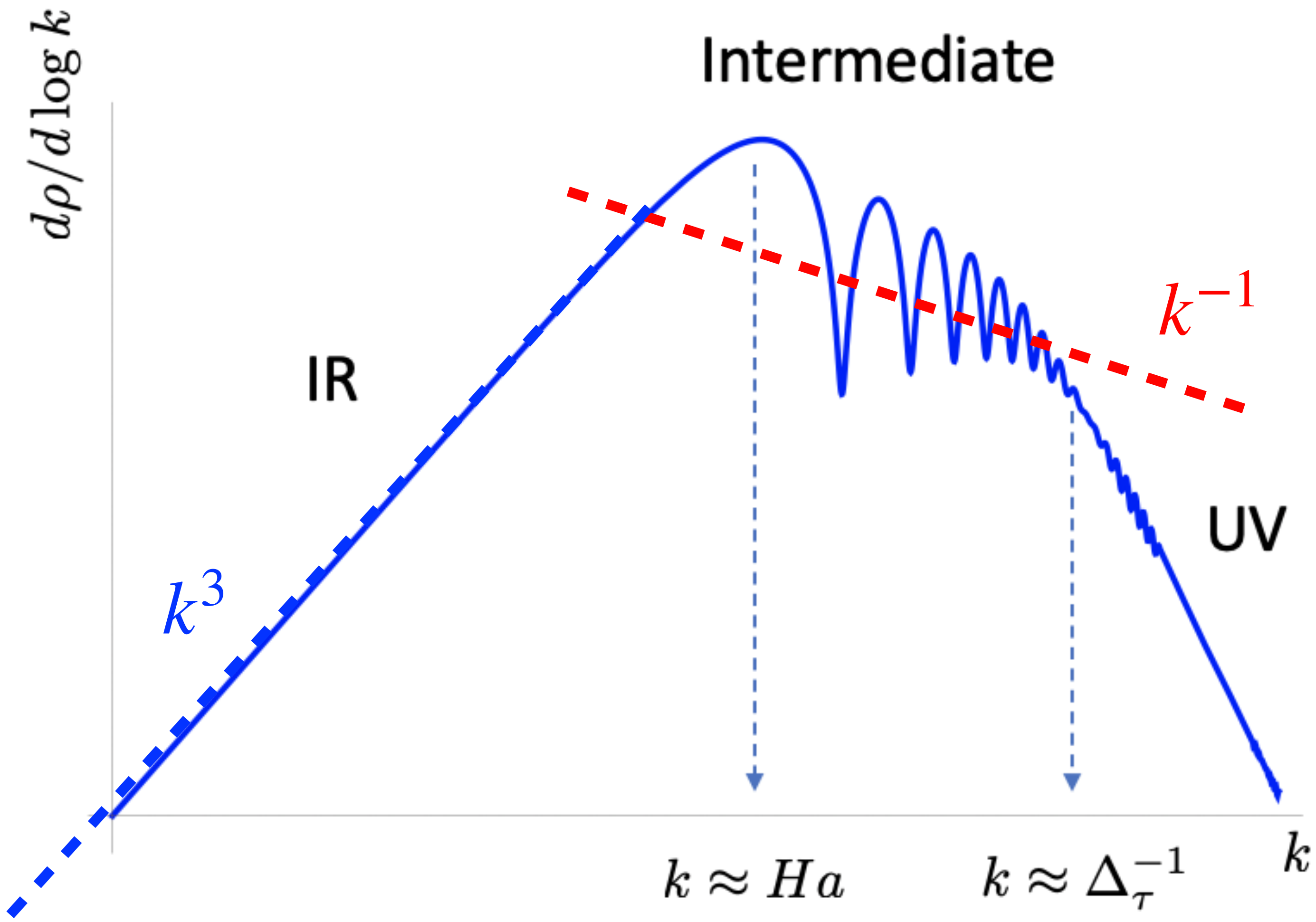


$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$



$$\frac{d\rho_{\mathrm{GW}}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

IR: $$h = \text{constant} \times \frac{\sin k\tau}{k\tau}$$



$$\frac{d\rho_{\mathrm{GW}}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

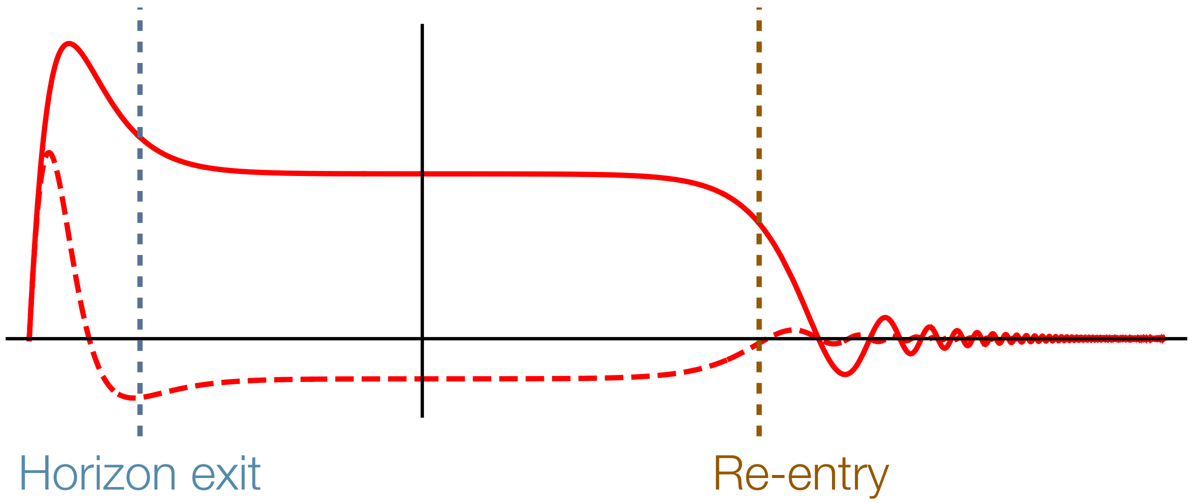
IR: $h = \text{constant} \times \dfrac{\sin k\tau}{k\tau}$

Intermediate: $h \propto \dfrac{\cos(k\tau_*)}{k^2} \dfrac{\sin k\tau}{k\tau}$ And, oscillations.

Oscillations

Intermediate

$\tau_*^{-1} < k < \Delta_\tau^{-1}$

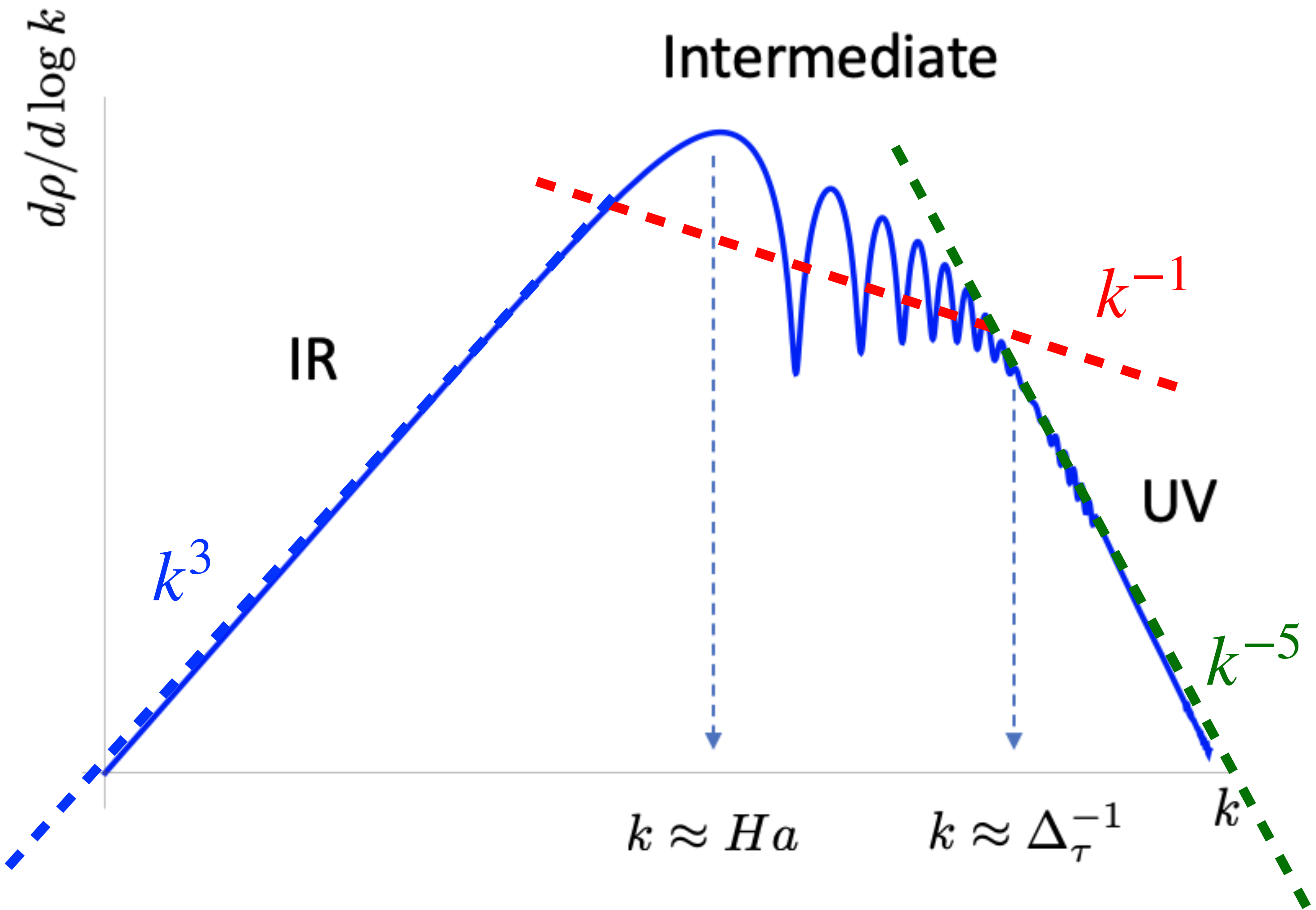


During inflation:

Mode starts inside horizon, oscillates till horizon exit.

➡ Amplitude depends on k.

➡ Leads to oscillatory pattern in frequency. $h \propto \dfrac{\cos(k\tau_*)}{k^2}$



$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

IR: $h = \text{constant} \times \dfrac{\sin k\tau}{k\tau}$

Intermediate: $h \propto \dfrac{\cos(k\tau_*)}{k^2} \dfrac{\sin k\tau}{k\tau}$

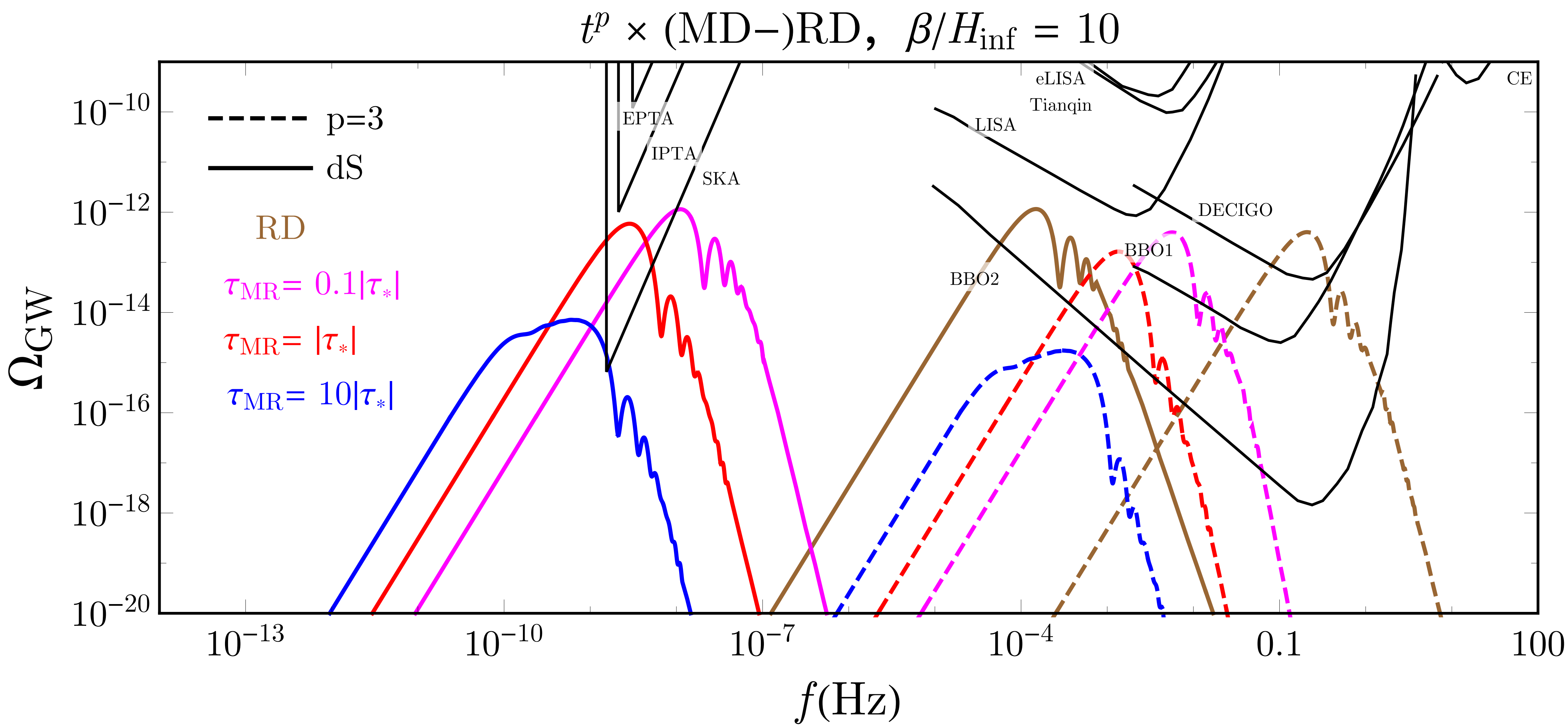
UV: $\dfrac{d\rho_{\rm GW}}{d\log k} \propto k^{-5}$

Details of source matters, determined by numerical simulation

Huber, Konstantin, 0806.1828

Cutting, Hindmarsh, Wier, 1802.05712

Haipeng An, KunFeng Lyu, Siyi Zhou, and LTW 2009.12381, 2201.05171

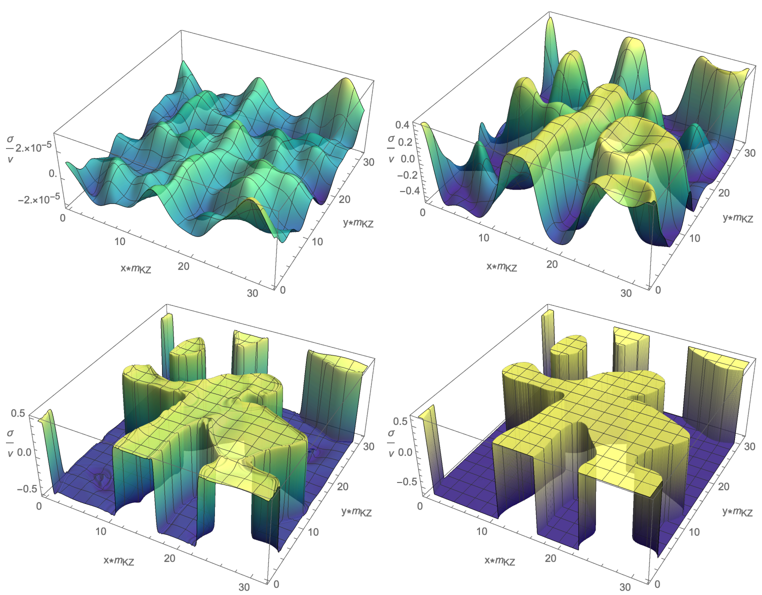


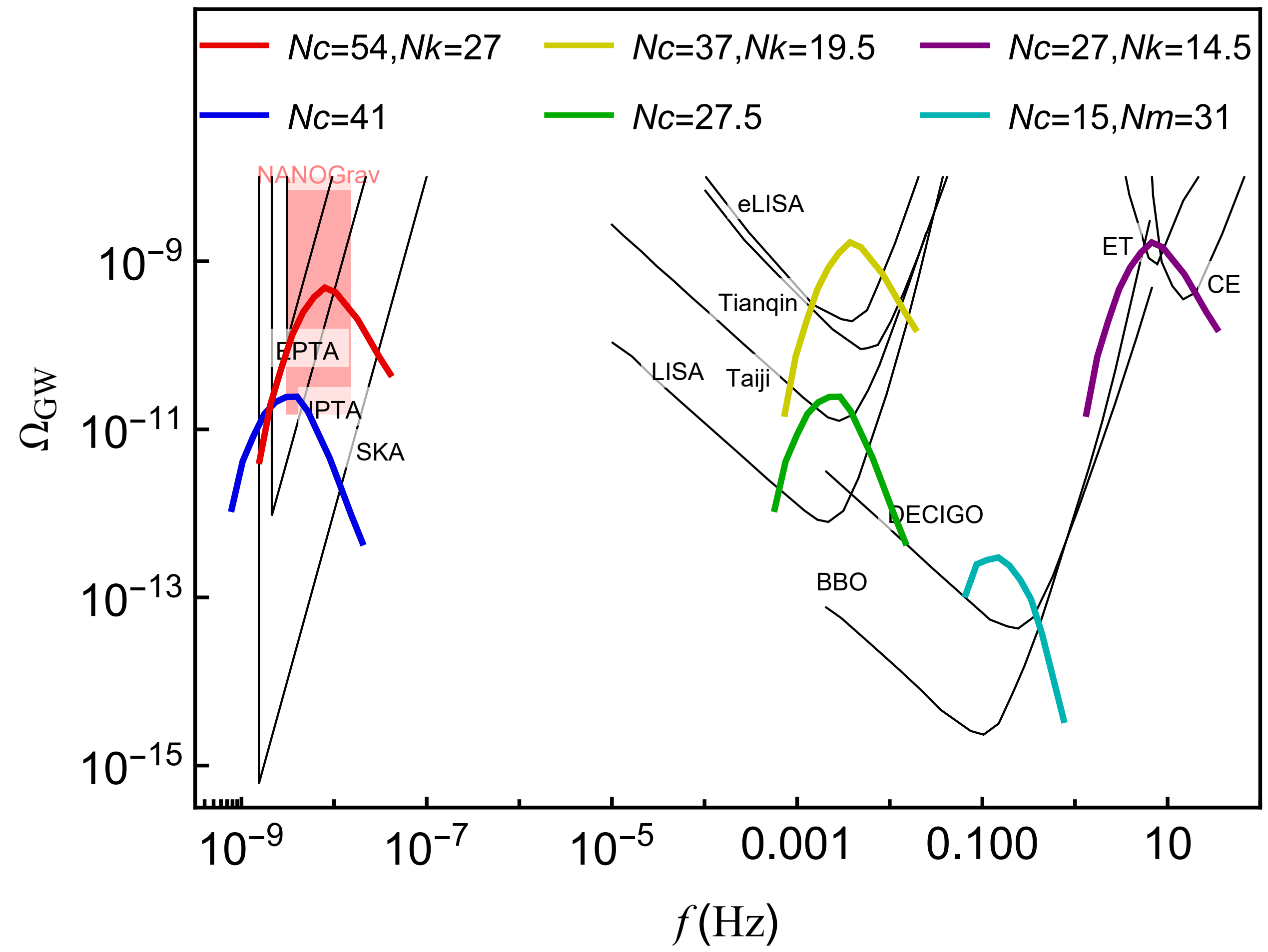
$$\Omega_{\rm GW}^{\rm max} \sim \Omega_R \times \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf\star}}\right)^2 \times \left(\frac{H_\star}{\beta}\right)^5 \tilde{\Delta} \times F(H_\star/H_r, a_\star/a_r, \cdots)$$

$$\approx 10^{-13} \times \left(\frac{\Delta\rho_{\rm vac}/\rho_{\rm inf\star}}{0.1}\right)^2 \times \left(\frac{H_\star/\beta}{0.1}\right)^5$$

From topological defects

H. An and C. Yang, 2304.02361

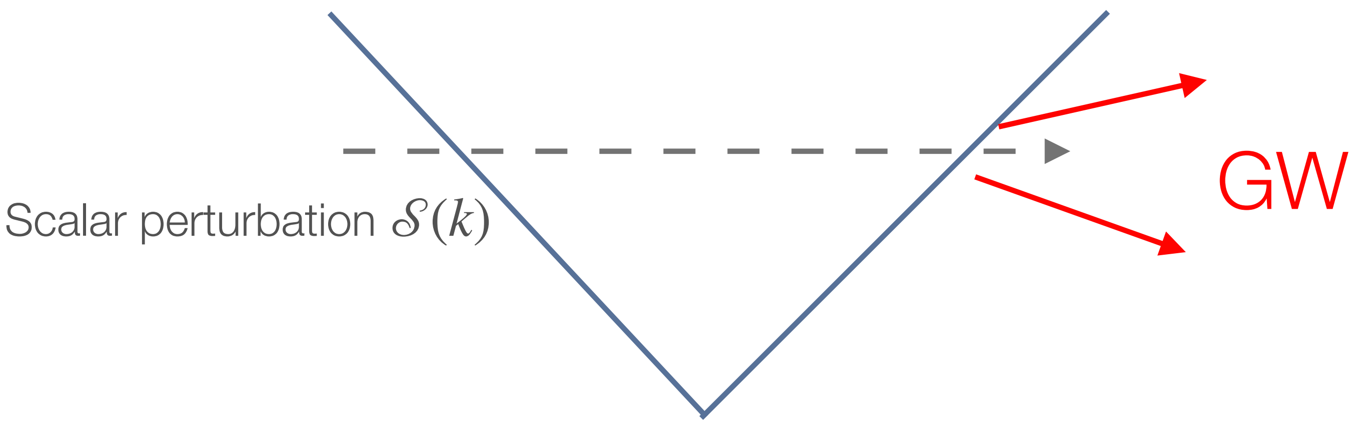




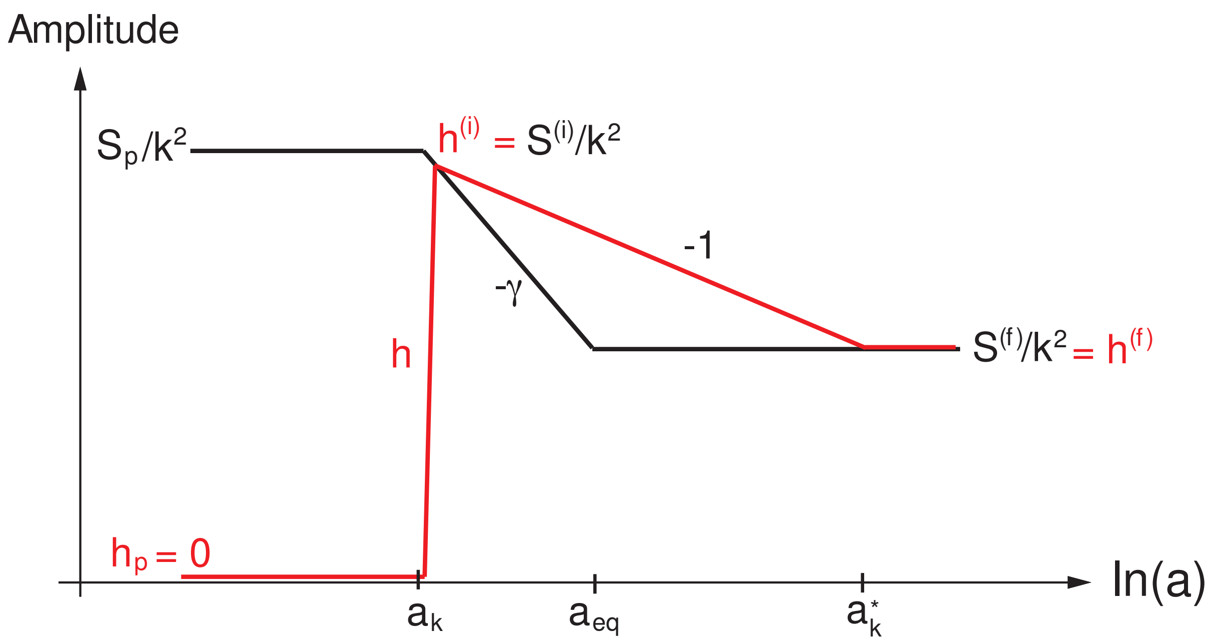
Domain wall from a 2nd order phase transition as source for GW.

Secondary GW

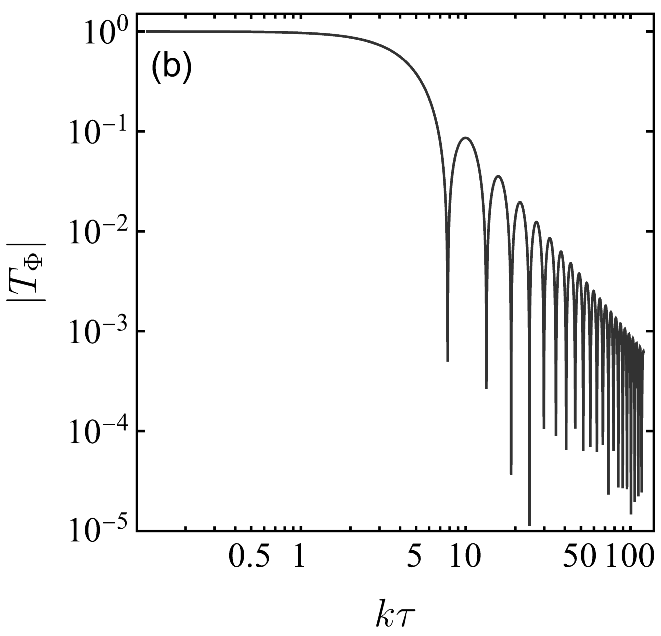
Secondary GW



$$h''_{\mathbf{k}} + 2\mathcal{H}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathcal{S}(\mathbf{k}, \eta)$$



Baumann, Steinhardt, Takahashi, hep-th/0703290



Modes enter horizon during RD, starts oscillate, and generates GW

Example 1: A spectator light scalar

R. Ebadi, S. Kumar, A. McCune, H. Tai, LTW, in progress

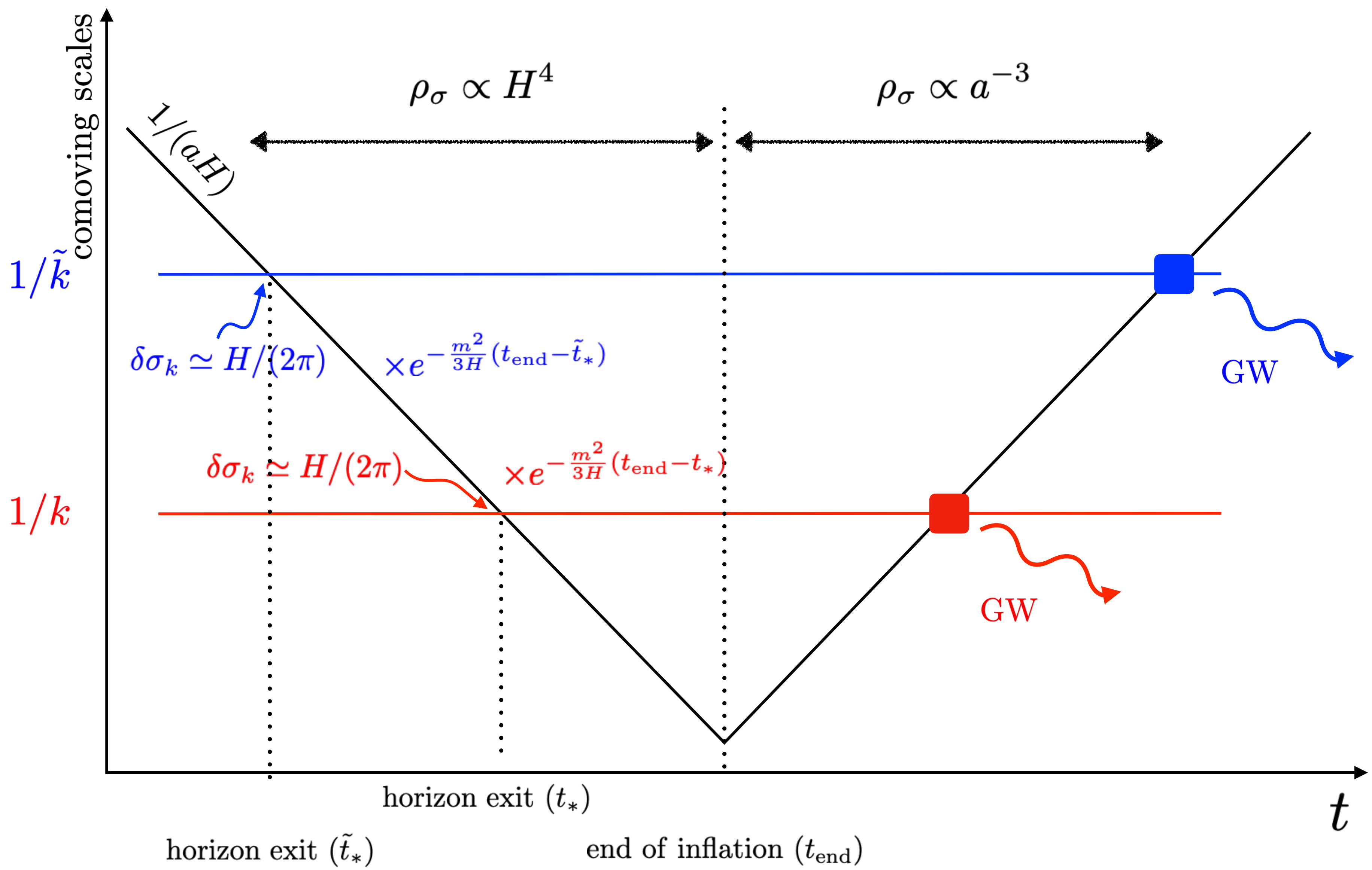
$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \frac{\lambda}{4}\sigma^4 \qquad \text{with } m < H$$

The spectrum of its fluctuation can be studied by stochastic method

Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right)\left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A}\left(\frac{k}{H}\right)^{\frac{2\Lambda_{\rm lowest}}{H}} \quad \text{for } k \ll H$$

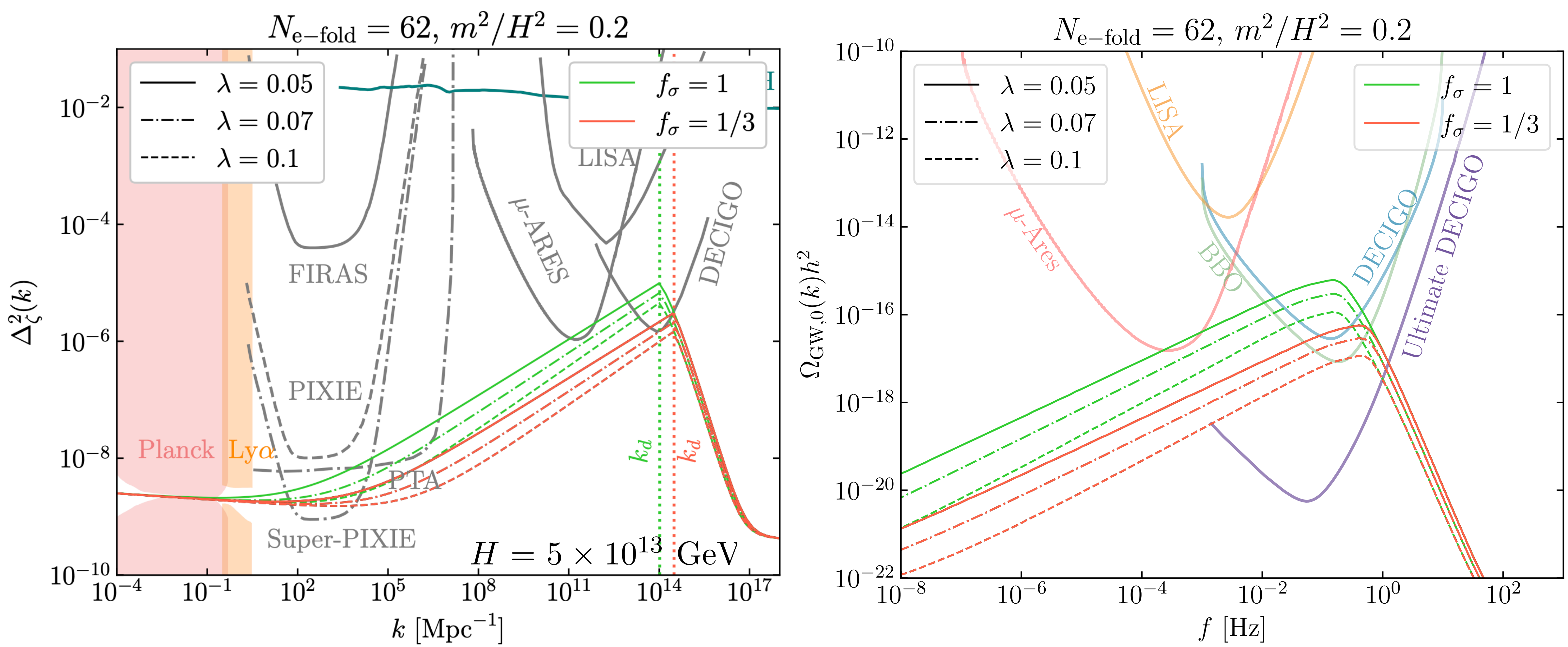
Blue tilt



More damping for longer wave-length (earlier exit)

Power spectrum, GW

Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048



Assuming the scalar behave similar to curvaton.
Becoming important before decay.

Example 2
1st order phase transition

Induced curvature perturbation:

$$\zeta_{\mathbf{q}} \approx \frac{H_{\rm inf}}{\boxed{\dot\phi_0}} \int \frac{d\tau'}{q^2\tau'} \left(\cos q\tau' - \frac{\sin q\tau'}{q\tau'} \right) \frac{c_m \phi_0 [\sigma^2(\tau')]_{\mathbf{q}}}{H_{\rm inf}^2 \tau'^2} \, .$$

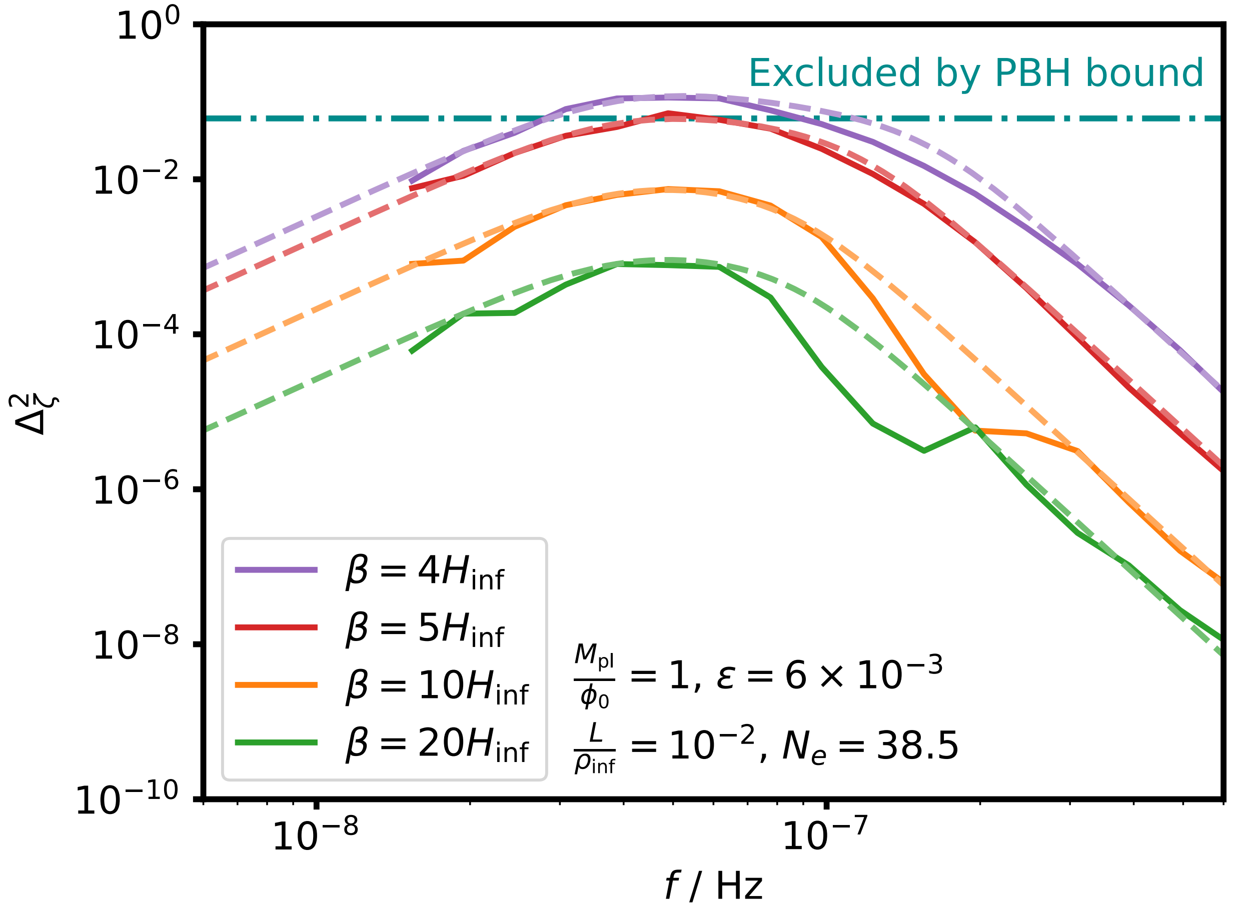
Can be enhanced by slow roll.

Example 2
1st order phase transition

Induced curvature perturbation:

$$\zeta_{\mathbf{q}} \approx \frac{H_{\rm inf}}{\boxed{\dot{\phi}_0}} \int \frac{d\tau'}{q^2\tau'}\left(\cos q\tau' - \frac{\sin q\tau'}{q\tau'}\right)\frac{c_m\phi_0[\sigma^2(\tau')]_{\mathbf{q}}}{H_{\rm inf}^2\tau'^2} \,.$$

Can be enhanced by slow roll.

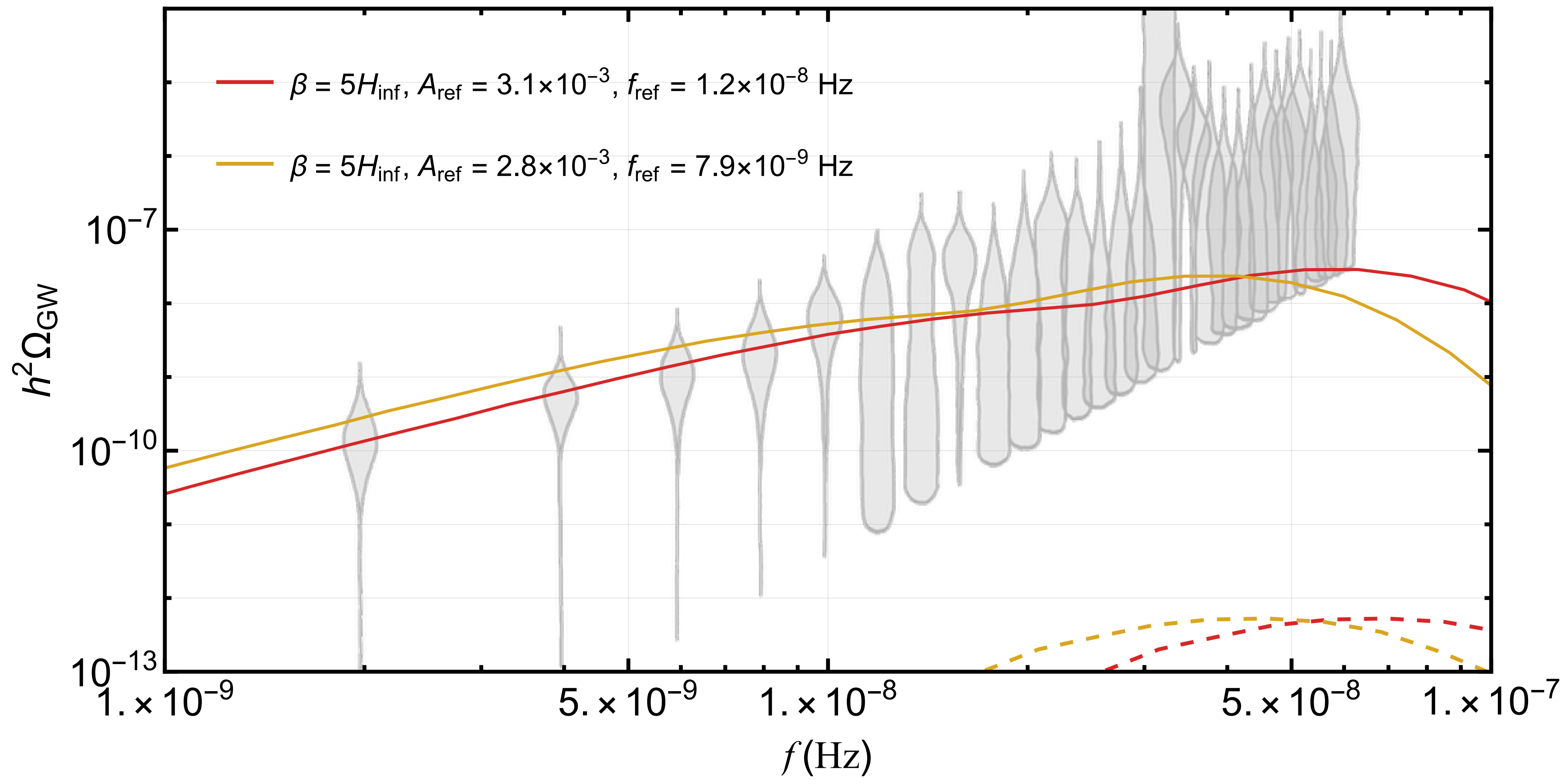


$$\Delta^2_\zeta(q) = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\rm pl}}{\phi_0}\right)^2\left(\frac{H_{\rm inf}}{\beta}\right)^3\left(\frac{L}{\rho_{\rm inf}}\right)^2\mathcal{F}\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

$$\mathcal{F}(x) = \frac{x^3}{1+(\alpha_1 x)^4+(\alpha_2 x)^9}$$

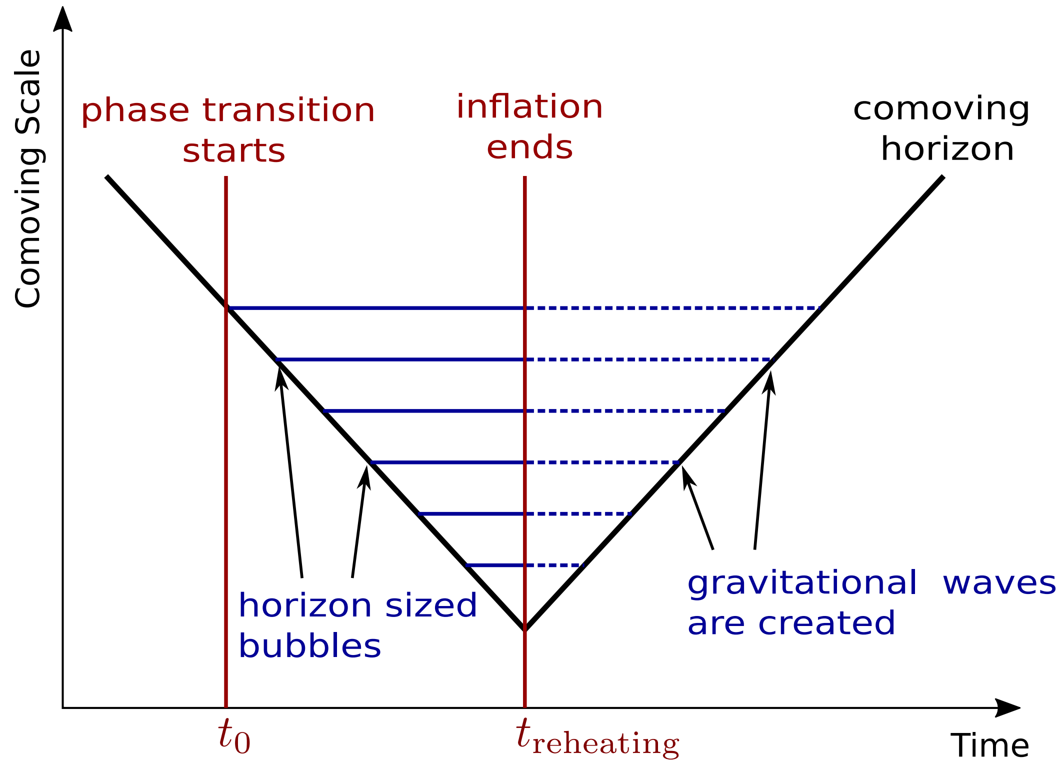
Could be interesting.

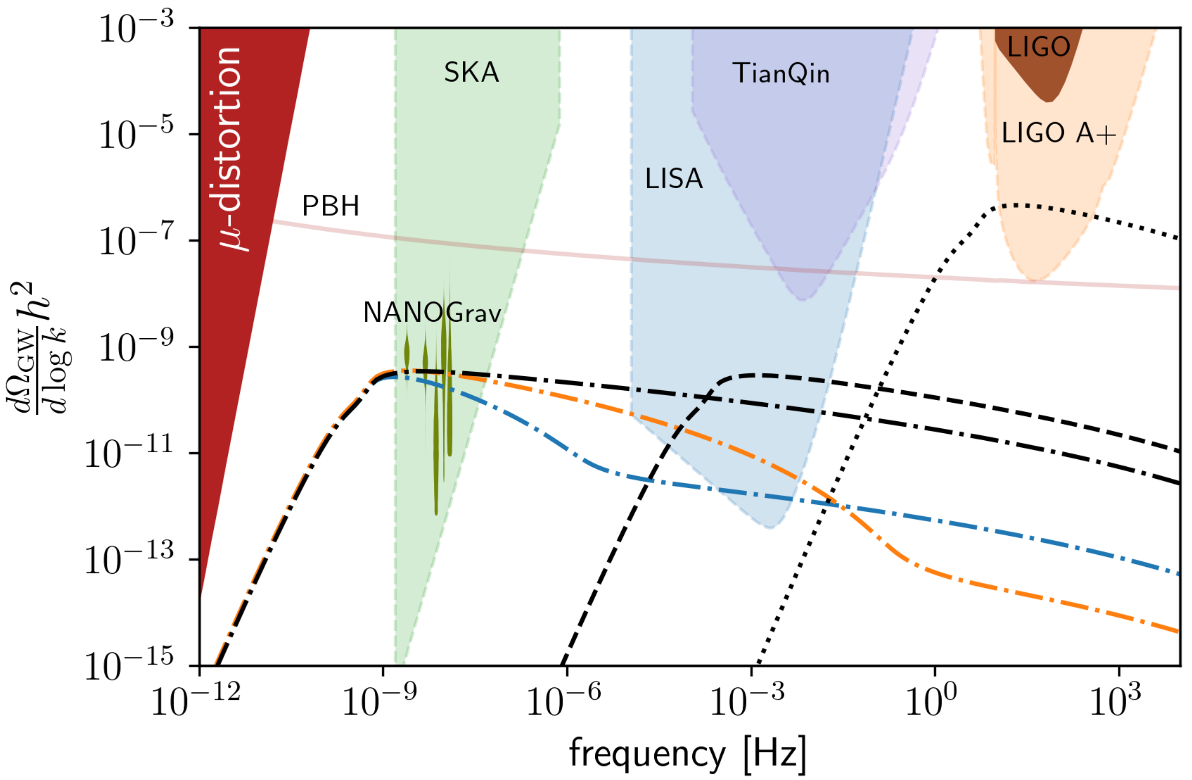
Haipeng An, Chen Yang, Boye Su, Hanwen Tai LTW, 2308.00070



3. Another interesting limit

Barir, Geller, Sun, Volansky, 2203.00693

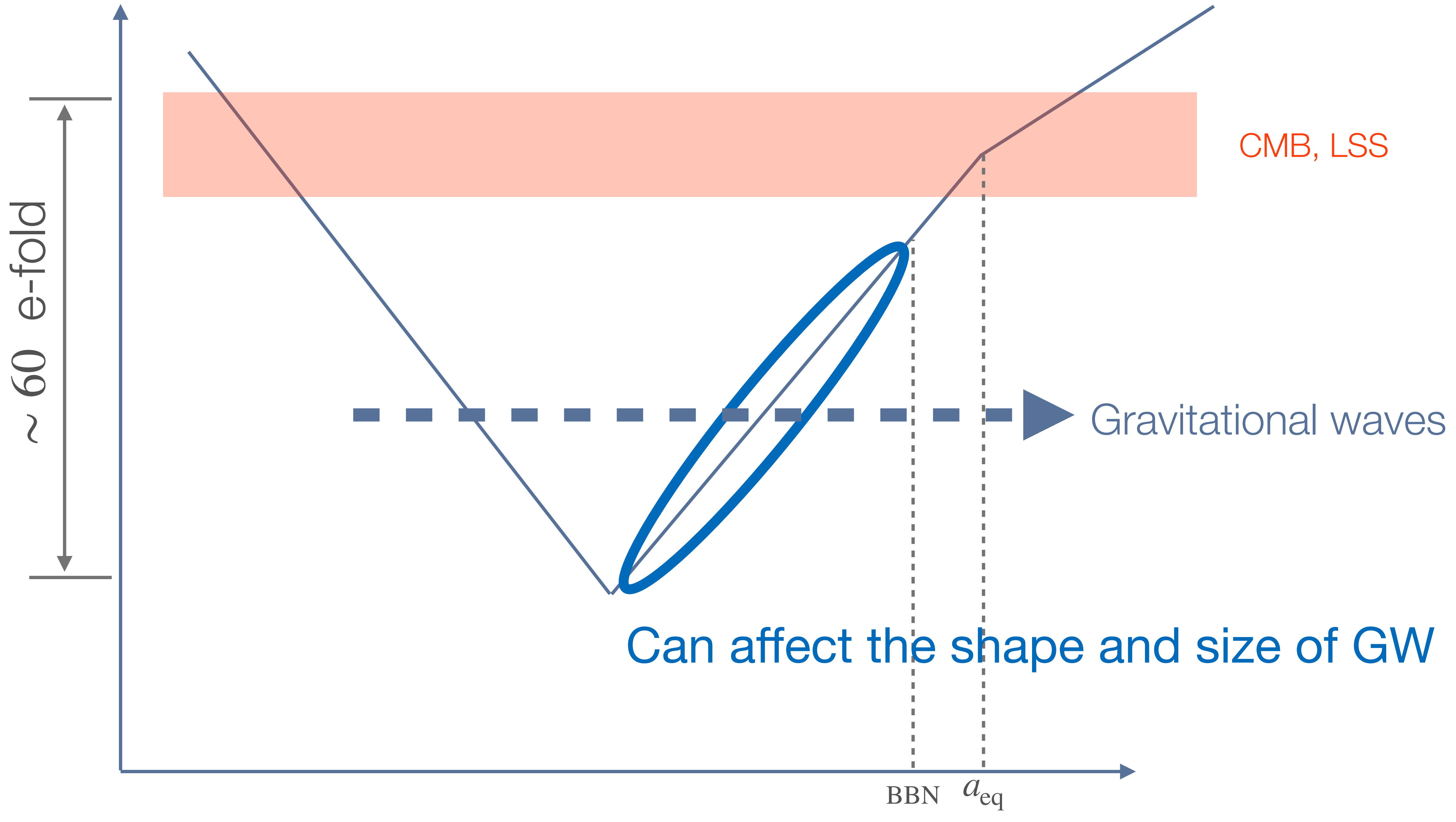




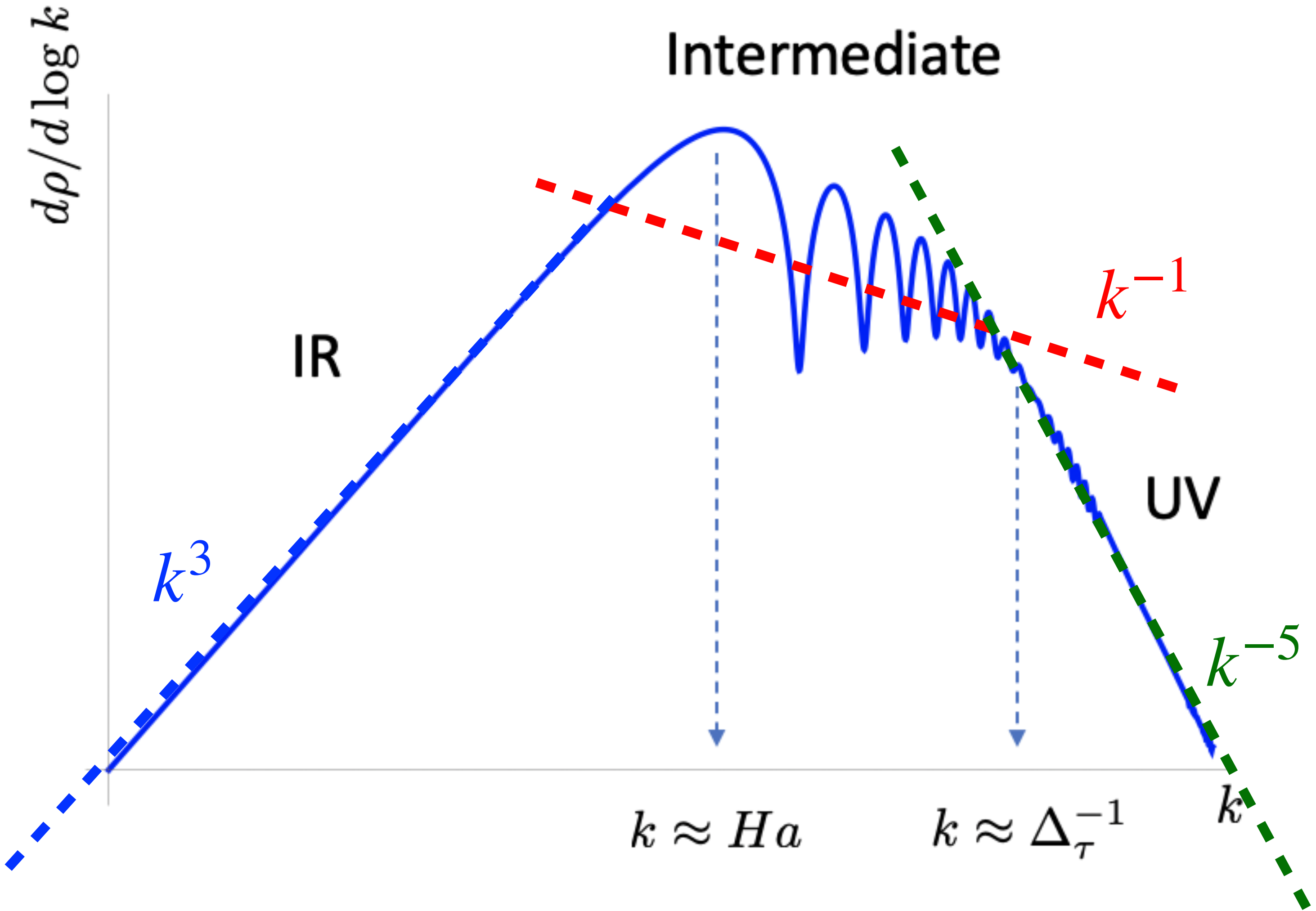
Large bubble does not percolate, generate large curvature perturbations
$\Rightarrow$ secondary gravitational wave at re-entry

Post-inflationary evolution.

Early universe



Example 1: phase transition

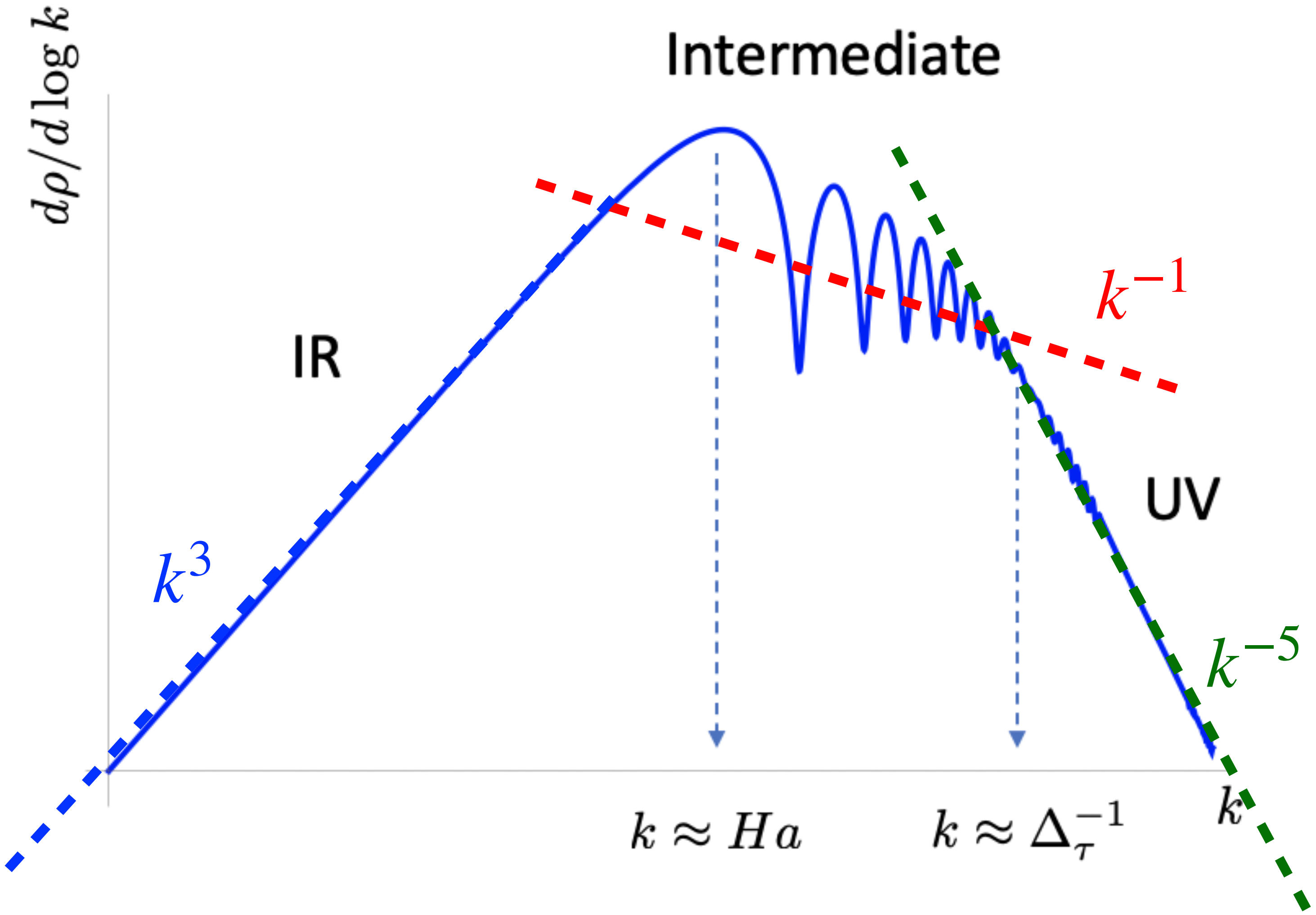


$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

Assumption: de Sitter - instant reheating, RD

$$\frac{d\rho_{\rm GW}^{\rm osc}}{d\log k} = \frac{2G_N |\tilde{T}_{ij}(0,0)|^2}{\pi V a^4(\tau) a^2(\tau_\star)} \left\{ \left[\tilde{\mathcal{E}}_0^i(k) \tilde{\mathcal{G}}_0^f(k) \right]^2 k^3 \left[1 + \mathcal{S}(k\Delta_\tau) \cos 2k(\tau_\star - \tau_0) \right] \right\}$$

Example 1: phase transition



$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

Assumption: de Sitter - instant reheating, RD

$$\frac{d\rho^{\rm osc}_{\rm GW}}{d\log k} = \frac{2G_N|\tilde{T}_{ij}(0,0)|^2}{\pi V a^4(\tau) a^2(\tau_\star)} \left\{ \boxed{\left[\tilde{\mathcal{E}}^i_0(k)\tilde{\mathcal{G}}^f_0(k)\right]^2} k^3 \left[1 + \mathcal{S}(k\Delta_\tau)\cos 2k(\tau_\star - \tau_0)\right] \right\}$$

Depending on
Time evolution

Scenarios of inflation and its aftermath

Scenarios of inflation

Parameterized by p

Quasi de Sitter: $a(\tau) = -\frac{1}{H\tau}$

Power law: $a(t) = a_0(t/t_0)^p$

Lucchin and Matarrese, 1985

$p \to \infty$, quasi de Sitter

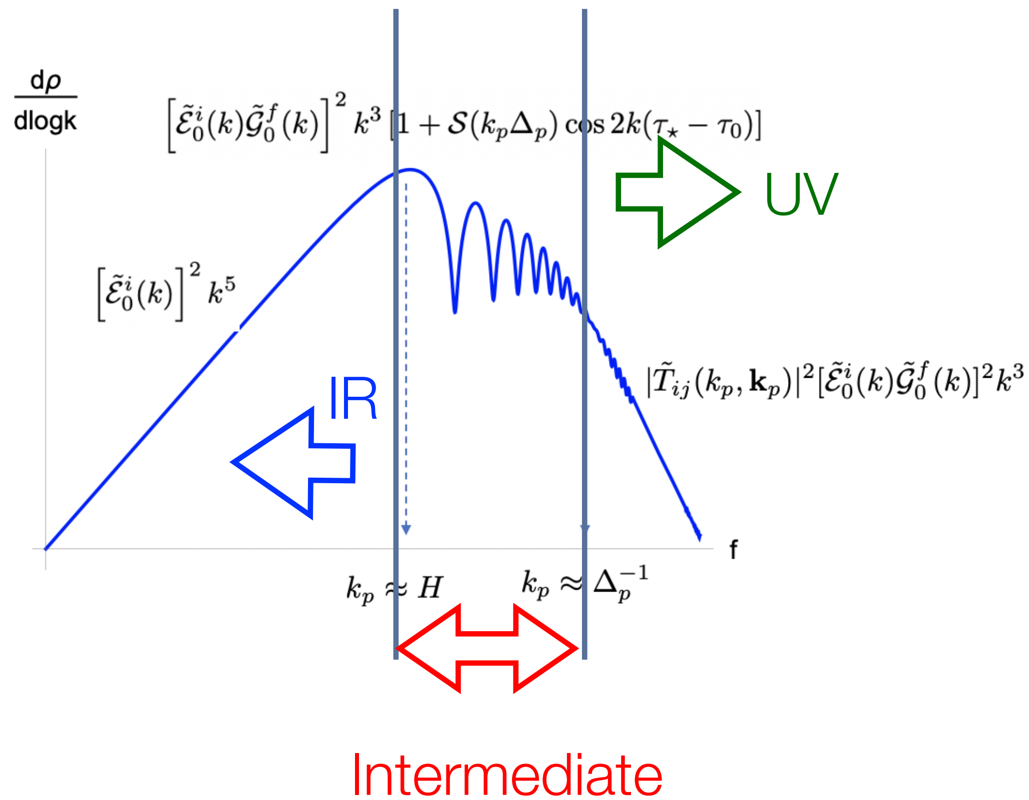
Scenarios after inflation:

Parameterized by $\tilde{p}$

$$a(t) \sim t^{\tilde{p}}$$

	w	$\rho(a)$	$\tilde{p}$	$\tilde{\alpha}$
MD	0	a^{-3}	2/3	-3/2
RD	1/3	a^{-4}	1/2	-1/2
Λ	-1	a^{0}	∞	3/2
Cosmic string	-1/3	a^{-2}	1	∞
Domain wall	-2/3	a^{-1}	2	5/2
kination	1	a^{-6}	1/3	0

Impact on spectrum



UV

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
dS	k^{-5}	k^{-7}	$k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	$k^{-3+2\frac{p}{1-p}}$	$k^{-5+2\frac{p}{1-p}}$	$k^{-1+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$

Intermediate

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
dS	k^{-1}	k^{-3}	$k^{1+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	$k^{1+2\frac{p}{1-p}}$	$k^{-1+2\frac{p}{1-p}}$	$k^{3+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$

IR

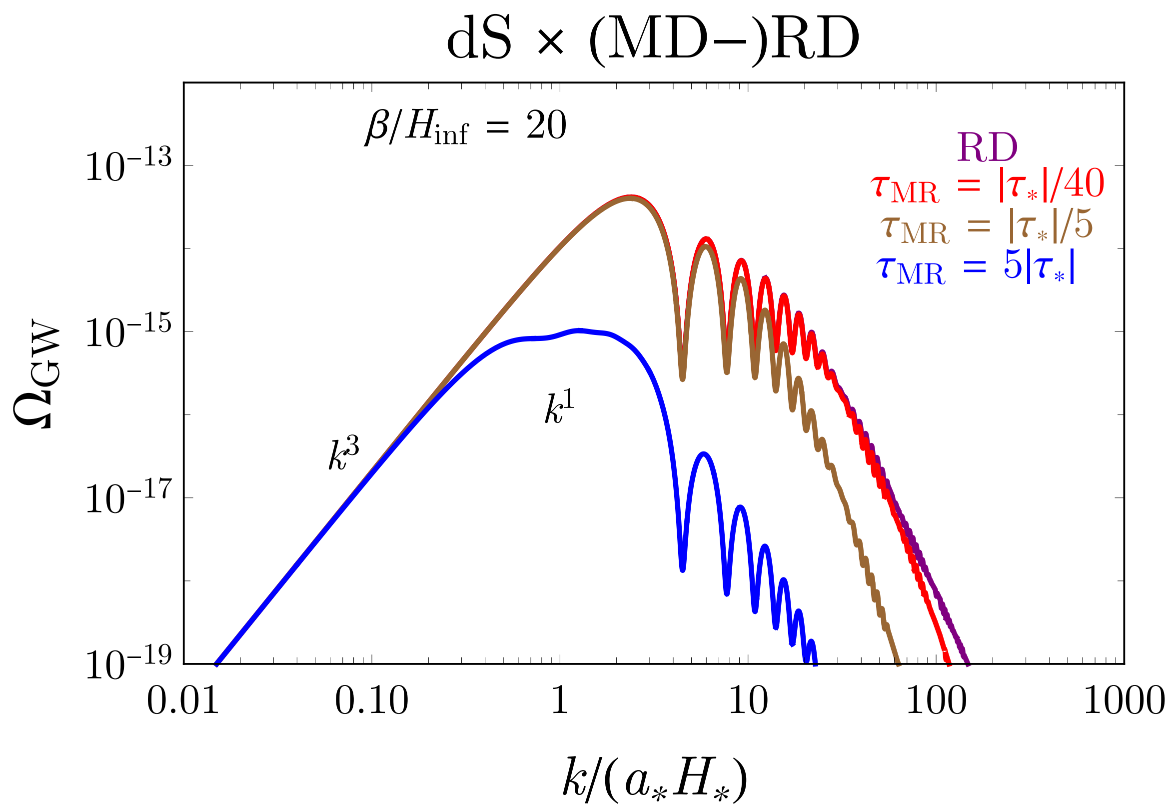
Scenarios after inflation →

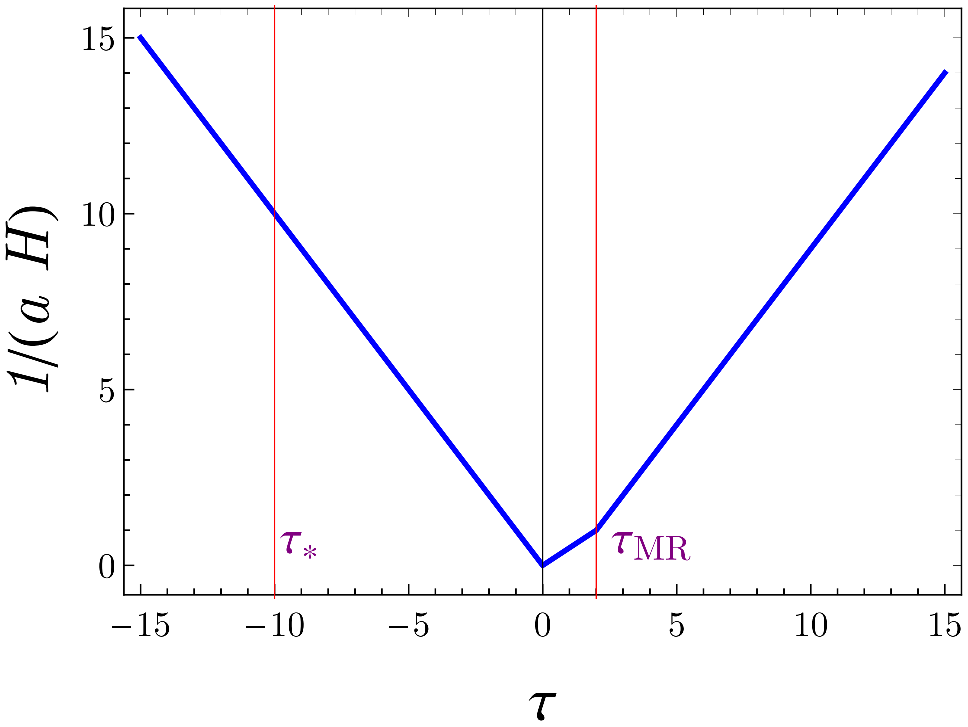
Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
dS	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$

Slopes sensitive to the evolution.

Comparing scenarios

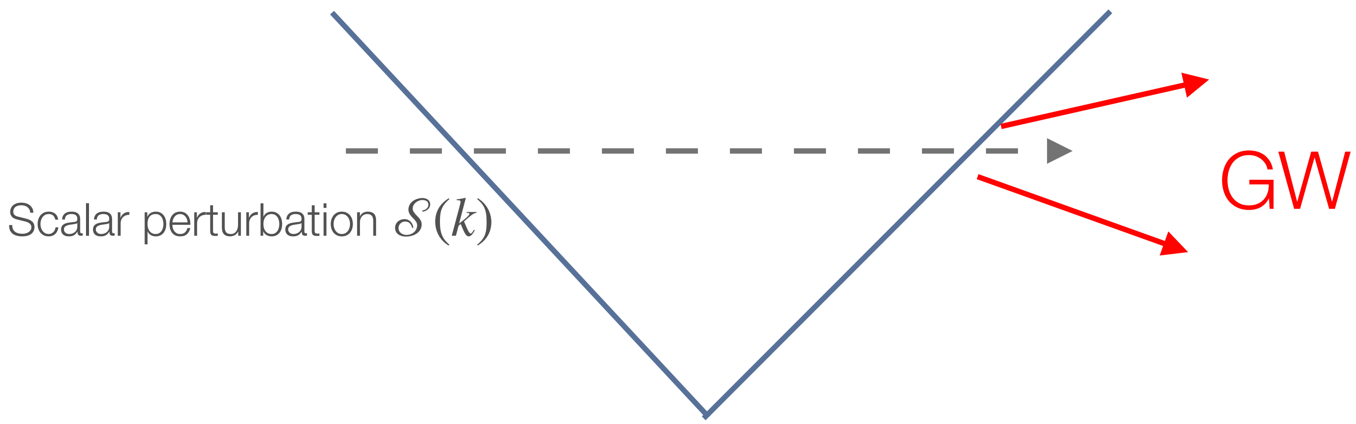




Scenarios after reheating.

$\tau_{\rm MR}$ = MD-RD transition

Example 2: Secondary GW

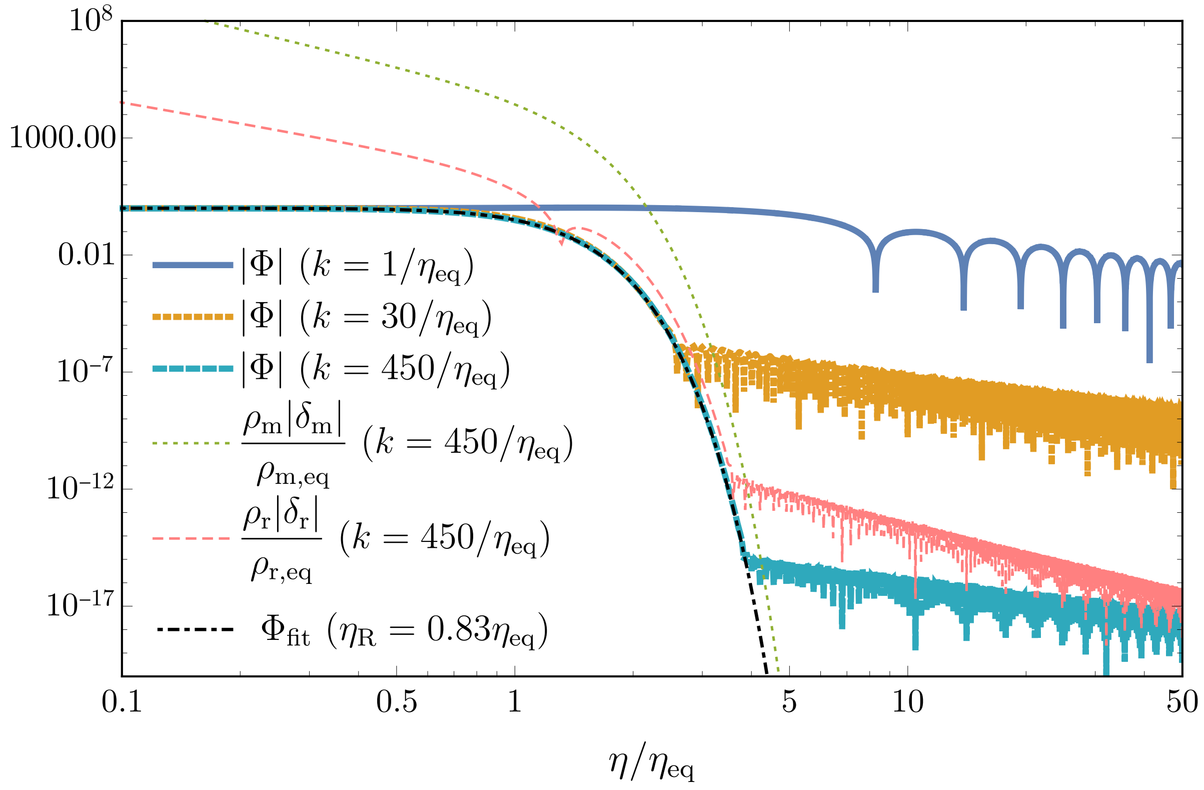


Sensitive to evolution after re-entry

Secondary GW

Early matter domination ⇒ Radiation

K. Inomata, 1904.12878, 1904.12879



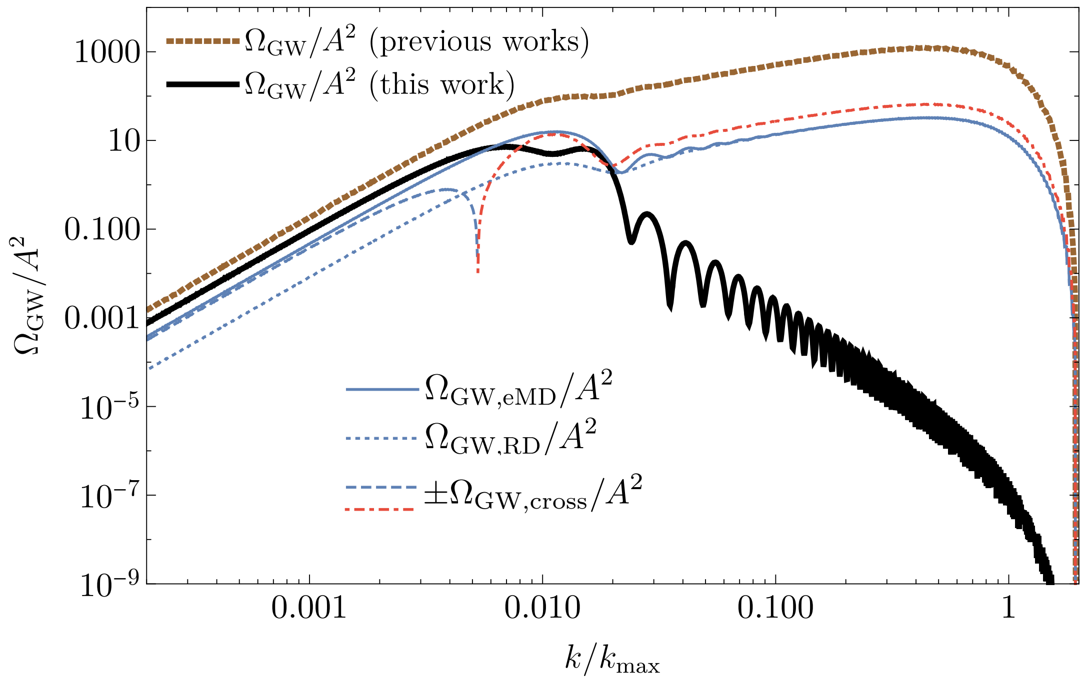
For modes entering in MD, no oscillation RD

Grav. potential approx constant in MD. Decay in transition.

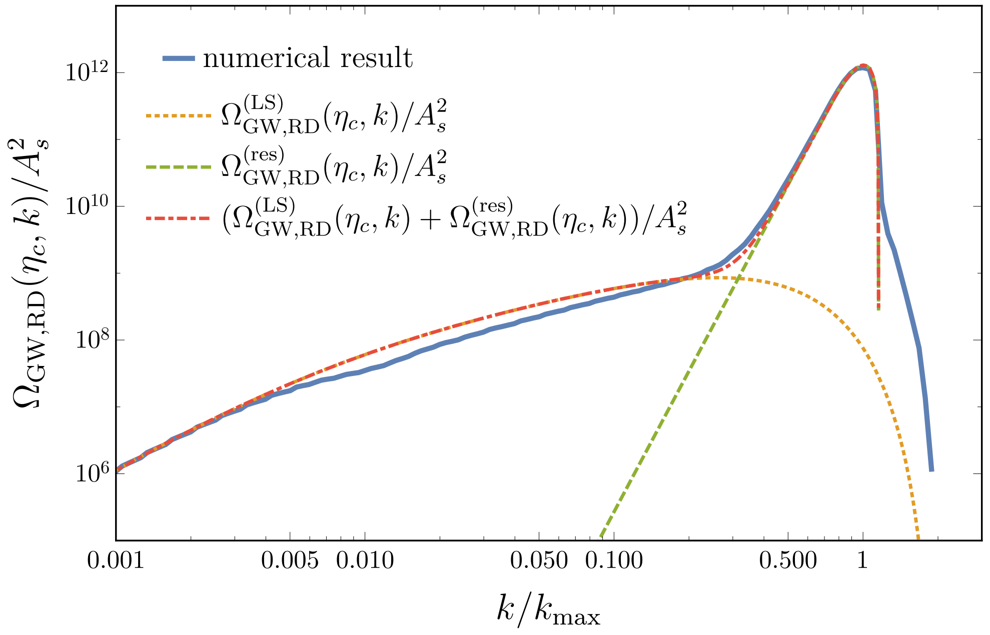
Secondary GW

Early matter domination ⇒ Radiation

K. Inomata, 1904.12878, 1904.12879



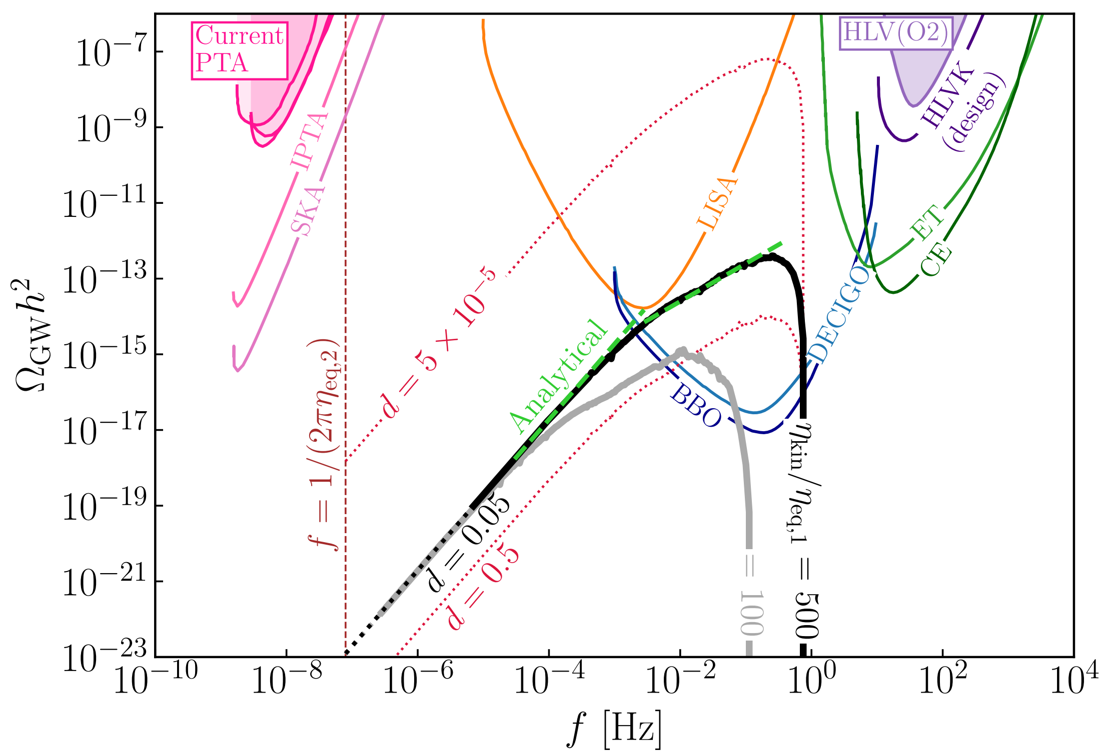
Gradual transition

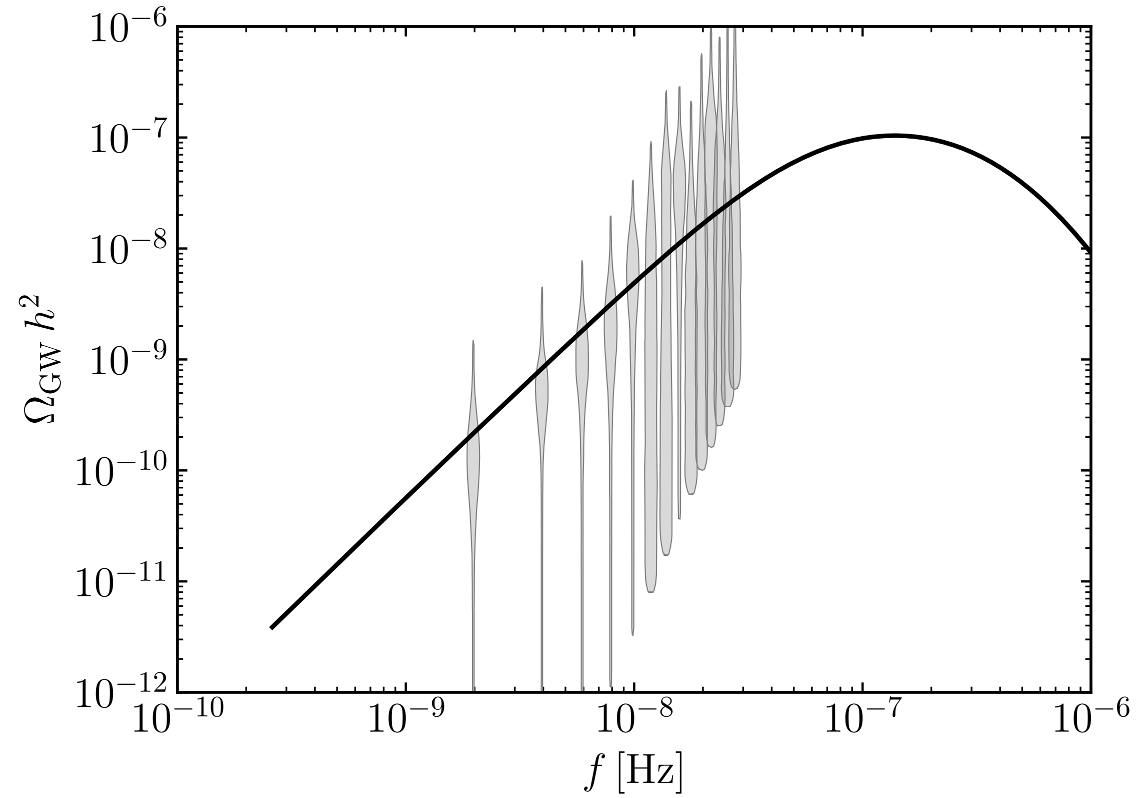


Sudden transition

Another example: Matter → kination

Harigaya, Inomata, Terada 2305.14242, 2309.00228

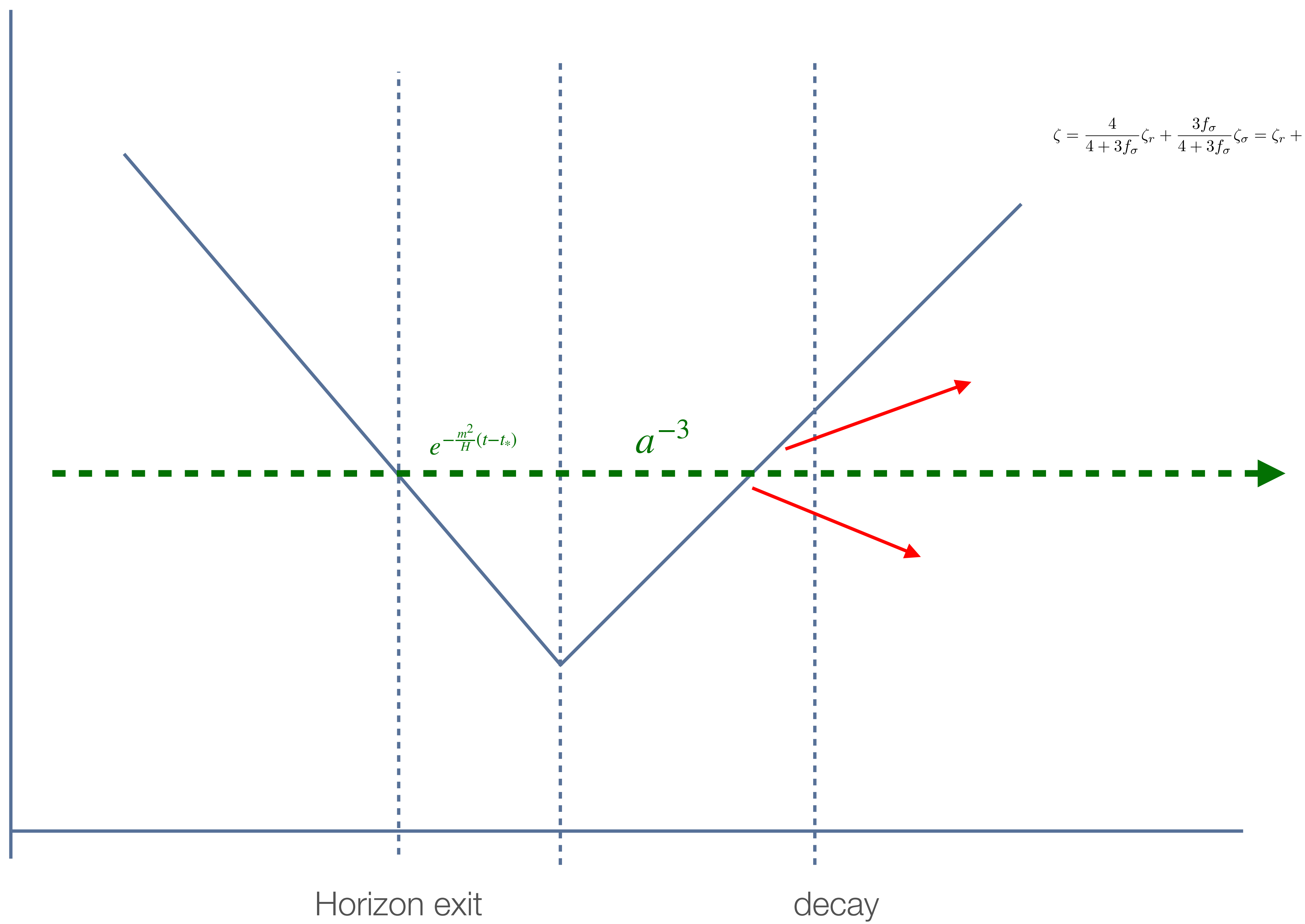




Conclusions

- GW will be a great tool in probing early universe, especially for epochs “invisible” through other means.
  - Long term prospect. Probably the only way to get these information.
- Inflation stage is a plausible place for interesting and observable GW signal can be generated.
  - Both primary and secondary GW.
  - Discovery and study its shape very informative.

extra

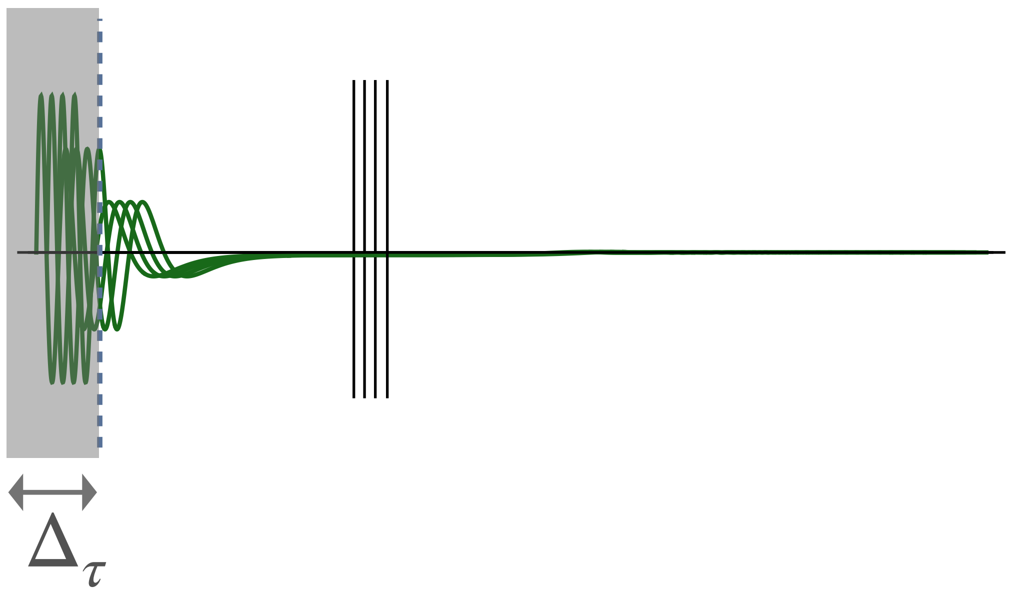


$$\zeta = \frac{4}{4+3f_\sigma}\zeta_r + \frac{3f_\sigma}{4+3f_\sigma}\zeta_\sigma = \zeta_r + \frac{f_\sigma}{4+3f_\sigma}S_\sigma$$

GW signal

$$h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

UV: $k > \Delta_\tau^{-1}$



Time scale of bubble collision $\approx \Delta_\tau$.

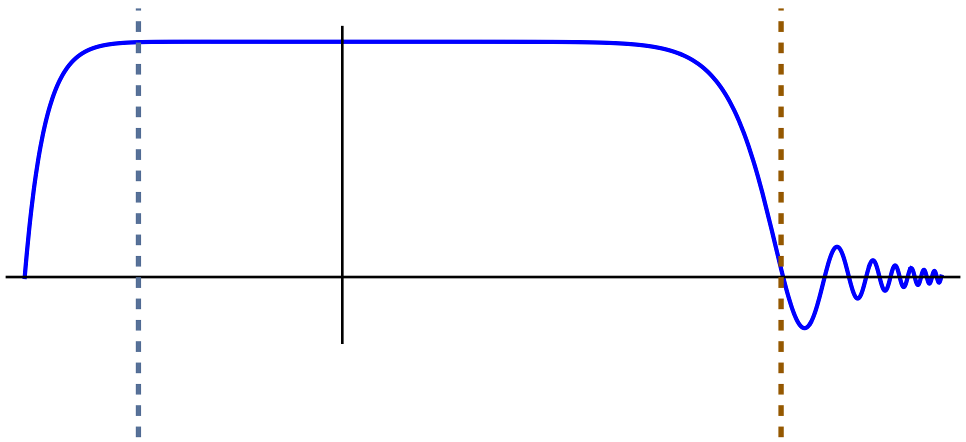
Oscillation pattern in frequency smeared out in this regime.

Spectrum depends on details of the source.

GW signal

$$h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

IR: $k < \tau_*$



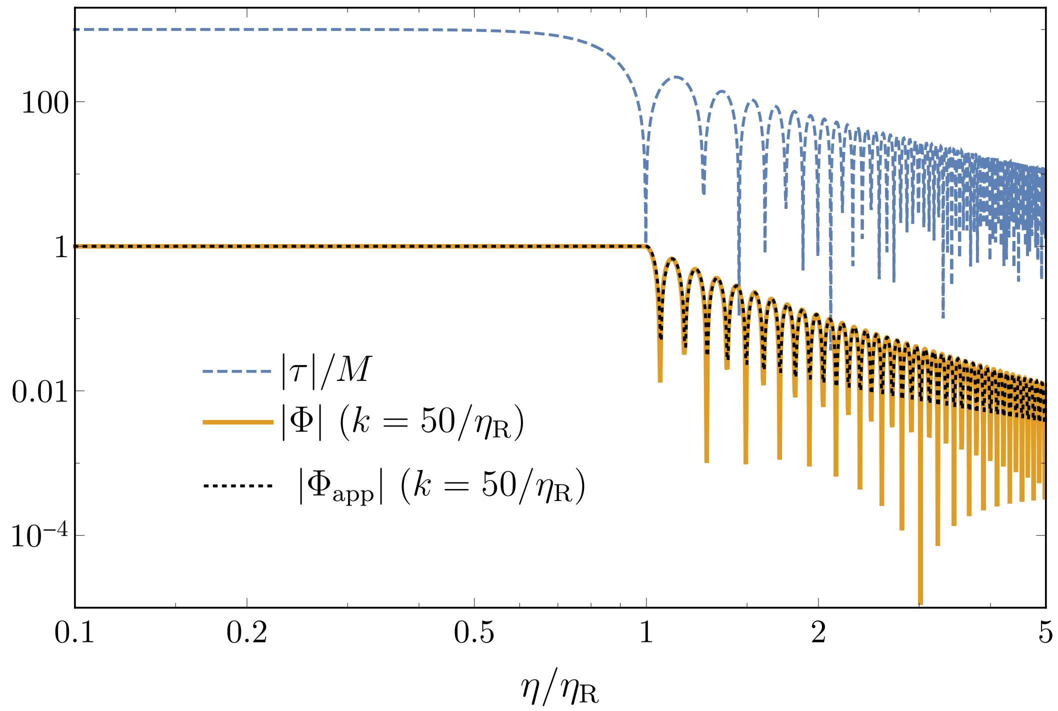
Mode outside horizon at the time of phase transition

No oscillation. Can treat the GW as if it is from a point source.

Gravitational potential

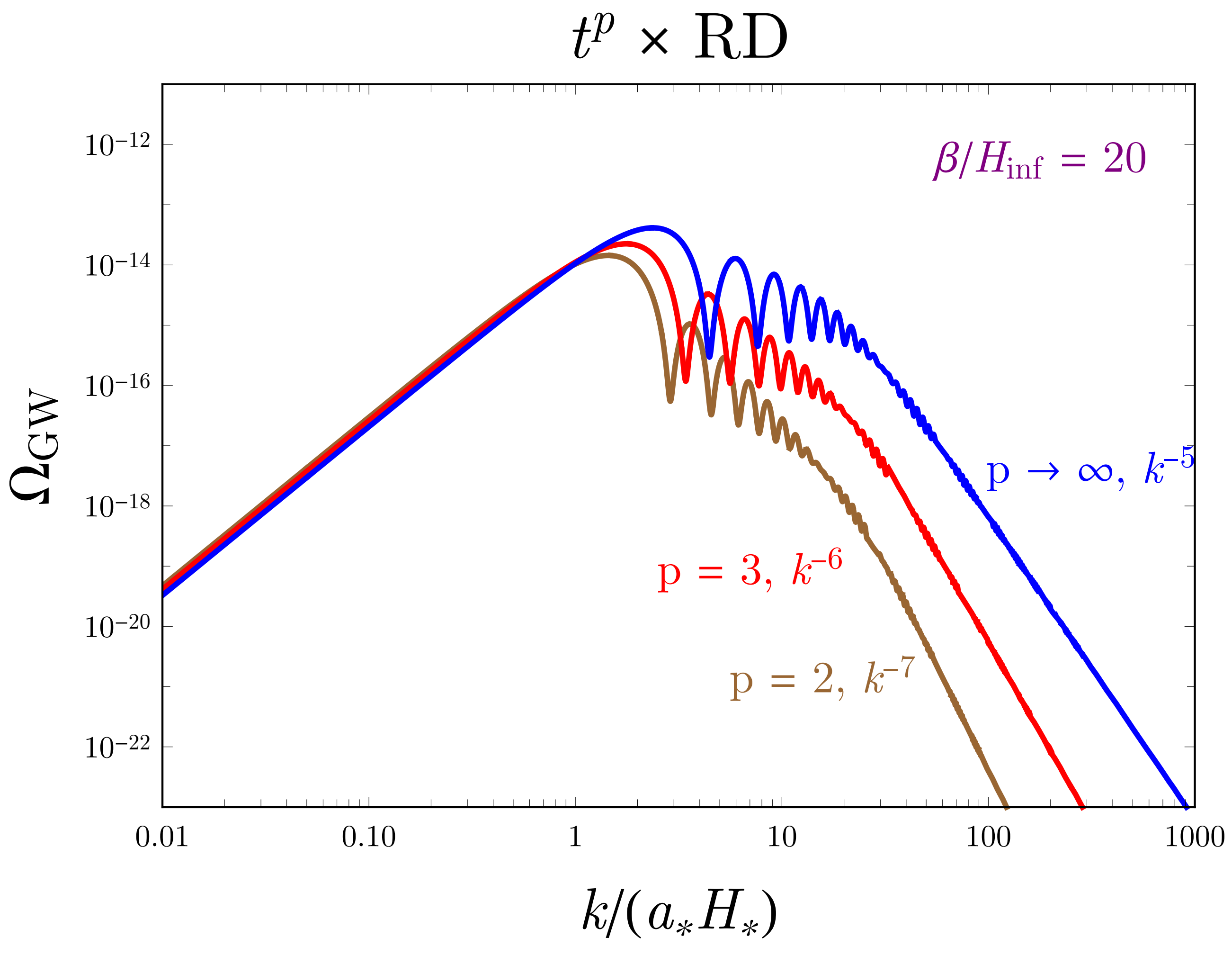
$$\Phi'' + 3(1+w)\mathcal{H}\Phi' + wk^2\Phi = 0$$

$$\Phi' = -\frac{k^2\Phi + 3\mathcal{H}^2\Phi + \frac{3}{2}\mathcal{H}^2\left(\frac{\rho_{\rm m}}{\rho_{\rm tot}}\delta_{\rm m} + \frac{\rho_{\rm r}}{\rho_{\rm tot}}\delta_{\rm r}\right)}{3\mathcal{H}}$$



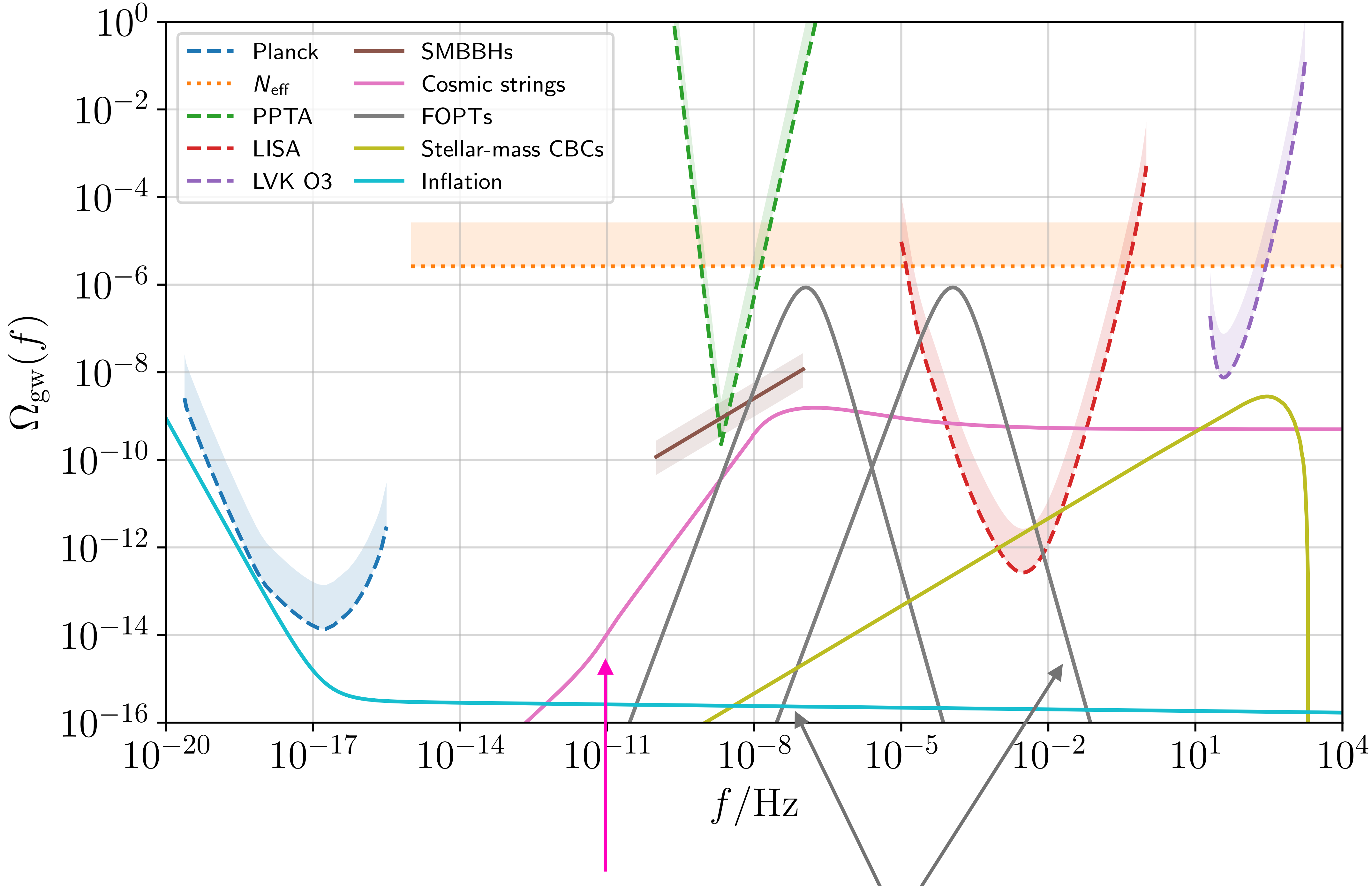
Sudden transition

Comparing scenarios



Different inflationary scenarios.
→ different slope in UV part.

Review by Renzini et al, 2202.00178



Cosmic string, $G\mu = 10^{-11}$

First order phase transition around EW scale

1st order phase transition

We consider the case:

Phase transition to complete within Hubble time. And, with O(1) of Hubble volume in new phase.

With parameterization

$$S_4(t) \simeq S(t_*) + \beta(t_* - t) + \ldots \qquad \beta = \left| \frac{dS_4}{dt} \right|$$

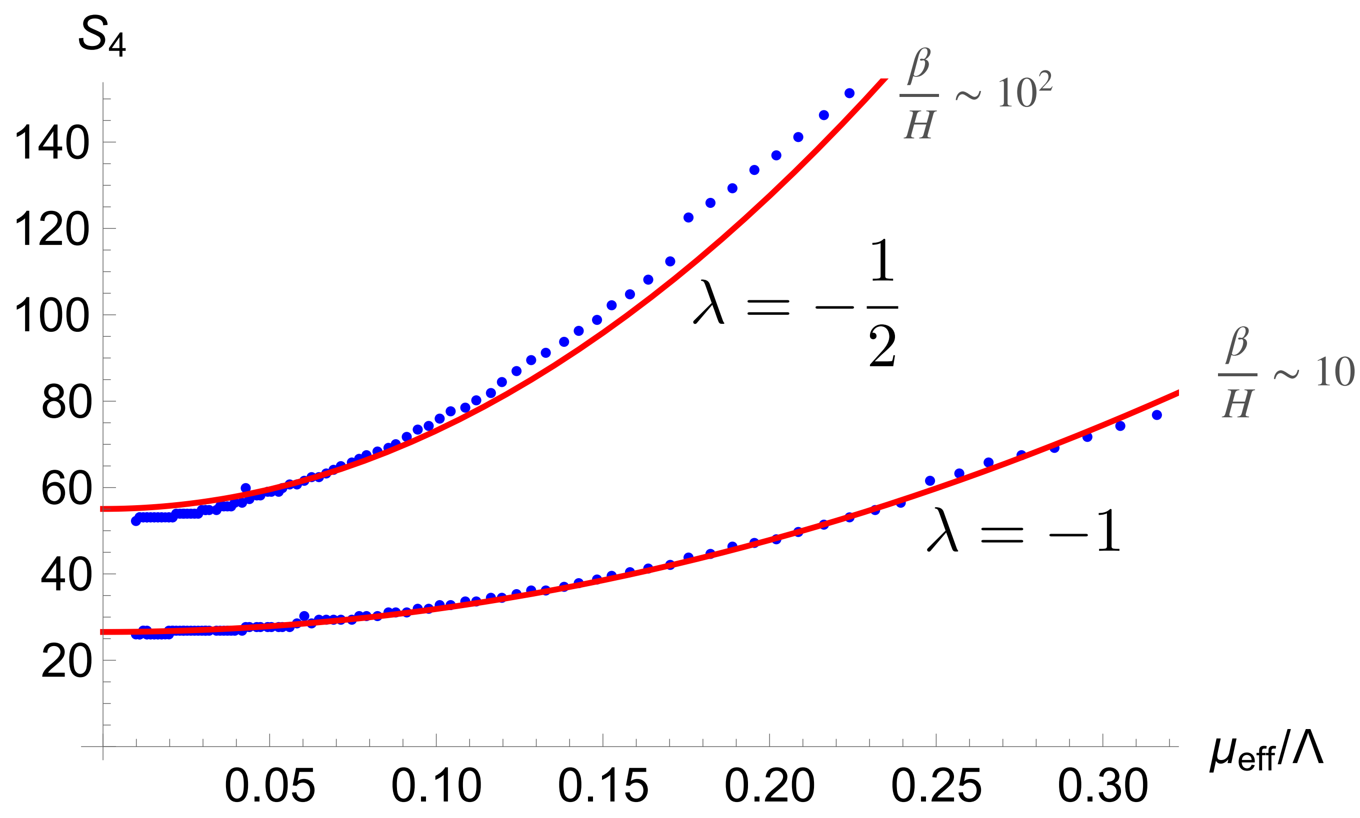
Guth and Weinberg, 83'

This can be satisfied with $\beta^4 \ll m_\sigma^4 \ll 3M_{\rm Pl}^2 H^2$

For GW signal in the opposite limit: J. Barir, M. Geller, C. Sun, T. Volansky, 2203.00693

In our toy model:

$$\frac{\beta}{H} = \left|\frac{dS_4}{d\log \mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm Pl}}{\phi\left(1 - m_\sigma^2/(c^2\phi^2)\right)} \qquad \left|\frac{dS_4}{d\log \mu_{\rm eff}^2}\right| \sim O(1)$$



Need:

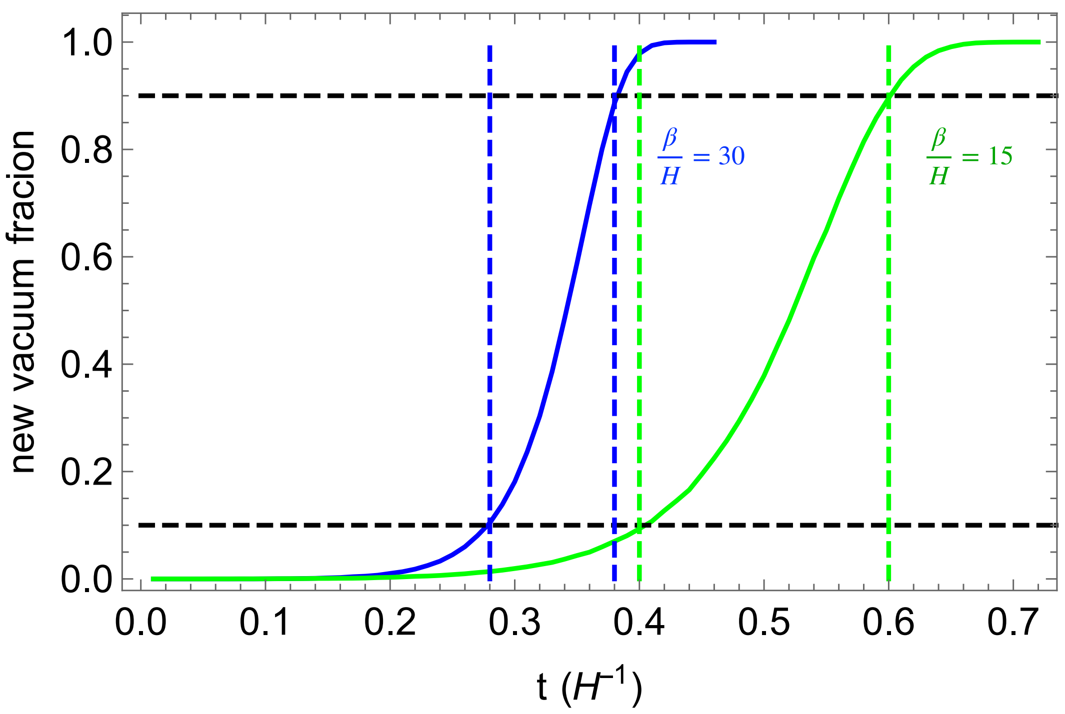
$$\beta^4 \ll m_\sigma^4 \ll 3M_{\rm Pl}^2 H^2$$

CosmoTransitions

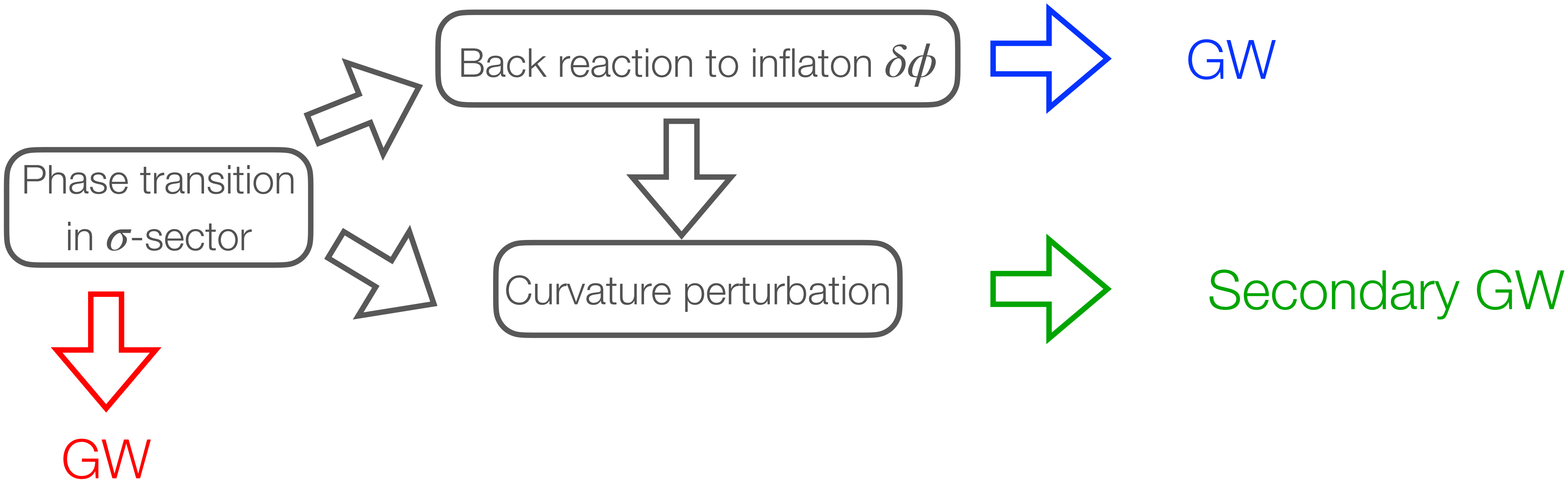
1st order phase transition

We want to have:
Phase transition to complete within Hubble time. And, with O(1) of Hubble volume in new phase. Guth and Weinberg, 83’

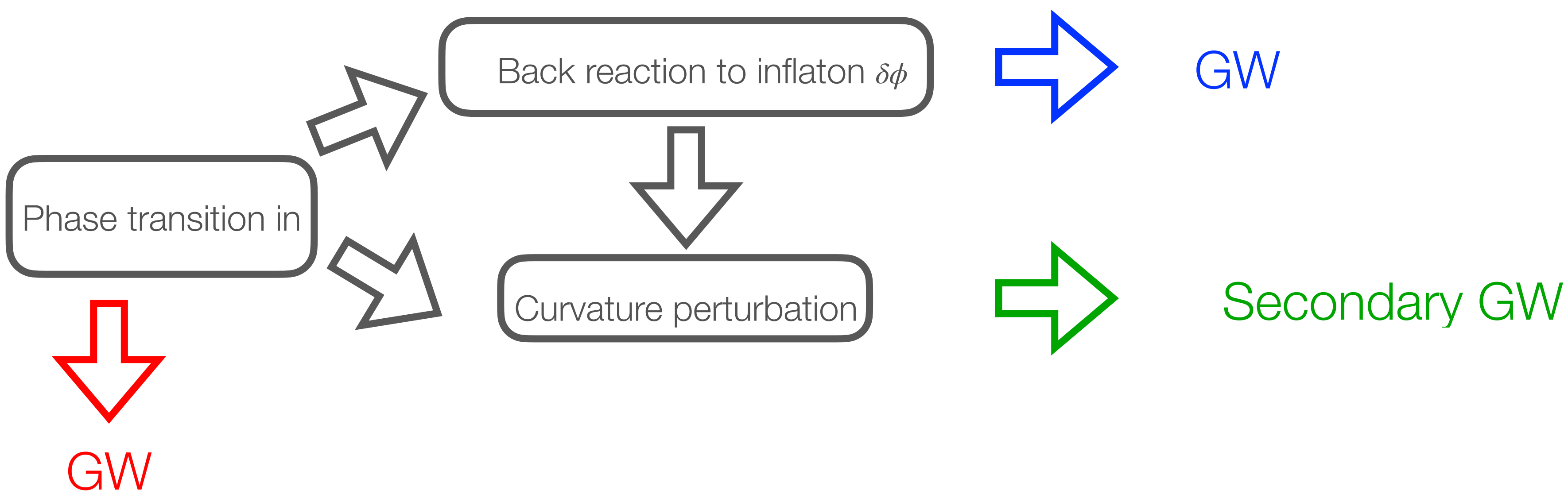
$$\beta^4 \ll m_\sigma^4 \ll 3M_{\rm Pl}^2 H^2 \qquad \beta \sim (10-100)\times H$$

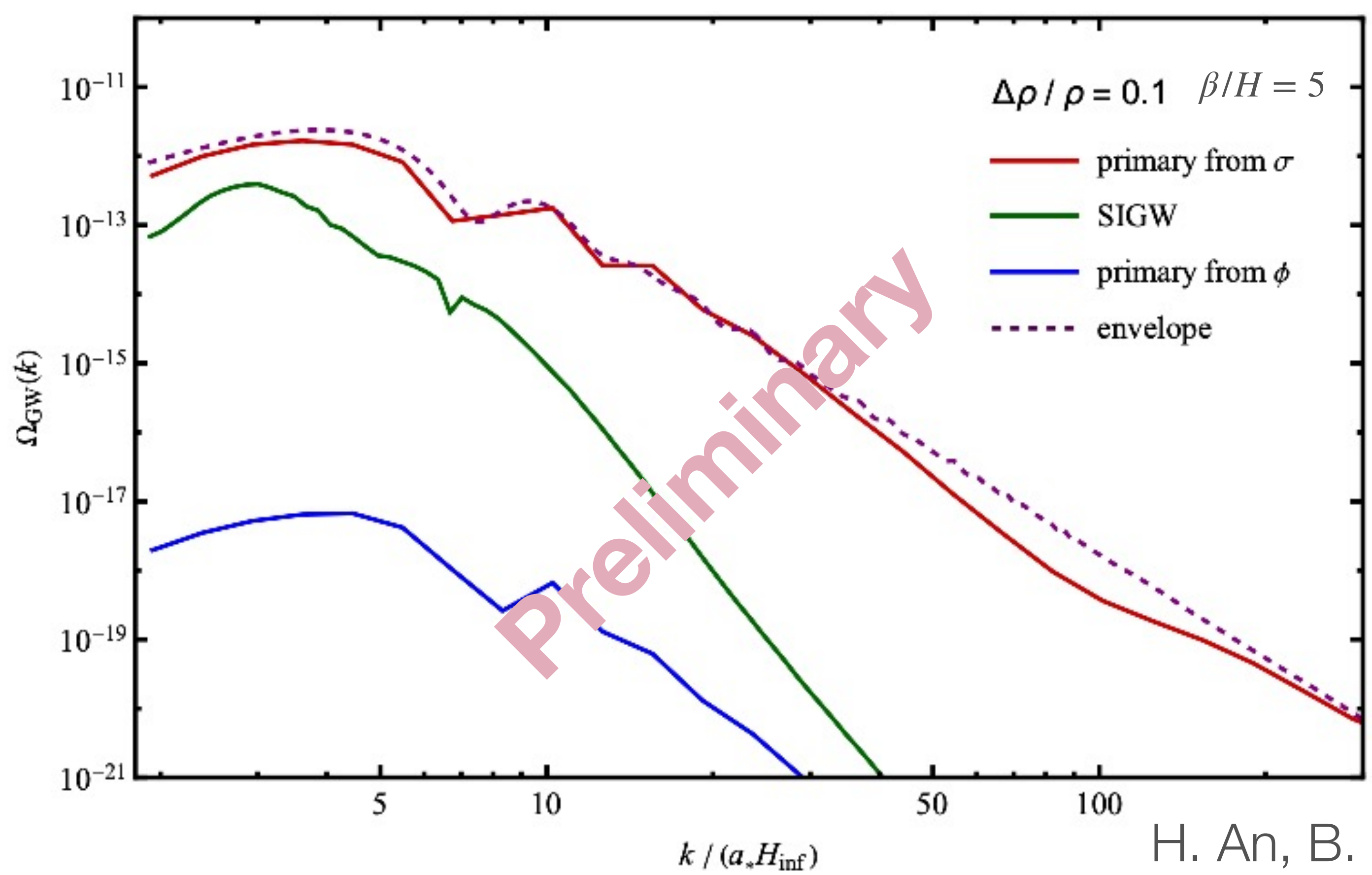


More complete picture



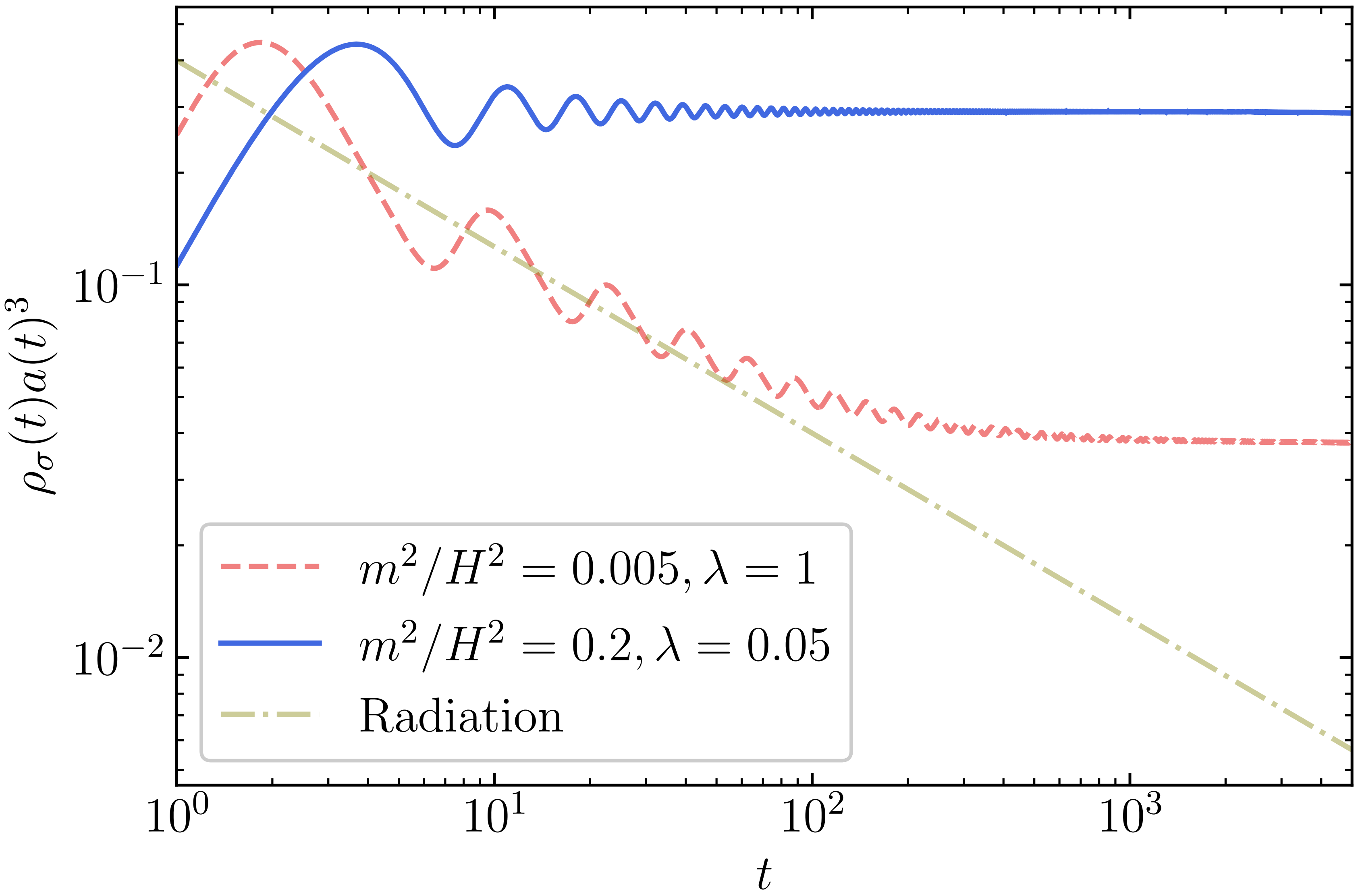
More complete picture





Also including a more careful numerical simulation of the spector (bubble) + inflaton system.

H. An, B. Su, H. Tai, LTW, C. Yang, in progress



$\rho_\sigma(t)a(t)^3$
10^{-1}
10^{-2}
10^0
10^1
10^2
10^3
t
$m^2/H^2 = 0.005, \lambda = 1$
$m^2/H^2 = 0.2, \lambda = 0.05$
Radiation

$$\zeta = -\Psi - H\frac{\delta\rho}{\dot{\rho}} = (1-f_\sigma)\zeta_{\rm r} + f_\sigma\zeta_\sigma = \zeta_{\rm r} + \frac{f_\sigma}{3}S_\sigma$$

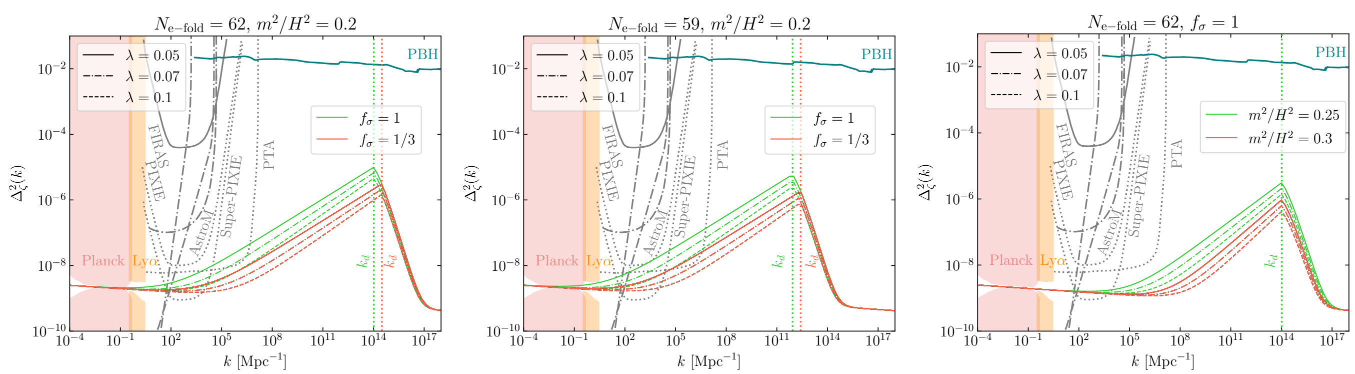
$$\zeta_{\rm r} = -\Psi + \frac{1}{4}\frac{\delta\rho_{\rm r}}{\rho_{\rm r}}\,, \quad \zeta_\sigma = -\Psi + \frac{1}{3}\frac{\delta\rho_\sigma}{\rho_\sigma}\,, \quad f_\sigma = \frac{3\rho_\sigma}{4\rho_{\rm r}+3\rho_\sigma}\,, \quad S_\sigma = \frac{\delta\rho_\sigma}{\rho_\sigma} - \frac{3}{4}\frac{\delta\rho_{\rm r}}{\rho_{\rm r}} = 3(\zeta_\sigma - \zeta_{\rm r})$$

$$P_\zeta = P_{\zeta_{\rm r}} + \left(\frac{f_\sigma}{3}\right)^2 P_{S_\sigma}$$

a. Benchmark 1. We focus on the first benchmark in eq. (55). For $m^2 = 0.2H^2$ and $\lambda \simeq 0.05 - 0.1$, we get $\langle V(\sigma)\rangle \approx 0.02H^4$ from eq. (41), implying $\langle V(\sigma)\rangle / V_k \approx 3\times 10^{-12}$ for $H = 5\times 10^{13}$ GeV. Assuming instantaneous reheating, and $\rho_{\rm end} \simeq V_k/100$, we see $f_\sigma \simeq 1$ for $a \simeq (1/3)\times 10^{10} a_{\rm end}$. As benchmarks, we assume σ decays when $f_\sigma = 1$ and $1/3$. Using $k_{\rm end} \approx 4\times 10^{23}$ Mpc^{-1}, we can then evaluate $k_d \approx 10^{14}$ Mpc^{-1} and $k_d \approx 3\times 10^{14}$ Mpc^{-1}, respectively. The result for the curvature power spectrum with these choices is shown in Fig. 3 (left).

b. Benchmark 2. We now discuss the second benchmark in eq. (55). We again choose $m^2 = 0.2H^2$ and $\lambda \simeq 0.05 - 0.1$, for which we get $\langle V(\sigma)\rangle \approx 0.02H^4$ from eq. (41). This implies $\langle V(\sigma)\rangle / V_k \approx 3\times 10^{-12}$ for $H = 5\times 10^{13}$ GeV, as before. The rest of the parameters can be derived in an analogous way, with one difference. During the reheating epoch, with our assumption $w \approx 0$, f_σ does not grow with the scale factor since the dominant energy density of the Universe is also diluting as matter. Accounting for this gives $k_d \approx 8\times 10^{11}$ Mpc^{-1} and $k_d \approx 3\times 10^{12}$ Mpc^{-1}, for $f_\sigma = 1$ and $1/3$, respectively, with the resulting curvature power spectrum shown in Fig. 3 (center).

c. Benchmark 3. This is same as the first benchmark discussed above, except we focus on $m^2 = 0.25H^2$ and $0.3H^2$ along with $f_\sigma = 1$. The result is shown in Fig. 3 (right).



1st order phase transition

Bubble nucleation rate: $\frac{\Gamma}{V} \simeq m_\sigma^4 e^{-S_4}$

m_σ: typical scale in the spectator sector

Efficient phase transition:

$$\int_{-\infty}^{t} dt' \frac{\Gamma}{V} \frac{1}{H^3} \simeq O(1) \;\rightarrow\; S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_\sigma^4}{H^4} \right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm Pl}} \frac{m_\sigma^4}{H^4} \right)$$

Phase transition is 1st order ($S_4 \gg 1$).

Assume spectator sector does not dominate energy density:

$$H^4 \ll m_\sigma^4 \ll 3 M_{\rm Pl}^2 H^2$$

Guth and Weinberg, 83'

$$\frac{\Gamma}{V} = e^{3Ht} C m_\sigma^4 e^{-S_4}$$

V: co-moving volume

$$R(t,t') = \frac{1}{H}(e^{-Ht'} - e^{-Ht})$$

Co-moving radius for bubble nucleated at t'

$$\mathcal{P}(t) = \exp\left[-\int_{-\infty}^{t} dt' \frac{4\pi}{3H^3} \times (e^{-Ht'} - e^{-Ht})^3 e^{3Ht'} C m_\sigma^4 e^{-S_4(t')}\right]$$

Fraction of space in false vacuum

For true vacuum to occupy an O(1) fraction:

$$\Gamma(t) \simeq \Gamma(t_\star) \mathrm{e}^{\beta(t-t_\star)},$$

$$\int_{-\infty}^{t} dt' \frac{4\pi}{3H^3} (e^{-Ht'} - e^{-Ht})^3 e^{3Ht'} C m_\sigma^4 e^{-S_4(t')} \sim \mathcal{O}(1)$$

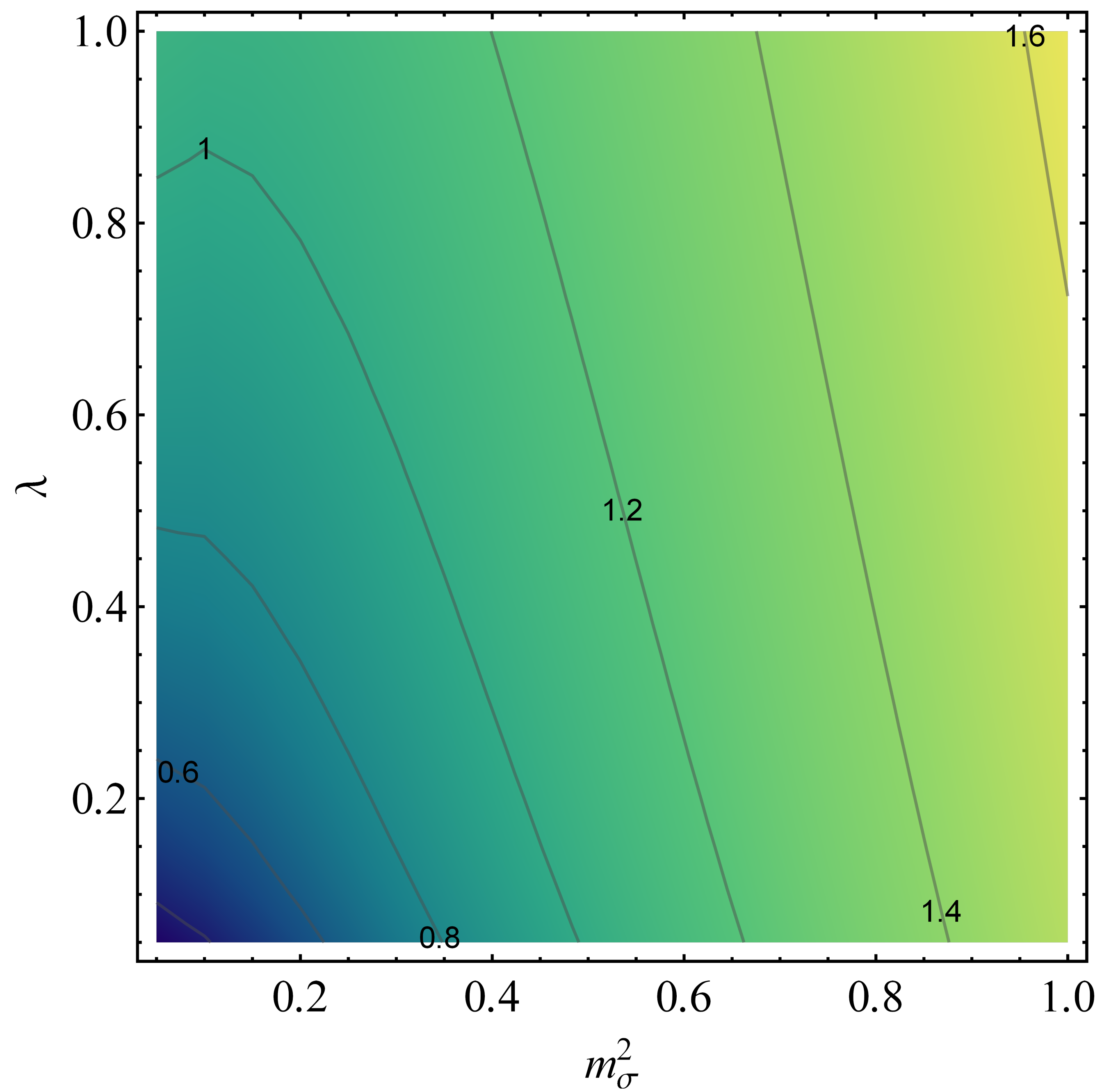
$$\beta \equiv -\left.\frac{\mathrm{d}S_b}{\mathrm{d}t}\right|_{t=t_\star}.$$

$$S_4(t') = S_4(t) + \frac{dS_4(t)}{dt}(t'-t) \equiv S_4(t) - \beta(t'-t)$$

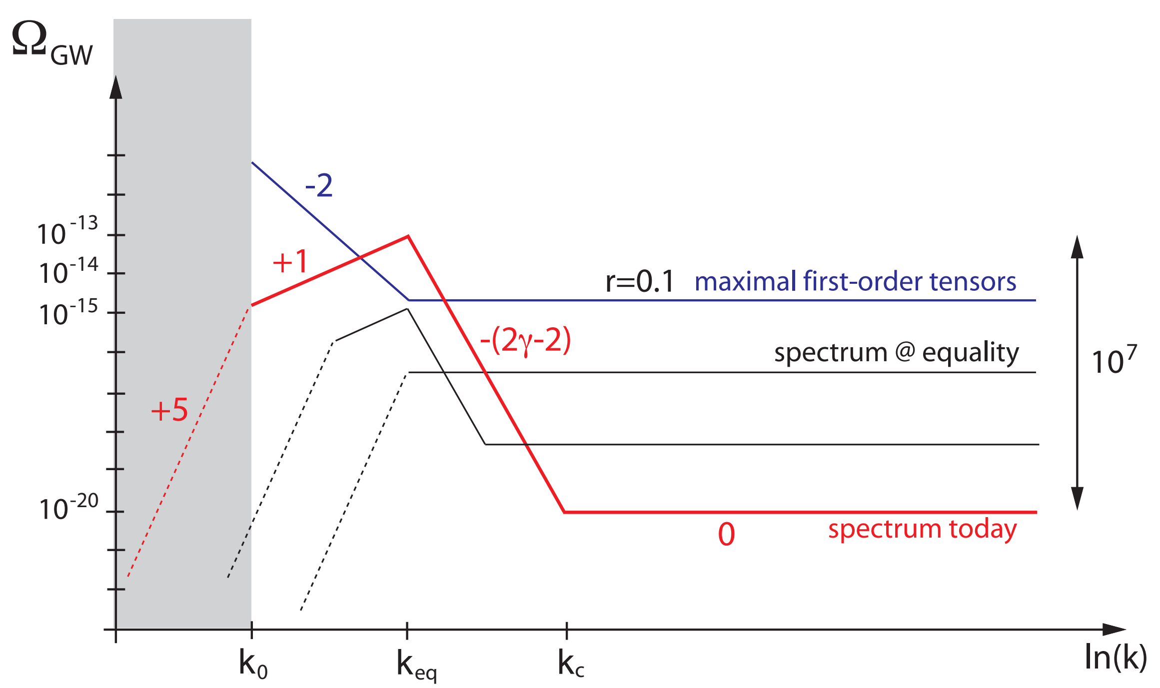
$$\begin{aligned}\mathcal{O}(1) &\sim C m_\sigma^4 e^{-S_4(t)} \frac{4\pi}{H^3} \int_{-\infty}^{t} dt' \left(1 - e^{-H(t-t')}\right)^3 e^{-\beta(t-t')} \\ &\approx C m_\sigma^4 e^{-S_4(t)} \frac{8\pi}{\beta(\beta+H)(\beta+2H)(\beta+3H)} \\ &\approx 8\pi C e^{-S_4(t)} \frac{m_\sigma^4}{\beta^4}, \qquad (25)\end{aligned}$$

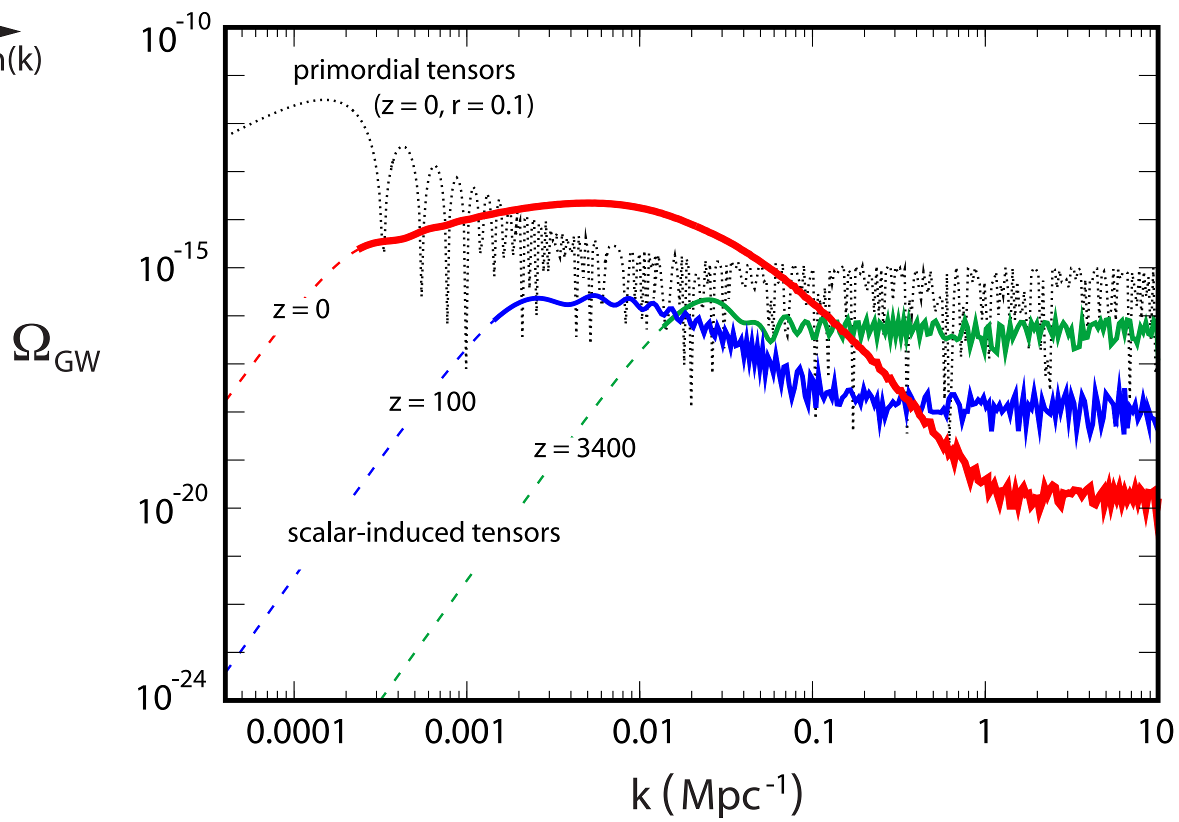
$$\Rightarrow \quad S_4(t_0) \approx \log\left(\frac{m_\sigma^4}{\beta^4}\right)$$

Amplitude of GW from light scalar

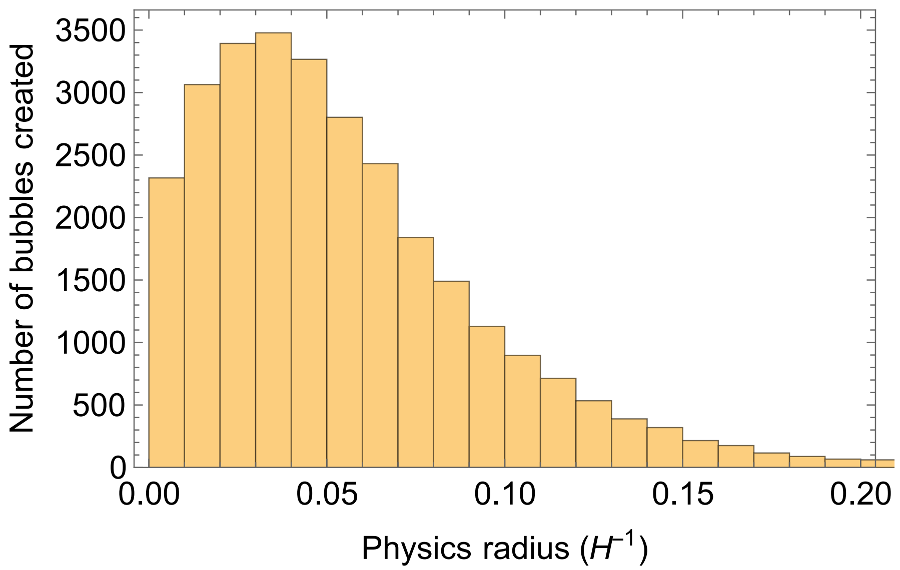


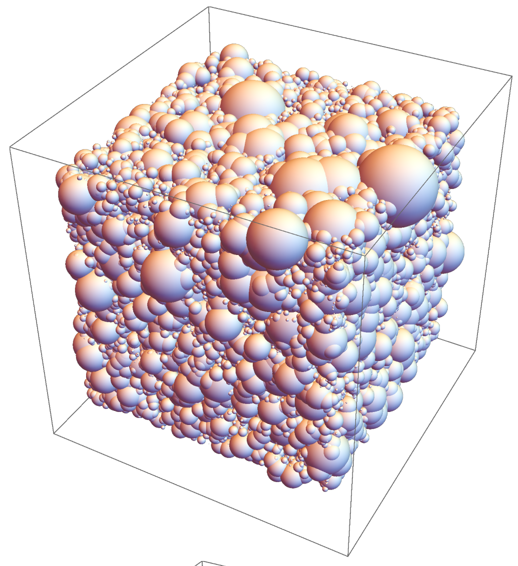
Secondary from $\Delta^2_{\mathcal{R}}$



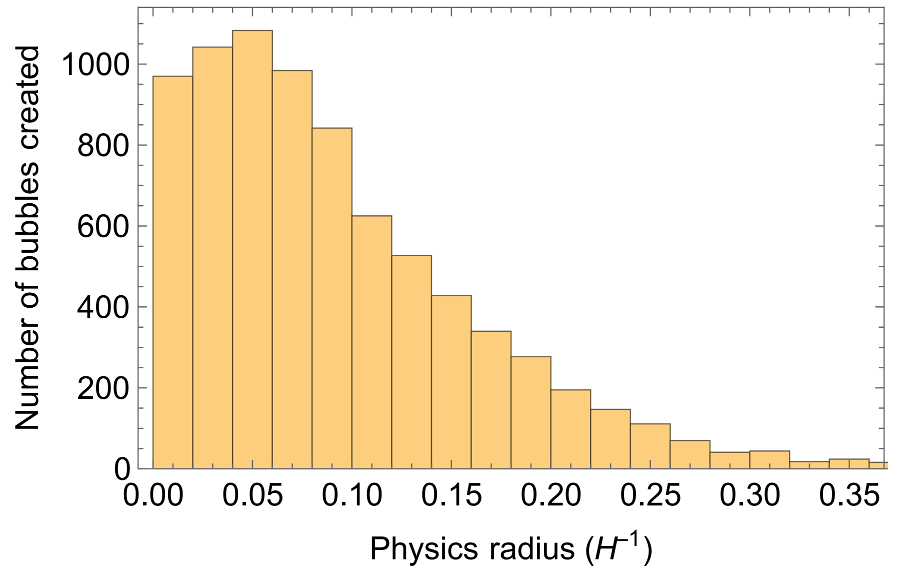


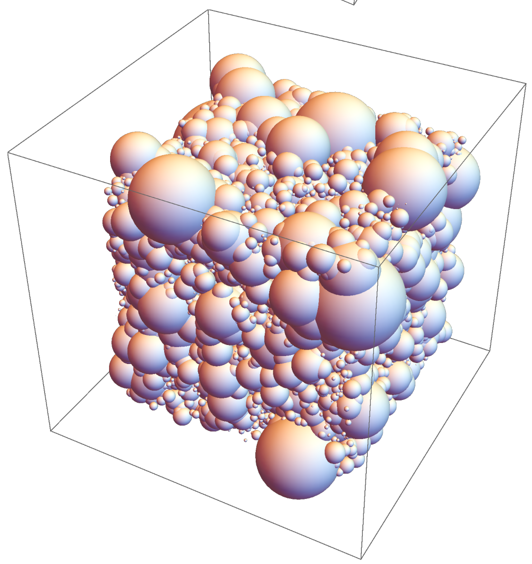
Numerical simulation of bubble percolation





$\beta/H = 30$





$\beta/H = 15$

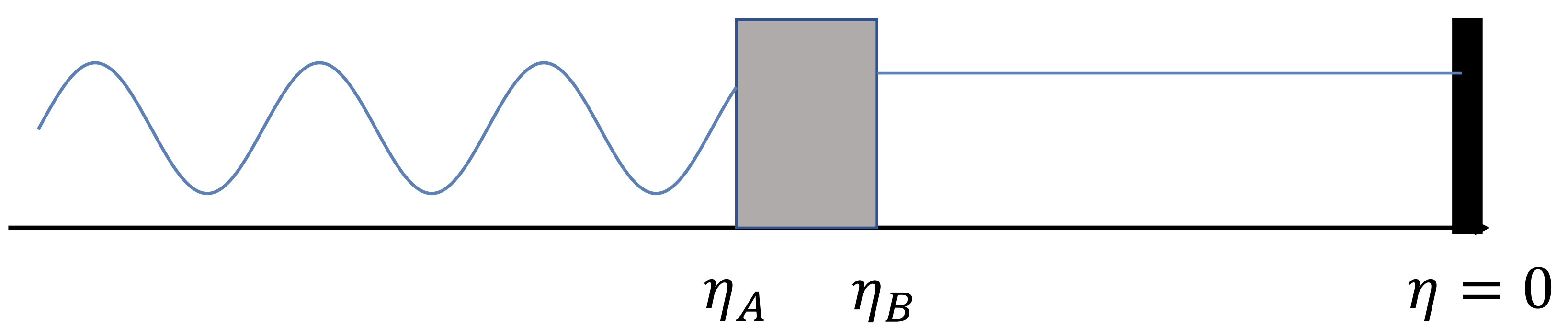
h^f in a generic inflation model

- Generic features

source

Model dependent

  - For $|k\tau'| > \eta_A$, $h^f = k^{-1}\cos k(\tau' - \tau_0')\tilde{\mathcal{G}}_0^f(k)$
  - For $|k\tau'| < \eta_B$, $h^f = \underbrace{\left[a(\tau')\int_{\tau'}^{0} a^{-2}(\tau_1)d\tau_1\right]}_{\text{Independent of k}}$



Generic features of GW spectrum

- $k_p \ll \Delta_p^{-1}$ $\quad \cos k_p t_p \to 1\ ,\ \ \sin k_p t_p \to 0$

$$\rho_{\rm GW}(\tau) = \int \frac{d^3k}{(2\pi)^3} \frac{8\pi G_N \left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2}{Va^4(\tau)a^2(\tau_\star)} \cos^2 k(\tau_\star - \tau_0)\tilde{T}_{ij}(0,\mathbf{k}_p)\tilde{T}^*_{ij}(0,\mathbf{k}_p)$$

$$\tilde{T}_{ij}(0,\mathbf{k}_p) = \int dt_p \tilde{T}_{ij}(\tau,\mathbf{k}_p)$$

$\langle \tilde{T}_{ij}\tilde{T}^*_{ij}\rangle_{k_p \ll \Delta_p^{-1}}$ independent of *k*. *Cai, Pi and Sasaki, 1909.13728*

- $k\Delta \ll 1 \ll |k\tau_\star|$, an oscillating feature in the GW spectrum

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{4G_N|\tilde{T}_{ij}(0,0)|^2}{\pi^2 Va^4(\tau)a^2(\tau_\star)} \left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3 \cos^2 k(\tau_\star - \tau_0)\right\}$$

Generic features of GW spectrum

- The UV part of the spectrum
  - $k_p \Delta_p \gg 1$, the oscillation pattern is completely smeared.

$$\frac{d\rho_{\rm GW}^{\rm UV}}{d\log k} = \frac{2G_N |\tilde{T}_{ij}(k_p, \mathbf{k}_p)|^2}{\pi^2 V a^4(\tau) a^2(\tau_\star)} \left\{ \left[\tilde{\mathcal{E}}_0^i(k) \tilde{\mathcal{G}}_0^f(k) \right]^2 k^3 \right\}$$

- The IR part of the spectrum $(\eta' > \eta_B, \text{ or } |\eta'| < |\eta_B|)$
  - $\tilde{\mathcal{G}}^f$ is flat, no oscillation parttern in the spectrum either,

$$\frac{d\rho_{\rm GW}^{\rm IR}}{d\log k} = \frac{4G_N |\tilde{T}_{ij}(0,0)|^2}{\pi^2 V a^4(\tau)} \left[\int_{\tau_\star}^0 a^{-2}(\tau_1) d\tau_1 \right]^2 \left\{ \left[\tilde{\mathcal{E}}_0^i(k) \right]^2 k^5 \right\}$$

First order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering and fast reheating

$$\Omega_{\mathrm{GW}}(k_{\mathrm{today}}) = \Omega_R \frac{H_{\mathrm{inf}}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\mathrm{inf}}}\right)\right]\frac{\Delta\rho_{\mathrm{vac}}}{\rho_{\mathrm{inf}}}\frac{d\rho_{\mathrm{GW}}^{\mathrm{flat}}}{\Delta\rho_{\mathrm{vac}} d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction

Redshift

$$\frac{f_{\mathrm{today}}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\mathrm{CMB}}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\mathrm{inf}}^2}{8\pi G_N}\right)\right]^{1/4}}$$

$$e^{-N_e}$$

N_e: e-folds before the end of inflation

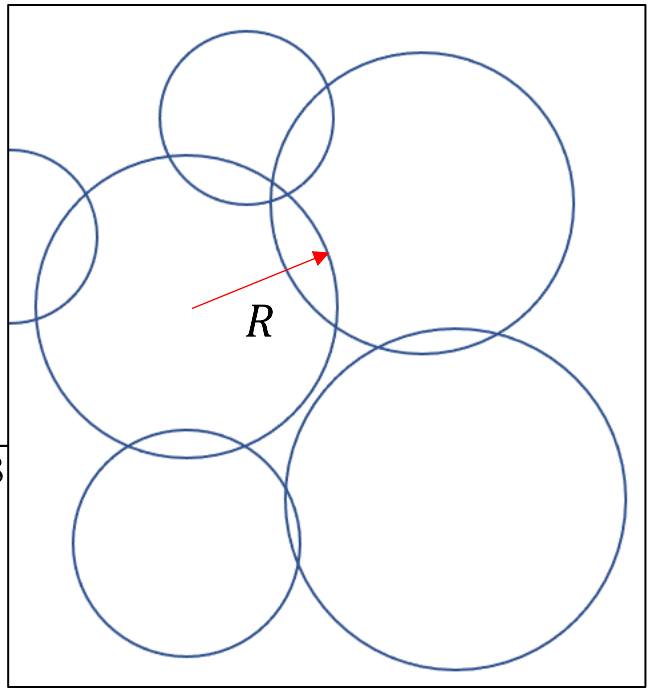
First order phase transition during inflation

- For phase transition to finish

$$R = \Delta_p \ll H_{\rm inf}^{-1}$$

$$\beta = \frac{dS_4}{dt} \sim \Delta_p^{-1} \gg H_{\rm inf}$$

$$\frac{d\rho_{\rm GW}^{flat}}{\Delta\rho_{\rm vac} d\log k_p} \approx \left(\frac{H_{\rm inf}}{\beta}\right)^2 \times \frac{\beta\ k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$



Huber and Konstandin, 0806.1828

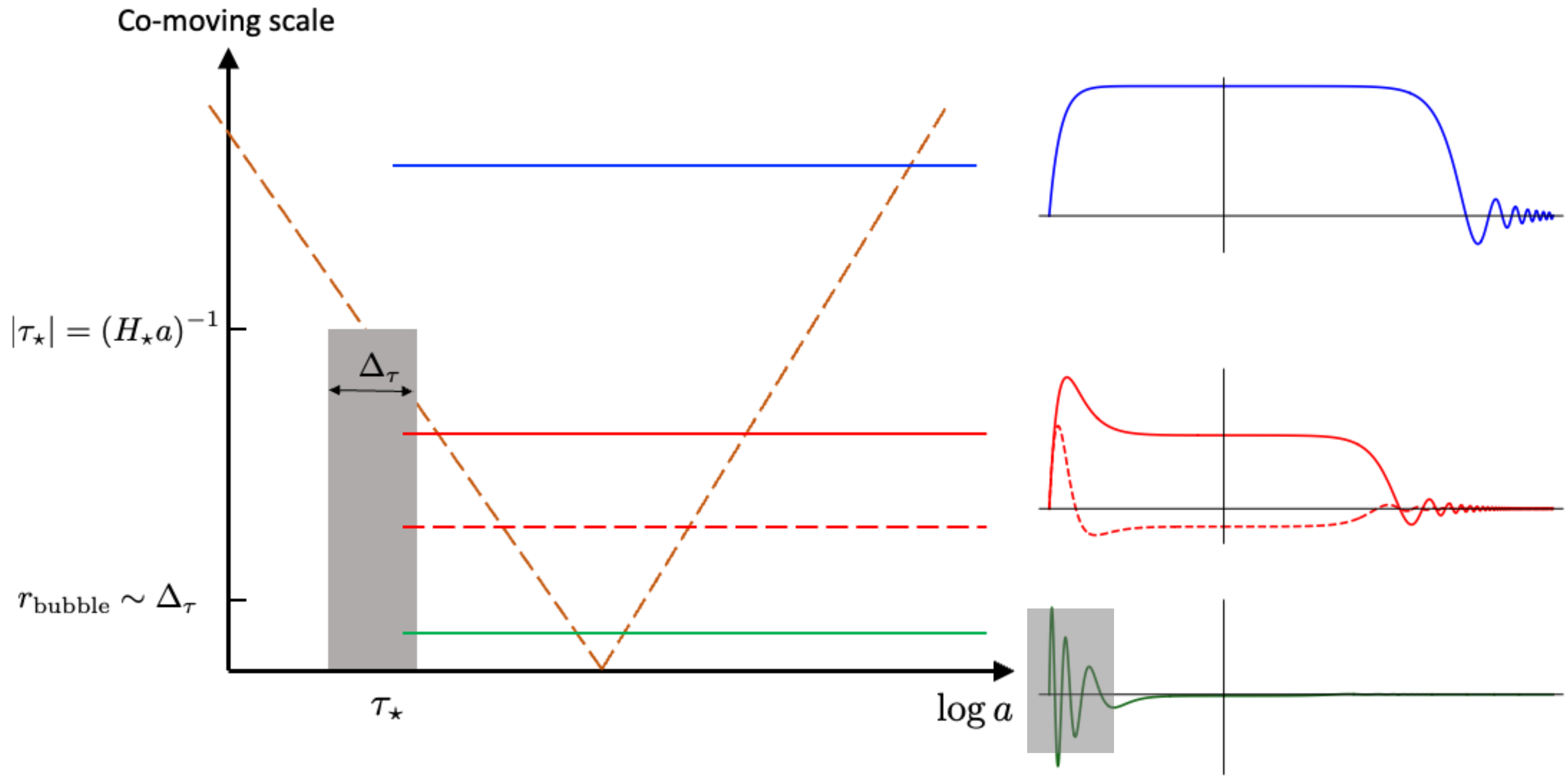
Examples

- Inflation models
  - Quasi-de Sitter inflation $\tilde{\mathcal{G}}_0^f = \left(-\frac{H}{k}\right)\,, \quad \eta_0' = 0$
  - t^p inflation $\tilde{\mathcal{G}}_0^f = a_0^{-1}(-k\tau_0)^{\frac{p}{1-p}}\frac{2^{\frac{p}{-1+p}}}{\sqrt{\pi}}\Gamma\left(\frac{3}{2}+\frac{1}{-1+p}\right)\,, \quad \eta_0' = \frac{\pi}{2-2p}$

  In t^p inflation, we have the slow-roll parameter $\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{p}$

$$\tilde{\mathcal{G}}_0^f \sim k^{-\frac{1}{1-\epsilon}}$$

- Evolution after inflation
  - In RD, $\tilde{\mathcal{E}}_0^i \sim k^{-1}$
  - In MD, $\tilde{\mathcal{E}}_0^i \sim k^{-2}$



Co-moving scale
$|\tau_\star| = (H_\star a)^{-1}$
Δ_τ
$r_{\rm bubble} \sim \Delta_\tau$
$\tau_\star$
$\log a$

Oscillations

$$\tau_*^{-1} < k < \Delta_\tau^{-1}$$