# Higgs-$R^2$ inflation and cosmological collider physics

Yohei Ema

University of Minnesota

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Based mainly on 2002.11739, 2102.12501, 2309.10841
with R. Jinno, K. Nakayama, K. Mukaida, J. van de Vis, S. Verner
See also 1609.05209, 1701.07665, 1907.00993 and 2008.01096.

# Outline

1. Higgs inflation, unitarity and $R^2$

2. Cosmological collider signatures

3. Summary

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# Higgs inflation

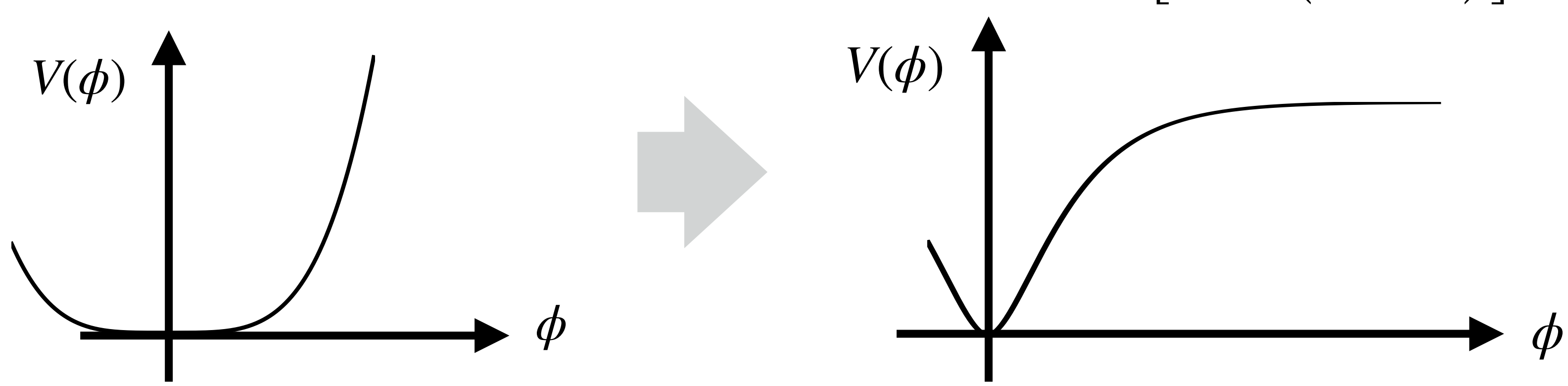
- Higgs inflation: standard model Higgs = inflaton. [Bezrukov, Shaposhnikov 07; ...]

$$\mathscr{L} = \frac{M_P^2}{2}R + |D_\mu\Phi|^2 + \underline{\xi|\Phi|^2 R} - \lambda|\Phi|^4$$ with $\Phi$: Higgs doublet.

- Non-minimal Higgs-gravity coupling "$\xi$" flattens the potential.

$V(\phi) \simeq \frac{\lambda}{4}\phi^4$

$V(\phi) \simeq \frac{\lambda M_P^4}{4\xi^2}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right)\right]^2$




CMB: $\xi \simeq 4\sqrt{\lambda} \times 10^4 \gg 1.$

(Focusing on metric formalism)

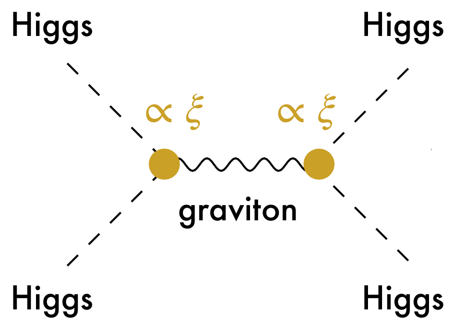
- Inflationary predictions: $n_s = 1 - \frac{2}{N_e}, \quad r = \frac{12}{N_e^2}.$

Reheating determines $N_e$, essential for inflationary prediction,

e.g. to distinguish between Higgs and $R^2$ inflation.

# Unitarity issue

- Higgs inflation is not consistent for $E > M_P/\xi \sim 10^{14}\,\mathrm{GeV}$. [Burgess+ 09,10; Barbon+09, Hertzberg10; Bezrukov+10]

$i\mathcal{M} =$ 

$\sim \dfrac{\xi^2 E^2}{M_P^2}$, $|\mathcal{M}|^2 \sim$ (scattering probablity) $> 1$ for $E > M_P/\xi$.

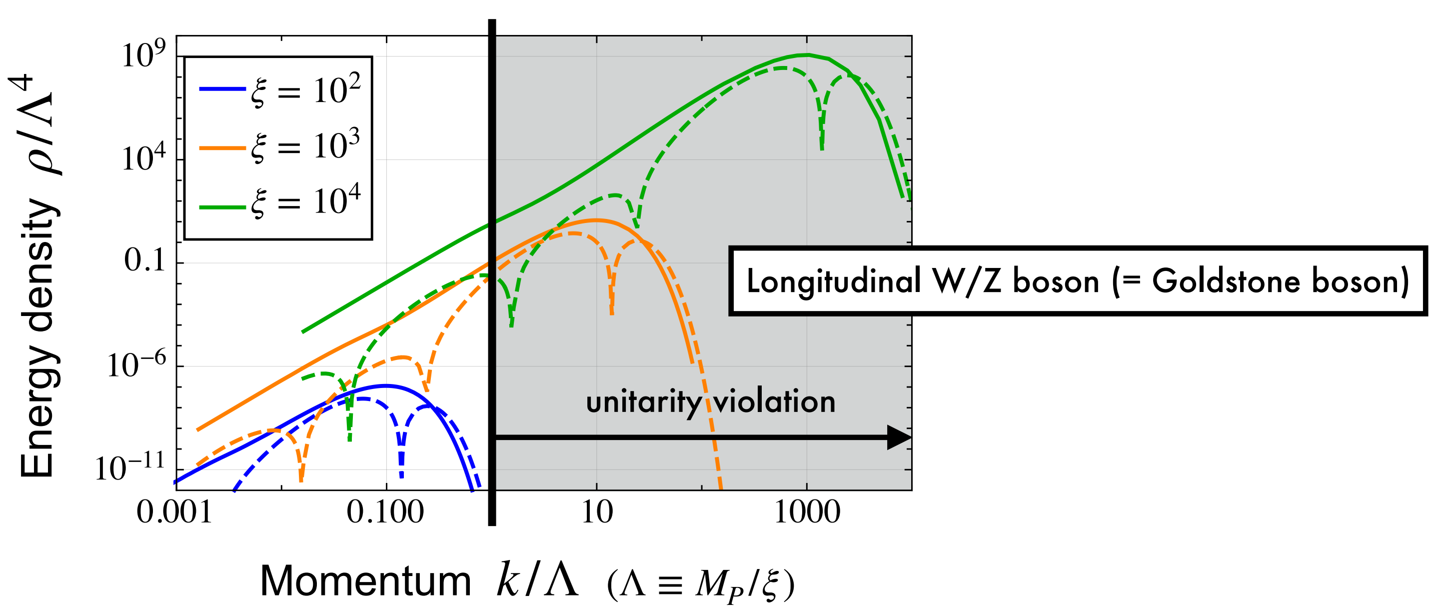
- Inflation energy scale exceeds this scale

$$\frac{M_P}{\xi} \sim 10^{14}\,\mathrm{GeV} < V_{\rm inf}^{1/4} \sim 10^{16}\,\mathrm{GeV}$$ (fixed from CMB),

although this by itself does not necessarily mean unitarity violation.

- Higgs inflation indeed violates unitarity during (p)reheating.

[YE, Jinno, Mukaida, Nakayama 16; Sfakianakis+ 18; YE, Jinno, Nakayama, van de Vis 21]




➡ theoretically inconsistent, UV completion necessary to determine $T_R$.

# Higgs-$R^2$ inflation

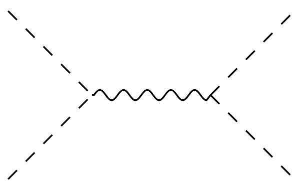
- Unitarity issue of Higgs inflation healed by $R^2$ term : "Higgs-$R^2$ inflation".

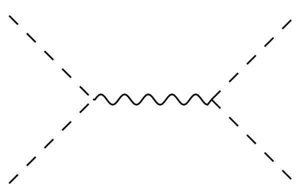
[YE 17, 19; Gorbunov+ 18]

$$\mathscr{L} = \frac{M_P^2}{2} R + \alpha R^2 + |D_\mu \Phi|^2 + \xi |\Phi|^2 R - \lambda |\Phi|^4 .$$

$R^2 \sim (\partial^2 g)^2$ introduces new degree of freedom "scalaron." $\begin{cases} \partial^2 \phi = 0 \rightarrow \text{2 initial conditions,} \\ \partial^4 \phi = 0 \rightarrow \text{4 initial conditions.} \end{cases}$

(no ghost here)

$i\mathscr{M}_{\text{Higgs}} =$  $\sim \dfrac{\xi^2 E^2}{M_P^2} > 1$ for $E > M_P/\xi$ : unitarity violation.

$i\mathscr{M}_{\text{Higgs}+R^2} =$  $+$ 

$\sim \dfrac{\xi^2 m_s^2}{M_P^2} \dfrac{E^2}{m_s^2 - E^2} < 1$ : unitarity always conserved

where scalaron mass $m_s^2 = \dfrac{M_P^2}{12\alpha} \lesssim \dfrac{M_P^2}{\xi^2}$, or $\alpha \gtrsim \xi^2$, needed.

- Higgs-$R^2$ inflation: no other operators generated below $M_P$ (except for Higgs mass and CC).

[YE 19; YE, Mukaida, van de Vis 20]

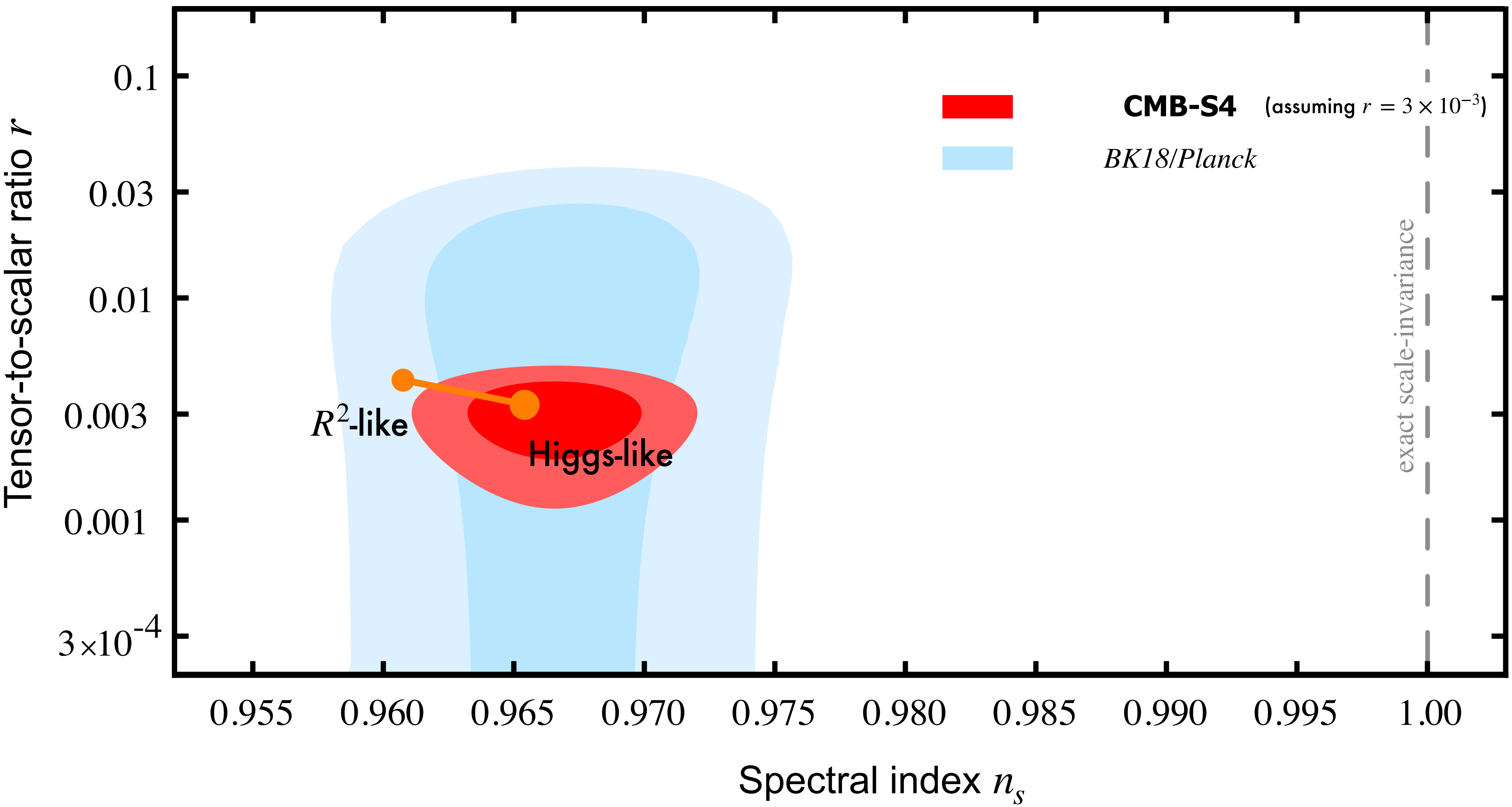
$\mathscr{L}_{\text{Higgs}}$ →(quantum correction) $\mathscr{L}_{\text{Higgs}+R^2}$ but $\mathscr{L}_{\text{Higgs}+R^2}$ →(quantum correction) $\mathscr{L}_{\text{Higgs}+R^2}$ .

($M_P \rightarrow \infty$ with $M_P/\xi$ : fixed, renormalizability of (spin-0 part of) quadratic gravity)

# Higgs-$R^2$ inflation: prediction

CMB requires $\dfrac{\xi^2}{\lambda} + 4\alpha \simeq 2 \times 10^9\,.$ $\begin{cases} \xi^2/\lambda \gg \alpha \to \text{inflaton: Higgs-like,} \\ \xi^2/\lambda \ll \alpha \to \text{inflaton: scalaron-like.} \end{cases}$

[Snowmass2021 Cosmic Frontier: CMB Measurements White Paper] (adapted by YE)



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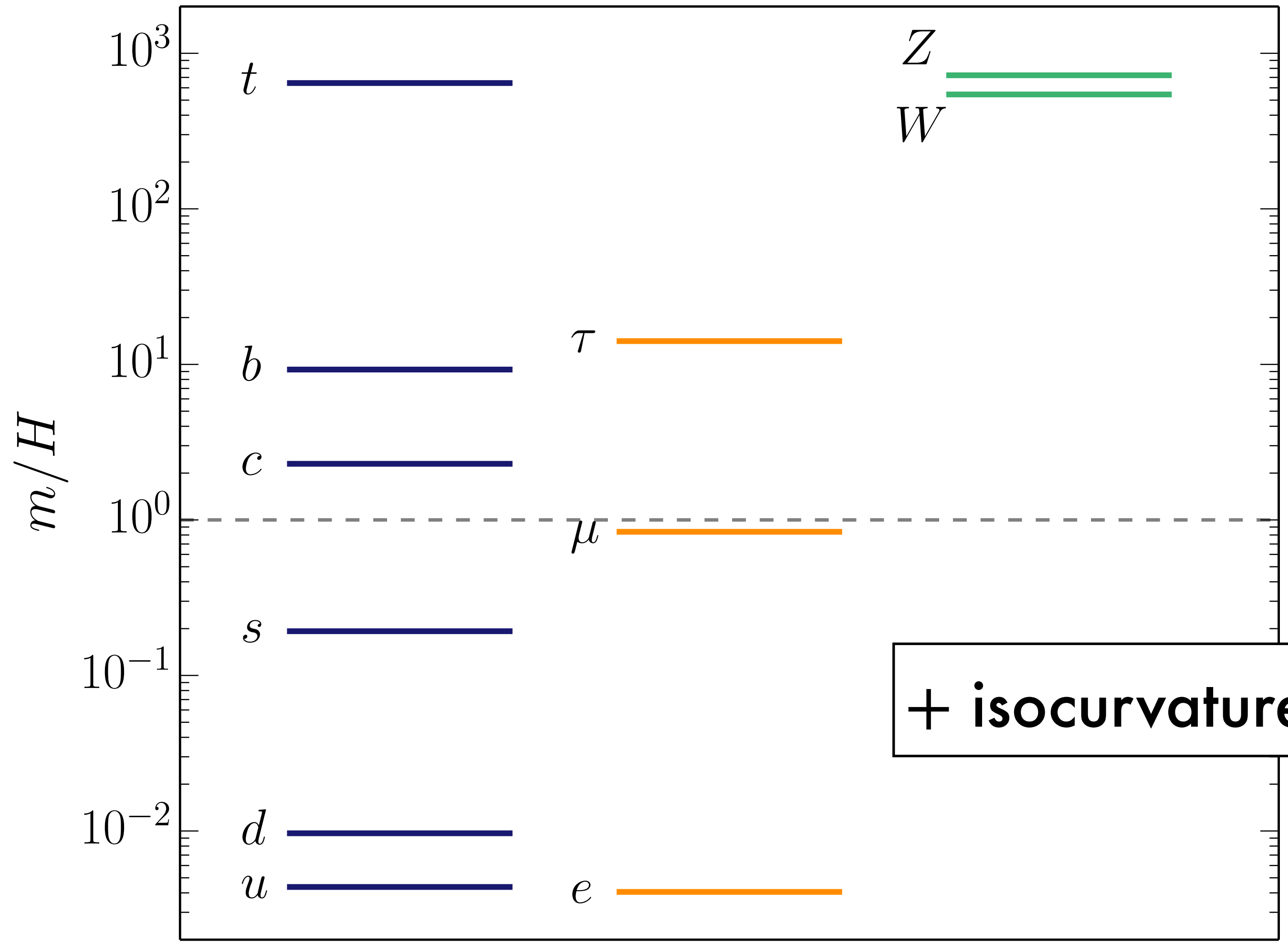
# SM mass spectrum

- SM mass spectrum rich and some as light as $H$ even though $h_0 \gg H$ during inflation.

[Chen+ 16]




+ isocurvature (= scalaron + Higgs).

- Inflaton couples to SM particles through Higgs/conformal factor in Higgs-$R^2$ inflation.

Any observable cosmological collider signatures?

SPOILER ALERT!

Yes, but only in somewhat a corner of parameter space.

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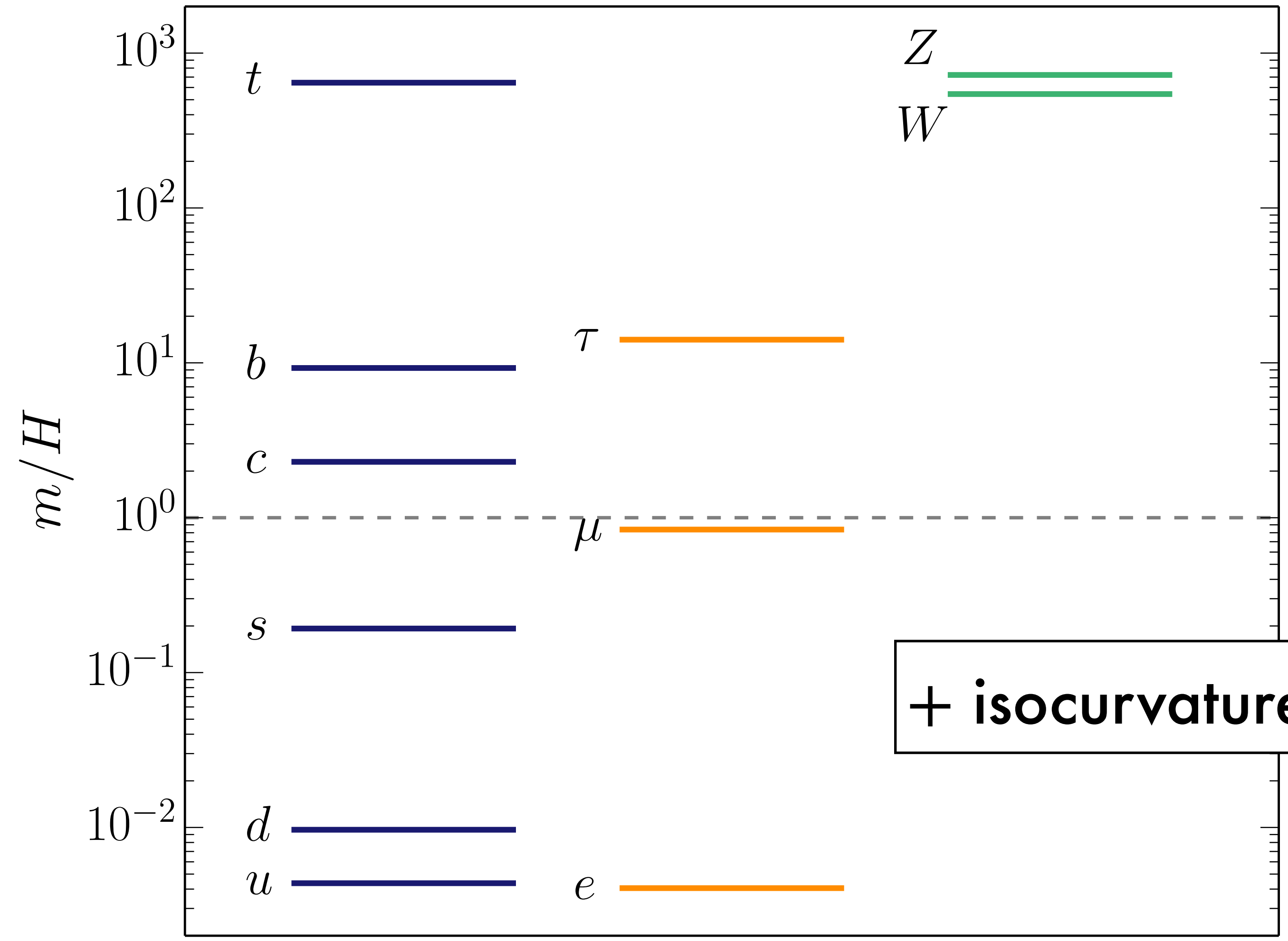


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SPOILER ALERT!

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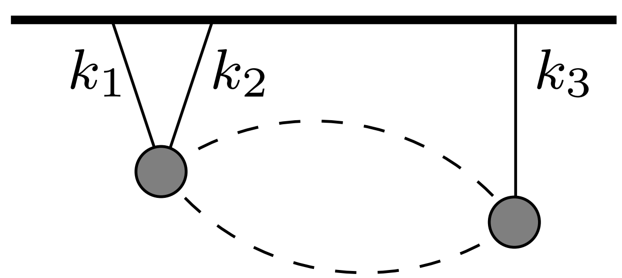
# Fermion and gauge boson

- Inflaton couples to SM fermion and gauge boson through the mass term:

$$S_{\rm int} = \int d^4x \sqrt{-g}\,\mathcal{O}_{\rm SM}\left[1 + c_1\frac{\varphi}{N_e M_P} + c_2\frac{\varphi^2}{2M_P^2}\right], \quad \mathcal{O}_{\rm SM} = -\,m\bar{\psi}\psi,\ \ \frac{m^2}{2}g^{\mu\nu}A_\mu A_\nu,$$

with $(c_1, c_2) = (\sqrt{6}/16, 1/6)$ for $\psi$ and $(\sqrt{6}/8, 1/3)$ for $A_\mu$.

- Cosmological collider signatures diagrammatically given by

$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle' =$ 

$\equiv (2\pi)^4 \dfrac{P_\zeta^2}{k_1^2 k_2^2 k_3^2} \times S_{\rm NG}\,.$

- Squeezed limit given by

$$S_{\rm NG} = \begin{cases} -\dfrac{3c_1c_2 H\dot{\phi}_0}{8\pi^4 N_e M_P^3}\dfrac{m^2}{H^2}C_{1/2}(\nu_-)\left(\dfrac{k_3}{k_1}\right)^{4-2\nu_-} + (\nu_- \to \nu_+):\ \text{fermion}, \\ -\dfrac{3c_1c_2}{8\pi^4}\dfrac{H\dot{\phi}_0}{N_e M_P^3}\dfrac{m^4}{H^4}C_1(\nu)\left(\dfrac{k_3}{k_1}\right)^{2-2\nu} + (\nu \to -\nu):\ \text{gauge boson}. \end{cases}$$

⇨ $S_{\rm NG} \lesssim \dfrac{H\dot{\phi}_0}{(2\pi)^4 N_e M_P^3} \sim 10^{-17}$: far too small to be observable. [YE, Verner 23]

# Isocurvature mode

- Action of adiabatic mode $\zeta$ and isocurvature mode $\chi$ given by

$$S = \int dt d^3x a^3 \left[ \frac{1}{2} \frac{\dot{\phi}_0^2}{H^2} \left( \dot{\zeta}^2 - \frac{1}{a^2} (\partial_i \zeta)^2 \right) + \frac{1}{2} \left( \dot{\chi}^2 - \frac{1}{a^2} (\partial_i \chi)^2 - m_\chi^2 \chi^2 \right) - \frac{2\dot{\theta}\dot{\phi}_0}{H} \dot{\zeta}\chi \right]$$

where $m_\chi^2 \simeq \frac{\xi(24\lambda\alpha + \xi(1+6\xi))}{\lambda\alpha} H^2 > 24\xi H^2, \quad \frac{\dot{\theta}}{H} \simeq \sqrt{\frac{3\xi}{2(4\lambda\alpha + \xi^2)}}.$

- Isocurvature mode heavy in "standard" case $\xi \gg 1$ but light for $\xi \sim \lambda\alpha \lesssim \mathcal{O}(0.1)$.

➡ Realizing the idea of "quasi-single field inflation": [Chen+ 09]

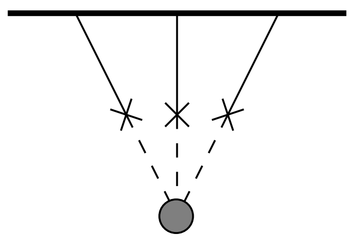
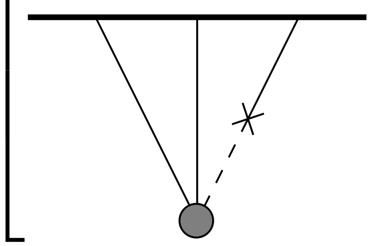
- Three point couplings from potential dominate:

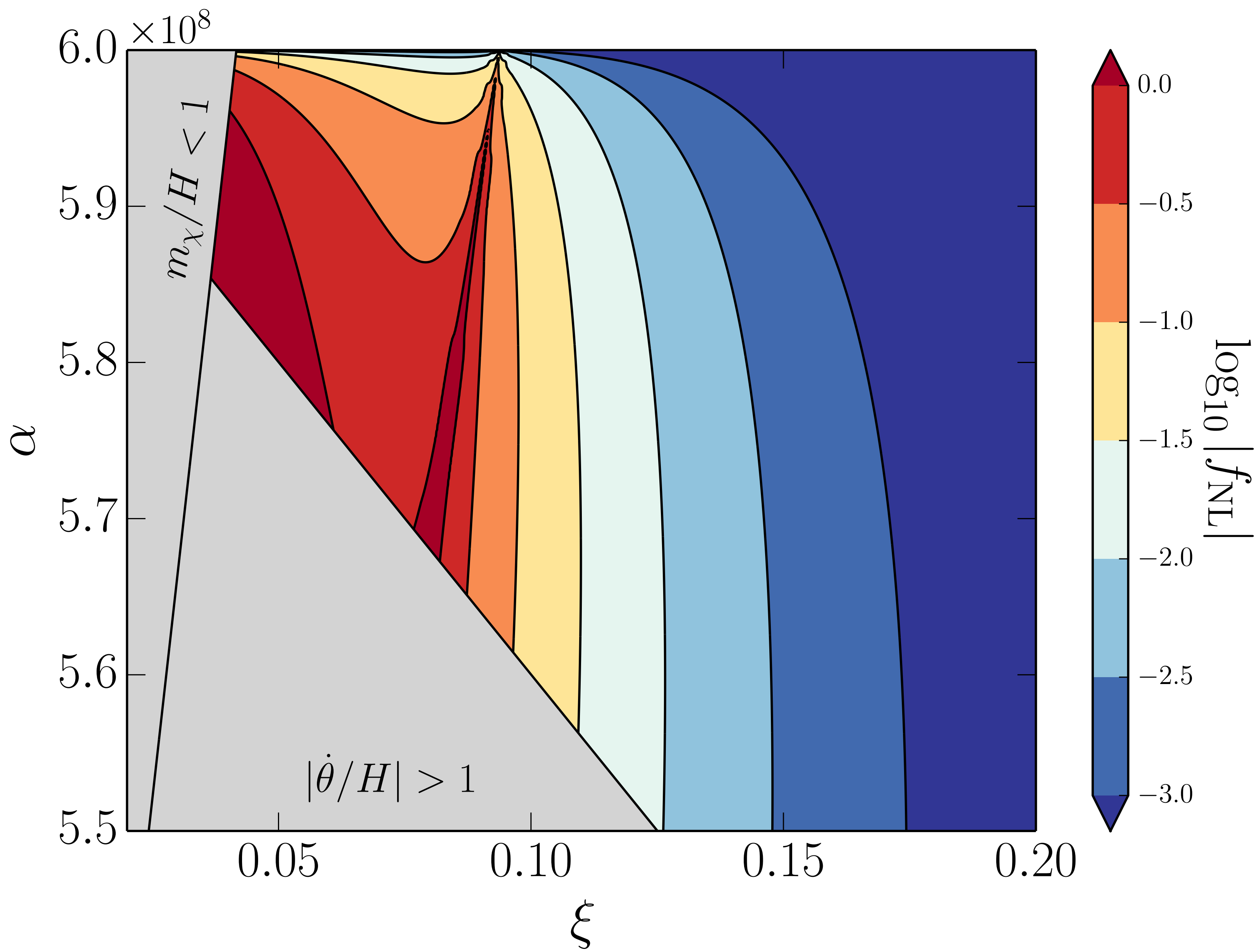
$$S_{\text{cubic}} = \int d\tau d^3x a^4 \left[ -\frac{1}{6} V_{N^3} \chi^3 - \frac{1}{2} V_{T^2N} \varphi^2 \chi \right], \quad V_{N^3} \sim V_{T^2N} \sim \frac{H}{\sqrt{\alpha}}$$

(for $\xi \sim \lambda\alpha \sim \mathcal{O}(1)$).

# Signal of isocurvature mode

$$-\frac{1}{6}V_{N^3}\chi^3-\frac{1}{2}V_{T^2N}\varphi^2\chi$$

$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle' =$  $+$ $\Big[$  $+\,(\text{perms.})\Big] \sim P_\zeta^{-1/2}\,\frac{\dot{\theta}^3}{H^3}\frac{V_{N^3}}{H},\quad P_\zeta^{-1/2}\frac{\dot{\theta}}{H}\frac{V_{T^2N}}{H}$




[YE, Verner 23]

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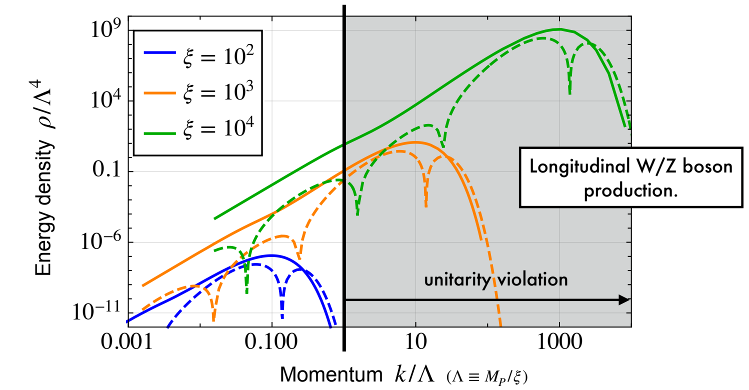
# Summary

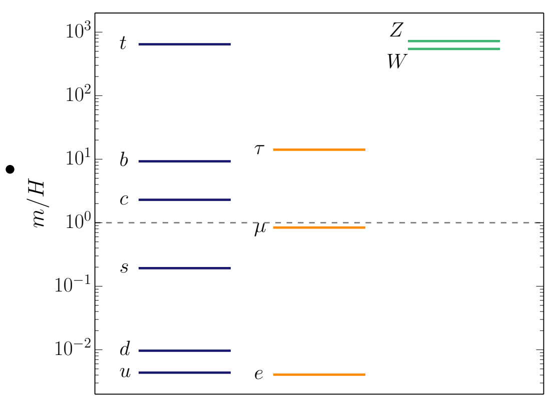
- Higgs inflation is appealing at first sight, as Higgs is the only scalar field within SM.

BUT unitarity violated during reheating:

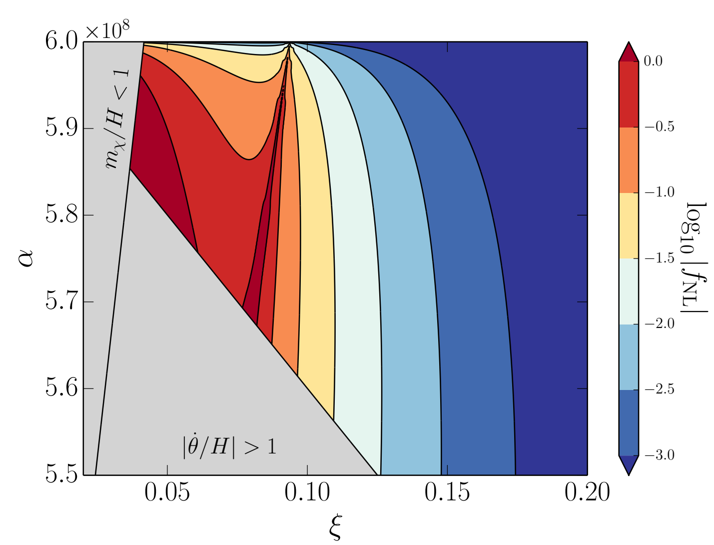



- Higgs-$R^2$ inflation UV-completes, with scalaron playing the role of "$\sigma$-meson".

- Rich SM mass spectrum and inflaton naturally couples to SM particles.




- Cosmological collider signature from the isocurvature mode can be sizable:

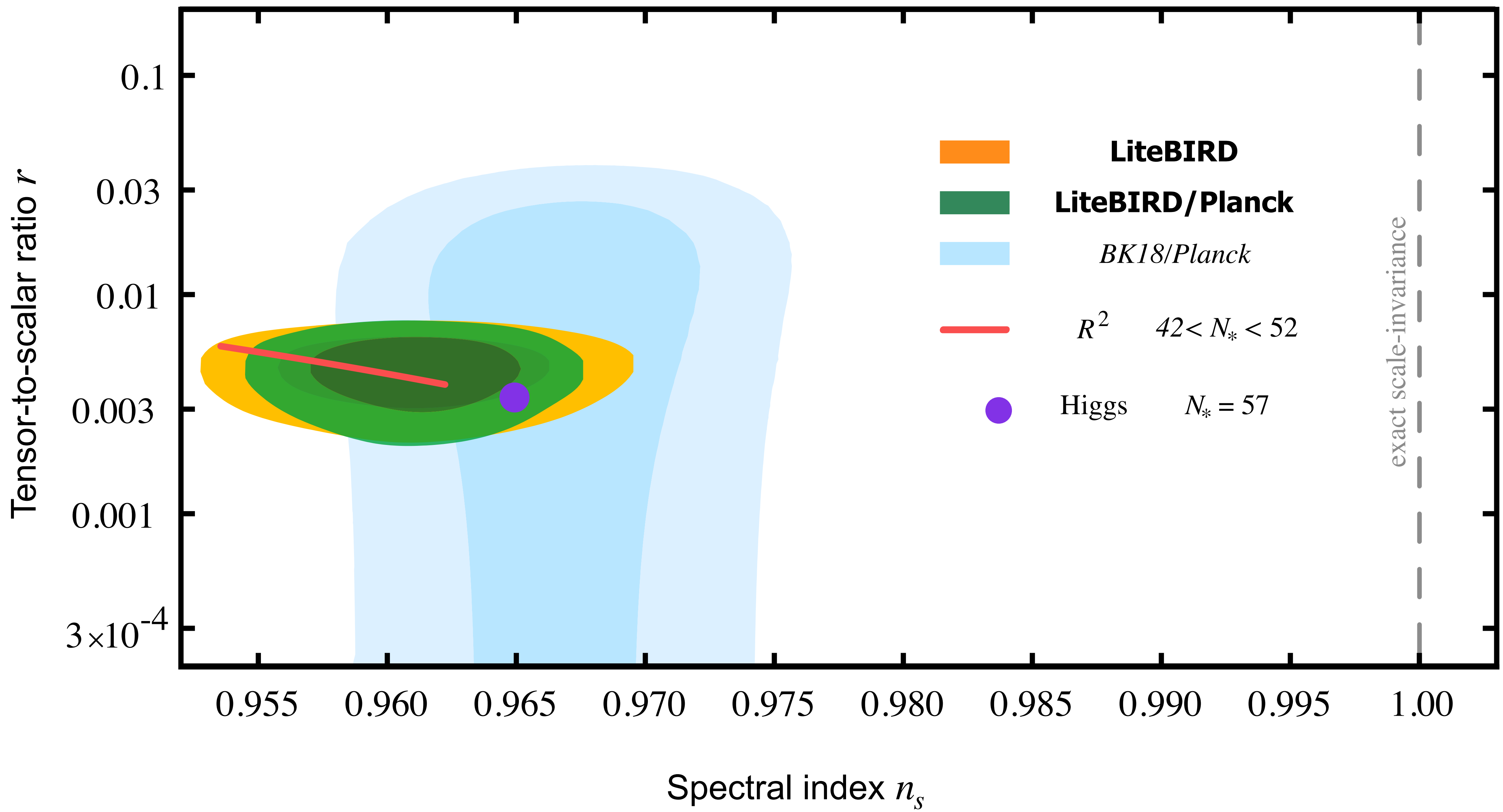


# Back up

# LiteBIRD

True values: $n_s = 0.961,\ r = 0.0046$ assumed.

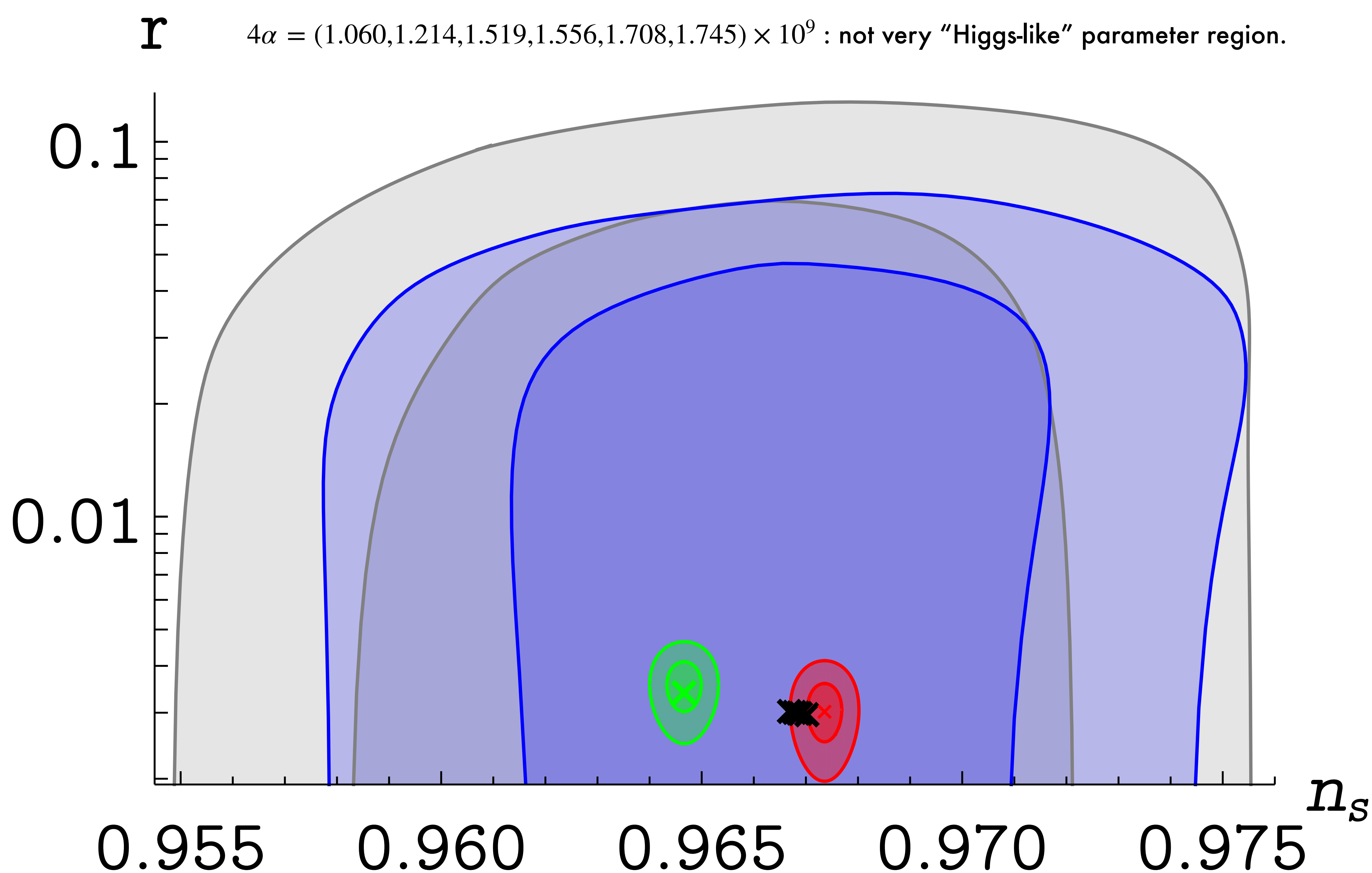
[LiteBIRD Collaboration 22] (adapted by YE)



# Higgs-$R^2$ inflation: reheating

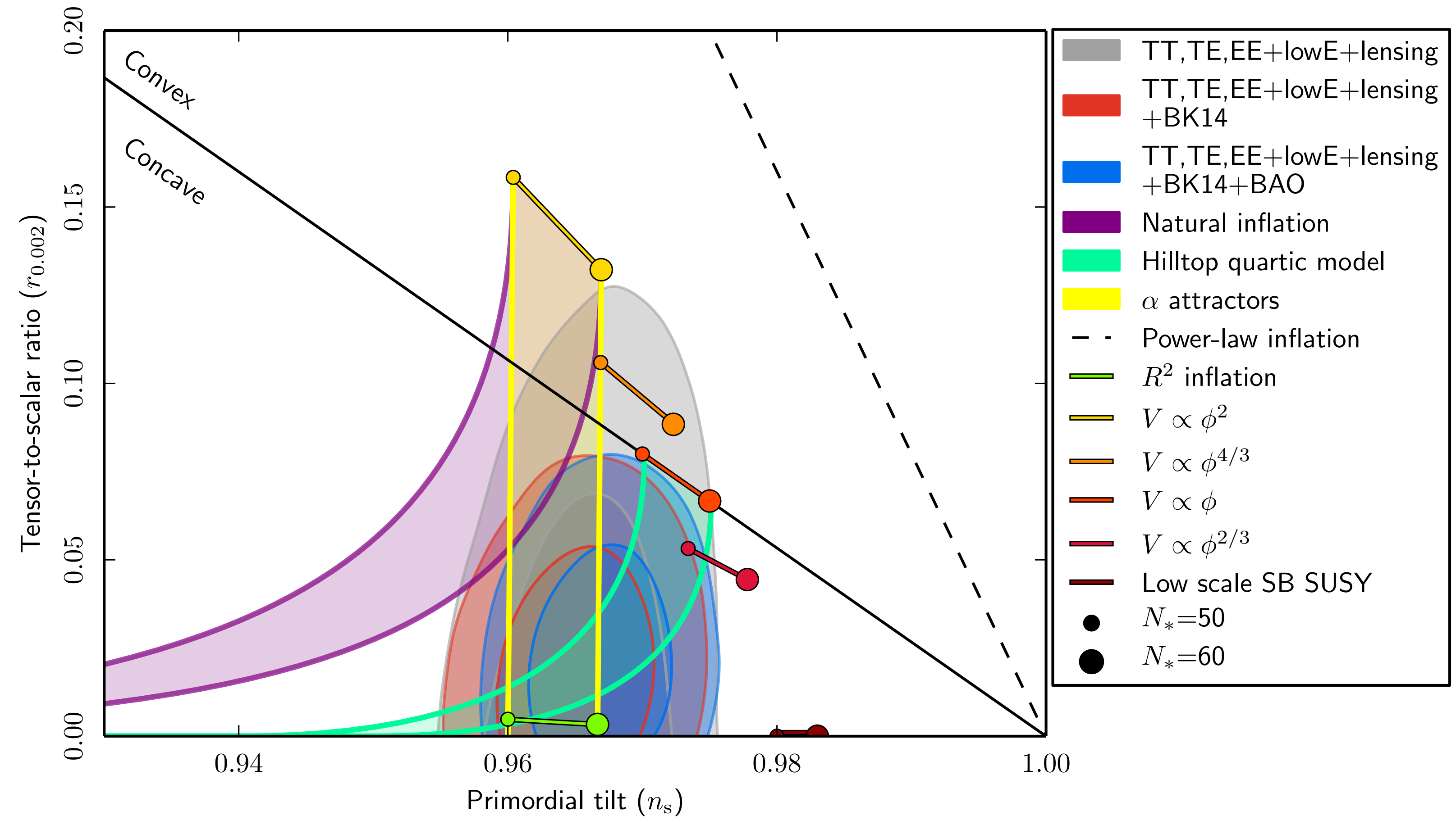
Reheating tends to be efficient → predicts large $N_e$.

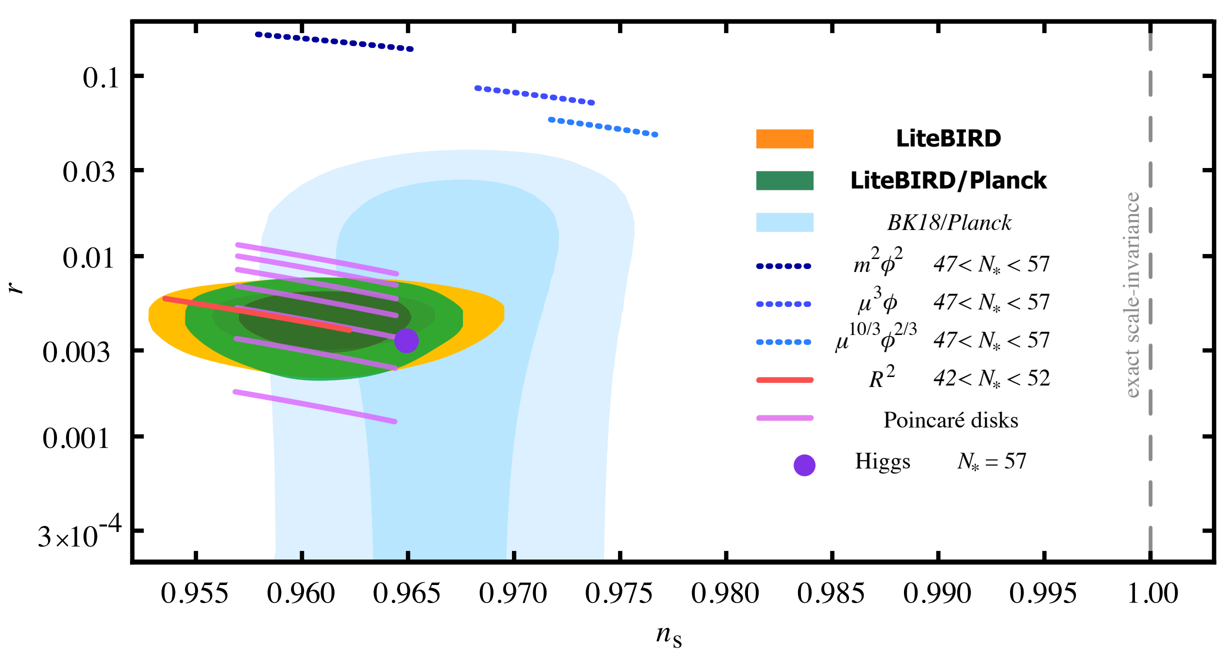
$4\alpha = (1.060, 1.214, 1.519, 1.556, 1.708, 1.745) \times 10^9$ : not very "Higgs-like" parameter region.

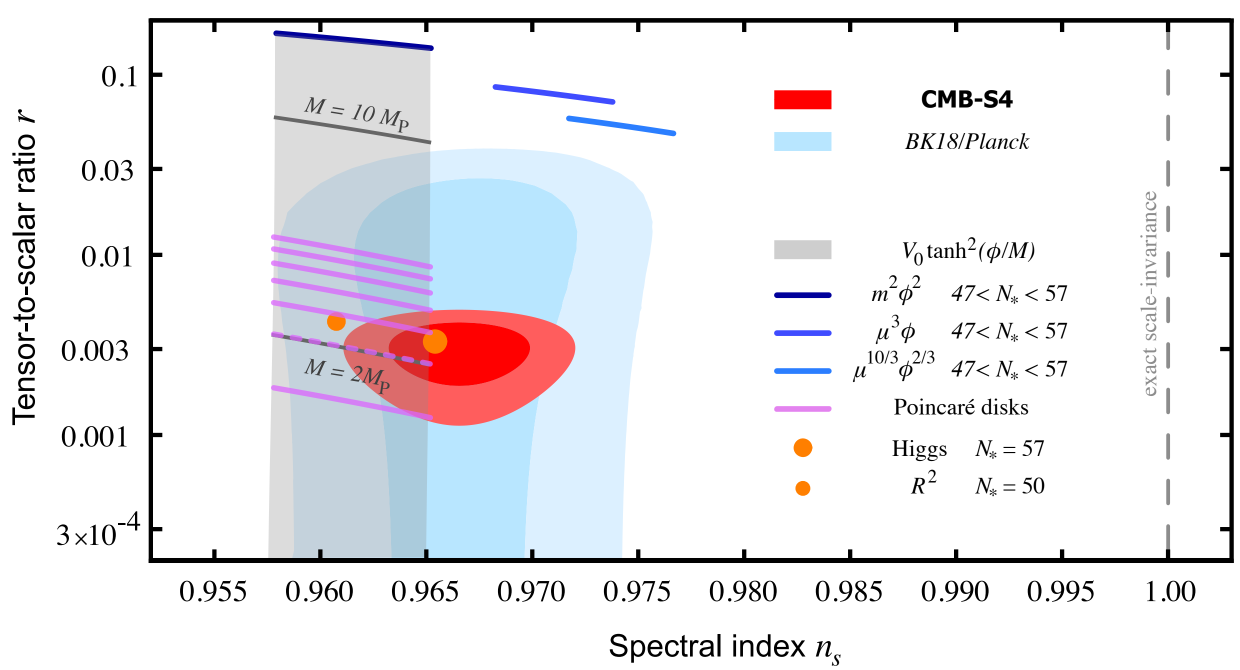



[Figure from Bezrukov, Shephered 20]

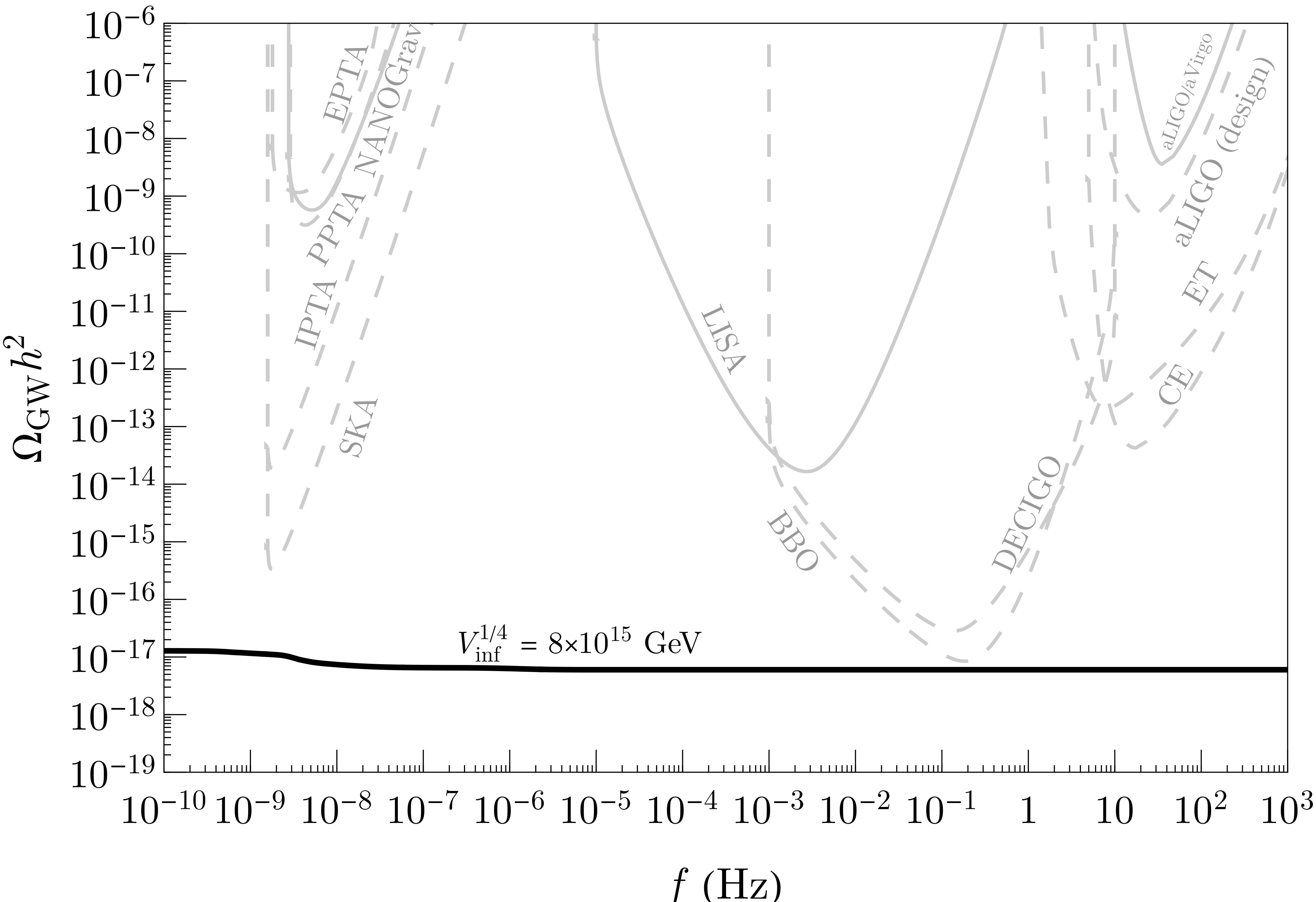
# Other inflation models



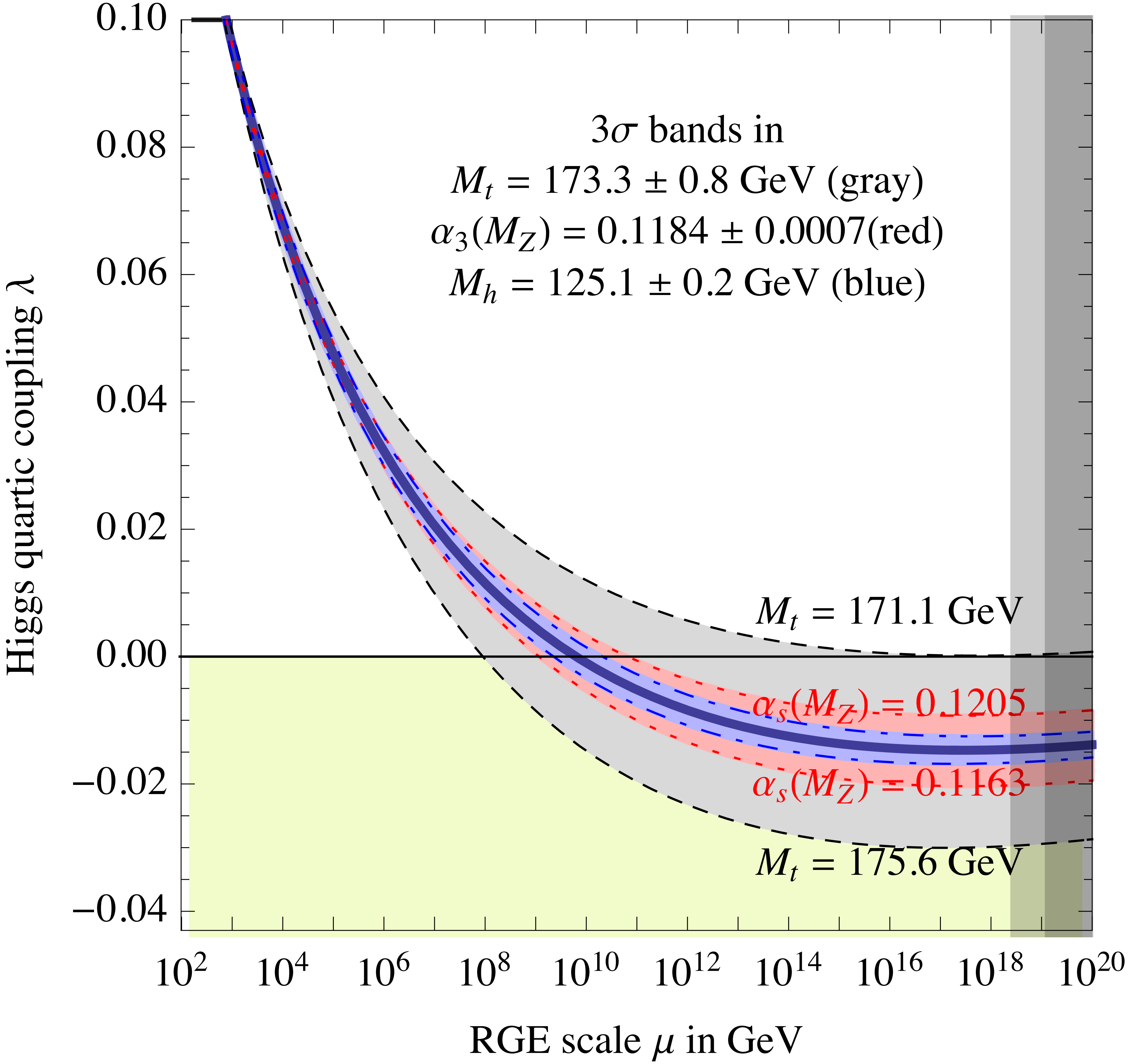



# Gravitational wave experiments




[Figure credit: Raymond Co]

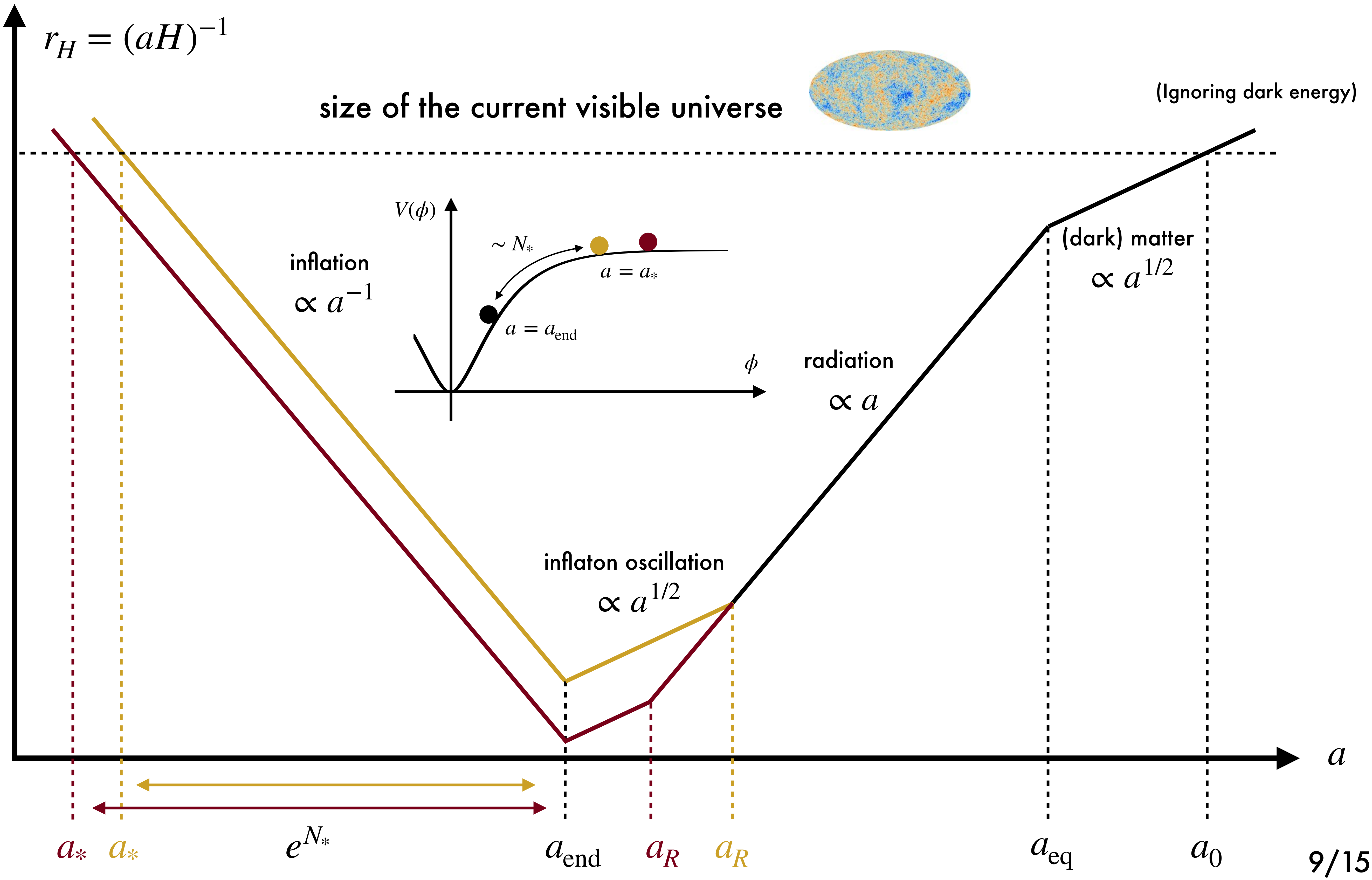
# Running of Higgs quartic coupling




[Buttazzo+ 13]

# Reheating

Cosmological perturbation depends on reheating temperature $T_R$ through $N_*$ .



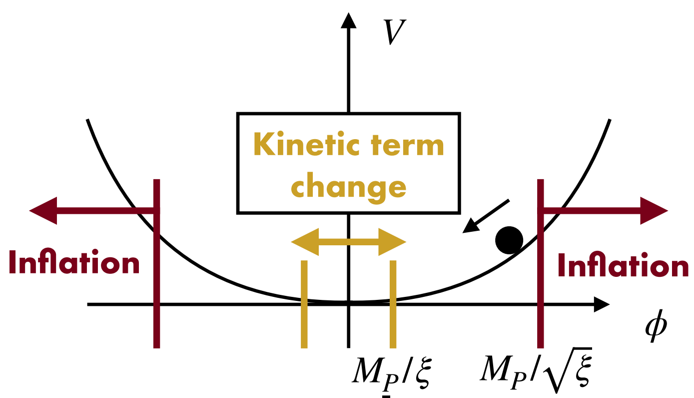
# Spiky oscillation after inflation

- Higgs fields have a non-trivial target space in Einstein frame:

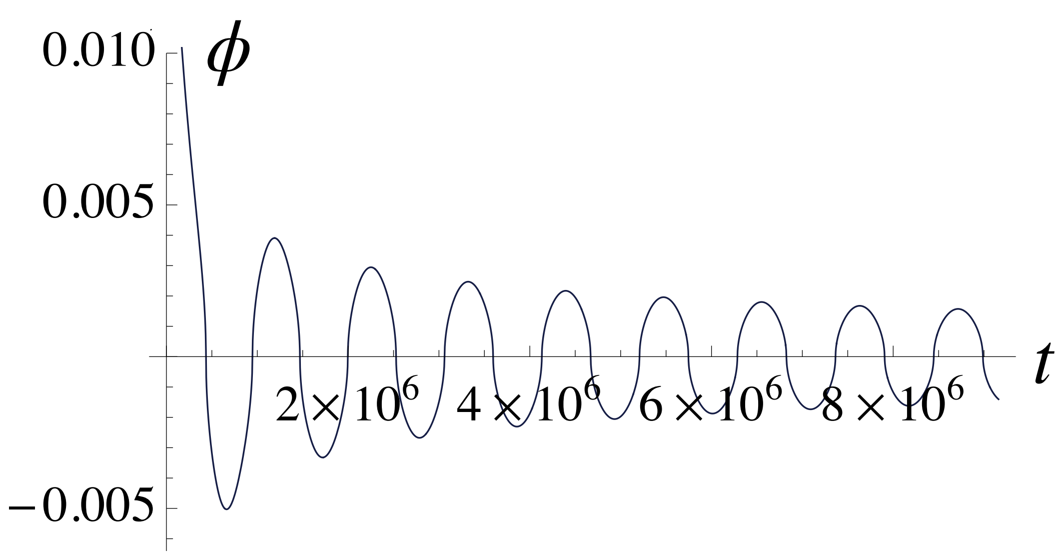
$$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{1}{2}h_{ab}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

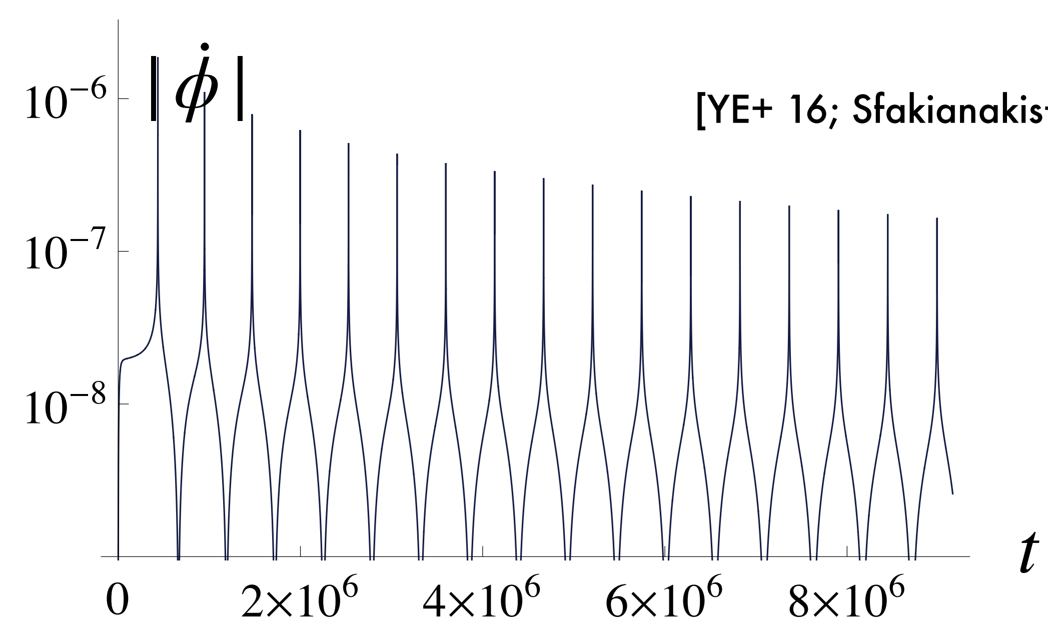
with $h_{ab} = \frac{1}{\Omega^4}\begin{pmatrix} \Omega^2 + \boxed{\frac{6\xi^2\phi^2}{M_P^2}} & \frac{6\xi^2\phi\chi}{M_P^2} \\ \frac{6\xi^2\phi\chi}{M_P^2} & \Omega^2 + \frac{6\xi^2\chi^2}{M_P^2} \end{pmatrix}$ with $\chi$ : NG mode(s) and $\Omega^2 = 1 + \xi\frac{\phi^2+\chi^2}{M_P^2}$ for HI.

- Kinetic term drastically changes for $|\phi| \lesssim M_P/\xi$,




➡ a "spiky" feature for $|\phi| \lesssim M_P/\xi$, causing unitarity violation.




[YE+ 16; Sfakianakis+ 18; Nguyen+ 19]

# Target space and unitarity

An easy-to-use condition of unitarity violation from target space

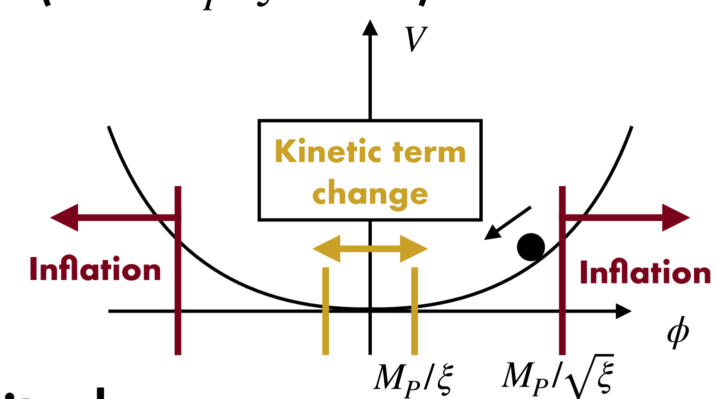
[YE, Jinno, Nakayama, van de Vis 21]

- NG boson has mass from target space curvature → feels spikes:

$$m_\chi^2 = \nabla^\chi V_\chi - \dot\phi^2 R^\chi{}_{\phi\phi\chi}\,, \qquad \text{e.g.} \left(1 + \frac{\chi^2}{\Lambda^2}\right)(\partial\phi)^2 \to m_\chi^2 = -\frac{\dot\phi^2}{\Lambda^2}.$$

- Inflaton motion changes for $|\phi| \lesssim \Lambda$ with curvature $R[h] \sim \Lambda^{-2}$ ($\Lambda \sim M_P/\xi$ for HI).

➡ typical momentum scale: $k_{\text{spike}} \sim (\Lambda/\dot\phi_{\text{origin}})^{-1}$ .




- Cut-off also $\sim \Lambda$ since the curvature affects e.g. scattering amplitudes.

- With energy cons. $\dot\phi^2_{\text{origin}} \sim V_{\text{inf}}$, unitarity violation $k_{\text{spike}} \gtrsim \Lambda$ translates to

$V_{\text{inf}} \gtrsim \Lambda^4$: simply compare inflation energy scale and cut-off.

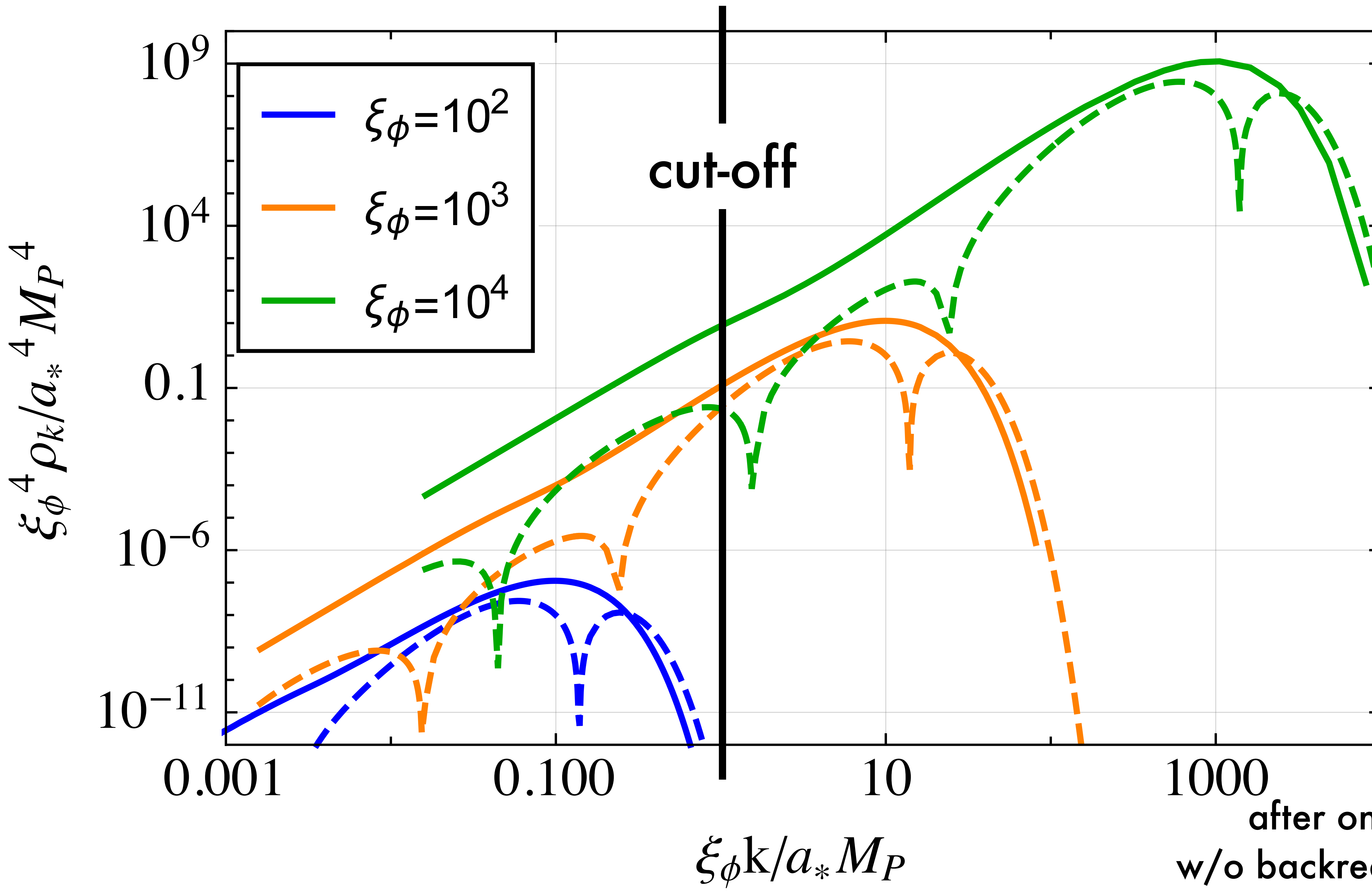
e.g. $V_{\text{inf}}/\Lambda^4 \sim \lambda\xi^2 \sim 10^{-9}\xi^4$ for HI → unitarity violation for $\xi \gtrsim 10^2$ .

- Applicable to other inflation models (can see e.g. running kinetic inflation violates unitarity).

# Numerical result

Numerical result confirms the previous estimation.

[YE, Jinno, Nakayama, van de Vis 21]




after one oscillation,
w/o backreaction to inflaton.

$\rho_\chi \sim k_{\rm spike}^4 \sim V_{\rm inf}^2/\Lambda^4 > V_{\rm inf}$ for $V_{\rm inf} > \Lambda^4$ → this production is fatal.

# Linear $\sigma$-model

[YE, Mukaida, van de Vis 20]

$$\mathscr{L} = \frac{M_P^2}{2}\left(1+\frac{\xi\phi_i^2}{M_P^2}\right)R + \alpha R^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4 .$$

Auxiliary field introduction + Weyl transformation

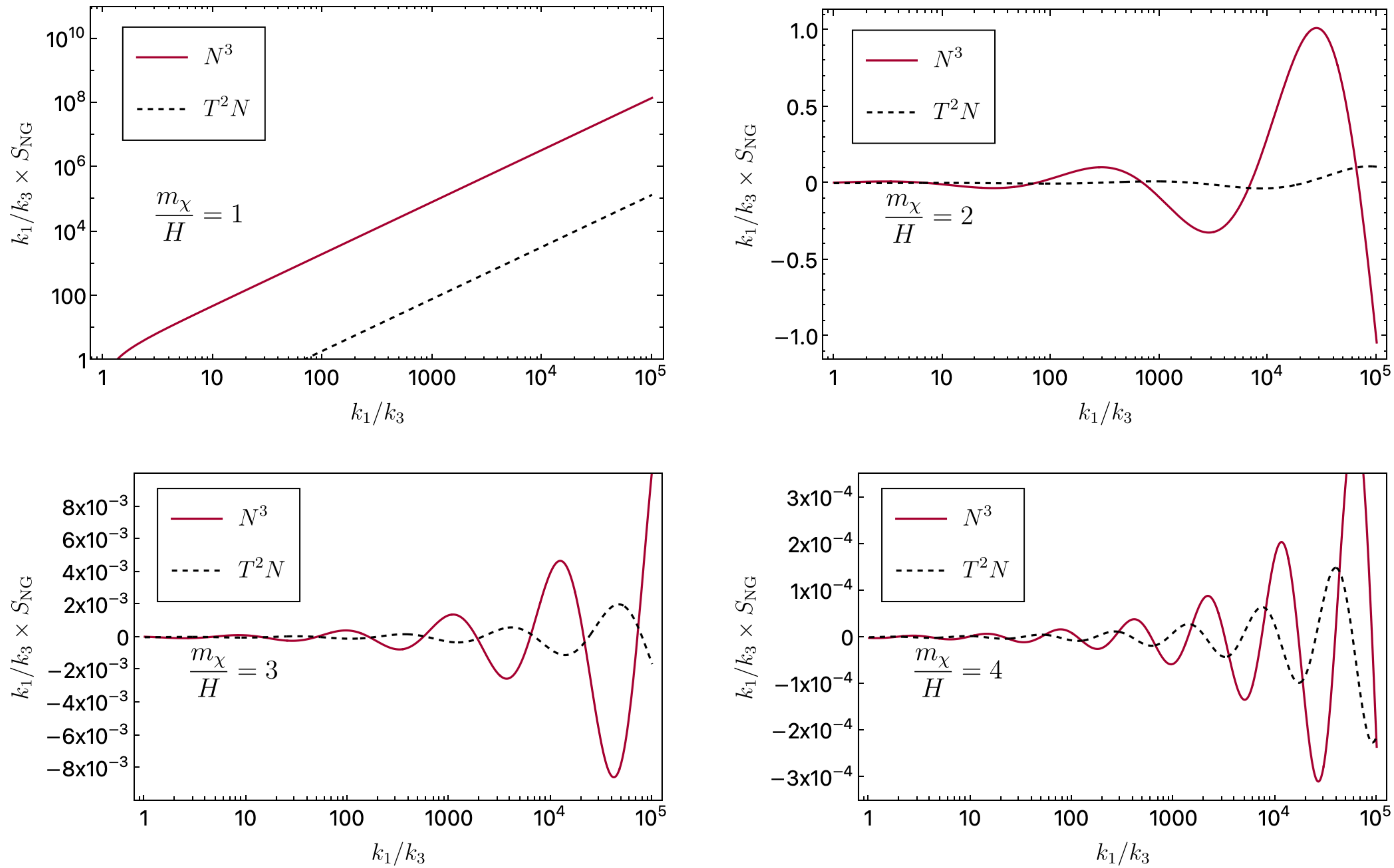
$$\mathscr{L} = \frac{M_P^2}{2}\left(1-\frac{\sigma^2}{6M_P^2}-\frac{\phi_i^2}{6M_P^2}\right)R + \frac{1}{2}\left[\left(\partial\phi_i\right)^2+(\partial\sigma)^2\right] - \frac{\lambda}{4}\phi_i^4 - \frac{1}{144\alpha}\left[\frac{3M_P^2}{2}-\left(\sigma+\sqrt{\frac{3}{2}}M_P\right)^2-\frac{6\xi+1}{2}\phi_i^2\right]^2 .$$

Flat kinetic term in the conformal frame!

| $\mathrm{O}(N)$ NLSM | Higgs inflation |
|---|---|
| pions $\pi_i$ | Higgs fields $\phi_i$, conformal mode of metric $\Phi$ |
| target space: $\pi_i^2+h^2=v^2, \quad \left(\pi_i, h\right)\in\mathbb{R}^{(N+1)}$ | target space: $\frac{6\xi+1}{2}\phi_i^2+\left(h+\frac{\Phi}{2}\right)^2=\frac{\Phi^2}{4}, \quad \left(\Phi,\phi_i,h\right)\in\mathbb{R}^{(1,N+1)}$ |
| sigma meson $\sigma$ | scalaron $\sigma$ |

# Cosmo collider signatures

[YE, Verner 23]



# Renormalizability of LSM

[YE, Mukaida, van de Vis 20]

- The LSM with the Higgs mass and the cosmological constant is renormalizable.

(= renormalizability of (spin-0 part of) quadratic gravity)

➡ One can compute the RGEs without any ambiguity!

1-loop:

$$\beta_{g_1}^{(1)} = \frac{41}{10} g_1^3, \quad \beta_{g_2}^{(1)} = -\frac{19}{6} g_2^3, \quad \beta_{g_3}^{(1)} = -7 g_3^3,$$

$$\beta_{y_t}^{(1)} = y_t \left[ \frac{9 y_t^2}{2} - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2 \right],$$

$$\beta_{\lambda}^{(1)} = \left(8\bar{\xi}^2 - 8\bar{\xi} + 2\right) \bar{\xi}^2 \lambda_\alpha^2 + 24\bar{\xi}^2 \lambda \lambda_\alpha + 24\lambda^2 - 6 y_t^4 + \frac{27 g_1^4}{200} + \frac{9 g_2^4}{8} + \frac{9}{20} g_1^2 g_2^2 + \left[ 12 y_t^2 - \frac{9 g_1^2}{5} - 9 g_2^2 \right] \lambda,$$

$$\beta_{\lambda_m}^{(1)} = 2\bar{\xi}\left(2\bar{\xi} - 1\right) \lambda_\alpha^2 - 8\bar{\xi}\lambda_m^2 + \lambda_m \left[ 4\bar{\xi}^2 \lambda_\alpha + 8\bar{\xi}\lambda_\alpha - 3\lambda_\alpha + 12\lambda + 6 y_t^2 - \frac{9 g_1^2}{10} - \frac{9 g_2^2}{2} \right],$$

$$\beta_{\bar{\xi}}^{(1)} = \bar{\xi} \left[ \left(4\bar{\xi}^2 + 4\bar{\xi} - 3\right) \lambda_\alpha + 12\lambda + 6 y_t^2 - \frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 \right],$$

$$\beta_{\lambda_\alpha}^{(1)} = \left(8\bar{\xi}^2 + 5\right) \lambda_\alpha^2,$$

$$\beta_{\lambda_\Lambda}^{(1)} = \frac{\lambda_\alpha^2}{2} - 2\lambda_\alpha \lambda_\Lambda - 16\bar{\xi}\lambda_\Lambda \lambda_m + 2\lambda_m^2.$$

* See 2008.01096 for an explicit form up to 2-loop.

- The Higgs mass and the CC are naturally at the scalaron mass scale = hierarchy problem.

➡ They do not affect inflationary dynamics, but (p)reheating??

- EW scale parameters can be related to inflationary scale parameters (with $\xi$ and $\alpha$).

# Spin-two sector

- Vacuum pol. diagrams contain divergences.

  ➡ Renormalized by $\mathscr{L}_{\text{c.t.}} = \alpha R^2 + \alpha_2 \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right)$.

  * We have no choice but including these terms.

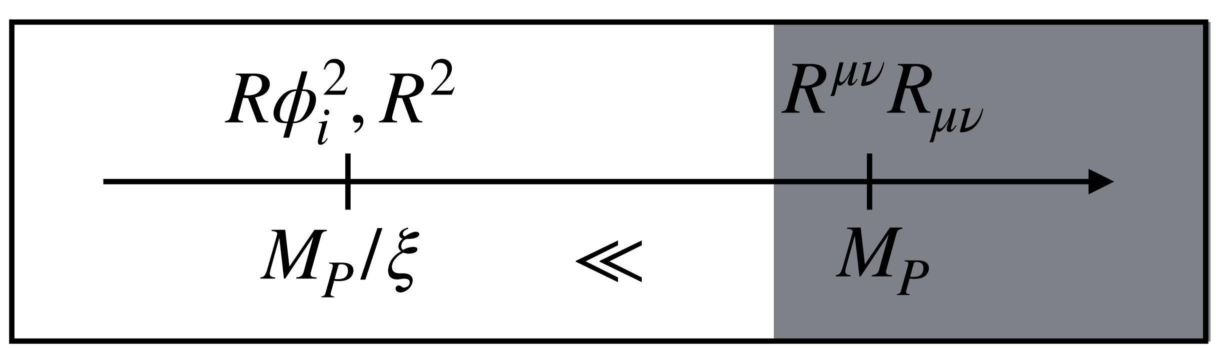
- Renormalization group equations:

$$\beta_\alpha \equiv \frac{d\alpha}{d \ln \mu} = - \frac{N_s}{1152\pi^2} (1 + 6\xi)^2, \quad \beta_{\alpha_2} \equiv \frac{d\alpha_2}{d \ln \mu} = - \frac{N_s}{960\pi^2}.$$

  The hierarchy $\alpha \sim \mathcal{O}(\xi^2) \gg \alpha_2 \sim \mathcal{O}(1)$ naturally exits.

- Alternatively, the coupling for the spin-2 is suppressed:

  $T_{\mu\nu} \ni \xi \left( \partial_\mu \partial_\nu - \eta_{\mu\nu} \Box \right) \phi_i^2$ ➡ $h^{\perp}_{\mu\nu} T^{\mu\nu}$: independent of $\xi$.



# Frame-independent target space

[YE, Mukaida, van de Vis 20]

- Naive definition solely by scalar fields is frame-dependent.

$$\begin{cases} \mathscr{L}_J = \frac{M_P^2}{2}\Omega^2 R + \underline{\frac{1}{2}\left(\partial\phi_i\right)^2} + \cdots, \quad \Omega^2 = 1 + \frac{\xi\phi_i^2}{M_P^2}, \\ \mathscr{L}_E = \frac{M_P^2}{2} R + \underline{\frac{1}{2\Omega^4}\left(\Omega^2\delta_{ij} + \frac{6\xi^2\phi_i\phi_j}{M_P^2}\right)\partial\phi_i\partial\phi_j} + \cdots . \end{cases}$$

Physics is frame-independent → a frame-independent definition is desirable.

- Frame-independent definition by including the conformal mode.

Metric decomposition: $g_{\mu\nu} = e^{2\varphi}\tilde{g}_{\mu\nu}, \ \ \mathrm{Det}\left[\tilde{g}_{\mu\nu}\right] = -1.$

$\Phi = \sqrt{6}M_P e^{\varphi}$: conformal mode.

Target space defined by $(\phi_i, \Phi)$: frame-independent!

∵ Weyl transformation = redefinition of $\Phi$ = coordinate transf. of target space.

# Higgs inflation as NLSM

[YE, Mukaida, van de Vis 20]

- Focus on the conformal mode of the metric as $g_{\mu\nu} = e^{2\varphi}\eta_{\mu\nu}$.

$$S = \int d^4x \left[ -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{6\xi+1}{2}\left(\frac{\Box\Phi}{\Phi}\right)\phi_i^2 - \frac{\lambda}{4}\phi_i^4 \right].$$

- Can be simplified by field redefinitions as

$$S = \int d^4x \left[ -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}\left(\partial h\right)^2 - \frac{\lambda}{4}\phi_i^4 \right],$$

where $h\left(\Phi, \phi_i\right) = \frac{1}{2}\left[\sqrt{\Phi^2 - 2\left(6\xi+1\right)\phi_i^2} - \Phi\right]$.

Interpreted frame-independently as NLSM.

- $\Phi$ is ghost-like but harmless.

* Similar to $A_0$ of $\mathrm{U}(1)$ gauge boson in the Lorentz gauge $\partial_\mu A^\mu = 0$:

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{2}\eta^{\alpha\beta}\partial^\mu A_\alpha \partial_\mu A_\beta = -\frac{1}{2}\left(\partial A_0\right)^2 + \frac{1}{2}\left(\partial A_i\right)^2.$$

# Scalaron as $\sigma$-meson

- Higgs inflation as NLSM: [YE, Mukaida, van de Vis 20]

$$\mathscr{L}_{\rm NLSM} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}\left(\partial h\right)^2 - \frac{\lambda}{4}\phi_i^4, \quad h = \frac{1}{2}\left[\sqrt{\Phi^2 - 2\left(6\xi+1\right)\phi_i^2} - \Phi\right].$$

Naturally imply $\sigma$-meson that linearizes the NLSM:

$$\mathscr{L}_{\rm LSM} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{144\alpha}\left[\frac{\Phi^2}{4} - \left(\sigma + \frac{\Phi}{2}\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 - \frac{\lambda}{4}\phi_i^4.$$

- It is identified as the scalaron:

$$\mathscr{L} = \frac{M_P^2}{2}\left(1 + \frac{\xi\phi_i^2}{M_P^2}\right)R + \alpha R^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 - \frac{\lambda}{4}\phi_i^4.$$

Auxiliary field introduction + Weyl transformation

$$\mathscr{L} = \frac{M_P^2}{2}\left(1 - \frac{\sigma^2}{6M_P^2} - \frac{\phi_i^2}{6M_P^2}\right)R + \frac{1}{2}\left[\left(\partial\phi_i\right)^2 + (\partial\sigma)^2\right] - \frac{\lambda}{4}\phi_i^4 - \frac{1}{144\alpha}\left[\frac{3M_P^2}{2} - \left(\sigma + \sqrt{\frac{3}{2}}M_P\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2.$$

Flat kinetic term in the conformal frame!

$g_{\mu\nu} = e^{2\varphi}\eta_{\mu\nu}$ + rescaling fields

$$\mathscr{L} = -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}\left(\partial\phi_i\right)^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{144\alpha}\left[\frac{\Phi^2}{4} - \left(\sigma + \frac{\Phi}{2}\right)^2 - \frac{6\xi+1}{2}\phi_i^2\right]^2 - \frac{\lambda}{4}\phi_i^4.$$

* Remember this identification is frame-independent.

# Why no ghost with $R^2$?

- Higher derivative in general involves ghost:

$$\pm\frac{1}{p^2(1-p^2/M^2)} = \pm\left(\frac{1}{p^2} - \frac{1}{p^2-M^2}\right).$$

two choices $\begin{cases} + : \text{high energy additional pole is ghost-like,} \\ - : \text{low energy pole is ghost-like.} \end{cases}$

- In the case of $R^2$, it is the low-energy pole that is ghost

This mode is unphysical, thanks to gauge (BRST) symmetry.

c.f. photon propagator in Feynman gauge: $-\frac{\eta_{\mu\nu}}{p^2} = \begin{cases} -\frac{1}{p^2} & (\mu,\nu)=(0,0), \\ +\frac{1}{p^2} & (\mu,\nu)=(i,i)\,. \end{cases}$

- In the case of $R_{\mu\nu}R^{\mu\nu}$, low-energy pole is physical ➡ spin-2 ghost problem.