Schrödinger's "Alarming" Phenomenon of Particle Creation in the Expanding Universe

Cosmological Gravitational Particle Production (CGPP)

The expansion of the universe creates particles from the vacuum

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October 2023
University of Florida

Inner Space/Outer Space Interface

Particle physics (**Inner Space**)
is required to explain the universe

- dark matter
- dark energy
- baryon asymmetry
- CMB fluctuations
- origin of structure

The universe (**Outer Space**)
is a particle accelerator

- big bang as particle accelerator
- limits on Beyond Standard Model physics
- long lifetime/path length
- stellar energy loss
- large B fields

Image credit: Chris Stabb

Inner Space/Outer Space Interface

Big Bang as accelerator for particle production assumes

1. at some point temperature larger than some mass scale m
2. particle interacts with SM plasma

BUT

1. Maximum temperature of the radiation-dominated universe is the "reheat" temperature after inflation, T_{RH}

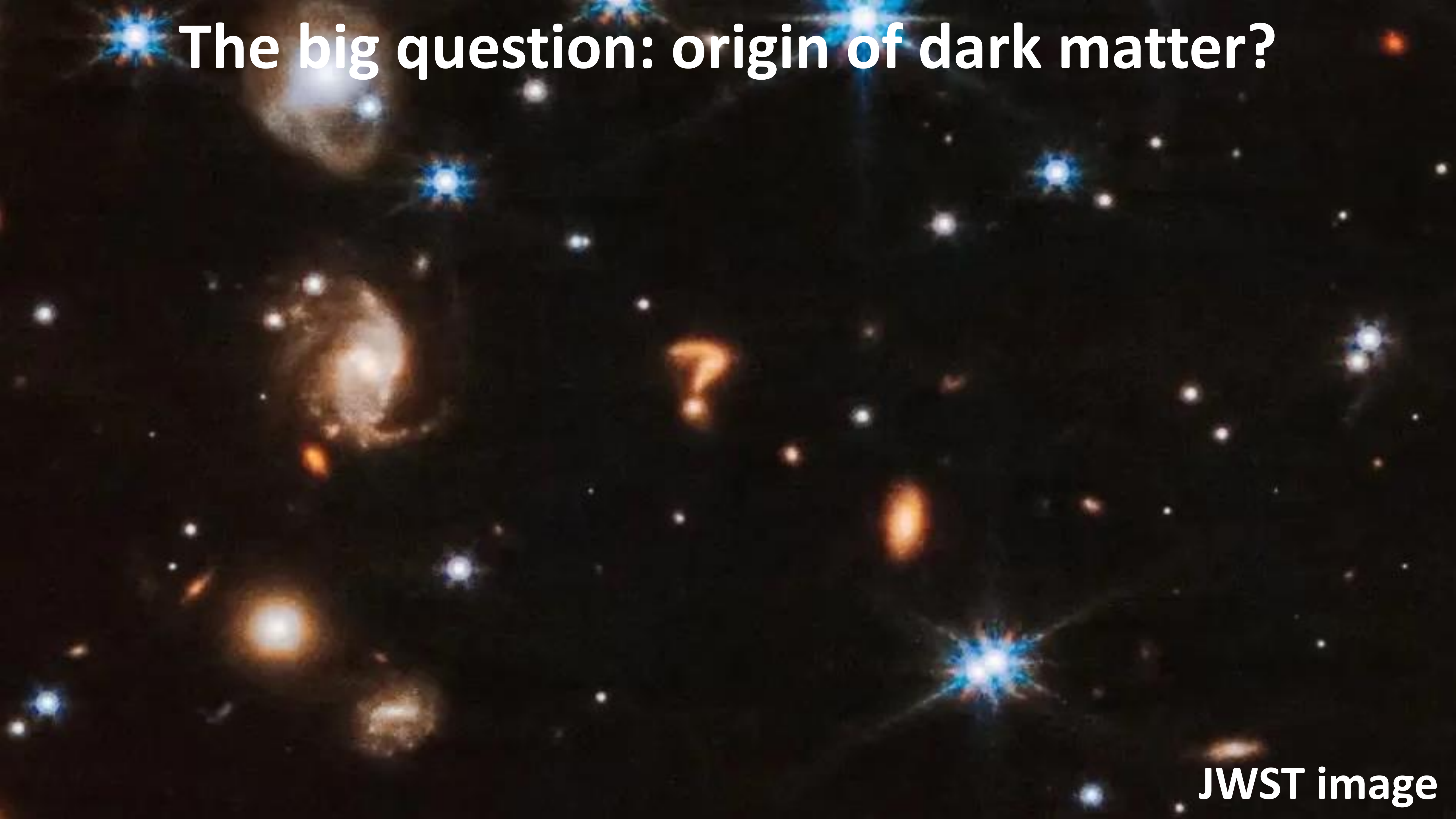
   T_{RH} may be as low as $8\ \mathrm{MeV}$ (to set stage for BBN)!

2. What about particles with no SM interactions (or) interactions too weak to be populated in the primordial soup?

   (No evidence that dark matter interacts with SM particles)

Image credit: Chris Stabb

The big question: origin of dark matter?



JWST image

Schrödinger's Alarming Times

1926: "Quantisierung als Eigenwertproblem," *Annalen der Physik*. **384**, 273

1927: Schrödinger visited U.S.
Found noise and dirt of New York "shattering."
Found Chicago worse, feared "bandits who spring with loaded guns from speeding autos."
Schrödinger departed UZH for Berlin.

1933: Nazis came to power. Schrödinger, marked by Nazis as "politically unreliable," departed Berlin for "exile" in Oxford. Nobel Prize.

1936: Schrödinger departed Oxford for Graz, Austria in a miscalculation of the political situation that was, in his words, an "unprecedented stupidity."

1938: March, Anschluss; 26 August Schrödinger dismissed; 14 September, Erwin & Anny left Graz for Rome with ten Marks, three suitcases, *sans* Nobel medal; met in Rome by Fermi; asylum in the Vatican.

1938: December, Schrödinger assumes position in Ghent, Belgium [ed. another stupidity].

1939: October, Schrödinger departed Belgium for Dublin.

Biographical info. from Walter Moore, *Schrödinger, Life and Thought* (Cambridge Univ. Press, 1992)

Schrödinger the Cosmologist

No. 3648, SEPT. 30, 1939 NATURE 593

LETTERS TO THE EDITORS

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NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 600.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Nature of the Nebular Red-Shift

FROM an investigation (to be published in *Physica*) of the proper vibrations of expanding spherical space, it follows that—in extremely good approximation—light is propagated with respect to co-moving co-ordinates irrespective of the expansion, except that (*a*) the time-rate of events is slowed down and (*b*) all energy portions decrease, both inversely proportional to the radius of curvature.

The slowing down secures the constancy of the velocity of light and entails the nebular red-shift, which from this point of view takes place *during the passage*. The attempt[1] to decide by observation, whether it is actually due to expansion, rests on two important formulæ, which follow from the new view with great ease. Let l be the linear diameter of a nebula *at the moment of emission* and χ its angular distance from the observer (linear distance divided by the circumference of space), then the angle $d\theta$ between two geodesics of space, pointing at the moment of emission from the observer to the ends of the diameter, is from pure geometry:

$$d\theta = \frac{l}{R \sin \chi}, \qquad (1)$$

R being the radius of curvature at the moment of emission. By the theorem quoted above, $d\theta$ is also the observed angular diameter of the nebula (Hubble and Tolman, equation 3).

Again, let the energy emitted by the nebula within an appropriately chosen unit of time be E_0. It will soon assume the shape of a spherical shell of thickness c (say). Let $R_{obs.}$ be the radius of space, when this shell reaches the observer. Its surface at this moment is, by pure geometry, $4\pi R^2_{obs.} \sin^2\chi$. By the theorem quoted above, its thickness then is $c\,R_{obs.}/R$ and its energy is $E_0\,R/R_{obs.}$. Hence its energy density ρ is

$$\rho = \frac{E_0}{4\pi c R^4_{obs.}} \cdot \frac{R^2}{\sin^2\chi}. \qquad (2)$$

ρ is a measure of the bolometric luminosity, observed outside the earth's atmosphere (Hubble and Tolman, equation 4).

My purpose in re-stating here these two important formulæ due to Tolman is to make the following remarks. Both l and E_0 refer to the moment of emission, which is different for two nebulæ observed simultaneously. Should l and E_0 exhibit a general dependence on R, then it would no longer be reasonable to regard them as *constants*, when equations (1) and (2) are combined (as they actually are) with the hypothesis of uniform spatial distribution of the nebulæ. For the latter, if admitted at all, has to apply to nebulæ which are intrinsically similar at the *same* moment of time—not at such moments as depend on the accidental position of our galaxy.

As regards l, the question is, whether we are inclined to assume (*a*) that the distances between the stars within a nebula behave, on the average, like the distances between two points of a rigid body—say, the ends of the Paris metre rod; or (*b*) like the distance between two distant nebulæ. Clearly the case of the stars is intermediate. To regard l as a constant means to decide for the first alternative. The second one would make l/R constant, giving formula (1) the same form as in the case of a non-recessional explanation of the red-shift (see Hubble and Tolman, equation 3′).

As regards E_0, the possible general decline of the nebular candle-powers has already been mentioned by Hubble and Tolman (see their concluding remarks). To the assumption that the same amount of energy is emitted during every second, there is a peculiarly simple alternative, namely, that the amounts of energy, which *have* been emitted during a second, *remain* equal. On account of the decay of travelling energy, this assumption would mean $E_0 \sim 1/R$, which reduces equation 2 to the same form as in the case of a non-recessional explanation of the red-shift (see Hubble and Tolman, equation 4′). I do not mean to suggest $E \sim 1/R$ particularly. I mention it in the way of an example.

These remarks detract nothing from the importance of deciding by observation how $d\theta$ and ρ actually behave, if the photographs are interpreted as assuming uniform spatial distribution. I understand that present evidence points to observed luminosities (ρ) decreasing with distance *not even quite as rapidly* as we should expect (with E_0 = const.) from the non-recessional explanation. If that is so, I should say they rather support the recessional explanation, in spite of its predicting a still more rapid decrease of the ρ's. The discrepancy, though greater, can here be removed by assuming the E_0's to decrease with time; an assumption which is very plausible in an expanding universe, which, on the whole, cools down; but not at all plausible in a static one.

E. SCHRÖDINGER.

7 Sentier des Lapins,
La Panne, Belgium.
July 31.

[1] Hubble, E., and Tolman, R. C., *Astrophys. J.*, 82, 302 (1935).

The Forbidden $^3P_0$–$^1D_2$ Line of O III in the Nebular Spectrum of Nova Herculis 1934

ALTHOUGH the two well-known lines of [O III] λ = 5007 A. ($^3P_2$—$^1D_2$) and λ = 4959 A. ($^3P_1$—$^1D_2$) are the most prominent features in the spectra of planetary nebulæ and novæ at the nebular stage, the third line of the triplet, corresponding to the $^3P_0$—$^1D_2$

In Belgium Schrödinger met big-bang cosmologist (and priest) Abbé Georges Lemaître.

Several previous interactions while at Oxford with astrophysicist Sir Arthur Stanley Eddington.

July 1939	*Nature of the Nebular Red-Shift*
August 1939	*The Proper Vibrations of the Expanding Universe*
1956	*Expanding Universes,* Cambridge Press

University of Chicago 1909 National Champions

Physica VI, no 9 October 1939

THE PROPER VIBRATIONS OF THE EXPANDING UNIVERSE

by ERWIN SCHRÖDINGER

§ 1. *Introduction and summary.* Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are its proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light. — E i n s t e i n s theory of gravitation imposes an a priori reason for assuming space to be, if closed, expanding or contracting; for this theory does not admit of a stable static solution. — The observed facts are, to say the least, not contrary to these assumptions.

This makes it imperative to generalize to expanding (or contracting) universes the investigation of proper vibrations, started for the the static cases (E i n s t e i n- and D e S i t t e r-universe) by the present writer and two of his collaborators [1]). The task is an easy one. The broad results are largely (in part even entirely) independent of the time-law of expansion. In the cases of main practical interest, i.e. with the present slow time rate of expansion and with wave lengths small compared with the radius of curvature of space (R), they are the following.

These are the broad results. A finer and particularly interesting phenomenon is the following.

The decomposition of an arbitrary wave function into proper vibrations is rigorous, as far as the functions of space (amplitude-functions) are concerned, which, by the way, are exactly the same as in the static universe. But it is known, that, with the latter, two frequencies, equal but of opposite sign, belong to every space function. *These two* proper vibrations cannot be rigorously separated in the expanding universe. That means to say, that if in a certain moment only one of them is present, the other one can turn up in the course of time.

Generally speaking this is a phenomenon of outstanding importance. With particles it would mean production or anihilation of matter, merely by the expansion, whereas with light there would be a production of light travelling in the opposite direction, thus a sort of reflexion of light in homogeneous space. Alarmed by these prospects, I have investigated the question in more detail. Fortunately the equations admit of a solution by familiar functions, if R is a *linear* function of time. It turns out, that in this case the alarming phenomena do not occur, even within arbitrarily long periods of time.

$e^{2\pi i\nu t}$ will re-assume (or approximately re-assume) the form $Ae^{2\pi i\nu' t}$ — and *not* $Ae^{2\pi i\nu' t} + Be^{-2\pi i\nu' t}$ — whenever $R(t)$, after an intermediate period of arbitrary variation, returns to constancy (or to approximate constancy). I can see no reason whatsoever for $f(t)$ to behave rigorously in this way, and indeed I do not think it does. There will thus be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to what in the introduction I called „the alarming phenomena''. They are certainly very slight, though, in two cases, viz. 1) when R varies slowly 2) when it is a linear function of time (see the following sections).

A second remark about the new concept of proper vibration is, that it is not always invariantly determined by the form of the universe. The separation of time from the spatial coordinates may succeed in a number of different space-time-frames. For De Sitters universe I know three of them. Besides the static one, for which P. O. Müller (l.c.) has redently given the proper vibrations, there is an expanding form with infinite R and an expanding form with finite R *). A proper vibration of one frame will not transform into a proper vibration of the other frame, for the separation of variables is destroyed by the transformation.

Schrödinger's two favorite phrases:

1. alarming phenomenon
2. mutual adulteration

Schrödinger was alarmed by creation of a *single* particle

per Hubble time $(H_0^{-1} \sim 10^{10}\ \text{yr})$

per Hubble volume $(H_0^{-3} \sim 10^{57}\ \text{km}^3)$

with Hubble energy $(H_0 \sim 10^{-33}\ \text{eV})$

Of all the circumstances faced by Schrödinger in 1939, why did this alarm him?

Why was Schrödinger alarmed?

- Appearance of particles from the vacuum sounded crazy.
- Technical issues with calculation:
  Quantum mechanical calculation (requires quantum field theory).
  Only create particles with mass less than expansion rate H (today $H_0 \sim 10^{-33}$ eV).
  Only create particles if violate Weyl Conformal Invariance ($T^{\mu}{}_{\mu} \neq 0$), so don't create photons.
  Would Schrödinger still have been alarmed?
- Schrödinger looked for (and found) a cosmological solution without mutual adulteration (not a very physical solution—Milne model).
- Conceptual challenge to Quantum Mechanics and/or General Relativity?
- Infinite particle creation in standard big bang at $t = 0$.
- (Sometimes should just follow the equations.)

Schrödinger's Alarming Phenomenon

"Outstanding importance"?

Schrödinger 1939: "Generally speaking this is a phenomenon of outstanding importance. With particles it would mean the production or annihilation of matter, merely by the expansion." [why would that be of "outstanding importance"?]

Forgotten in 40s, 50s, 60s (by Schrödinger also).

"Great Cosmological Significance"?

Leonard Parker Thesis 1966. In 1968 paper: "…for the early stages of a Friedmann expansion it [particle creation] may well be of great cosmological significance, especially since it seems inescapable if one accepts quantum field theory and general relativity." [no speculation as to the "great cosmological significance"]

Schrödinger's Alarming Phenomenon

Other interest in CGPP in the 1970s (mostly regarded as a curiosity).

US: Parker, Ford, Fulling, Allen, Friedman, Wald, …
Soviet Union: Zel'dovich, Starobinski, Grishchuk, Grib, Mostepanenko, Lukash, … (CGPP in CCCP)
UK: Bunch, Davies, Birrell, Hawking, …

In 1970s Zel'dovich explored CGPP to explain why the universe is homogeneous and isotropic.

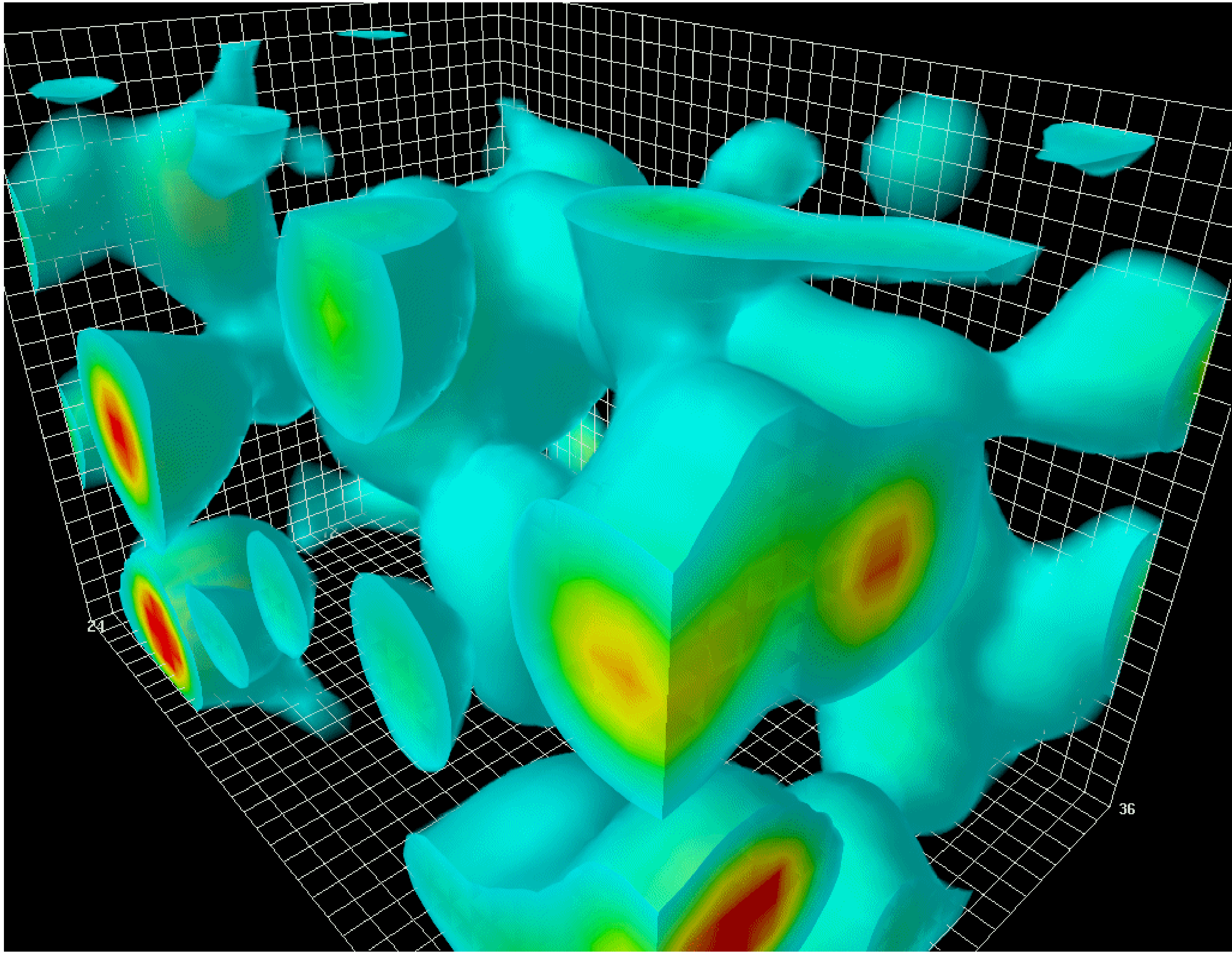
Finally, great cosmological significance in the 1980s (inflation):

Sasaki, Kodama, **Mukhanov & Chibisov**, Vilenkin, Linde, Abbott, Wise, Lyth, Salopek, Bond, …

Why mutual adulteration (particle creation)?

Quantum Vacuum Full of Stuff

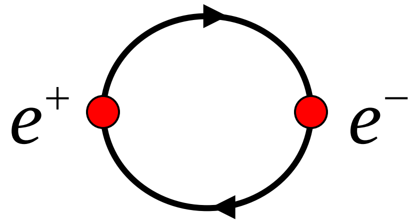
Quantum uncertainty principle
→ quantum vacuum contains
“virtual particles."



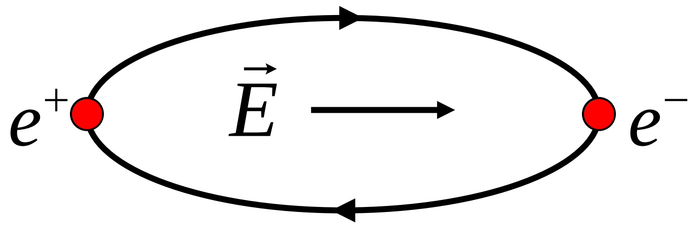
Derek Leinweber, University of Adelaide

Disturbing the Quantum Vacuum with an External Field

Electric Field → Particle creation

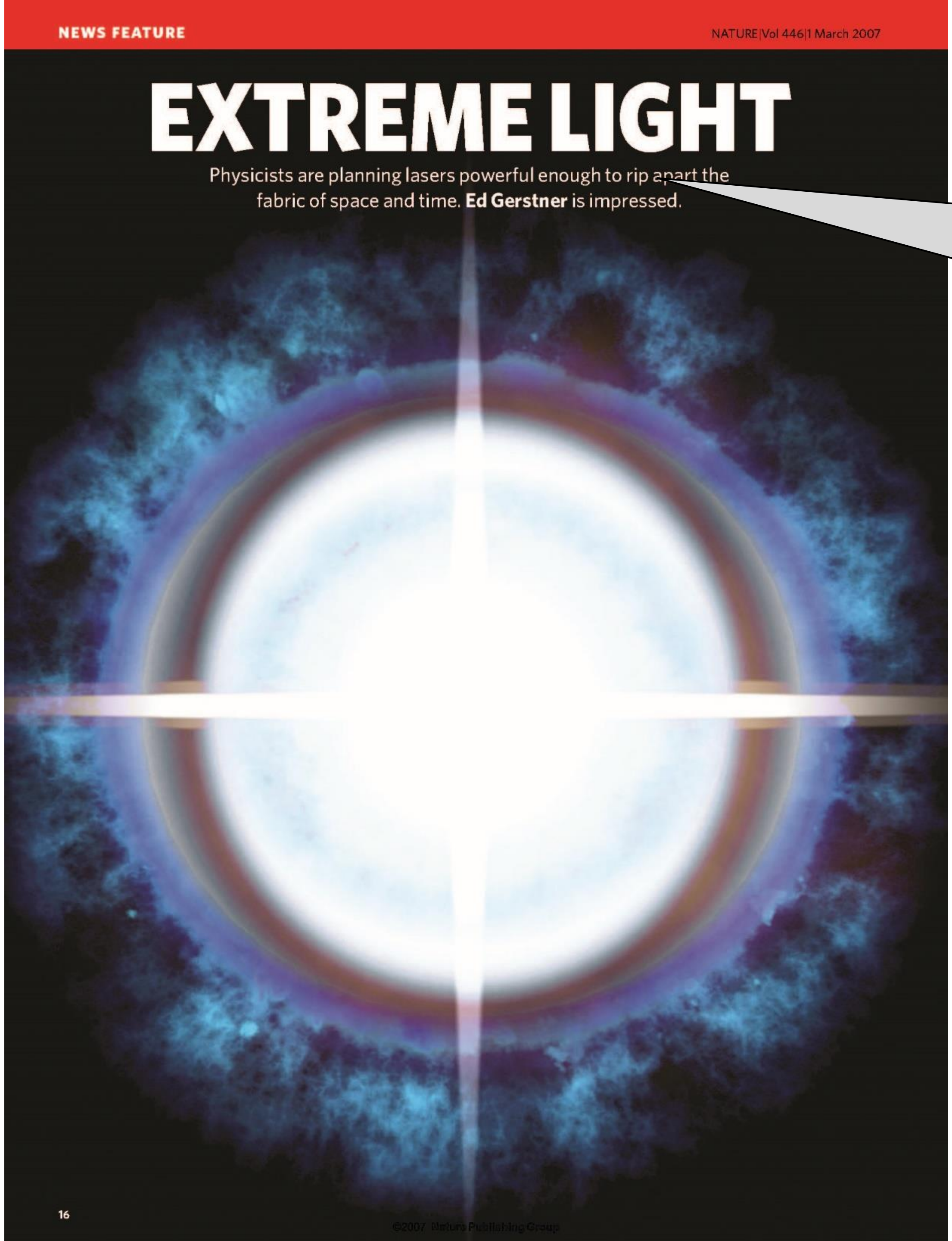


Particle creation if energy gained in acceleration from E-field over a Compton wavelength exceeds the particle's rest mass.



$$\left|\vec{E}_{\text{crit}}\right| = \frac{m_e^2 c^3}{e\hbar} \approx 10^{16}\ \text{V cm}^{-1}$$

Sauter (1931); Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)



NATURE, Vol 446/1 March 2007

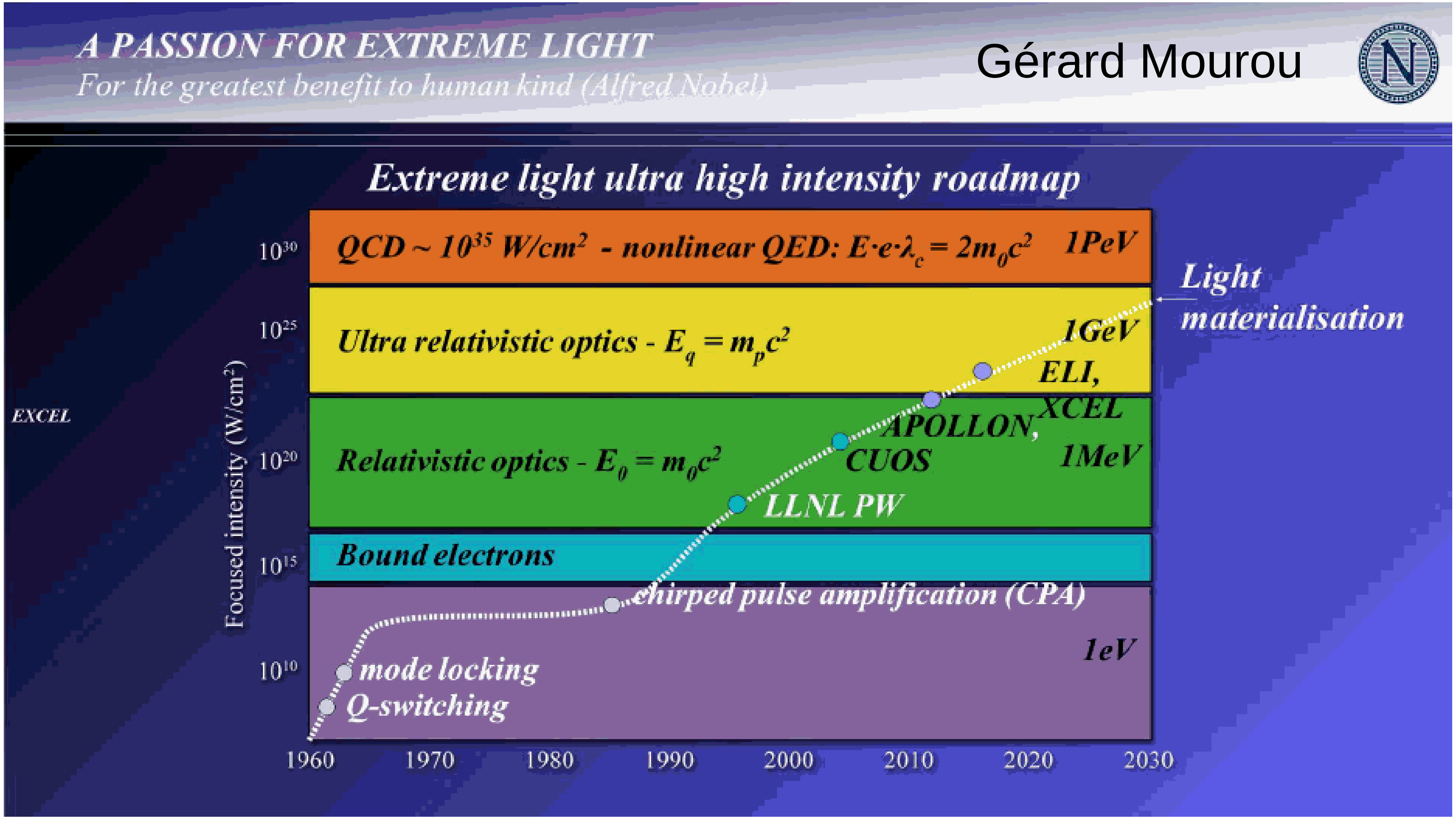
Physicists are planning lasers powerful enough to rip apart the fabric of space and time.

“We’re going to change the index of refraction of the vacuum and produce new particles.”

Gérard Mourou

$$I_C \approx \frac{c}{8\pi}\left|\vec{E}_{\text{crit}}\right|^2 \approx 10^{30}\ \text{W cm}^{-2}$$

Disturbing the Quantum Vacuum with an External Field



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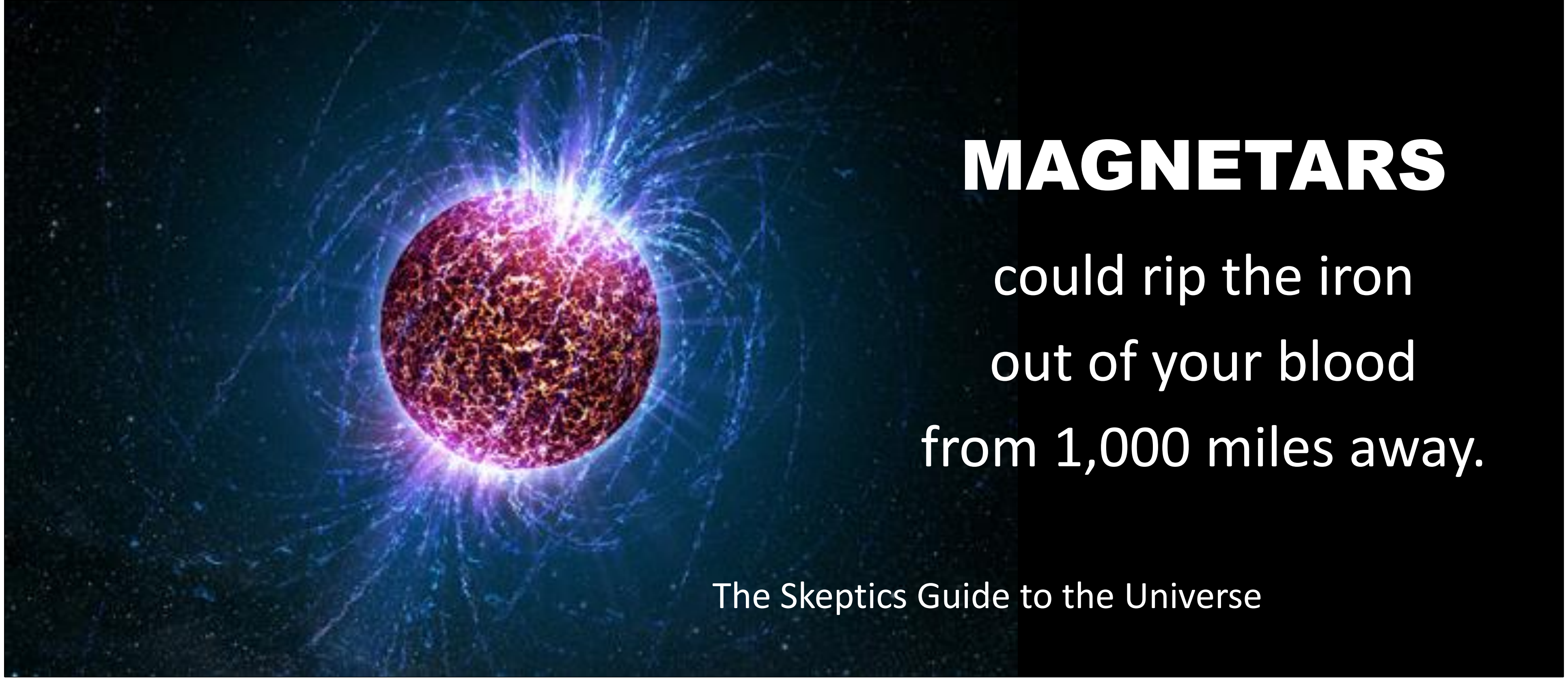


$$\left|\vec{B}_{\mathrm{crit}}\right| = \frac{m_e^2}{e} \approx 5 \times 10^{13}\,\mathrm{G}$$

Crab pulsar	3×10^{13} G
Magnetars	10^{14} □ 10^{15} G

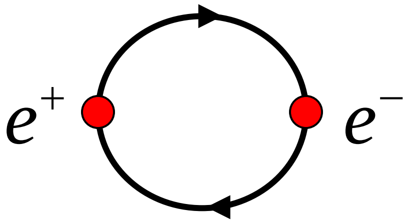
Strong magnetic fields imply existence of strong electric fields.

Many unexplained phenomena associated with pulsars, magnetars, etc.

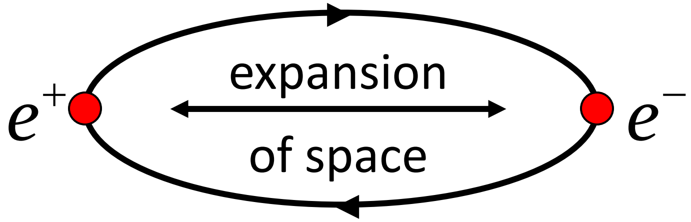
Damour & Ruffini

Disturbing the Quantum Vacuum with an External Field

Expanding universe → Particle creation



Particle creation if energy gained in acceleration from expansion over a Compton wavelength exceeds the particle's rest mass.



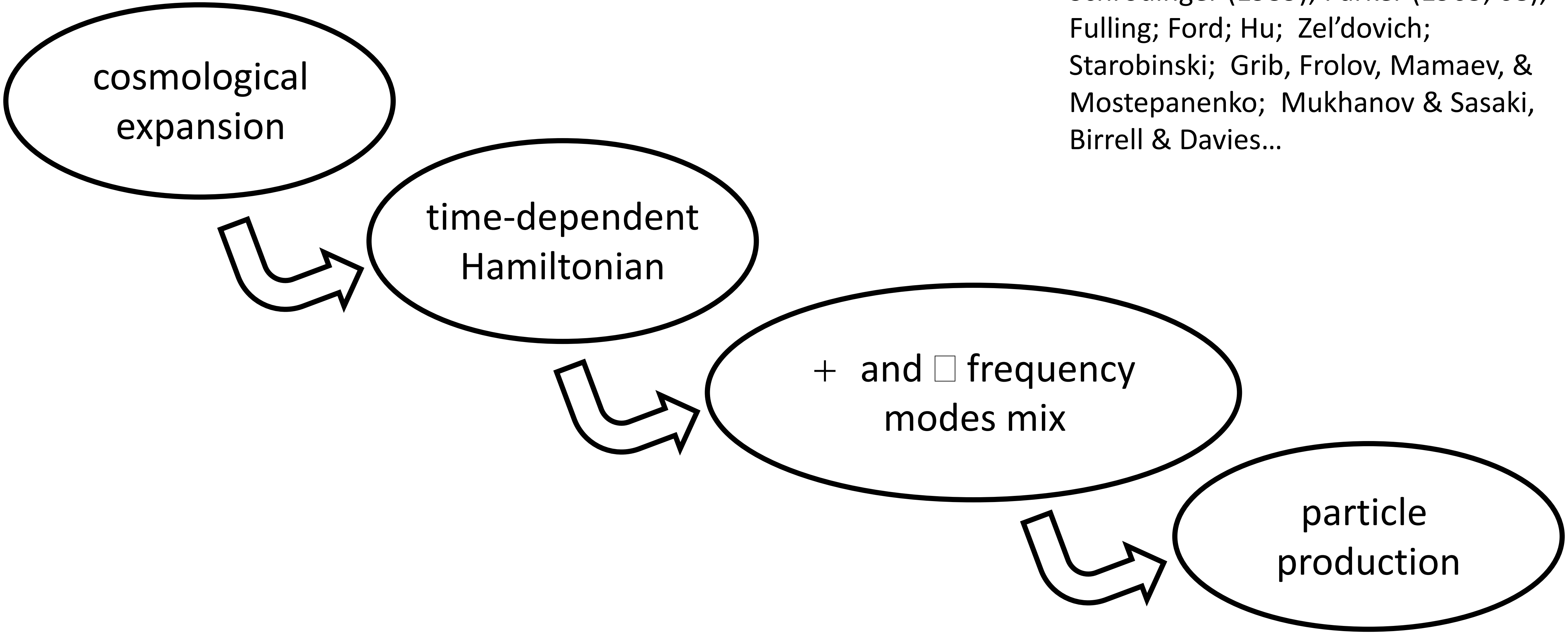
$\mathrm{v} = c$ at Hubble radius

$$H_{\text{crit}} = m$$

CGPP via Expansion of the Universe

- In Minkowskian QFT, a particle is an IR of the Poincaré group.
- But, expanding universe not Poincaré invariant.
- Notion of a “particle” is approximate.

In the early days:
Schrodinger (1939); Parker (1965, 68); Fulling; Ford; Hu; Zel’dovich; Starobinski; Grib, Frolov, Mamaev, & Mostepanenko; Mukhanov & Sasaki, Birrell & Davies…



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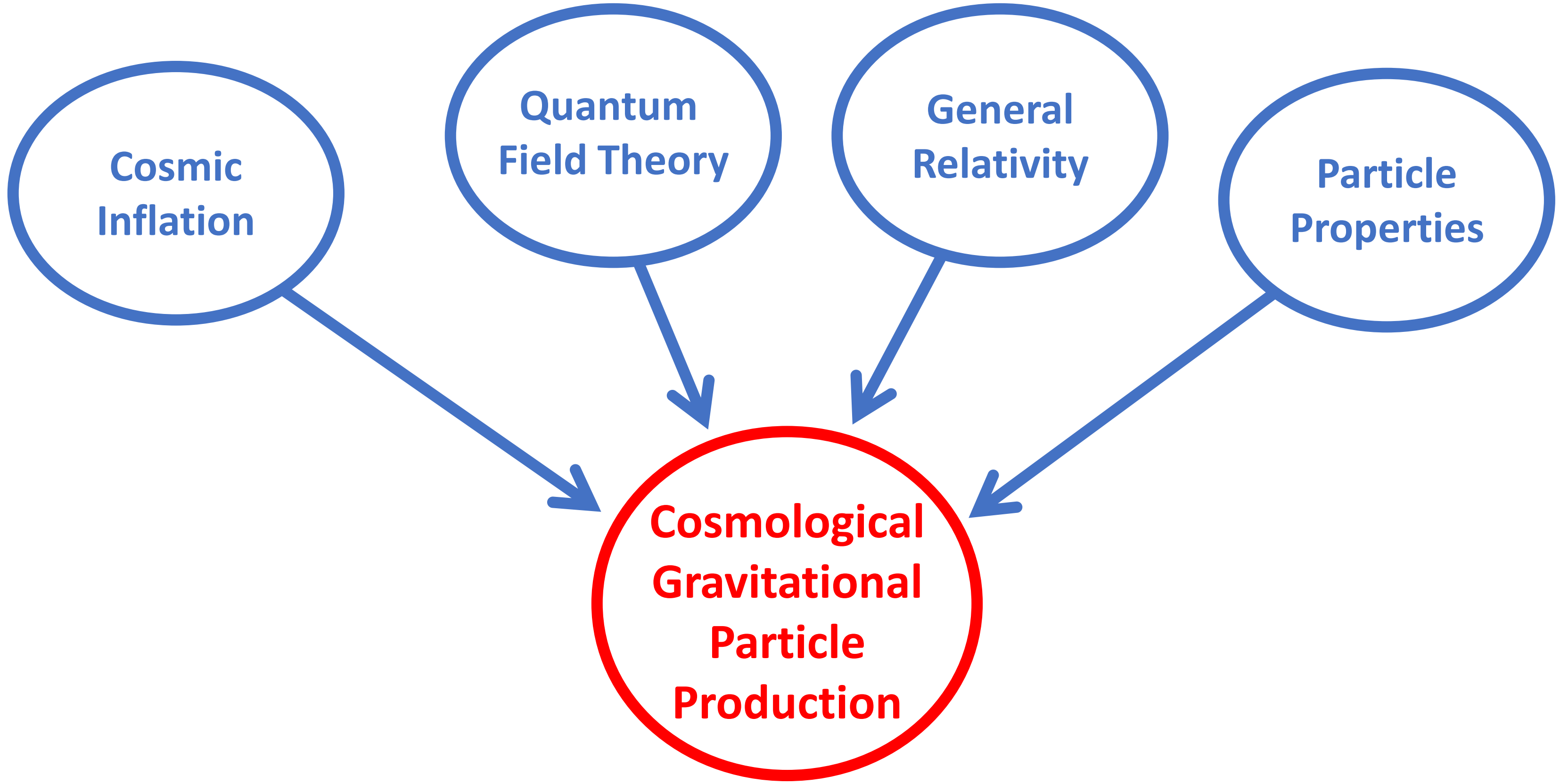
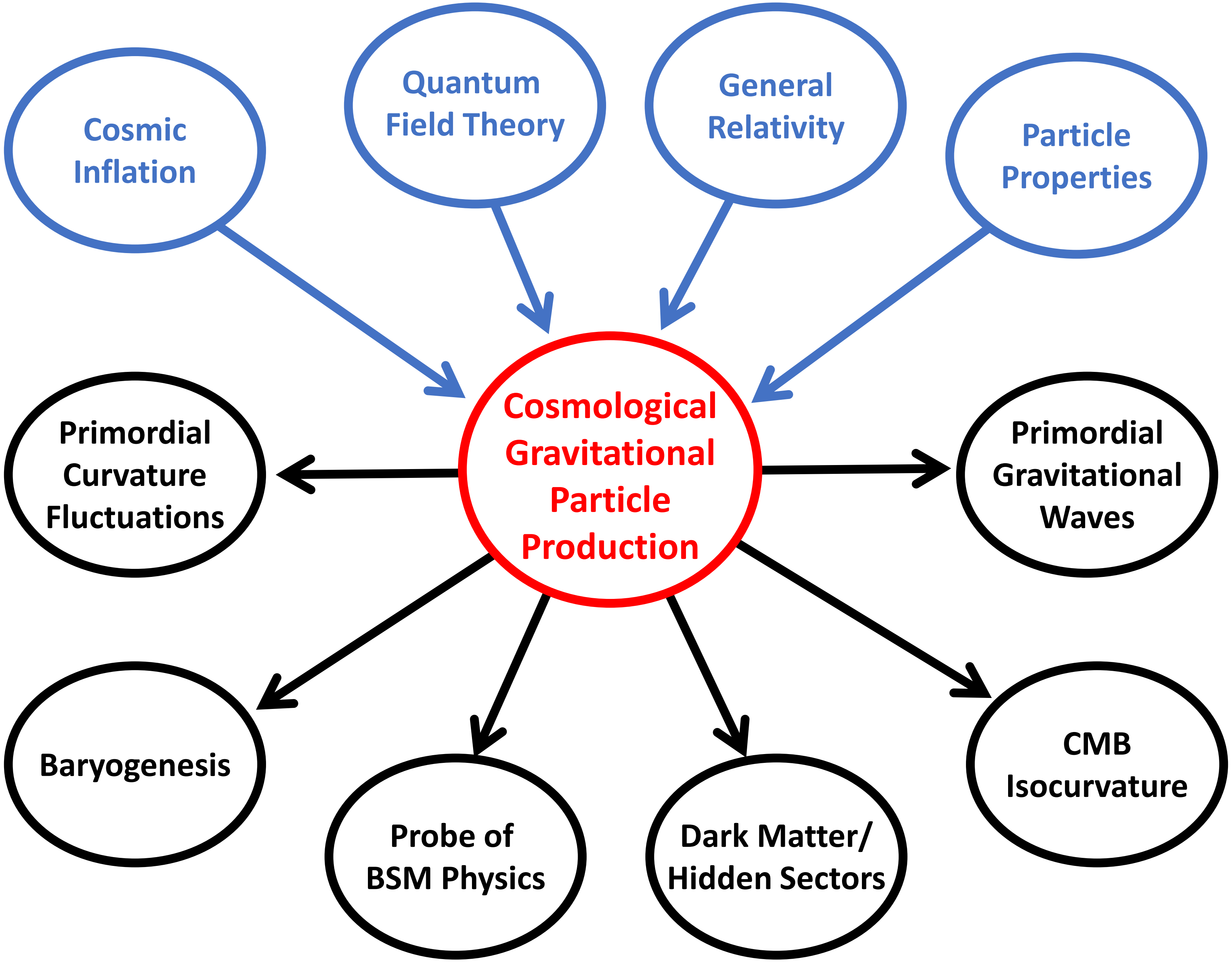
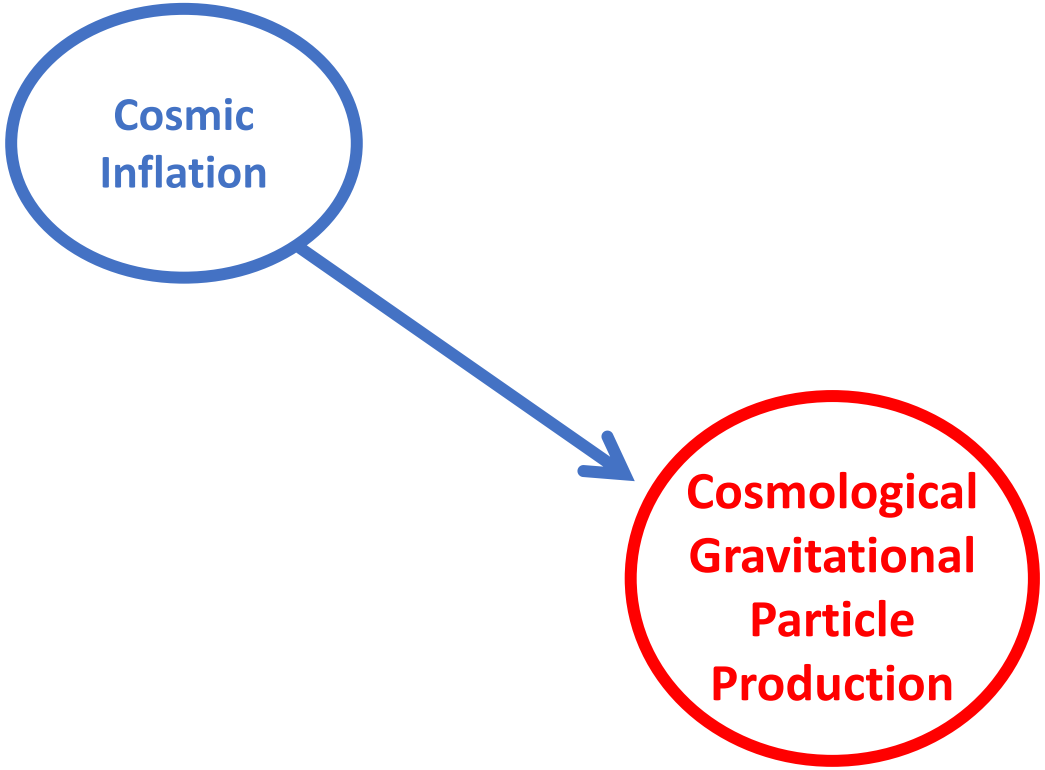

Cosmic Inflation
Quantum Field Theory
General Relativity
Particle Properties
Cosmological Gravitational Particle Production

Output


Cosmic Inflation
Quantum Field Theory
General Relativity
Particle Properties
Cosmological Gravitational Particle Production
Primordial Curvature Fluctuations
Primordial Gravitational Waves
Baryogenesis
Probe of BSM Physics
Dark Matter/ Hidden Sectors
CMB Isocurvature



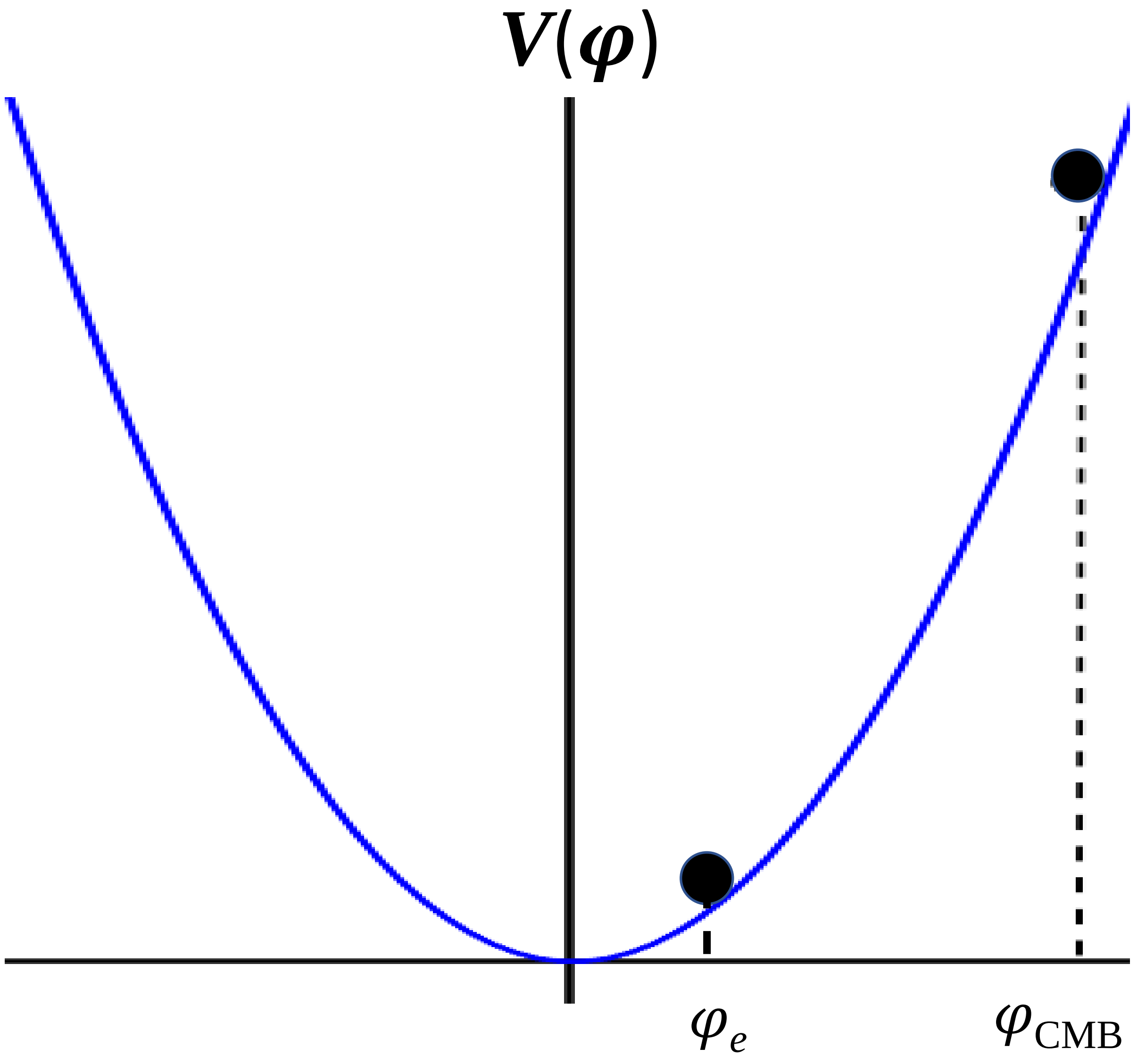
We have many models of cosmic inflation. <u>Qualitative</u> results insensitive to model.

∃ Standard Inflationary Picture, but not Standard Inflationary Model

There is a “simple” inflationary model:
single-field with Quadratic Model:

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2$$

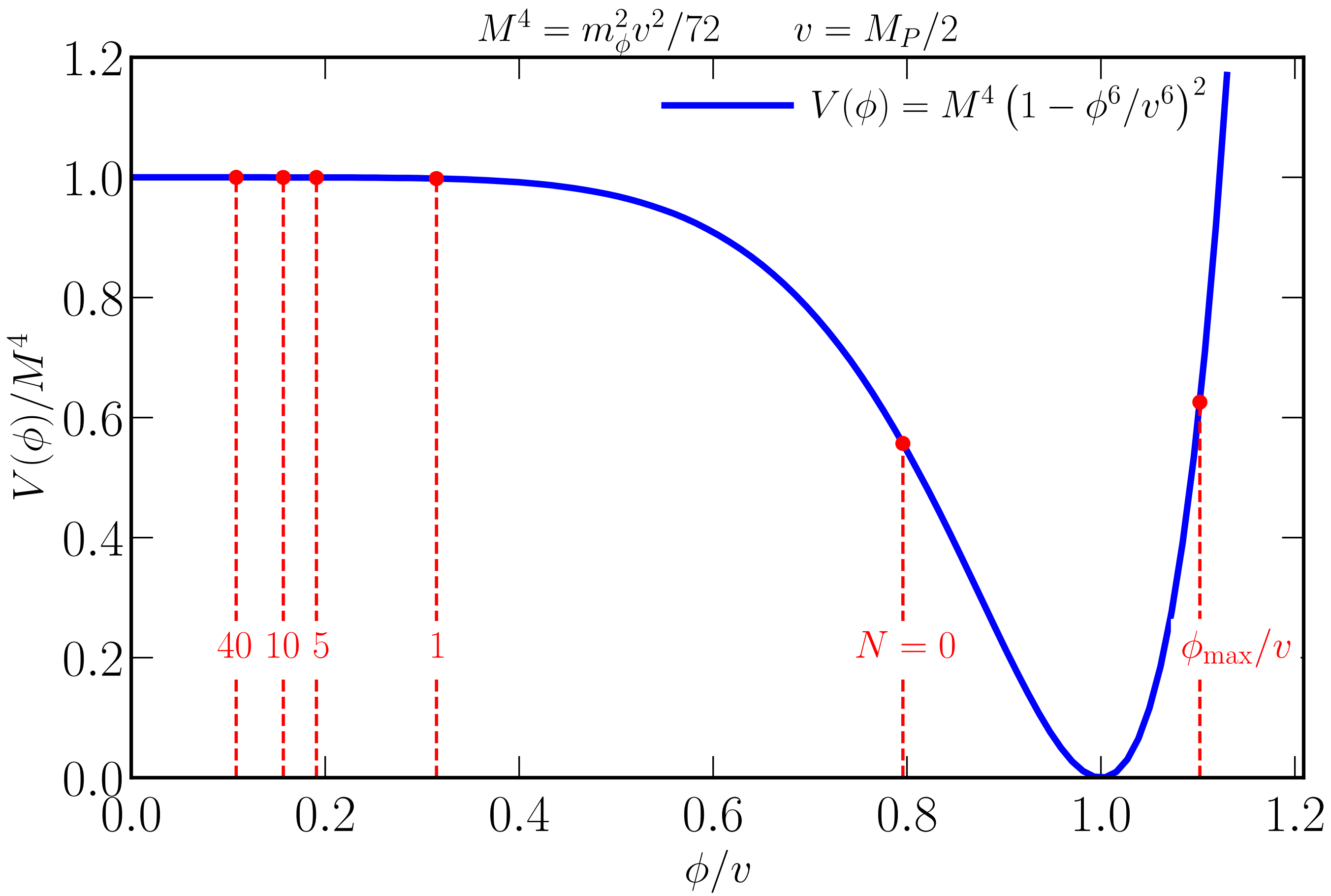
This simple model ruled out by CMB measurements. *But* CMB measurements probe inflaton potential 60 or so e-folds before the end of inflation. We will often be interested in inflaton potential near the end or after inflation ends when φ is close to the minimum of its potential and quadratic description may be a good approximation.



∃ Standard Inflationary Picture, but not Standard Inflationary Model

Also, recent studies employing Hilltop Model (Basso, Chung, EWK, Long)

$$V(\varphi) = M^4 \left(1 - \varphi^6/v^6\right)^2$$

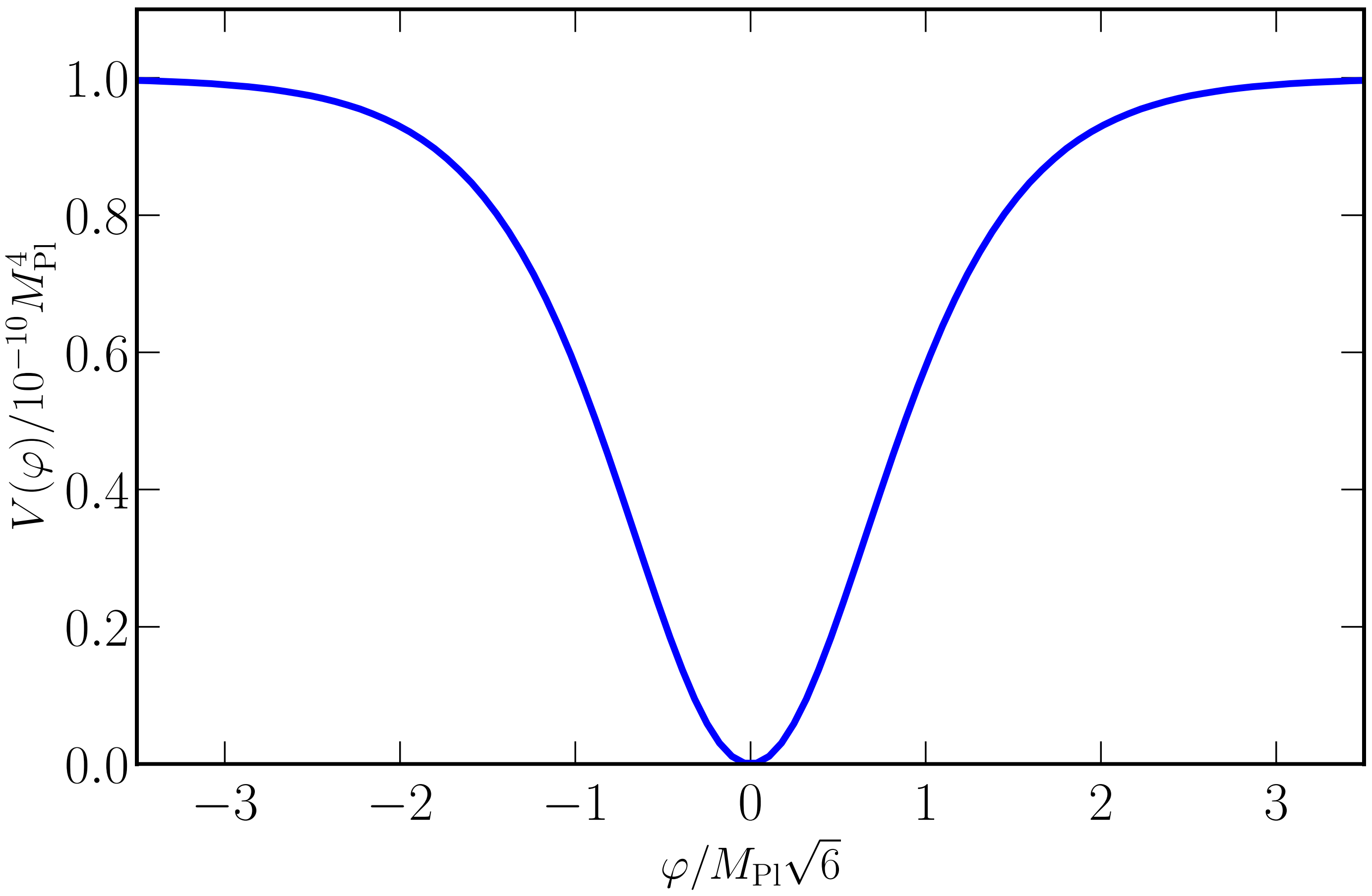


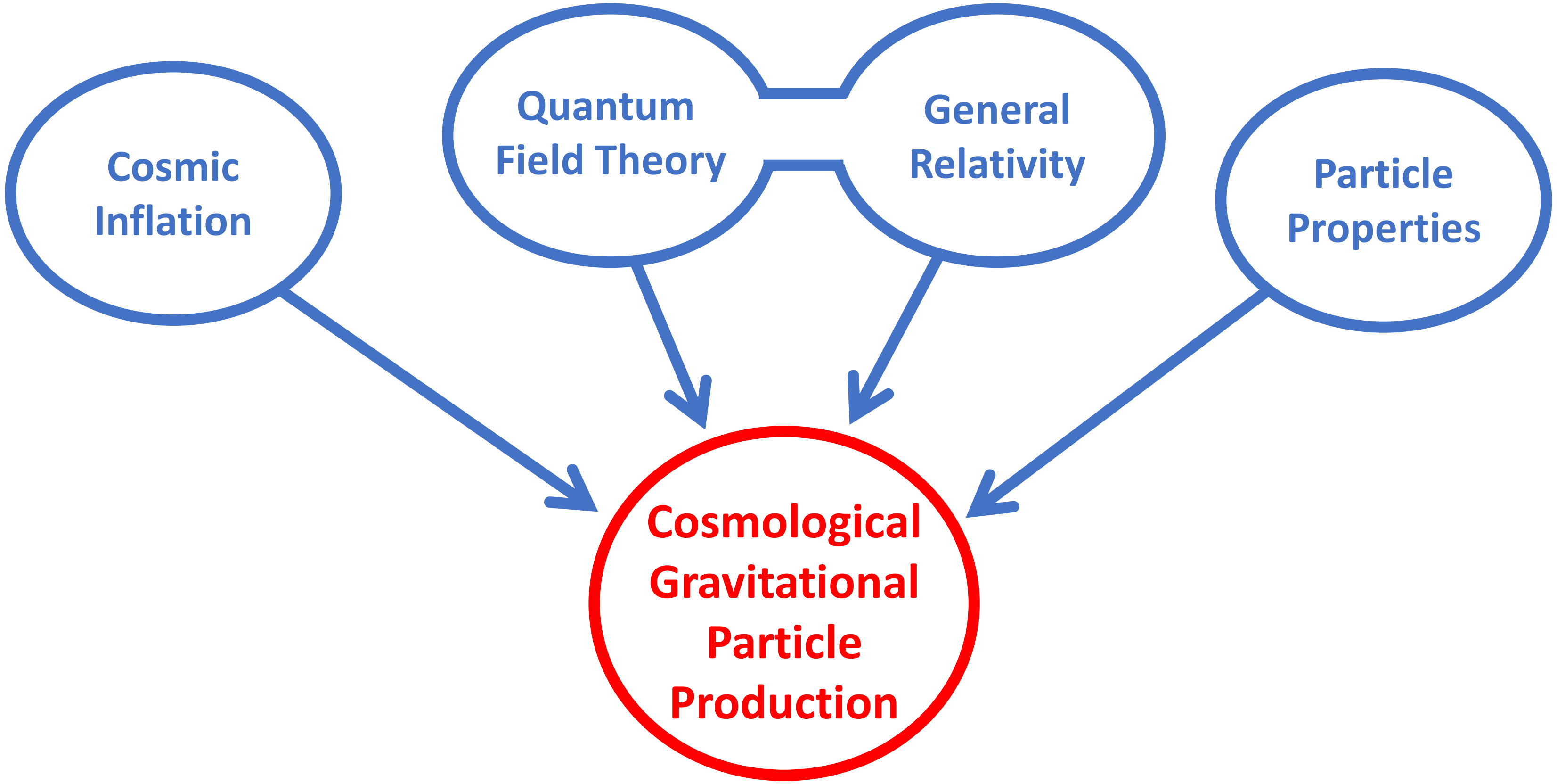
and rapid-turn inflation models (hyperbolic inflation, angular inflation, racetrack inflation, orbital inflation,…) with two fields (EWK, Long, McDonough, Payeur 2023)

$\exists$ Standard Inflationary Picture, but not Standard Inflationary Model

T-Model

$$V(\varphi) = 10^{-10} M_{\rm Pl}^4 \tanh^2(\varphi/\sqrt{6}M_{\rm Pl})$$



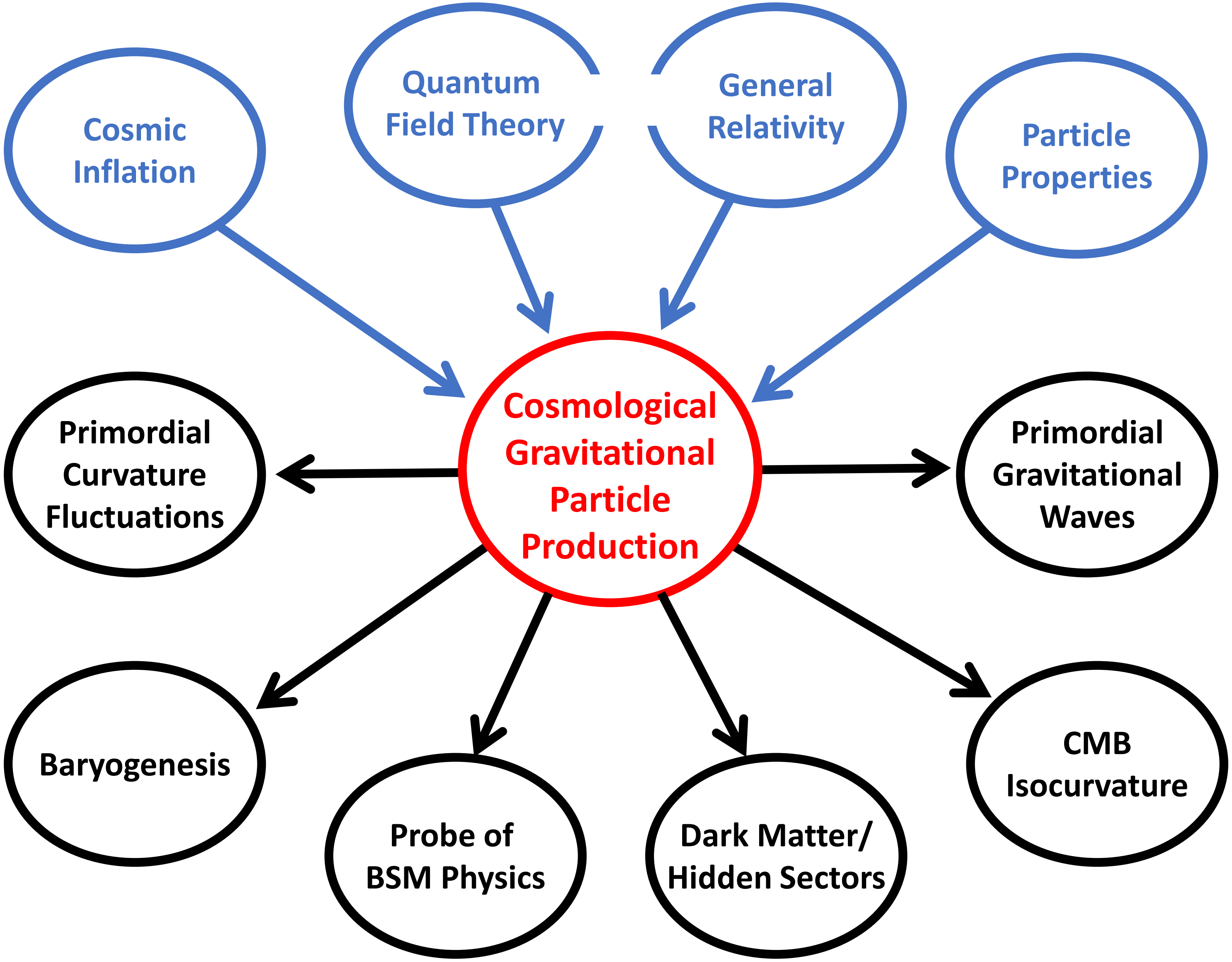


We have many models of cosmic inflation. Qualitative results insensitive to model.

Involves Quantum Field Theory in curved spacetime.

Depends on particle's spin, mass, and gravitational coupling.

Representation	Particle	**1-point function Dark Matter**	**2-point function CMB Isocurvature**	**3-point function CMB Nongaussian**
(0,0)	Conformally Coupled Scalar $\xi = 1/6$ (use as template)	Kuzmin & Tkachev (99)	Expected to be very small (blue)	Chung & Yoo (13)
(0,0)	Minimally Coupled Scalar $\xi = 0$ (e.g., inflaton)	Kuzmin & Tkachev (99)	Chung, EWK, Riotto, & Senatore (05)	
(1/2,0) ⊕ (0,1/2)	"Dirac" Fermion	Chung, EWK, & Riotto (98)	Expected to be very small (blue)	
(1/2,1/2)	de Broglie-Proca Vector	Graham & Mardon (16); Ahmed, Grzadkowski,& Socha (20); EWK & Long (21)		
(1,0) ⊕ (0,1)	2-Form (Pseudo) Vector (e.g., Kalb-Ramond)	Capanelli, Jenks, EWK, & McDonough (23)		
(1/2,1) ⊕ (1,1/2)	Rarita-Schwinger Fermion (e.g., gravitino)	EWK, Long, & McDonough (21)		
(1,1)	Fierz-Pauli (massive graviton)	EWK, Liang, Long, Rosen (23)		
Higher-spin bosons		Jenks, Koutrolikos, McDonough, Alexander, Gates (23)		



We have many models of cosmic inflation. <u>Qualitative</u> results insensitive to model.

Involves Quantum Field Theory in curved spacetime.

Depends on particle's spin, mass, and gravitational coupling.

Primordial gravitational waves and curvature perturbations via CGPP during inflation.

DM with mass from μeV to GUT scale & populate hidden sectors.

CGPP can be a probe of BSM physics.

Cosmological Gravitational Particle Production

Background Gravitational Field

- Assume FLRW homogeneous/isotropic cosmology ⇒ one dynamical variable: scale factor $a(t)$
- Assume spatially-flat metric $ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$
- Work in conformal standard time η: $a\, d\eta = dt$, metric is $ds^2 = a^2(\eta)\left(d\eta^2 - d\vec{x}^2\right)$
- Geometry conformal to Minkowski space
- Assume begin in inflation (quasi-de Sitter), transition to matter-dominated phase dominated by coherent oscillations of inflaton field, then transition to a radiation-dominated hot big-bang after decay of inflaton field.

Scalar field in FLRW background

Covariant action for spectator scalar field Φ (not the inflaton)

$$S[\Phi(x), g_{\mu\nu}(x)] = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}m^2\Phi^2 + \frac{1}{2}\xi R\Phi^2\right]$$

Gravity enters the picture

ξ is a dimensionless constant: $\xi = 0$ minimal coupling; $\xi = 1/6$ conformal coupling.

In principle, ξ could be anything (and presumably there is RGE).

In spatially-flat FLRW background in conformal time with rescaled field $\phi = a\,\Phi$

$$S[\phi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\mathbf{x}\left[\frac{1}{2}(\partial_\eta\phi)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m_{\text{eff}}^2\phi^2\right]$$

Time-dependent effective mass

$$m_{\text{eff}}^2(\eta) = a^2(\eta)\left[m^2 + \left(\frac{1}{6} - \xi\right)R(\eta)\right]$$

cosmological expansion ⇒
time-dependent background field ⇒
time-dependent Hamiltonian for spectator field

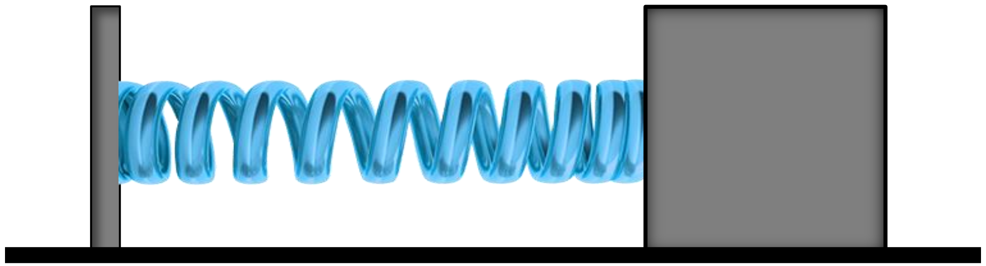
CGPP Through Expansion of the Universe

Expansion of the universe causes explicit time dependence in action for "spectator" fields.

Initial State ~ Minkowski (early-time) vacuum may not evolve to

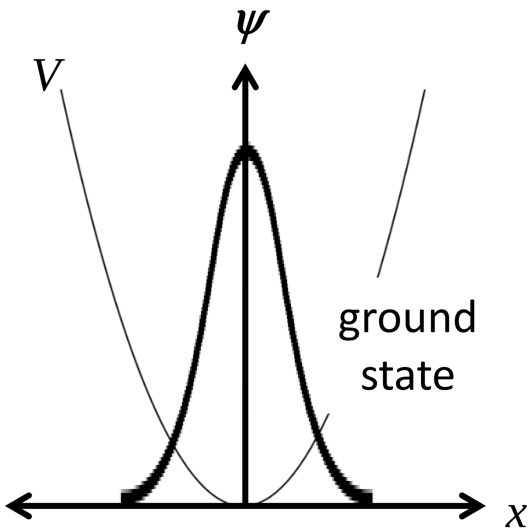
Final State ~ Minkowski (late-time) vacuum, but to an excited state populated by particles.

Think of a harmonic oscillator

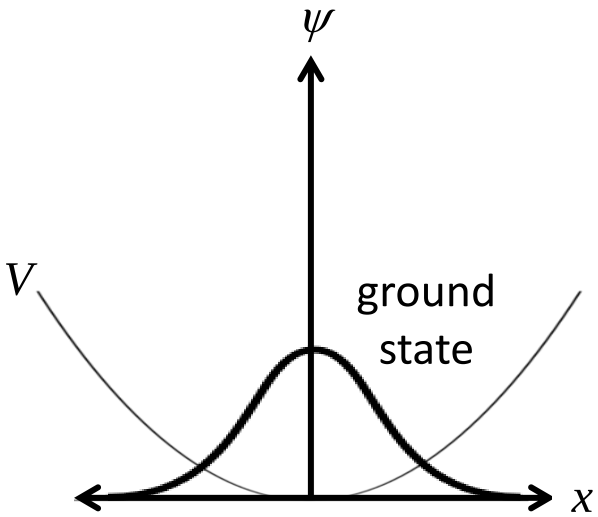


$$\ddot{x}(t)+\omega^2(t)\ x(t)=0$$

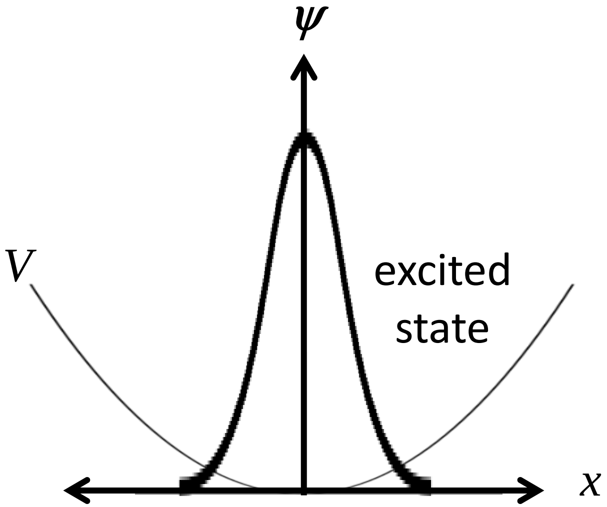
Initial State



Spring constant varied slowly (adiabatically)



Spring constant varied abruptly (nonadiabatically)



Scalar field in FLRW background

Fourier modes of ϕ (denoted as χ_k) obey wave equation: $\partial_\eta^2 \chi_k(\eta) + \omega_k^2 \chi_k(\eta) = 0$

Solutions to wave equation for mode functions include both + and – frequency terms

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{-i\int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}}\, e^{+i\int \omega_k(\eta) d\eta} \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

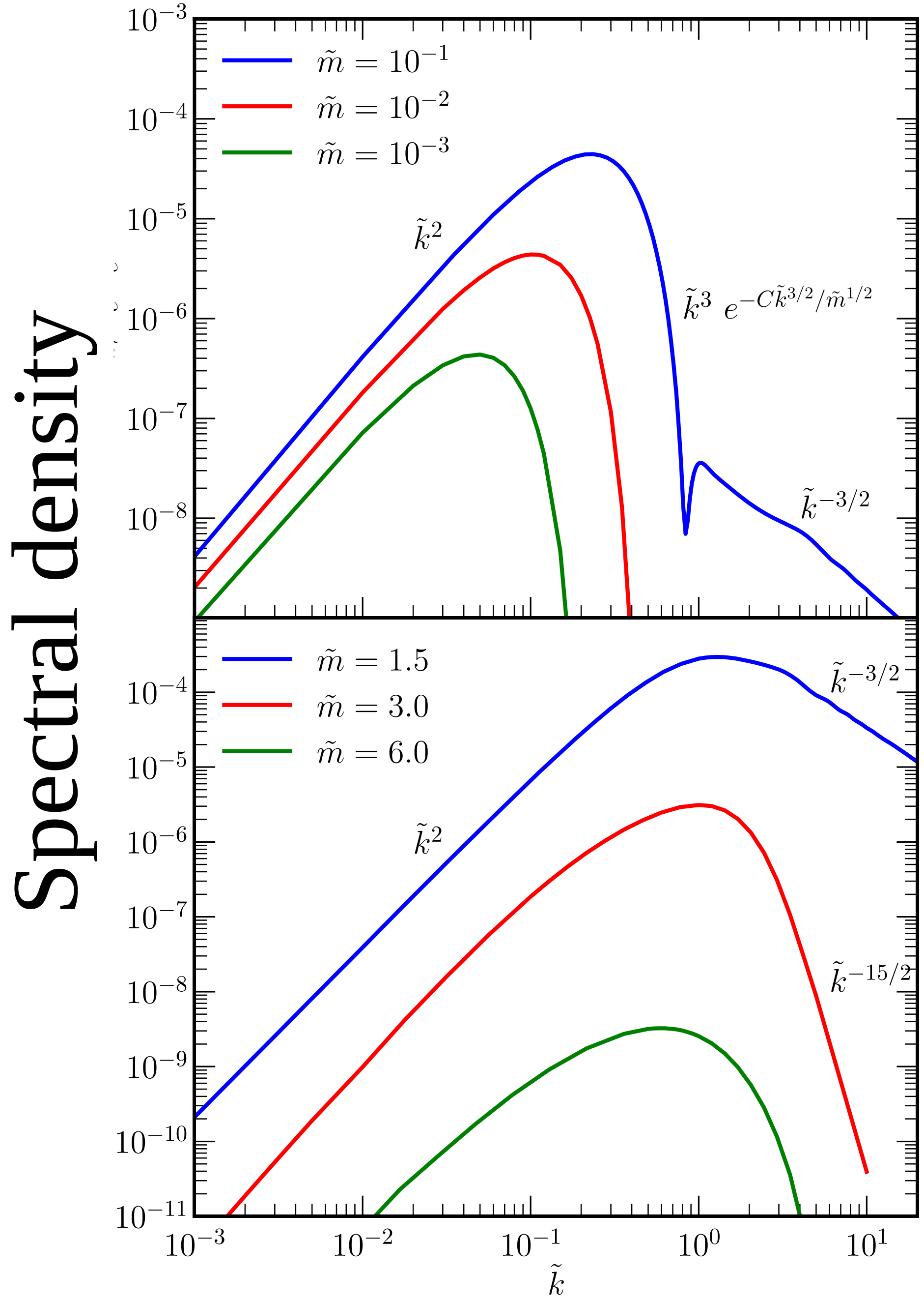
If start with only positive frequency modes, $|\alpha_k| = 1$ & $|\beta_k| = 0$, evolution of the universe will generate negative frequency modes (particles), β_k 0.

Comoving number density of particles at late time is

$$a^3 n = \int \frac{dk}{k} \underbrace{\frac{k^3}{2\pi^2} |\beta_k|^2}$$

n_k = spectral density

Quadratic Inflaton Potential for Conformally-Coupled Scalar: $\xi = 1/6$



$$\tilde{m} = m/H_e \qquad \tilde{k} = k/a_e H_e$$

$\tilde{k} < 1$ Crosses Hubble radius during inflation
Spectral density $\propto k^2$

$\tilde{k} > 1$ Always sub-Hubble radius
Spectral density exponentially or power-law damped

$\tilde{m} < 1$ Production more efficient as m increases

$\tilde{m} > 1$ Production less efficient as m increases (expect exponential suppression)

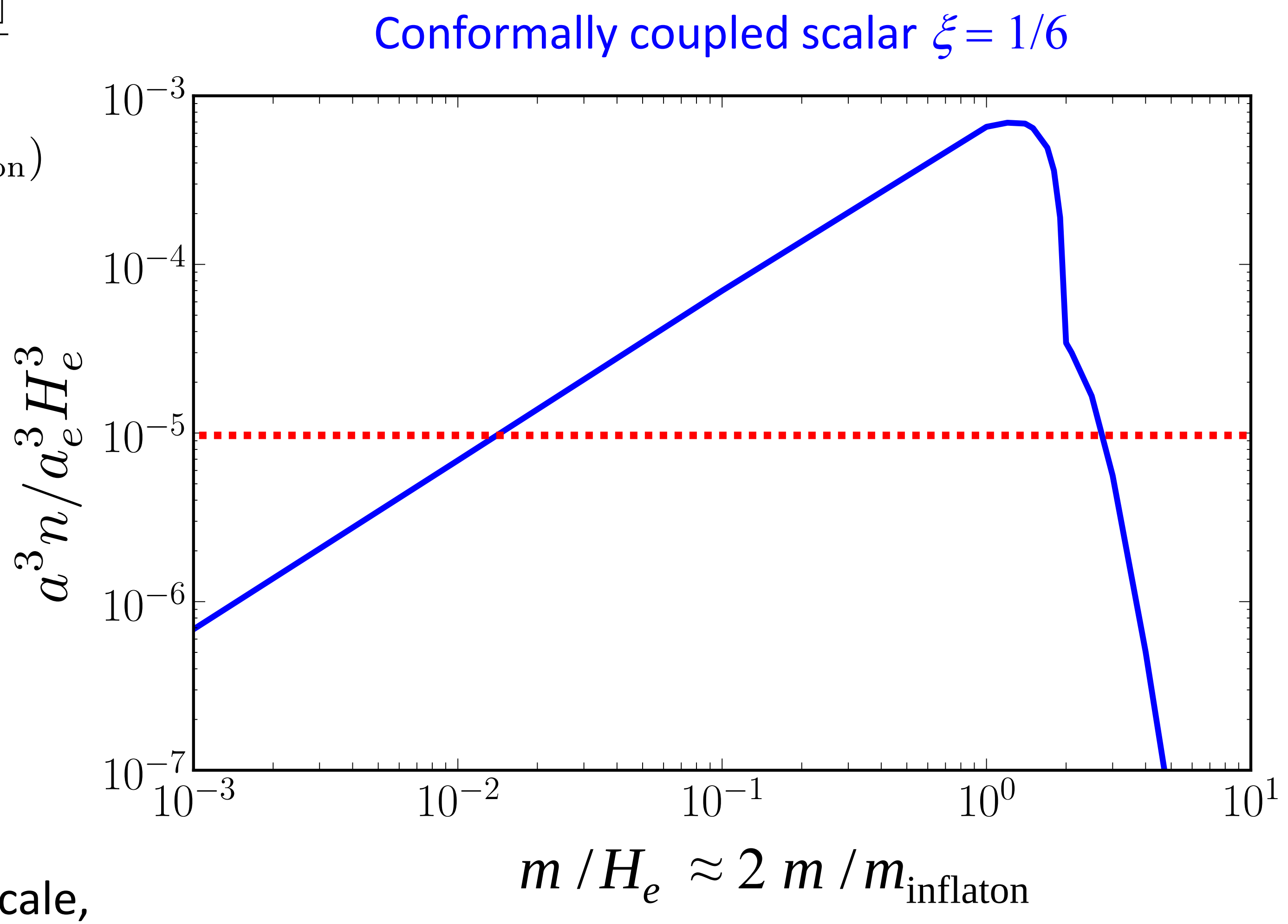
Blue spectrum

Quadratic Inflaton Potential for Conformally-Coupled Scalar: $\xi = 1/6$

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right)\frac{[na^3/\,a_e^3H_e^3]}{10^{-5}}$$

$$\sim \left(\frac{m}{10^{11}\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\mathrm{GeV}}\right) \qquad (m \lesssim m_{\mathrm{inflaton}})$$

- Calculation assumes inflationary model (quadratic, which is ruled out).
- But general picture holds in other models since action occurs around end of inflation.
- Don’t know, but $H_e \approx 10^{11}$ GeV and $T_{\mathrm{RH}} \approx 10^9$ GeV are “common.”
- If stable and dark matter, $\Omega h^2 = 0.12$ $\rceil$ $m \approx H_e$.
  Could have been anything! WIMPZILLA miracle!
- Perhaps inflation scale represents new physics scale, stable particle at that mass scale natural DM candidate.

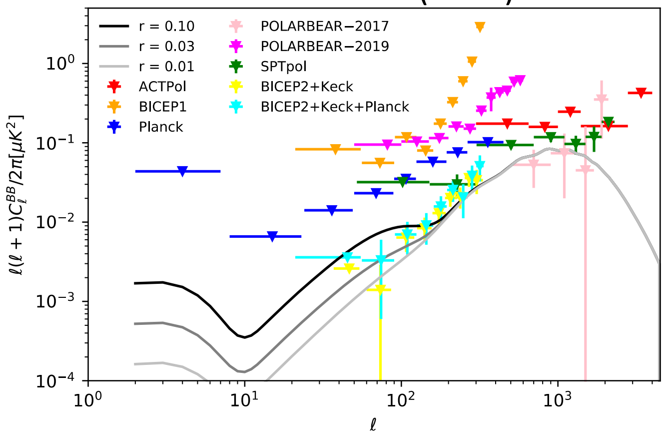


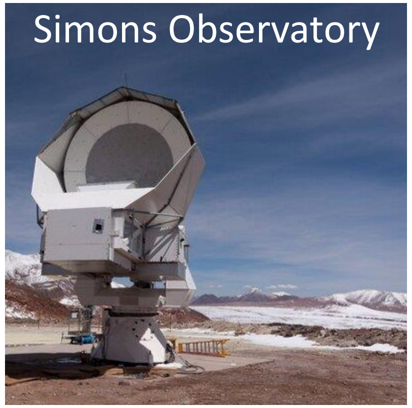
Conformally-coupled scalar WIMPZILLA DM candidate
if $m_\chi = \mathcal{O}(m_{\mathrm{inflaton}})$

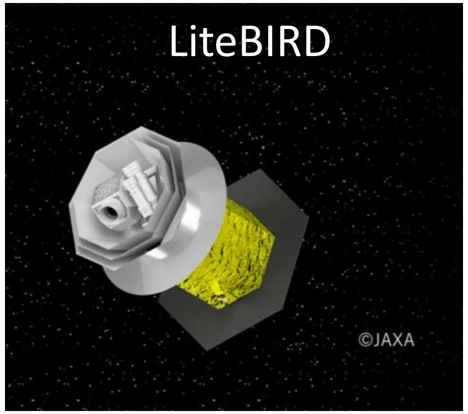
CGPP & Dark Matter

- Inflation indicates a new mass scale
- In most models, $m_{\text{inflaton}} \approx H_{\text{inflation}} \approx 10^{12} - 10^{14}$ GeV?
- $H_{\text{inflation}}$ potentially detectable via primordial gravitational waves in CMB

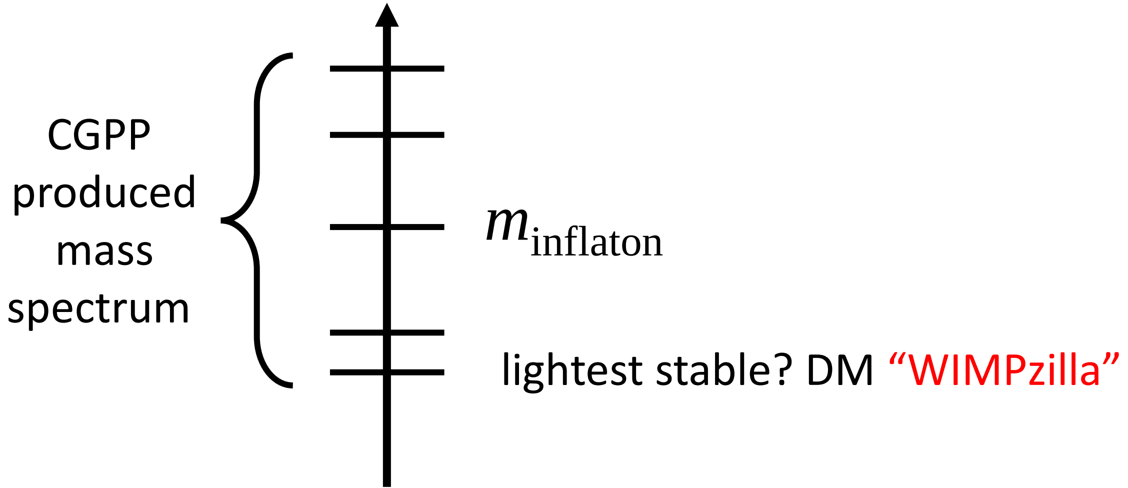
Battistelli et al. (2022)







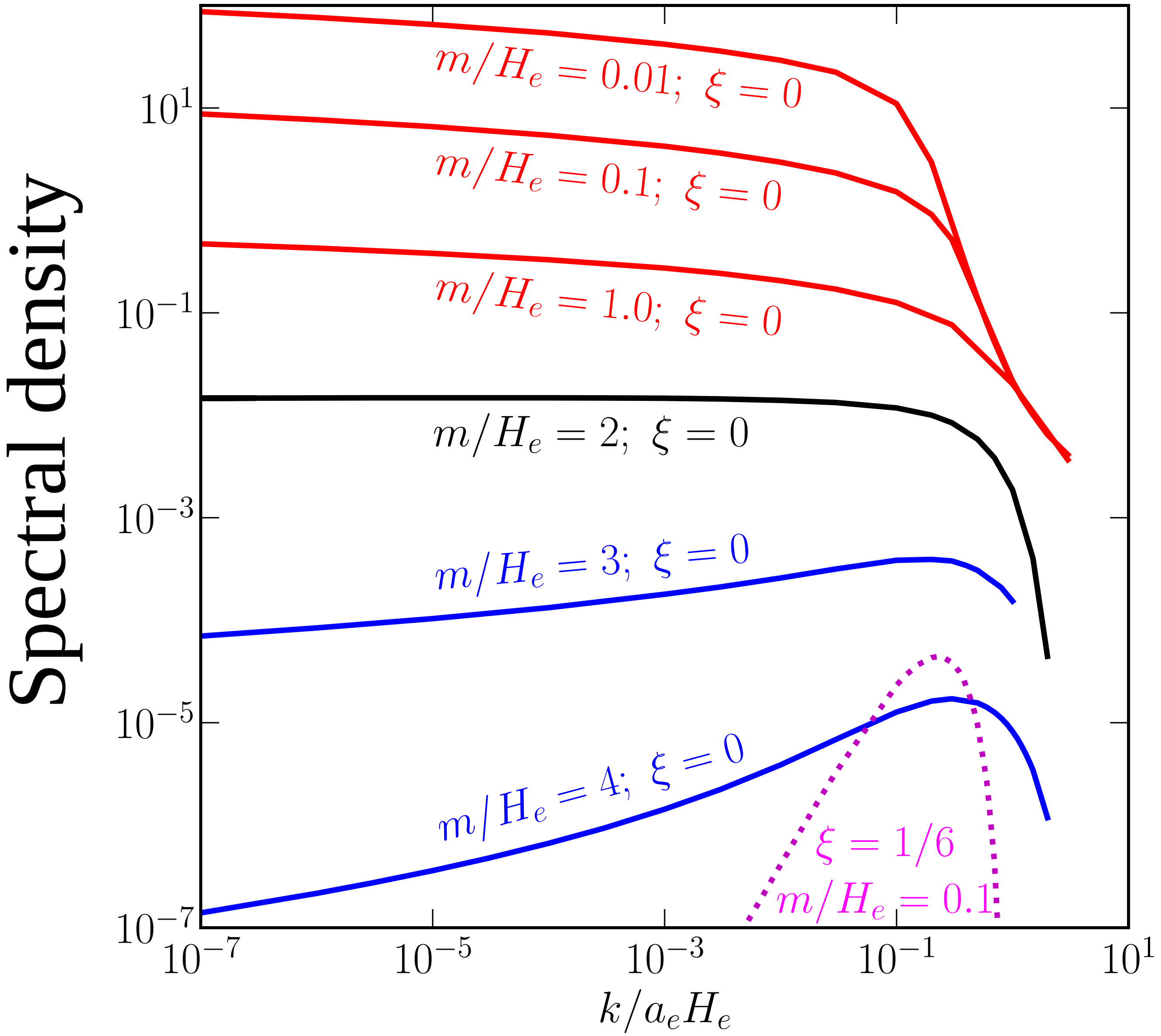
- Expect other particles with mass $\approx m_{\text{inflaton}}$



WIMPzilla™ is a very friendly, very massive* dark-matter candidate

* Very massive ⇒ too massive to be a cold thermal relic ($\gtrsim 200$ TeV)

Quadratic Inflaton Potential for Minimally-Coupled Scalar: $\xi = 0$



Red Spectrum leads to dangerous isocurvature fluctuations

Stable, minimally-coupled scalars are disallowed if $m \lesssim$ few H_e

The Red Spectrum Menace

- “Isocurvature” perturbations are perturbations in the composition of the mass-energy density.

- The perturbations in the radiation energy density set by quantum fluctuations in the inflaton field.

- If dark matter has a CGPP origin, perturbations in dark matter uncorrelated with perturbations in radiation.

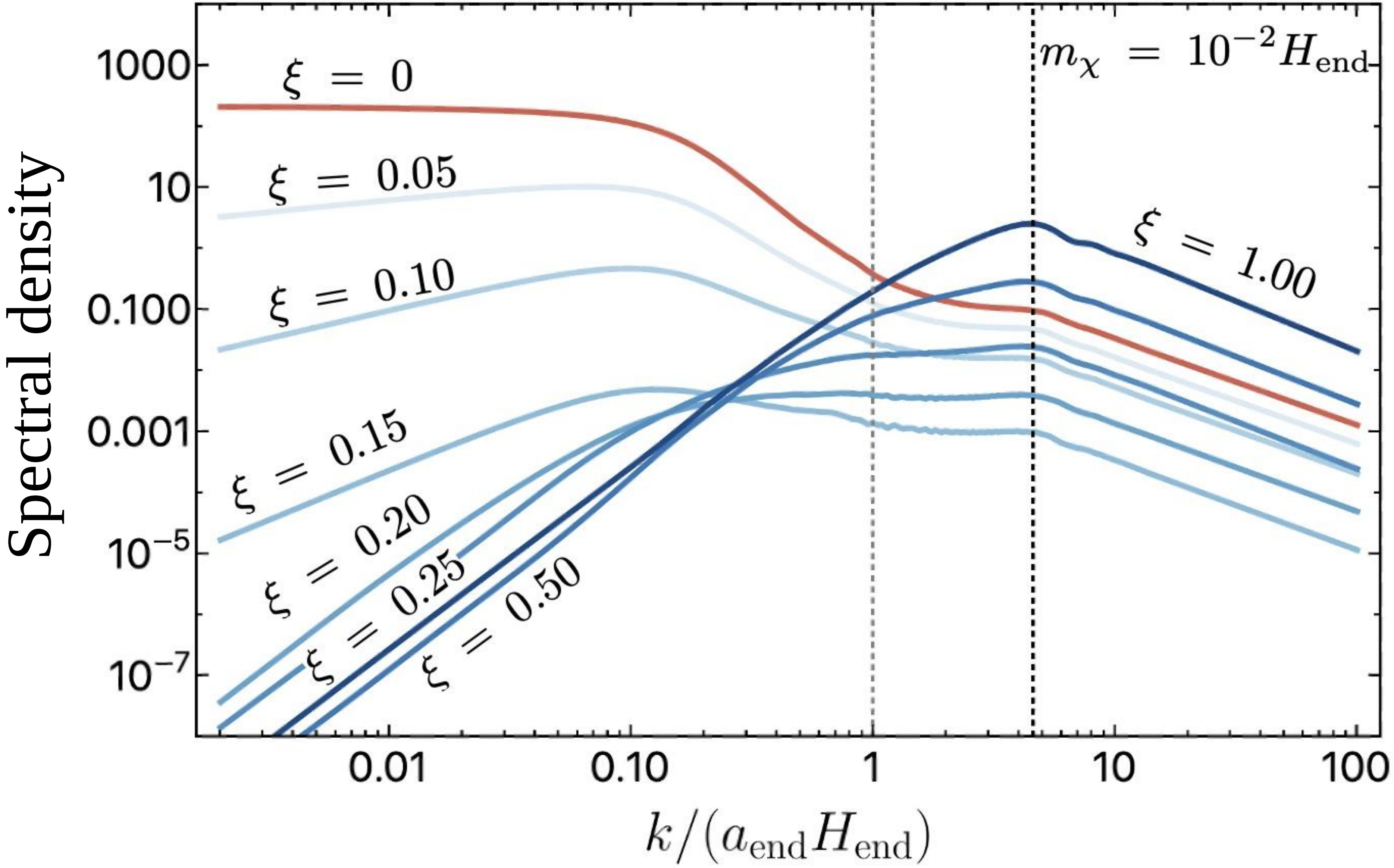
- CMB anisotropy measurements limit size of isocurvature contributions.

- If spectrum red (e.g., $\xi = 0$), isocurvature perturbations detectable on CMB scales.

- If spectrum blue (e.g., $\xi = 1/6$), safe from annoying observational constraints.

Chung, EWK, Riotto, Senatore (2005)

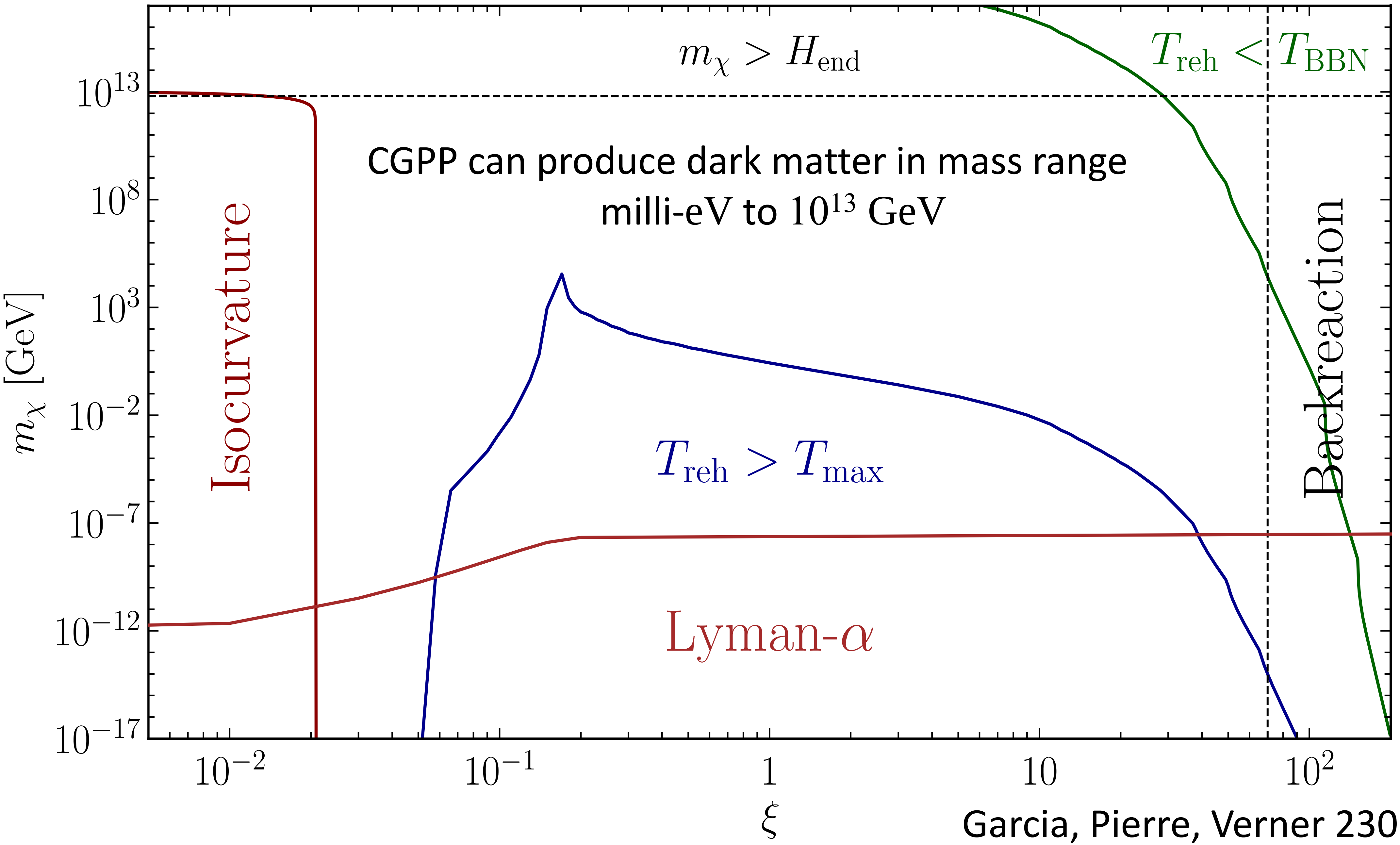
Model-T inflation (Kallosh & Linde): $V(\varphi) = 10^{-10} M_{\rm Pl}^4 \tanh^2(\varphi/\sqrt{6} M_{\rm Pl})$



Garcia, Pierre, Verner
2305.14446

also EWK, Long, McDonough, Payeur 2211.14323

Model-T inflation (Kallosh & Linde): $V(\varphi) = 10^{-10} M_{\rm Pl}^4 \tanh^2(\varphi/\sqrt{6} M_{\rm Pl})$



Garcia, Pierre, Verner 2305.14446

Dirac field ψ in FRW background

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\mathrm{GeV}}\right)^2\left(\frac{T_{\rm RH}}{10^9\mathrm{GeV}}\right)\ \frac{[na^3/\,a_e^3H_e^3]}{10^{-5}}$$

$$\sim \left(\frac{m}{10^{11}\mathrm{GeV}}\right)^2\left(\frac{T_{\rm RH}}{10^9\mathrm{GeV}}\right) \qquad (m \lesssim m_{\rm inflaton})$$

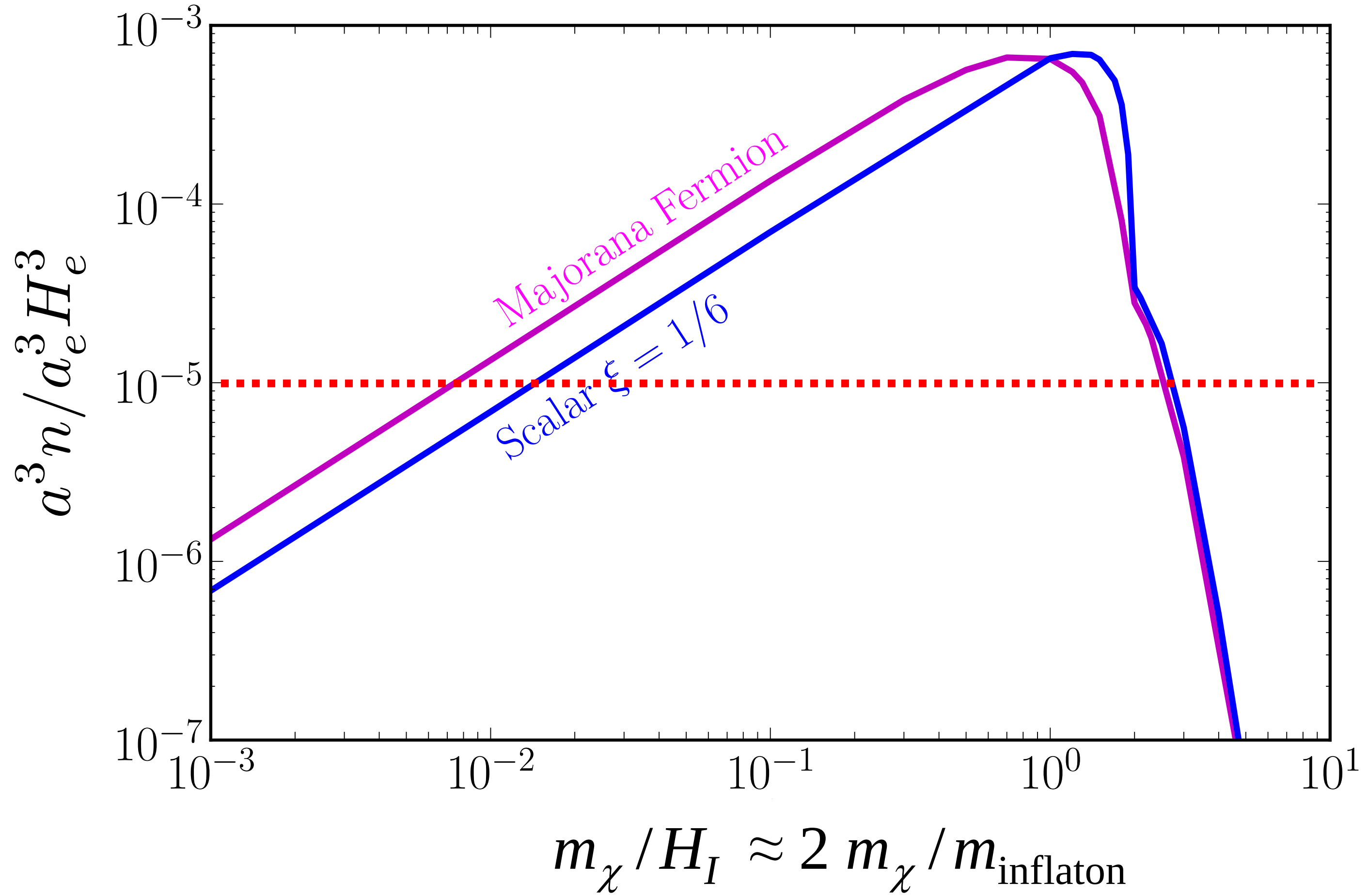
Dirac Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta)\end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m\end{pmatrix}\begin{pmatrix} u_A(\eta) \\ u_B(\eta)\end{pmatrix}$$

Dispersion relation same as conformally-coupled scalar

Blue spectrum: no isocurvature issues

Dirac WIMPZILLA DM candidate for $m = \mathcal{O}(m_{\rm inflaton})$



Fields with Spin > 1/2

For bosons, $\omega_k(\eta)$ tells all:

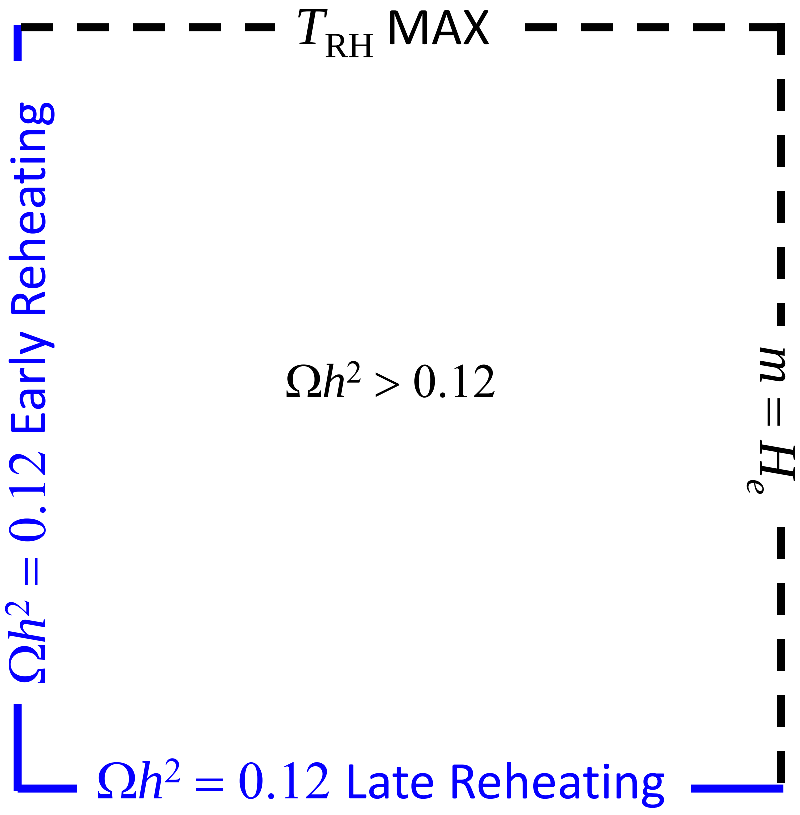
$$\omega_k^2(\eta) = \begin{cases} k^2 + a^2(\eta)m^2 + (\frac{1}{6} - \xi)a^2(\eta)R(\eta) & s=0 \\ k^2 + a^2(\eta)m^2 \quad \text{Like conformally-coupled scalar: in massless limit no production} & s=1 \quad \lambda=\pm 1 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}\frac{k^2 a^2(\eta)R(\eta)}{k^2 + a^2(\eta)m^2} + 3\frac{k^2 a^4(\eta)H^2(\eta)m^2}{(k^2 + a^2(\eta)m^2)^2} \quad \text{Interesting (i.e., complicated)} & s=1 \quad \lambda=0 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}a^2(\eta)R(\eta) \quad \text{Like minimally-coupled scalar; graviton in massless limit} & s=2 \quad \lambda=\pm 2 \\ k^2 + a^2(\eta)m^2 + \frac{1}{6}\frac{a^2(\eta)(2k^2 + a^2(\eta)m^2)R(\eta)}{k^2 + a^2(\eta)m^2} - \frac{a^2(\eta)k^2(2k^2 - a^2(\eta)m^2)H^2(\eta)}{(k^2 + a^2(\eta)m^2)^2} & s=2 \quad \lambda=\pm 1 \\ \text{way, way too long to show} & s=2 \quad \lambda=0 \end{cases}$$

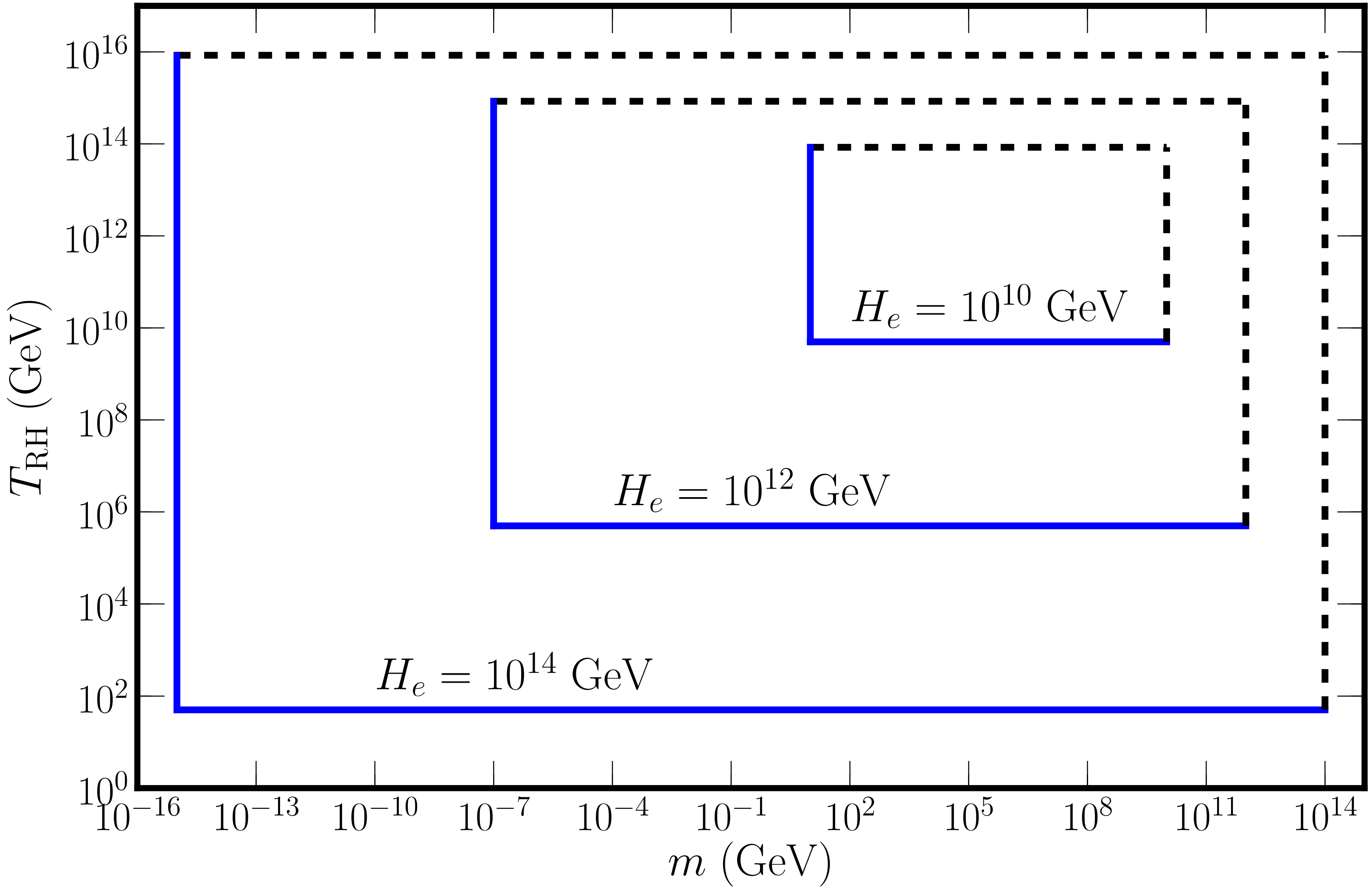
de Broglie—Proca field in FLRW background

$$S[A_\mu(x), g_{\mu\nu}(x)] = \int d^4x\,\sqrt{-g}\,\left[-\tfrac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + \tfrac{1}{2}m^2 g^{\mu\nu}A_\mu A_\nu - \tfrac{1}{2}\xi_1 R g^{\mu\nu}A_\mu A_\nu - \tfrac{1}{2}\xi_2 R^{\mu\nu}A_\mu A_\nu\right]$$

- Two possible nonminimal terms
- Transverse mode behaves like conformally-coupled scalar
- Longitudinal mode more complicated
- For some choices of (ξ_1, ξ_2) kinetic term can be negative leading to ghost-like action
- CGPP of longitudinal mode dominates transverse mode

de Broglie—Proca field in FLRW background





Early/Late:

$$T_{\rm RH} = 8.4 \times 10^8 \left(\frac{m}{\rm GeV}\right)^{1/2} \rm GeV$$

Very light (μeV) DM from GPP

or

Very massive (10^{14} GeV) DM from GPP

Graham, Mardon, & Rajendran

Ahmed, Grzadkowski, & Socha

EWK & Long

Rarita—Schwinger field ψ_μ in FRW background

EWK, Long, McDonough PRD **104**, 075015 (2021);
PRL **127** 13, 131603 (2021)

"Dirac" Equation in FRW:

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

$s = 3/2;\ \lambda = \pm 3/2$ (same as $s = 1/2$)

$$i\partial_\eta \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & (C_A + iC_B)k \\ (C_A - iC_B)k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(\eta) \\ u_B(\eta) \end{pmatrix}$$

nonzero for gravitino

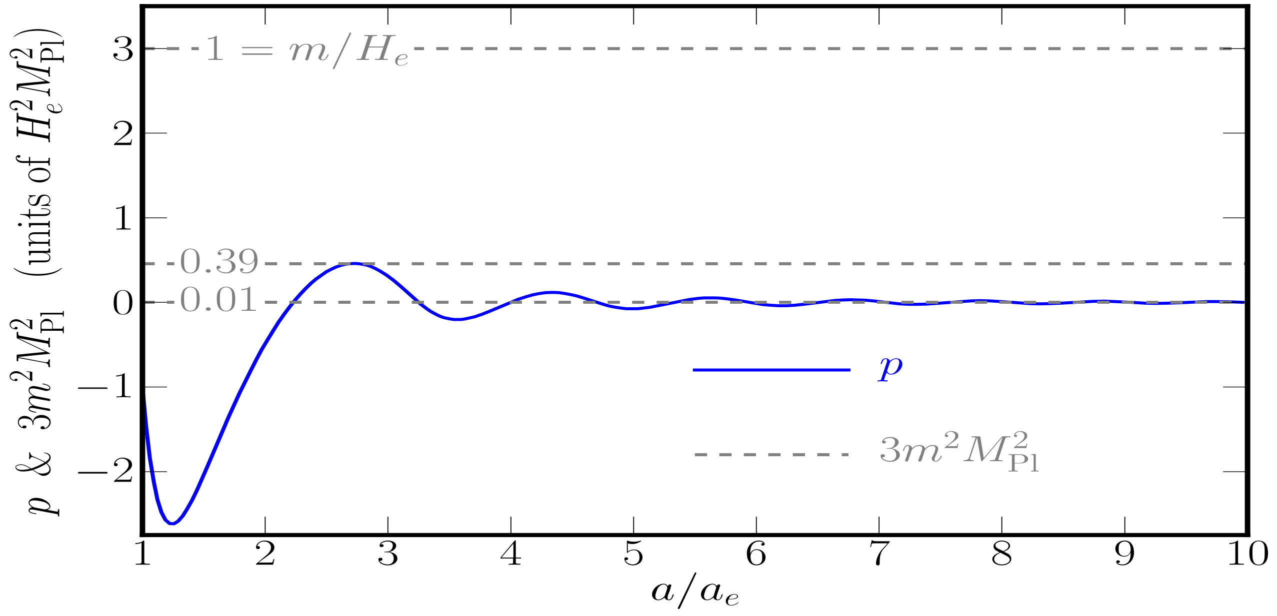
$s = 3/2;\ \lambda = \pm 1/2$
C_A & C_B functions of (H, m, R, $\partial_\eta m$)
$C_A^2 + C_B^2 = c_s^2 =$ sound speed

New feature: $c_s = \dfrac{\left|p(\eta) - 3m^2 M_{\mathrm{Pl}}^2\right|}{\rho(\eta) + 3m^2 M_{\mathrm{Pl}}^2}$ time-dependent effective sound speed!

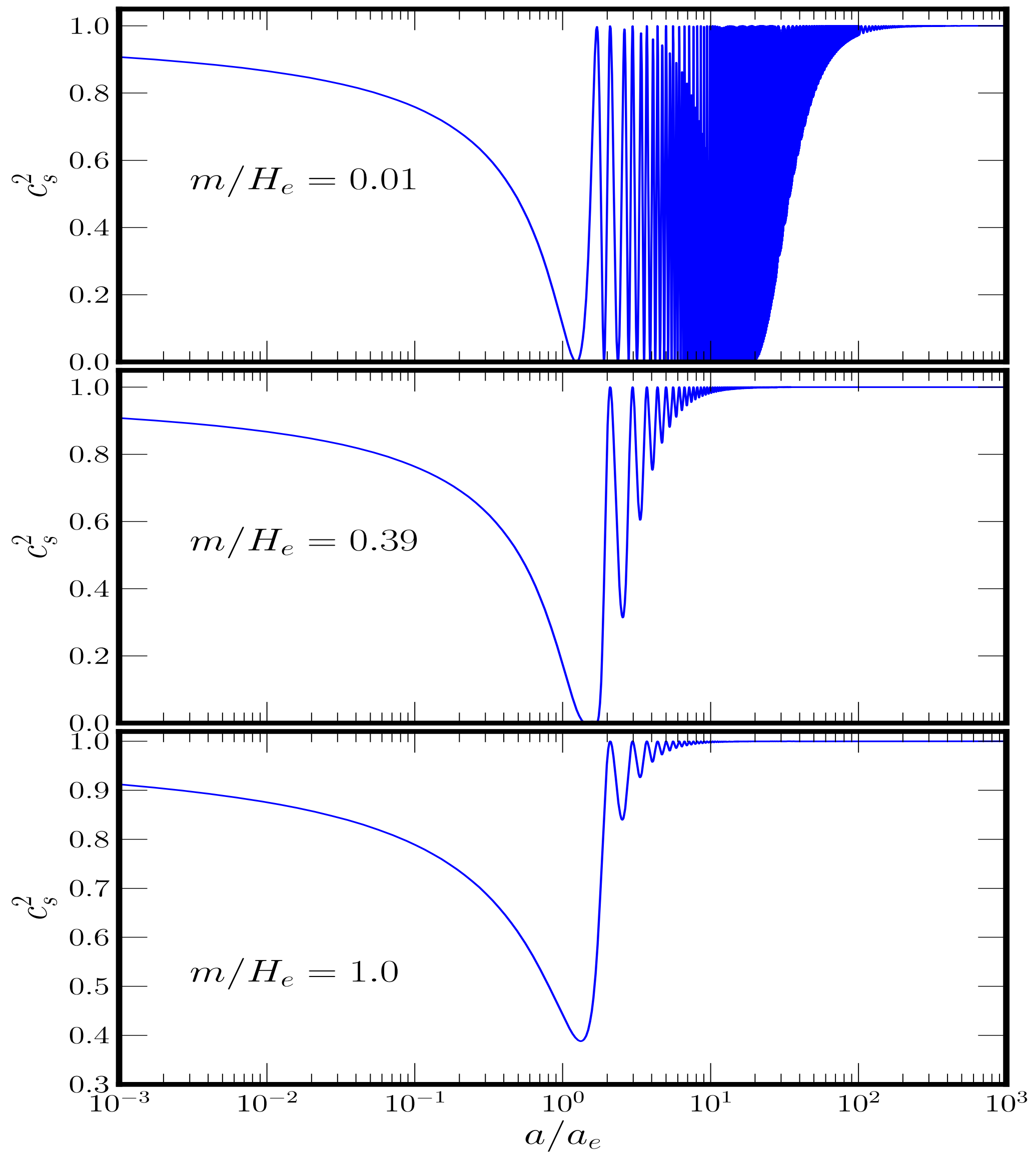
Can vanish when $p = 3m^2 M_{\mathrm{Pl}}^2$!!

Rarita—Schwinger field ψ_μ in FRW background

$$c_s = \frac{\left|p(\eta) - 3m^2 M_{\rm Pl}^2\right|}{\rho(\eta) + 3m^2 M_{\rm Pl}^2}$$



vanishing sound speed



Sound speed will vanish (perhaps many times) if $m < 0.39\ H_e$
(assumes harmonic potential after inflation)

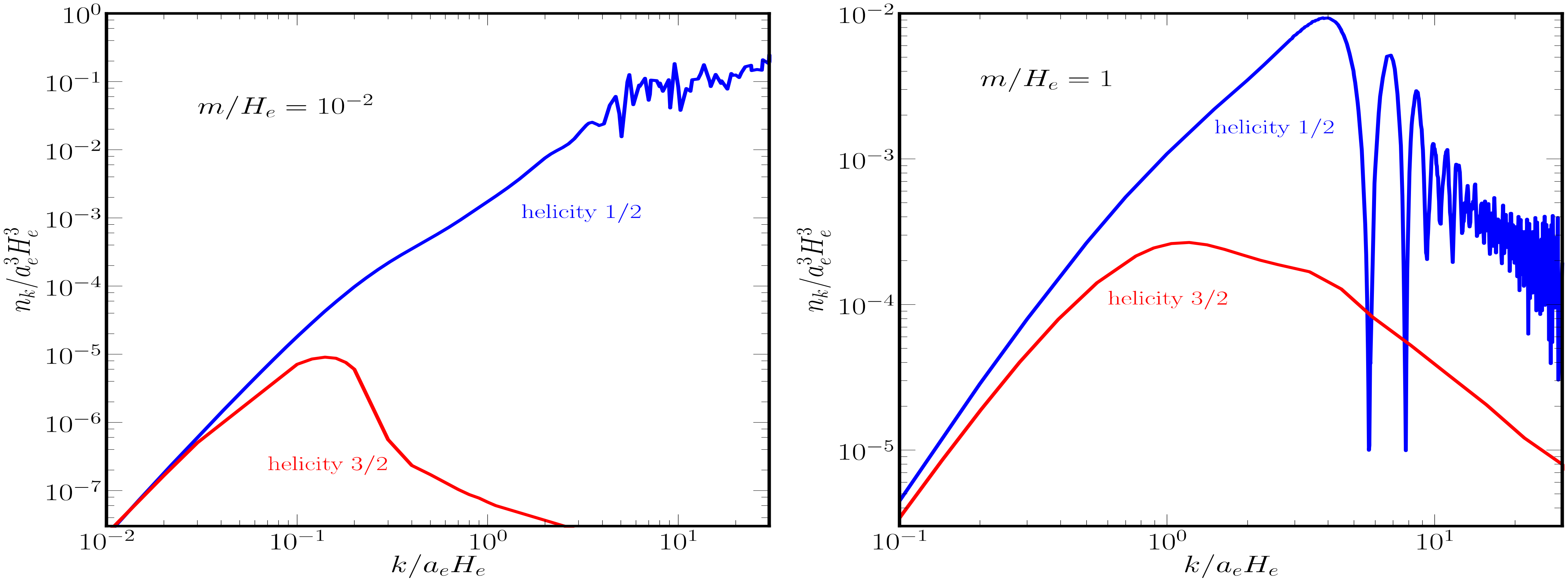
Rarita—Schwinger field ψ_μ in FRW background

Dispersion relation is $\omega_k^2(\eta) = c_s^2 k^2 + a^2(\eta) m^2$

Usual case: $c_s^2 = 1 \Rightarrow \omega_k(\eta) = k$ and <u>constant</u> for $k \Rightarrow \infty$

GPP depends on changing $\omega_k(\eta)$, so no production of high-k modes!

If $c_s^2 = 0$: as $k \Rightarrow \infty$, $\omega_k(\eta)$ is independent of k, production of high-k modes unsuppressed!



Rarita—Schwinger field ψ_μ in FRW background

Supergravity employs spin-3/2 field (gravitino, inflation, …), the superpartner to graviton.

Catastrophic production of gravitinos dependent on model.

For models with a single chiral superfield gravitino mass is time dependent ($\partial_\eta m \neq 0$).

$c_s = 1$ at all times $\Longrightarrow$ no catastrophic production

For models with multiple chiral superfields (most modern models)

c_s depends on relative orientation of inflaton direction & susy breaking

$c_s = 0$ in models with a nilpotent superfield and orthogonal constraint KKLT

mixing between the goldstino & inflatino may avoid the catastrophe (explicit calculation needed)

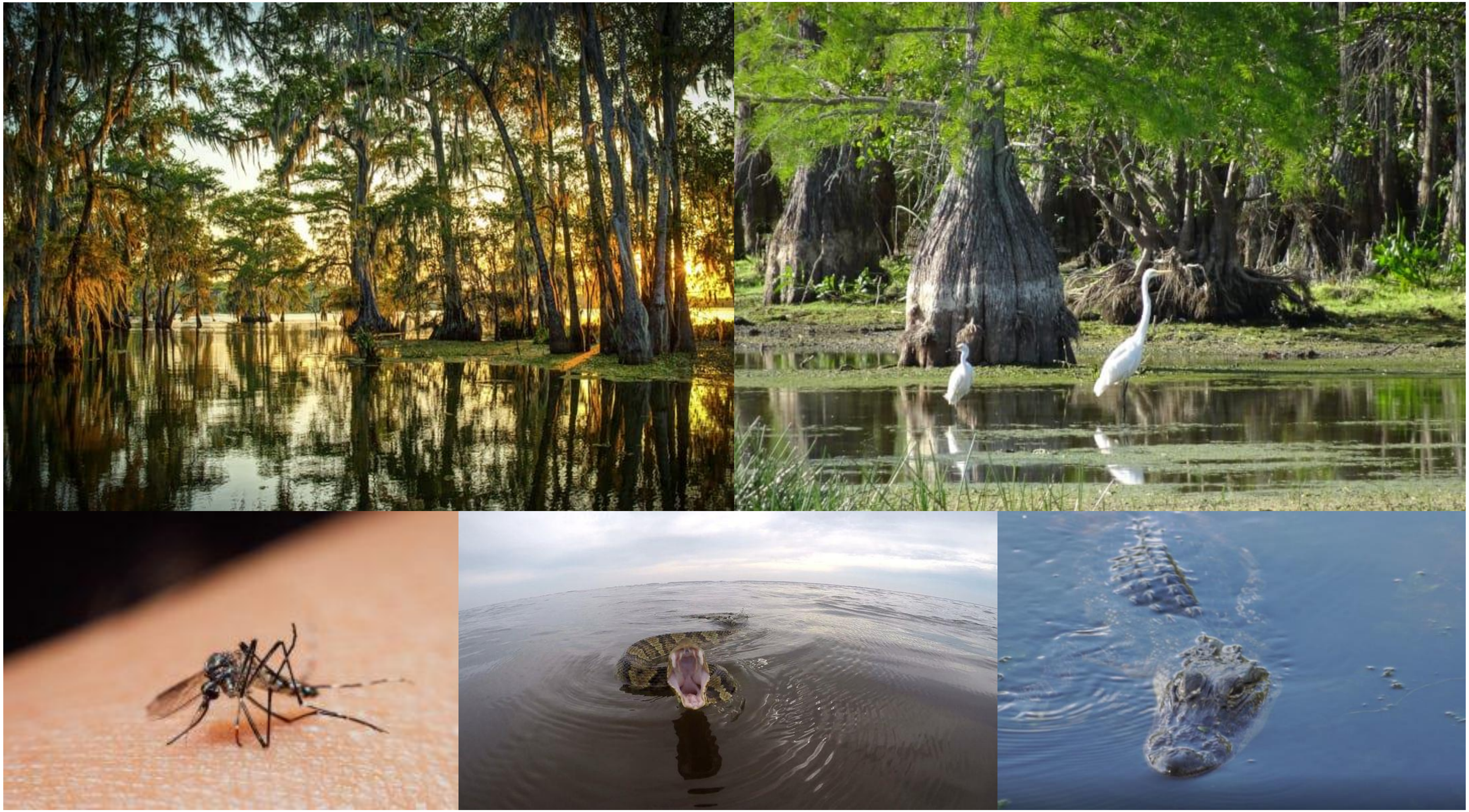
Dudas, Garcia, Mambrini, Olive, Peloso, & Verner (2021); Antoniadis, Benakli, & Ke (2021)

GGP may provide constraints on SUGRA model building.

Models with $c_s = 0$ are in a SWAMPLAND! Kolb, Long, & McDonough (2021)

https://louisianaswamp.com/

A swamp can be beautiful and teeming with life (that will sting, bite, or eat you)



Massive Spin-2 Fields

- EWK, Ling, Ling, Rosen, *JHEP* **05** (2023) 181 [2302.04390](2302.04390)

- Assume there is a massive spin-2 field in addition to the massless graviton.

- Construction of such a model is not straightforward because of ghosts (Andrew Long's talk).

- Massive spin-2 field at quadratic order has decoupled scalar, vector, and tensor components

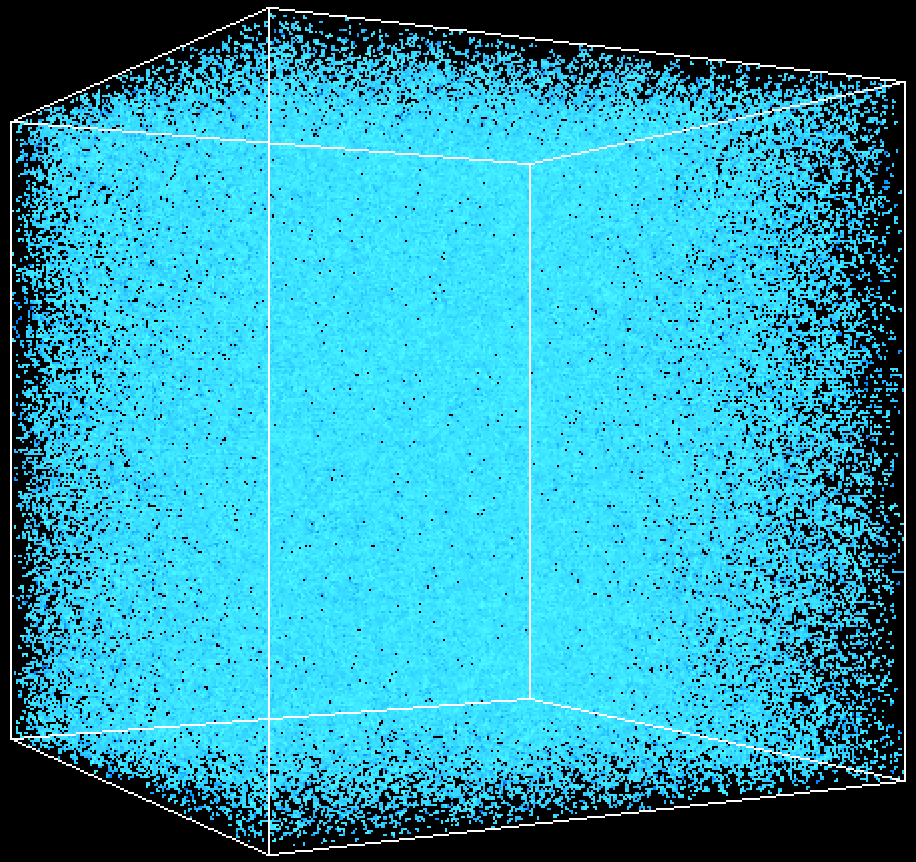
- Dispersion relation for Tensor component: $\omega_k^2(\eta) = k^2 + a^2(\eta)m^2 + \frac{1}{6}a^2(\eta)R(\eta)$

- If $m = 0$, mode equation for gravitational wave propagating on an FRW background

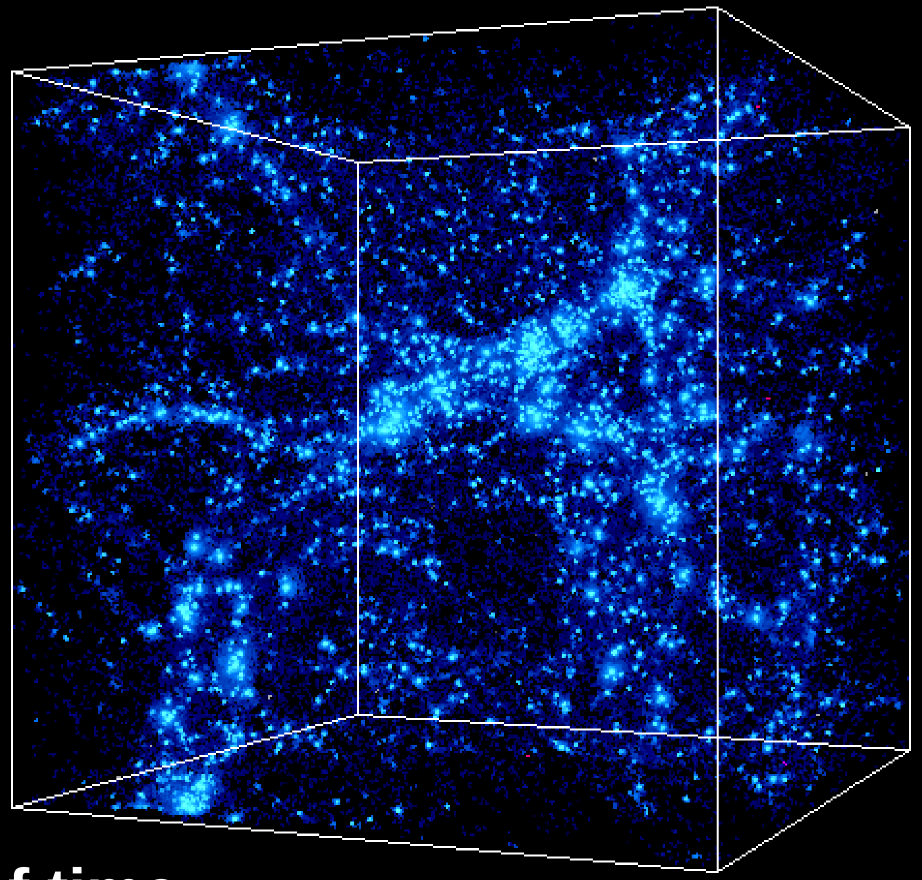
- CGPP also responsible for generation of curvature perturbations, also responsible for CMB curvature perturbations, which, in turn, are responsible for seeds of structure.

Primordial Seeds of Structure

Primordial Universe



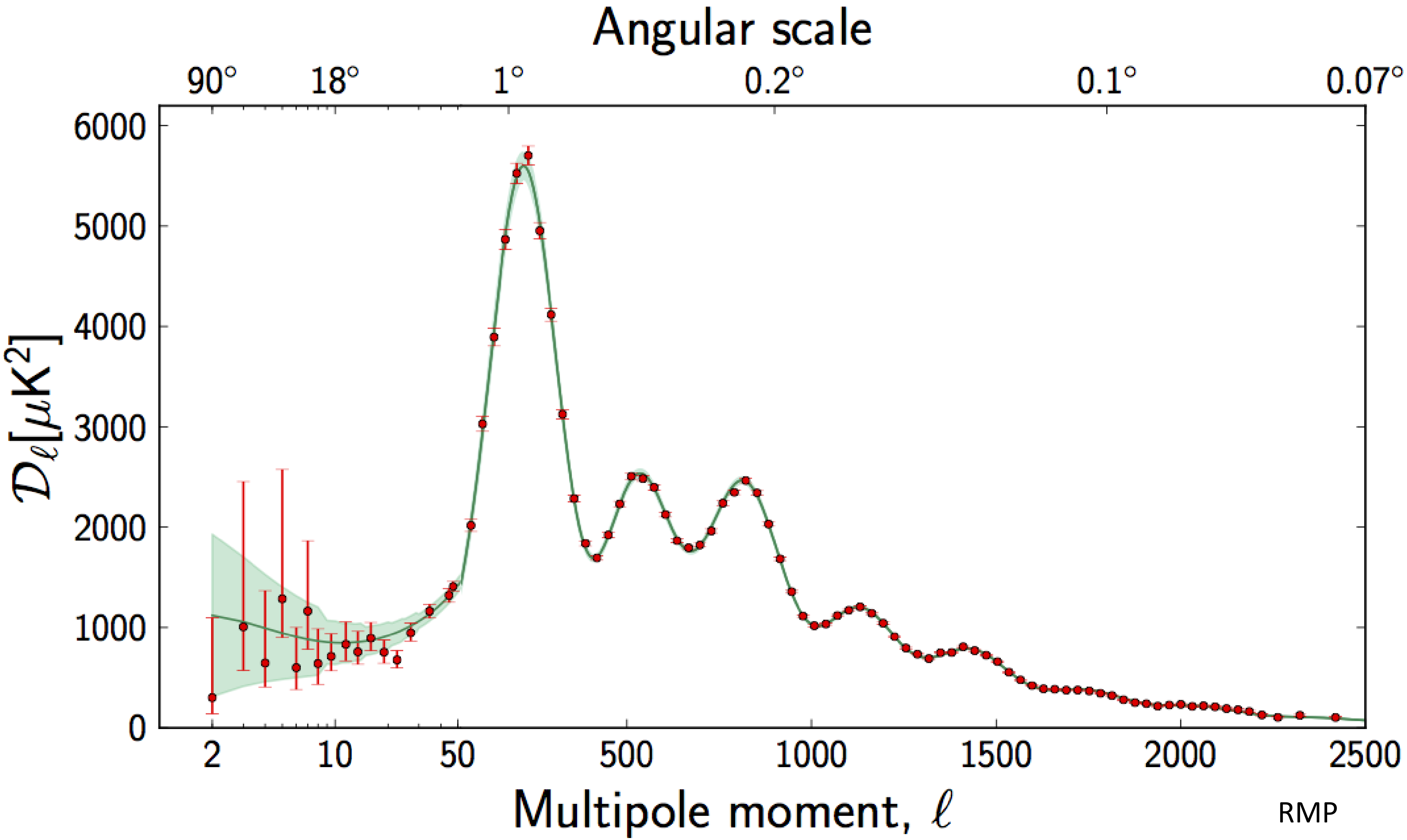
Today



Andrey Kravtsov, Chicago

If you can look into the seeds of time
And say which grain will grow and which will not,
Speak then to me, who neither beg nor fear
Your favours nor your hate. – ***Macbeth (Banquo)***

CMB Fluctuations



RMP

Inner Space/Outer Space Interface

Quantum processes
on a scale of 10^{-28} cm
are responsible for

cosmic structures
on a scale of 10^{+28} cm

The Quantum and the Cosmos!

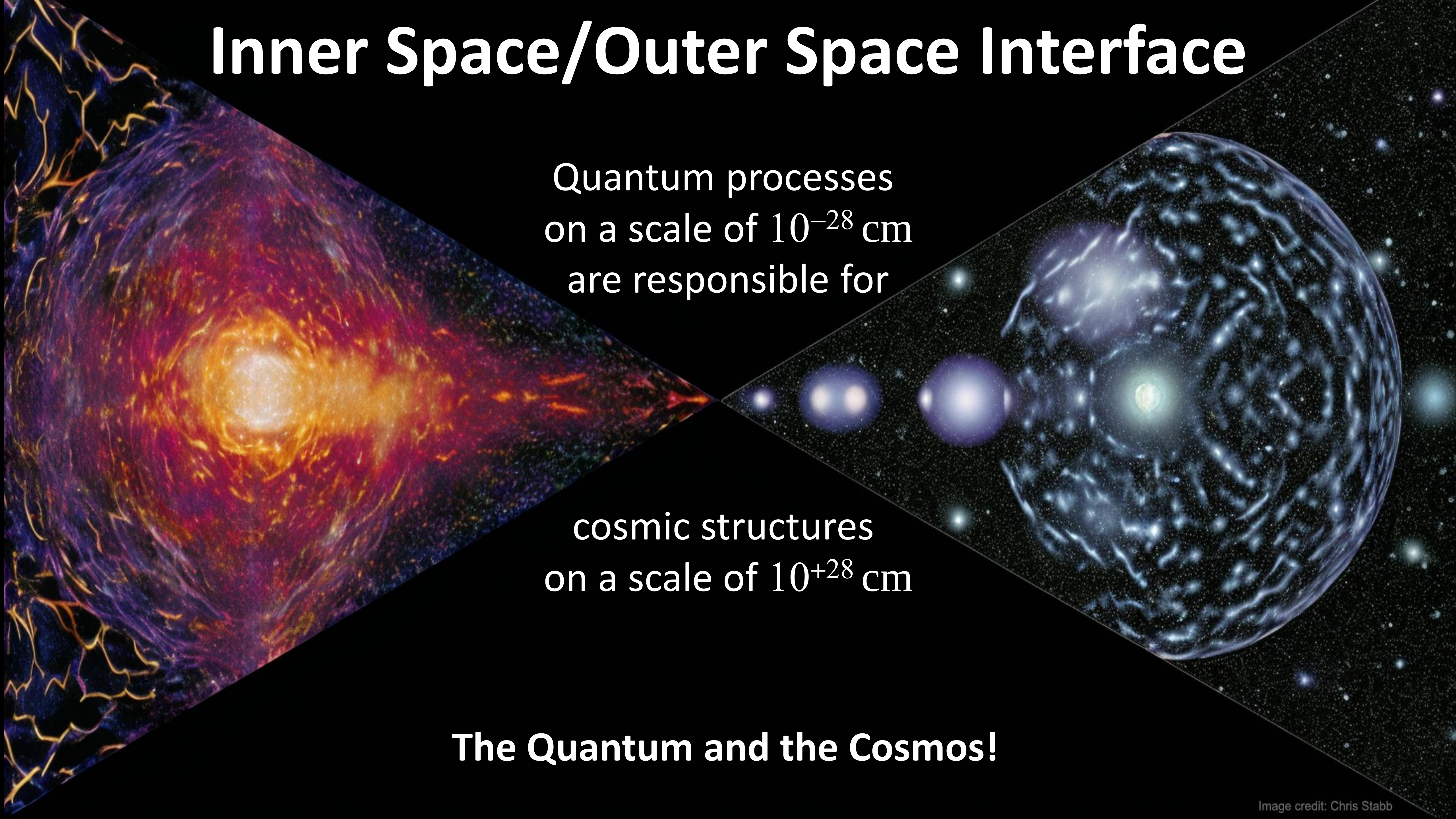
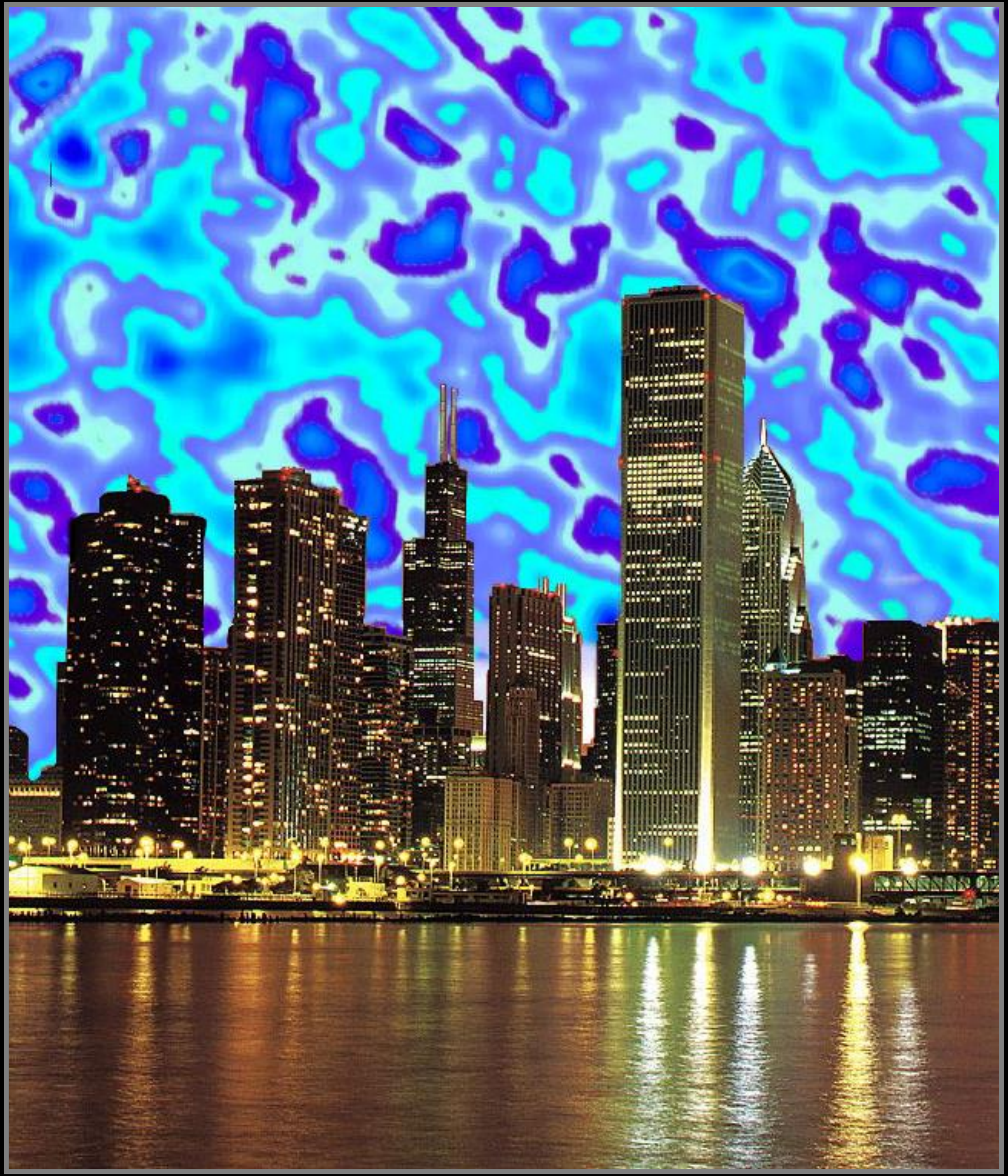


Image credit: Chris Stabb

A pattern
of vacuum
quantum
fluctuations



$$\hbar \to 0$$

Finally, Summary: CGPP can produce DM & constrain BSM physics!

Dark matter might have only gravitational interactions (that's all we really "know")

If so, dark matter must have a gravitational origin.

Cosmological Gravitational Particle Production promising.

Scalars:

Conformally-coupled: promising DM candidate if $m \approx H_e$ (WIMPZILLA miracle).

Minimally-coupled: not promising DM candidate, exclude stable particles with $m \lesssim$ few H_e.

If allow $2\times10^{-2}\ \rho\ \xi\ \rho\ 10^2$ DM candidate in mass range milli-eV to 10^{13} GeV.

Dirac fermions:

Like conformally-coupled scalars; promising DM candidate if $m \approx H_e$ (WIMPZILLA miracle).

de Broglie—Proca vectors:

DM candidate could be very light (μeV) or very massive (H_e).

Rarita-Schwinger fermions:

Catastrophic production if c_s vanishes. Implications for models of supergravity.

Gravitinos: EWK, Long, McDonough (2021); Dudas, Garcia, Mambrini, Olive, Peloso, Verner (2021)

Fierz-Pauli tensors:

FRW-generalization of the Higuchi bound; DM relic abundance.

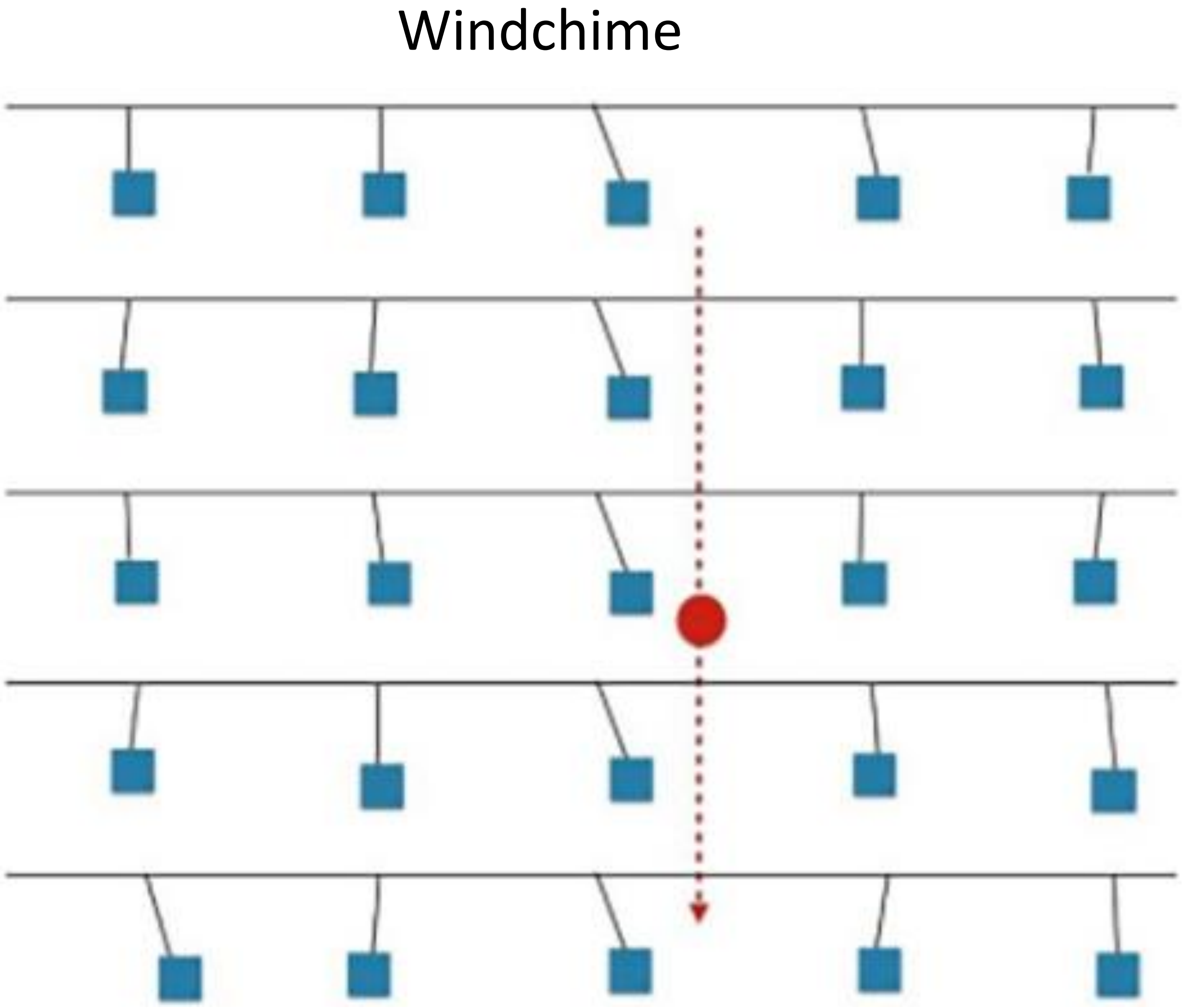
Spin greater than 2: Jenks, Koutrolikos, McDonough, Alexander, Gates

Much Recent Work … Many Open Roads

- Complete CGPP for higher-spin fields
- Fully explore Rarita-Schwinger = Gravitino
- Massive particles from K-K reduction in SUGRA/Strings
- Understand what it means to have ghosts
- Develop CMB implications
- Dark matter as Kalb-Ramond-Like-Particle (KRLP)? Leah Jenks' talk.
- Long-lived massive particles from CGPP
  - Baryo/leptogenesis?
  - Entropy generation?
  - ….
- Direct detection?

Windchime: Detect WIMPzillas with only gravitational coupling

"Gravitational Direct Detection of Dark Matter" Carney, Ghosh, Krnjaic, Taylor arXiv: 1903.00492



$$\mathrm{SNR}^2 = 10^4 \left(\frac{M_\chi}{1\ \mathrm{mg}}\right)^2 \left(\frac{M_D}{1\ \mathrm{mg}}\right)^2 \left(\frac{1\ \mathrm{mm}}{d}\right)^4$$

Meter-scale detector

Billion microgram to milligram sensors

Lattice spacing millimeter to centimeter

Detect DM of mass greater than Planck mass

How about 10^{-6} Planck mass?

Coming soon-ish, to a *Reviews of Modern Physics* Near You

Cosmological gravitational particle production and its implications for cosmological relics

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The University of Chicago,
5640 S. Ellis Ave., Chicago,
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[2] *Department of Physics and Astronomy,*
Rice University, Houston,
Texas 77005 USA

The focus of this review is the phenomenon of particle production in the early universe solely by the expansion of the universe, with particular attention to the possibility that the created particle species could be the dark matter. We will treat particle production by cosmological expansion for particles of spin 0, 1/2, 1, 3/2, and 2, and comment on the possibility of larger spins. For the early-universe evolution of the background spacetime we assume an initial inflationary phase, followed by a transition to a matter-dominated phase, eventually transiting to a radiation-dominated phase. We review the two basic requirements for particle production by the expansion of the universe: 1) the contribution to the matter action from the particle must violate conformal invariance (the trace of the matter stress-energy tensor involving the new field must be nonzero), and 2) the mass of the particle must not be too much in excess of the expansion rate of the universe during inflation. In this review we specialize to a Friedman-Lemaître-Robertson-Walker cosmological model, and calculate the spectrum of particles resulting from the expansion of the universe. We summarize the criteria for the resulting density of particles to be sufficient to account for the dark matter, as well as discuss several other cosmological implications. We then mention other mechanisms for cosmological particle production through gravity: particle production from the standard-model plasma through graviton exchange, particle production through black-hole evaporation, and particle production through a misalignment mechanism.

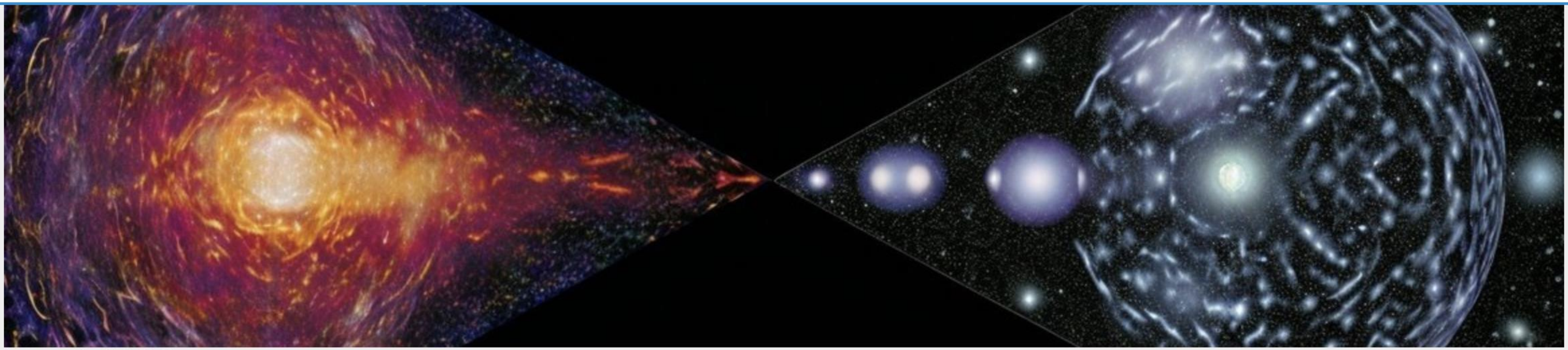

Image credits: Chris Stabb

QUANTUM ASPECTS OF INFLATIONARY COSMOLOGY

24 June - 19 July 2024

Andrew Long, Edward (Rocky) Kolb, Jun´ichi Yokoyama, Rachel Rosen, Viatcheslav (Slava) Mukhanov

QUANTUM ASPECTS OF INFLATIONARY COSMOLOGY

Overview

Participants

Schedule

Room reservation

Astrophysical and cosmological observations have revealed a wealth of information about the structure, composition, and evolution of the Universe. Although we can classify the ingredients that compose the Universe today, we don't yet know their origin. Their genesis must have been the early stages of the big bang and involved particle physics beyond the standard model. This MIAPbP program is centered around topics that sit at the connection between particle physics and cosmology:

1) cosmological inflation,

2) the end of inflation,

3) cosmological relics, and

4) gravitational particle production.

How did quantum fluctuations of the inflaton field provide the seeds for structure on cosmological scales? Did other fields play a role during inflation? How did their quantum fluctuations imprint on cosmological observables

Thanks to my collaborators in 25 years of CGPP: (Chung, EWK, Riotto, *PRD* **59** (1998) 023501)

Ivone Albuquerque, Edward Basso, Christian Capanelli, Daniel Chung, Patrick Crotty, Michael Fedderke, Gian Giudice, Lam Hui, Leah Jenks, Siyang Ling, Andrew Long, Evan McDonough, Toni Riotto, Rachel Rosen, Leo Senatore, Alexi Starobinski, Igor Tkachev, Mark Wyman

Schrödinger's "Alarming" Phenomenon of Particle Creation in the Expanding Universe

Cosmological Gravitational Particle Production (CGPP)

The expansion of the universe creates particles from the vacuum

Rocky Kolb
University of Chicago

October 2023
University of Florida

Metric Perturbations About Minkowski Spacetime

Start with EH action: $S[g_{\mu\nu}] = \int d^4x \sqrt{-g}\; \frac{M_P^2}{2}\, R[g]$

Linearize about Minkowski spacetime: $g_{\mu\nu} \to \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}$ $\qquad h = \eta^{\mu\nu} h_{\mu\nu}$

$$S[h_{\mu\nu}] = \int d^4x \left[-\tfrac{1}{2}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \nabla_\mu h^{\nu\lambda}\nabla_\nu h^\mu{}_\lambda - \nabla_\mu h^{\mu\nu}\nabla_\nu h + \tfrac{1}{2}\nabla_\mu h \nabla^\mu h\right]$$

$$\delta S[h_{\mu\nu}] = \int d^4x \left[-\tfrac{1}{2} m^2 \left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

Fierz-Pauli mass term

Boulware—Deser Ghost

Boulware and Deser (1972) pointed out that Fierz-Pauli tuning breaks down with generic nonlinear extensions of Fierz-Pauli, and a sixth ghostly degree of freedom arises (zombie ghost?).

Once thought that all Lorentz-invariant massive gravity theories were ghostly, until …

… de Rahm-Gabadadze-Tolley (dRGT) developed a ghost-free massive gravity theory in 2010.

dRGT introduced second "reference" metric, taken to be Minkowski. Metrics interact via potential $V(\mathbb{X};\beta_n)$.

Extended/completed to general metric by Hassan & Rosen → ghost-free bigravity (2011).

This is our starting point.

$$S=\int \mathrm{d}^4x\left[\frac{M_g^2}{2}\sqrt{-g}\,R[g]+\frac{M_f^2}{2}\sqrt{-f}\,R[f]-m^2M_*^2\sqrt{-g}\,V(\mathbb{X};\beta_n)+\sqrt{-g}\,\mathcal{L}_g(g,\phi_g)+\sqrt{-f}\,\mathcal{L}_f(f,\phi_f)\right]$$

Kinetic terms for f and g + dRGT potential + Matter Lagrangians