Electroweak Topology and its Cosmological Implications

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Electroweak symmetry breaking

$$\Phi \equiv \langle \hat{\Phi} \rangle = v \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Electroweak phase: $\Phi = 0$

Maxwell phase: $\Phi = v \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = 1$$

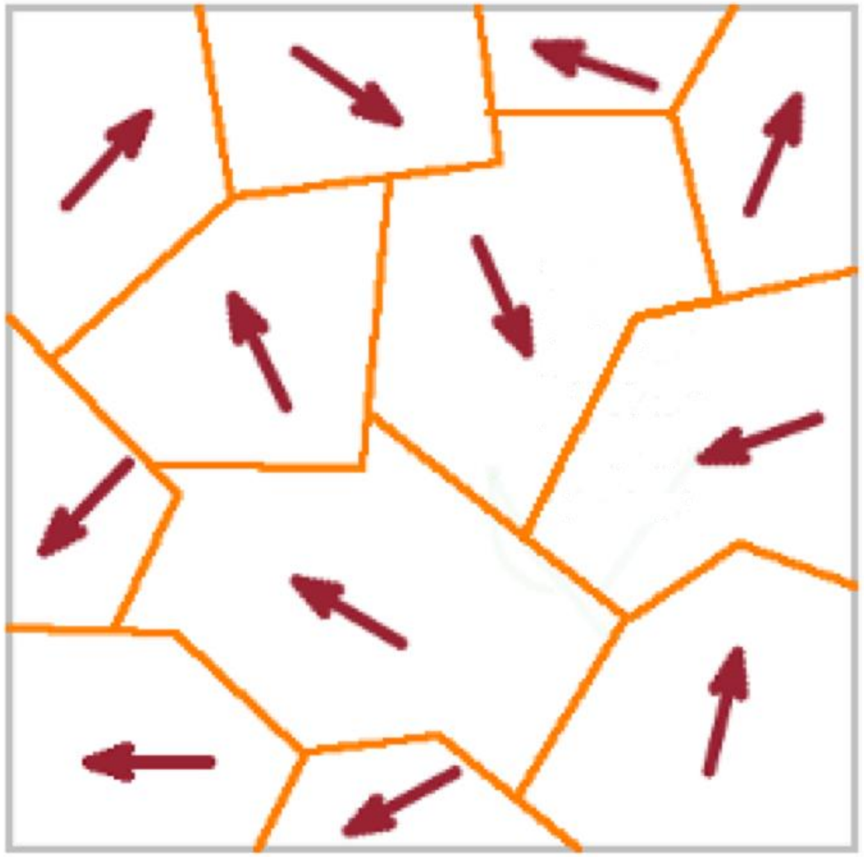
(S^3)

During electroweak symmetry breaking, **Φ** acquires random values consistent with the constraints. I.e. **Φ** can lie at any point on a three-sphere.

Kibble mechanism

Kibble, 1976

During electroweak symmetry breaking, **Φ** acquires independent random values on a three-sphere in well-separated spatial regions.



Basis for topological defect formation in cosmology and condensed matter systems.

E.g. He-3 with spectacular success.

Ruutu et al, 1995

What are the consequences of the Kibble mechanism in electroweak theory?

Embed Maxwell in Electroweak

Maxwell gauge field, A, is a linear combination of the W and Y gauge fields.

$$A_\mu = \sin\theta_w \hat{n}^a W^a_\mu + \cos\theta_w Y_\mu \qquad \hat{n}^a \equiv -\frac{\Phi^\dagger \sigma^a \Phi}{\Phi^\dagger \Phi}$$

The naive field strength is $A_{\mu\nu} = \sin\theta_w \hat{n}^a W^a_{\mu\nu} + \cos\theta_w Y_{\mu\nu}$

but the non-Abelian gauge field strength is quadratic in the gauge fields and, even in unitary gauge, the usual linear relation between field strength and gauge field is broken.

’t Hooft, 1974

Instead, define the electromagnetic field strength in a gauge invariant way and by requiring the Maxwellian relation in unitary gauge,

$$\begin{aligned} A_{\mu\nu} &= \sin\theta_w \hat{n}^a W^a_{\mu\nu} + \cos\theta_w Y_{\mu\nu} - i\frac{2\sin\theta_w}{g\eta^2}\left(D_\mu\Phi^\dagger D_\nu\Phi - D_\nu\Phi^\dagger D_\mu\Phi\right) \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g\eta^2}\left(\partial_\mu\Phi^\dagger\partial_\nu\Phi - \partial_\nu\Phi^\dagger\partial_\mu\Phi\right) \end{aligned}$$

TV, 1991

E&M from Electroweak

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g\eta^2}\left(\partial_\mu\Phi^\dagger\partial_\nu\Phi - \partial_\nu\Phi^\dagger\partial_\mu\Phi\right)$$

Now the magnetic field has an extra term, $\mathbf{B} = \nabla\times\mathbf{A} - i\frac{2\sin\theta_w}{gv^2}\nabla\Phi^\dagger\times\nabla\Phi$

Even if A=0, the magnetic field need not vanish.

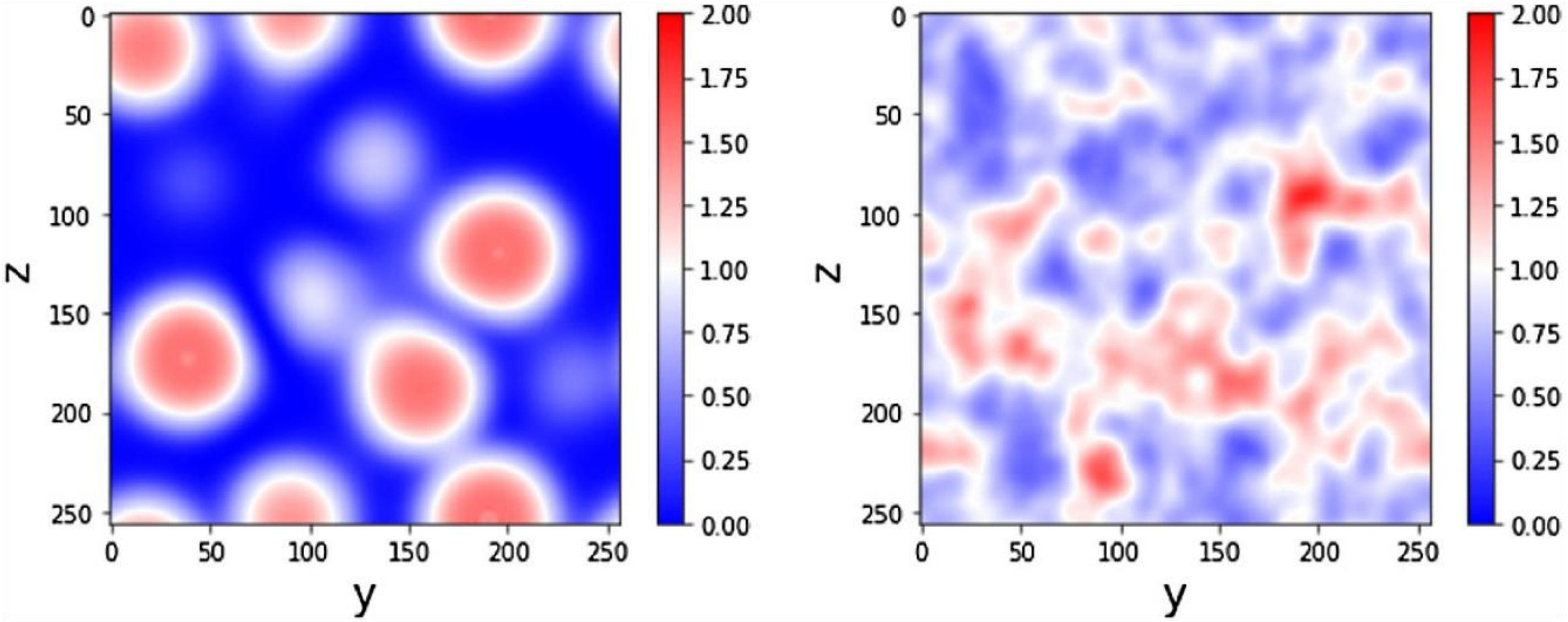
Random distributions of **Φ** can lead to a stochastic magnetic field.

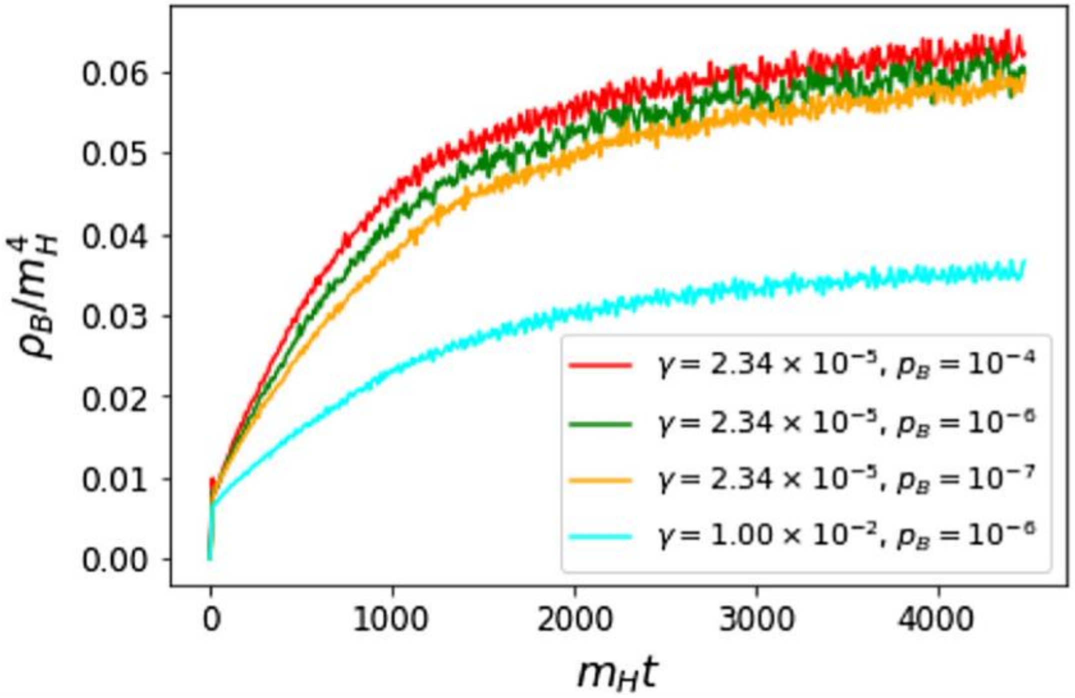
Direct simulations of EWSB

Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo
Mou, Saffin & Tranberg

(bubbles of **Φ** perturbations)

Zhang, Ferrer & TV



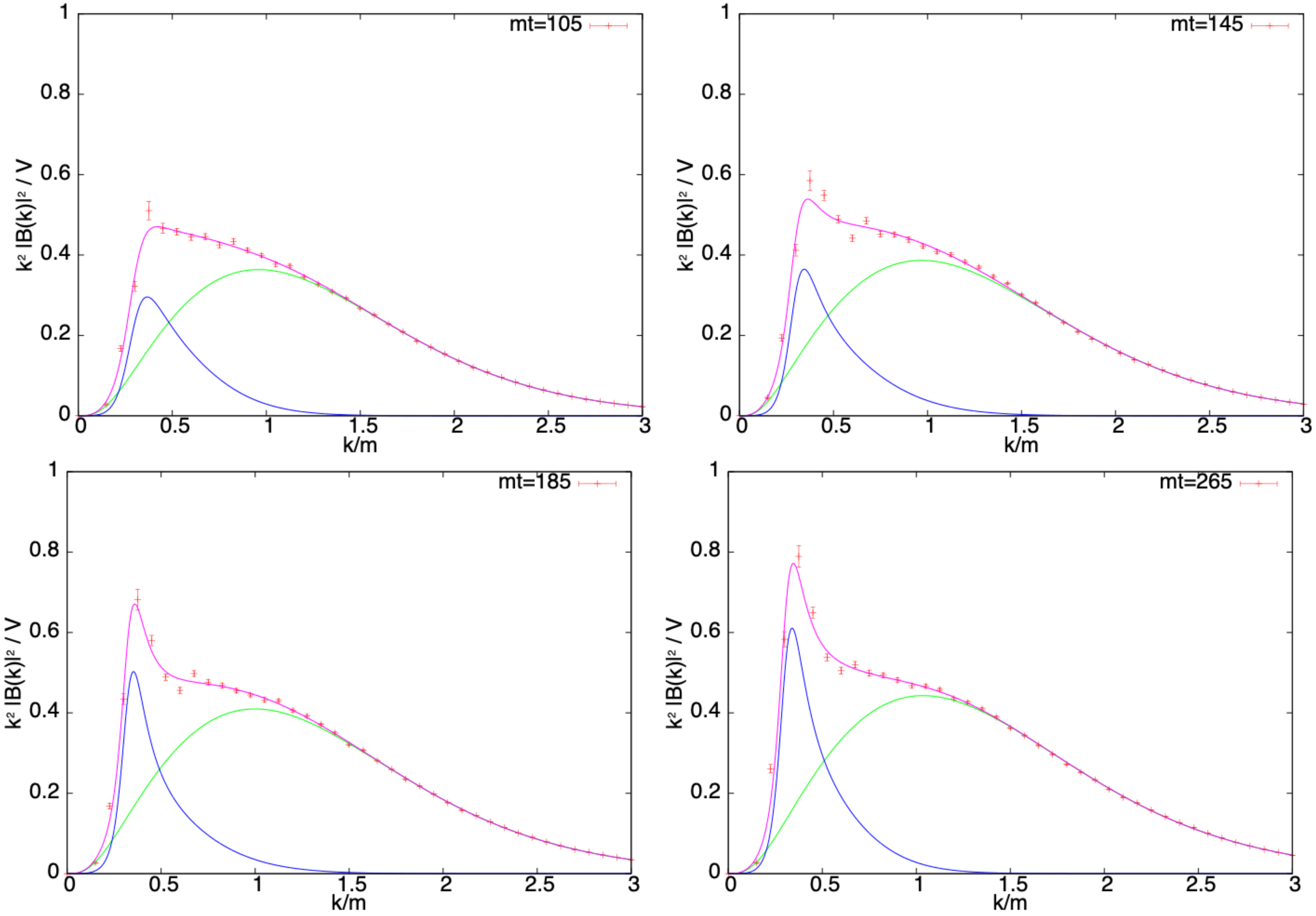


$$\left.\frac{\rho_B}{\bar{\rho}}\right|_{t_f} \approx 6\%$$

(bosonic degrees of freedom only)

B Spectrum

Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo



Pink=total

Green=thermal

Blue=total-thermal

Peak at small k.

Similar results from other simulations.

Zhang, Ferrer & TV, 2019

Analytical estimate

$$\mathbf{B} = \nabla \times \mathbf{A} - i\frac{2\sin\theta_w}{g}\nabla\hat{\Phi}^\dagger \times \nabla\hat{\Phi}$$

Contribution to volume-averaged magnetic field:

TV, 2021

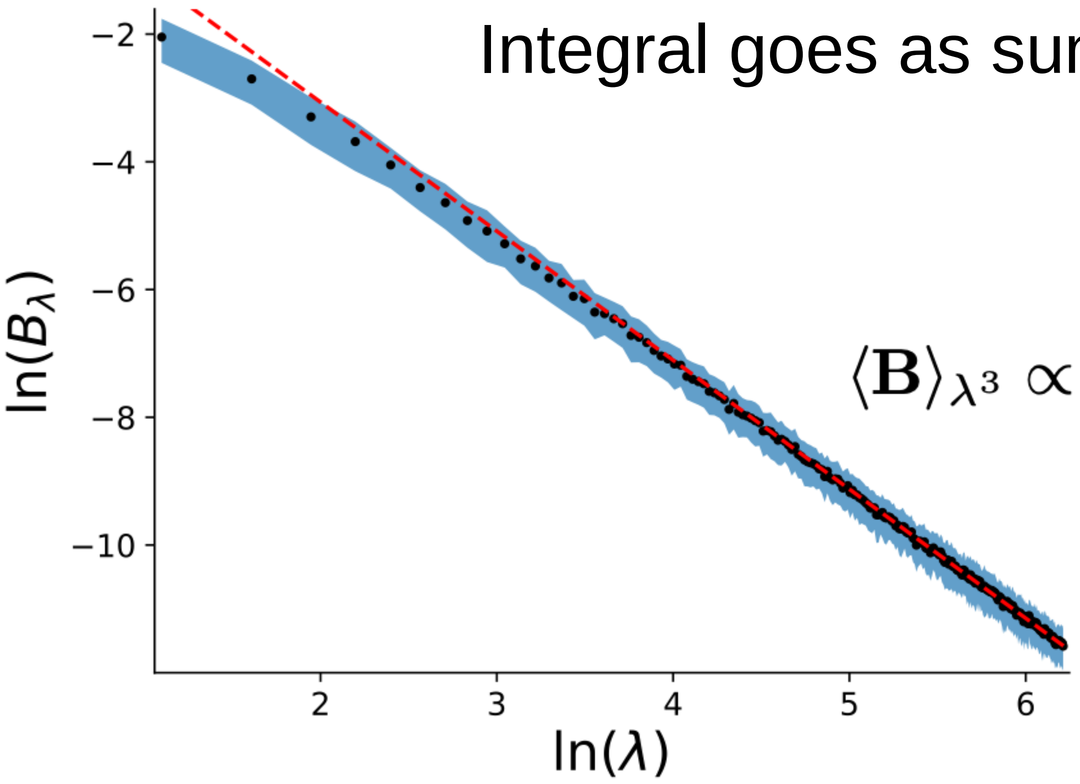
area~$\boldsymbol{\lambda}^2$

$$\langle\mathbf{B}\rangle_V = \frac{1}{V}\int_V d^3x\,\mathbf{B} = -i\frac{2\sin\theta_w}{gV}\int_{\partial V} d\mathbf{S} \times (\hat{\Phi}^\dagger\nabla\hat{\Phi})$$

volume~$\boldsymbol{\lambda}^3$

random on scale L

Integral goes as sum of ($\boldsymbol{\lambda}$/L)^2 random terms.



$$\langle\mathbf{B}\rangle_{\lambda^3} \propto \frac{1}{\lambda^2}$$ or, $$\mathbf{B}(k) \propto k^2$$

Correlator

Correlation function for **any** isotropic, homogeneous, divergenceless vector field:

Monin & Yaglom, 1975

$$\langle b_i^*(\mathbf{k})b_j(\mathbf{k}')\rangle = \left[\frac{E_M(k)}{4\pi k^2}(\delta_{ij} - \hat{k}^i\hat{k}^j) + i\epsilon_{ijl}k^l\frac{H(k)}{8\pi k^2}\right](2\pi)^6\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

$$\langle \mathbf{B}\rangle_{\lambda^3} \propto \frac{1}{\lambda^2}$$ or, $$\mathbf{B}(k) \propto k^2$$

$$\text{magnetic helicity} = \int d^3x\,\mathbf{A}\cdot\mathbf{B}$$

$$E_M(k) \sim \frac{4\rho_{EW,B}}{k_*}\left(\frac{k}{k_*}\right)^3 \qquad H(k) \sim 0$$

Evolution simulations draw initial conditions assuming Gaussian distribution for Fourier amplitudes and uniformly distributed Fourier phases.

Summary: Magnetized Universe

Fractional cosmic energy density in magnetic fields:

$$\Omega_B(t_{EW}) \sim 1\%$$

(also by counting degrees of freedom)

with spectrum:

$$B_\lambda(t_{\rm EW}) \sim 2\sqrt{\rho_{EW,B}} \left(\frac{k}{k_*}\right)^2, \quad k \le k_*$$

$$E_M(k) \sim \frac{4\,\rho_{EW,B}}{k_*} \left(\frac{k}{k_*}\right)^3, \quad k \le k_*$$

What is k_*?

k_*

Simulations suggest some large coherence scale but are limited by dynamic range.

Instead first try to understand the source of the magnetic field after EWSB.

Recall: $$\mathbf{B} = \nabla \times \mathbf{A} - i\frac{2\sin\theta_w}{g}\nabla\hat{\Phi}^\dagger \times \nabla\hat{\Phi}$$

In a local patch, the first term can cancel the second term to give zero magnetic field.

However, if the second term has non-zero divergence it cannot be compensated by the first term and there is a topological obstruction in going to unitary gauge, i.e. magnetic monopoles.

Electroweak monopoles

Nambu, 1977

There are several ways to see the existence of electroweak monopoles.

First is that to minimize energy for a given spatial dependence of the Higgs, the gradient energy has to be minimized: $D_\mu \Phi \sim 0$

For the gauge fields to compensate gradients of the Higgs, the Higgs must lie on gauge orbits. The gradient terms fiber the three-sphere vacuum manifold into S^2xS^1, i.e. the vacuum manifold is a Hopf fibered S^3.

Gibbons, Ortiz, Ruiz Ruiz & Samols, 1992

Topology of S^2 gives monopoles; topology of S^1 gives strings.

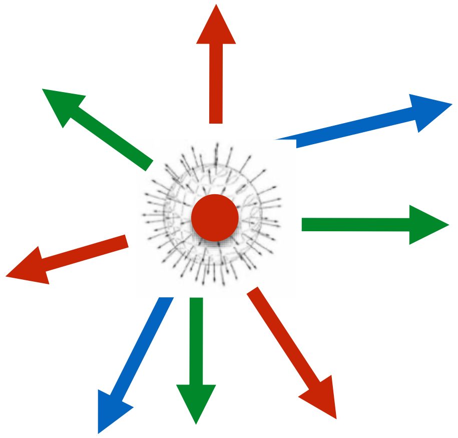
Simplest to see in semilocal case (g=0). **TV & Achucarro, 1991**

Electroweak monopoles contd.

Another way to see the existence of magnetic monopoles is to consider the 3-vector:

$$\vec{n} = \Phi^\dagger \vec{\sigma} \Phi$$

Monopoles are due to hedgehog configurations of **n**.

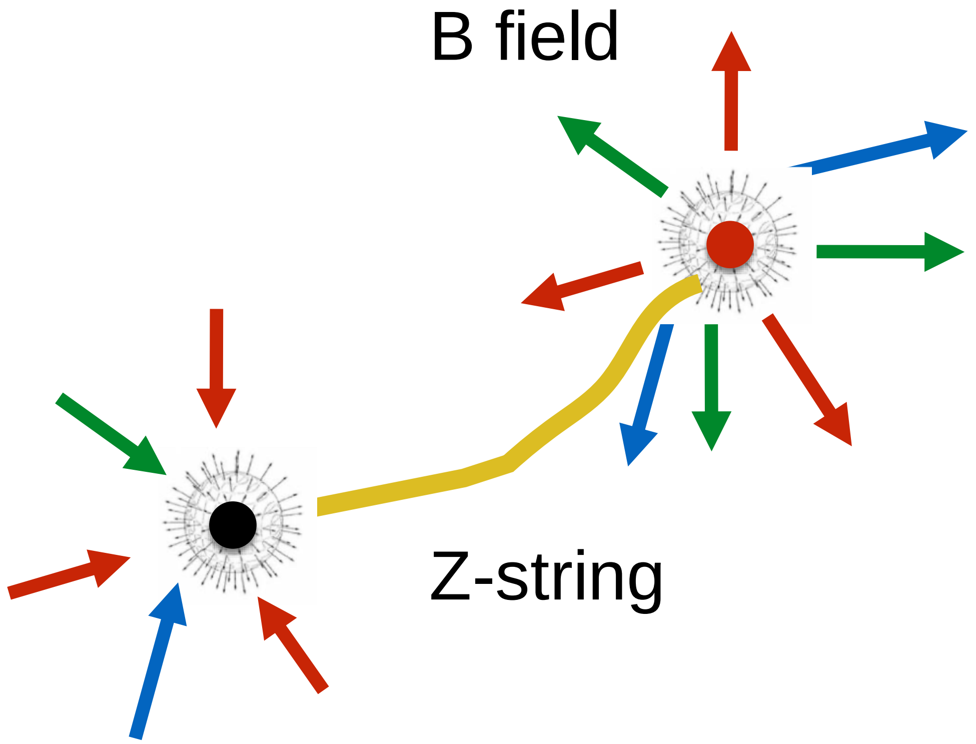


Topology dictates that **n** must vanish in the interior of a hedgehog configuration. Vanishing **n** implies vanishing Higgs which, in turn, is at the top of the Higgs potential.

Electroweak Z-strings

Z-strings occur due to winding of a residual U(1) phase of the Higgs after fixing **n** (illustrated by the color of an **n** arrow). Equivalently, winding around the S^1 fiber.

An electroweak monopole is attached to a Z-string.



- ⋆ Z is orthogonal to A.
- ⋆ Topology only tells us about Higgs zeros, not classical solutions.
- ⋆ Confining Z-string can break by monopole-antimonopole nucleation (similar to QCD strings).

“Magnetic monopoles connected by strings” or “dumbbells”

Monopole-string distribution

Teerthal Patel & TV, 2021

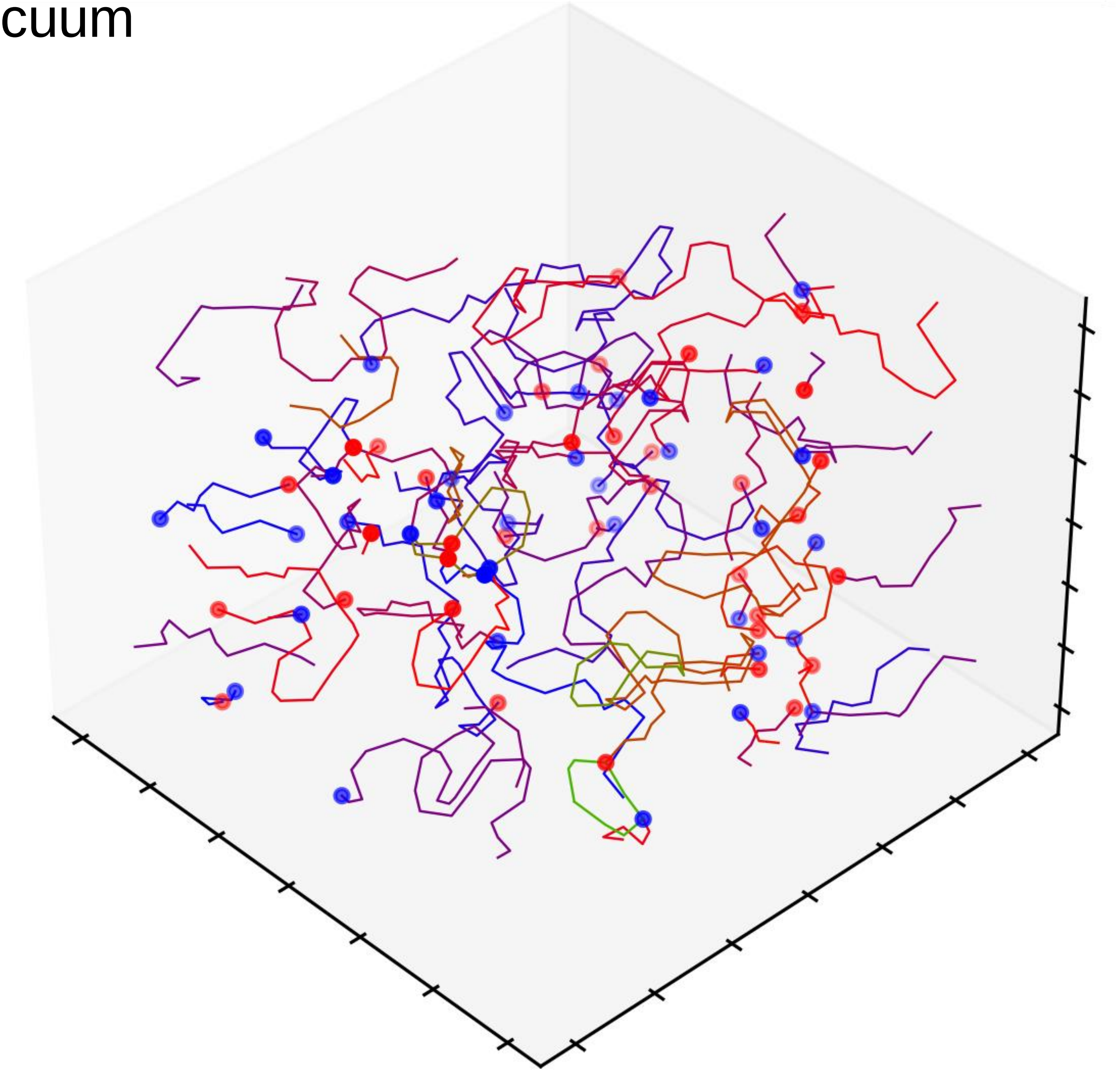
Throw random Higgs field on a lattice with a uniform measure on the vacuum manifold.

Construct **n**.

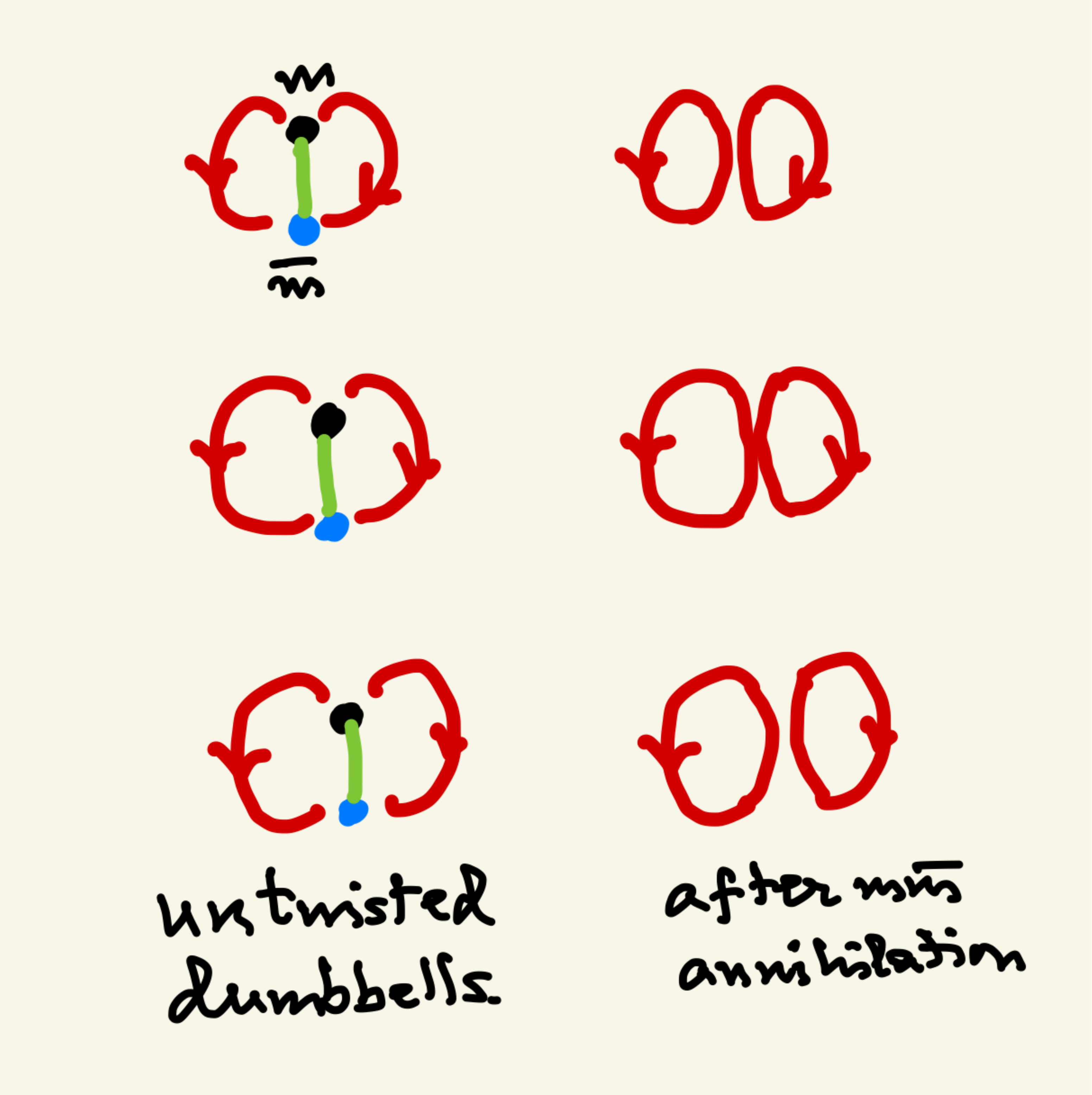
Find hedgehogs in **n** and locate monopoles and antimonopoles.

Determine winding of residual phase (reasonably tricky).

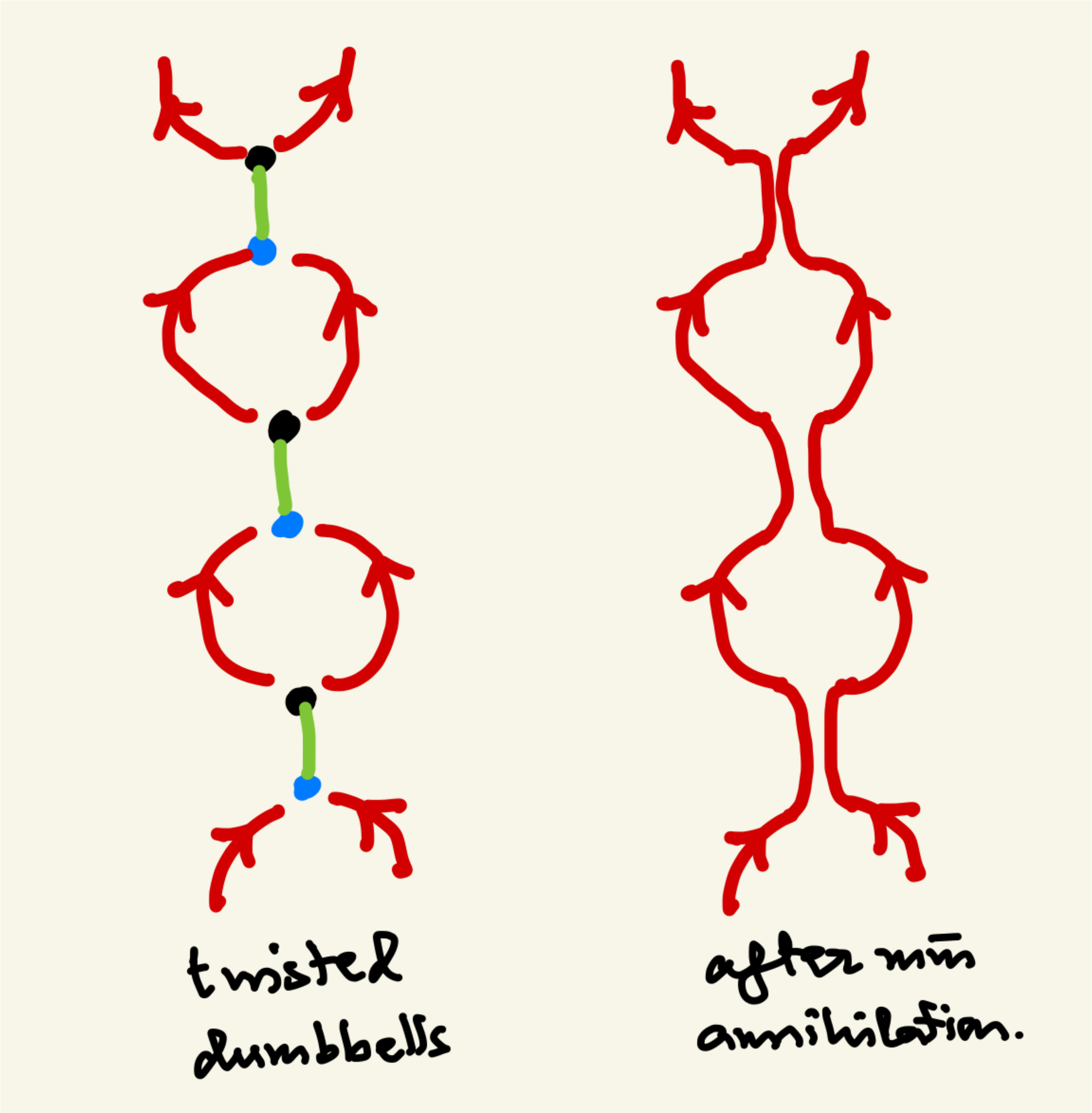
Locate strings where winding is non-trivial.



Gas of untwisted dumbbells

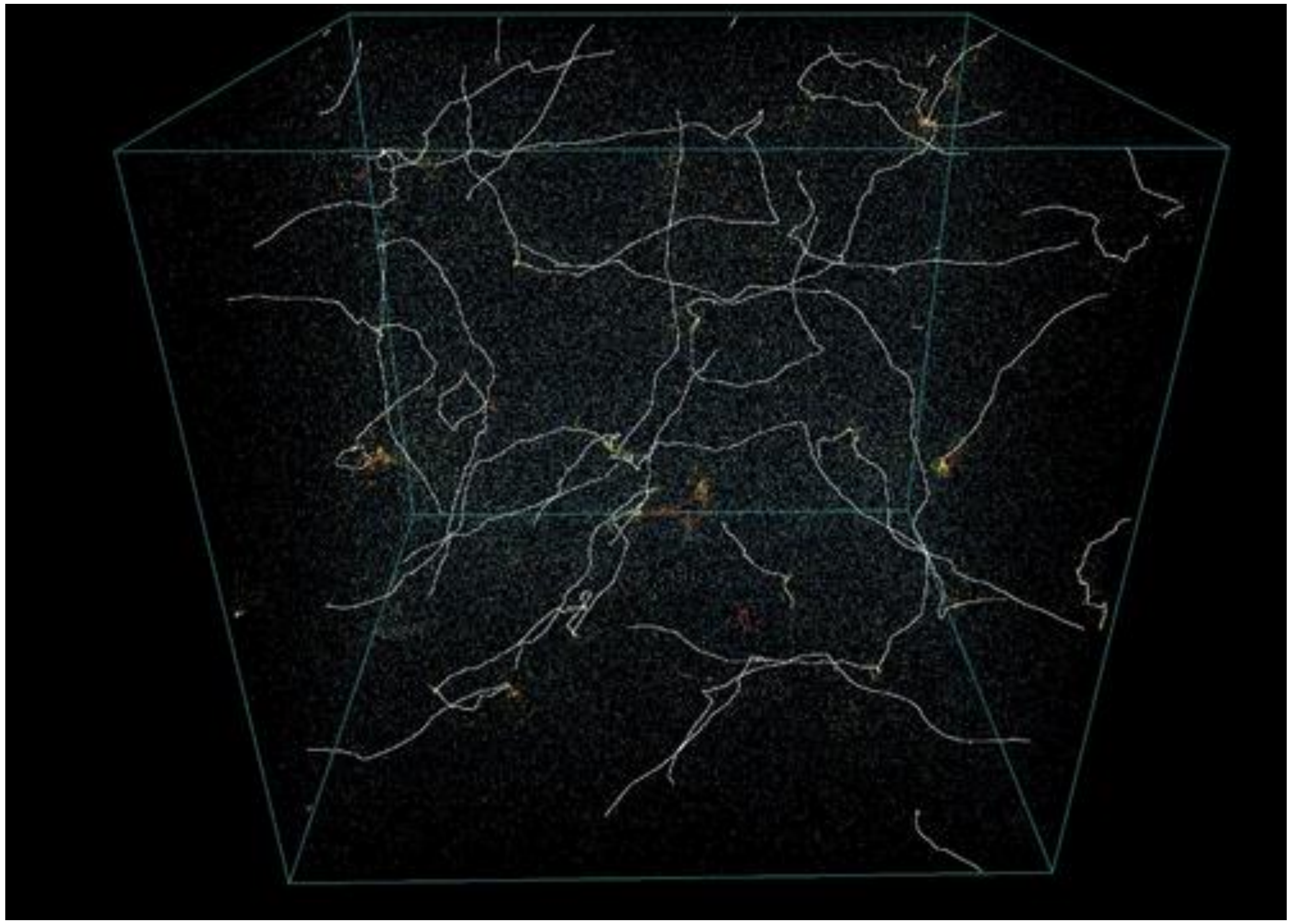


Gas of twisted dumbbells



After dumbbell annihilation

If cosmic strings are a guide, a large fraction of the magnetic field energy is on the largest length scales.



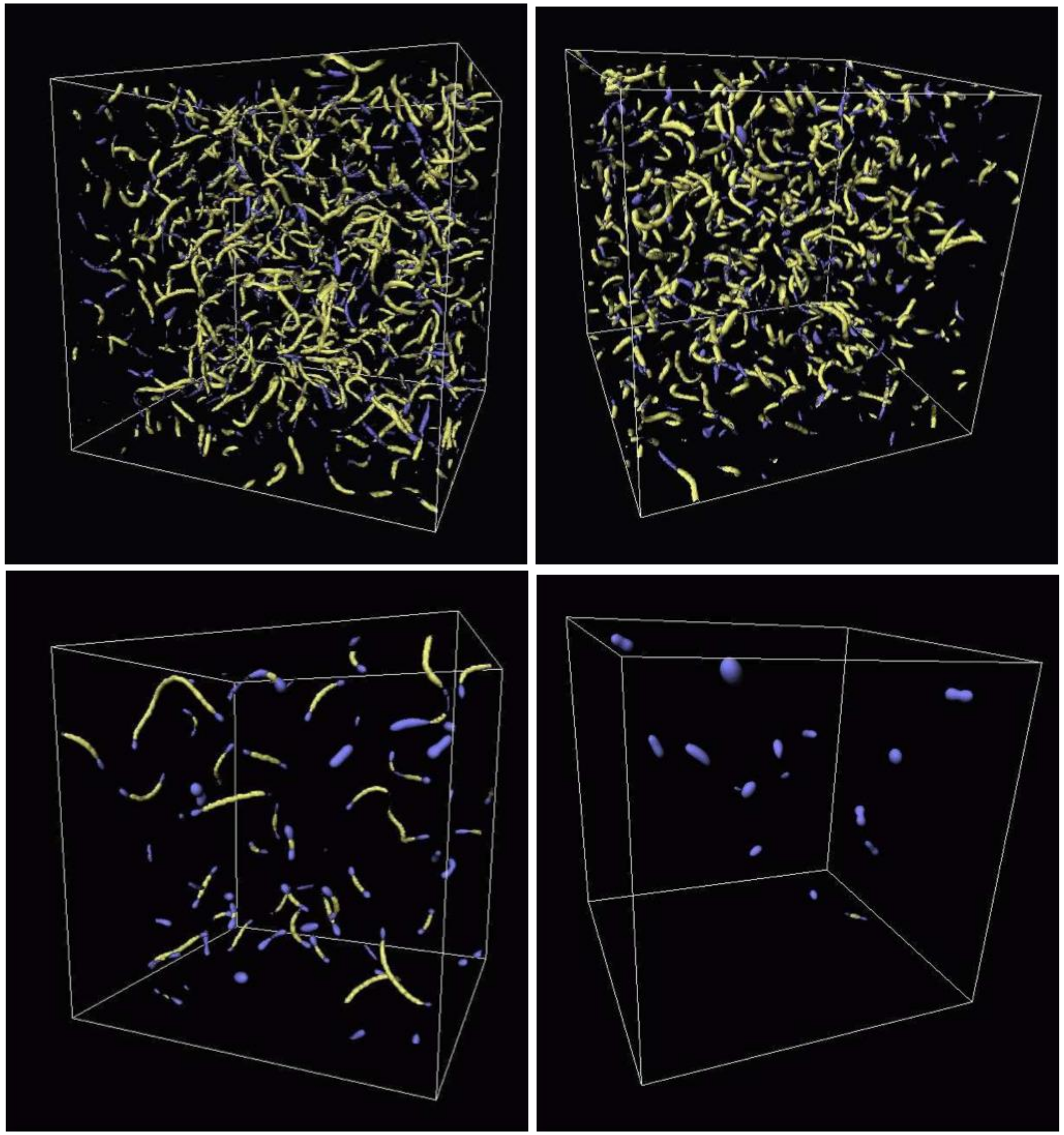
U(1) strings

~80% of the initial energy is in infinite strings. **TV & Vilenkin, 1984**

Magnetic field lines random walk in 3D and never close on themselves.

k_* is presumably set by the horizon scale.

Dumbbells at large Weinberg angle, small Higgs mass



Urrestilla, Achucarro, Borrill & Liddle, 2002

Standard model but with:

$$m_H \lesssim m_Z$$

$$\sin^2 \theta_w \approx 0.995$$

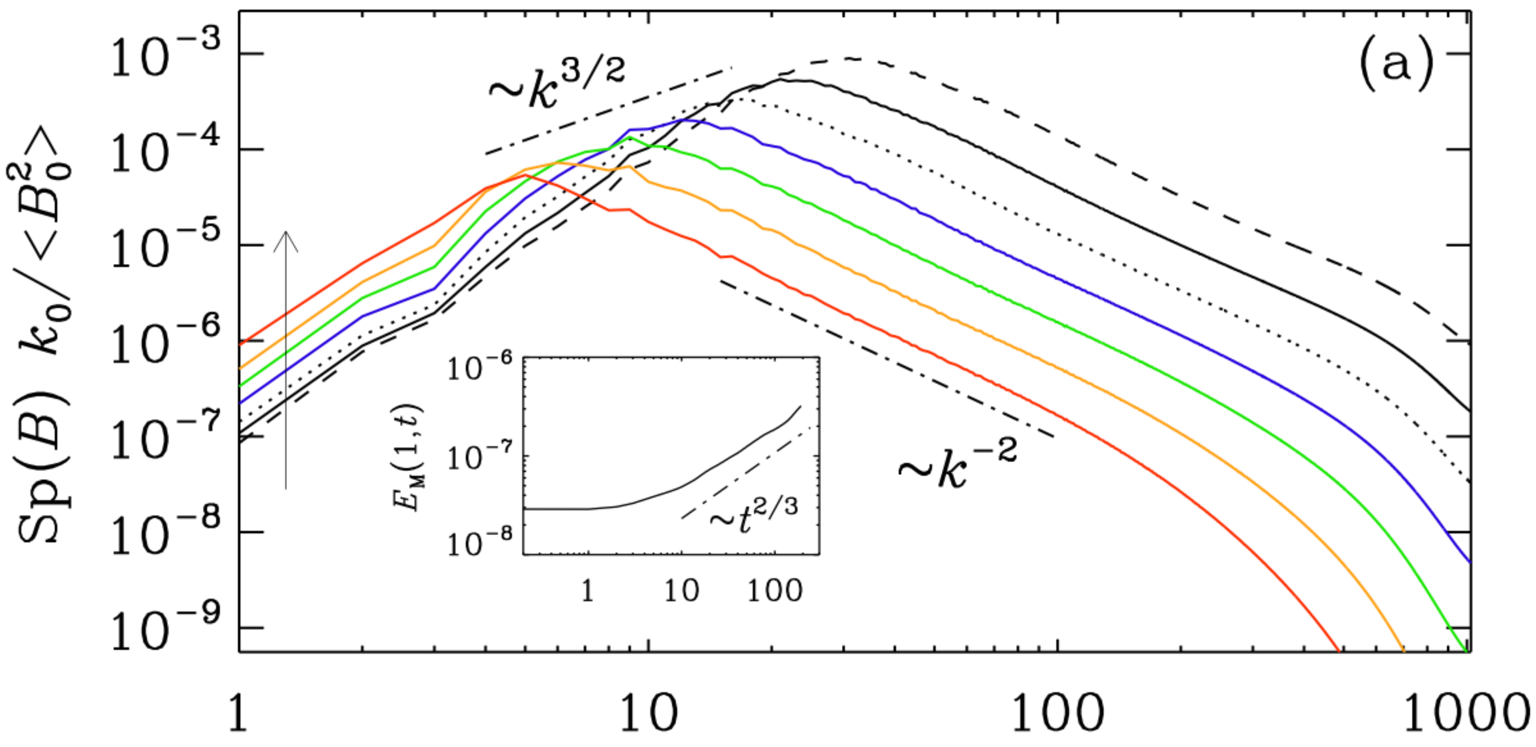
yellow=Z magnetic
blue=A “magnetic” (w/o Higgs term)

Magnetic Field Evolution

Many experts: Banerjee, Brandenburg, Hosking, Jedamzik, Kahniashvili, Schekochihin, Sigl, Subramanian,…

k^3 evolution

Brandenburg, Sharma & TV, 2023



Hosking integral (measures magnetic helicity fluctuations) is conserved.

Magnetic Fields at the present epoch

Non-helical:

Only the long wavelength tail of the distribution can survive dissipation.

$$B_{1\,\mathrm{kpc}} \sim 10^{-18}\,\mathrm{G}$$

(1 kpc dissipation scale)

Helical:

$$B_{10\,\mathrm{kpc}} \sim 10^{-11}\,\mathrm{G}$$

(would be consistent with blazar observations)

$$\text{magnetic helicity} = \int d^3x\, \mathbf{A} \cdot \mathbf{B}$$

Other electroweak effects

“Chirality from tau decays”, TV & Vilenkin

“Chiral magnetic effect”, Vilenkin

Fermionic sector: plasma, chirality.

Joyce & Shaposhnikov,….

CP violation and helicity: how much? in what interactions?

Helicity fluctuations: …

“Helicity from baryogenesis”, Cornwall; Vachaspati

“Hosking integral”, Hosking & Scheckochihin

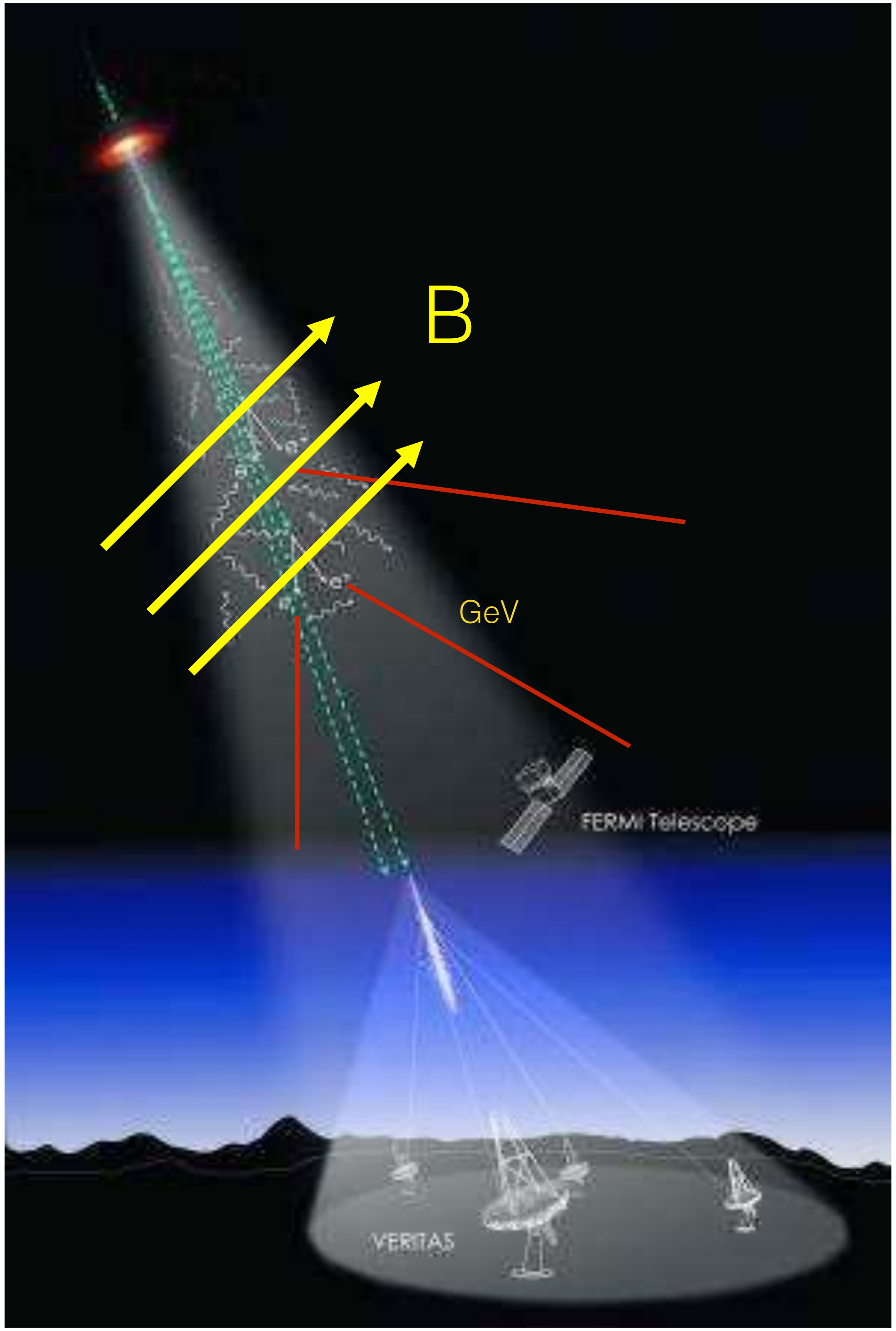
…..

Statistics of magnetic fields? Non-Gaussian? Random phases?

What are the initial conditions from electroweak epoch that are suitable for MHD evolution?

Blazar Lower Bound

Blazar Cascades + **B**



Credit: Nina McCurdy and Joel R. Primack/UC-HiPACC

TeV blazars lead to GeV electromagnetic cascades.

GeV flux gets spread out by magnetic field and becomes too dilute to distinguish from background for strong enough B.

Missing GeV photons attributed to e.g.* $B > 10^{-16}$ Gauss magnetic field with 1 Mpc coherence.

*Stronger fields on shorter length scales can also disperse the cascade.

Magnetic field lower bounds:

Neronov & Vovk, 1006.3504
Tavecchio, Ghisellini, Foschini, Bonnoli, Ghirlanda & Coppi, 1004.1329
Dolag, Kachelriess, Ostapenko, Tomas, 1009.1782
Dermer, Cavadini, Razzaque, Finke, Chiang & Lott, 1011.6660
Essey, Ando & Kusenko, 1012.5313
Taylor, Vovk & Neronov, 1101.0932
Huan, Weisberger, Arlen & Wakely, 1106.1218
Takahashi, Mori, Ichiki, Inoue & Takami, 1303.3069
Finke et al, 1510.02485
Ackermannn et al (Fermi-Lat), 1804.08035
Podlesnyi, Dzhatdoev & Galkin, 2204.11110
Acciari et al (MAGIC), 2210.03321
…

Plasma instability timescale debate:

Broderick, Chang & Pfrommer, 1106.5494, …
Schlickeiser, Ibscher & Supsar, Ap. J. 758, 102 (2012).
Miniati & Elyiv, 1208.1761
Batista, Saveliev & Dal Pino, 1904.13345
…

Conclusions

- ★ Electroweak physics is well-established since the discovery of the Higgs.
- ★ Electroweak symmetry breaking leads to magnetic fields with significant energy density. Coherence scale may be reasonably large.
- ★ Predictions for present day magnetic fields depends on the helicity and evolutionary details.
- ★ If the magnetic field is helical, or if the evolution is significantly different from the “standard” picture, magnetic fields generated at the electroweak epoch are consistent with blazar bounds.
- ★ Magnetic field observations may be a window into electroweak physics and magnetic helicity may provide hints for accelerator physics (CP violation).

Electroweak to Maxwell

Electroweak: W_i^1, W_i^2, W_i^3, Y_i all massless

<Higgs> ↓ at T~100 GeV (10^{15} K), t~1 ns
— electroweak plasma

Weak; E&M: $\{W_i^+, W_i^-, Z_i^0\}, A_i$ photon massless

What are the cosmological consequences of the transition from electroweak to Maxwell?

Electroweak physics

At the electroweak epoch, electroweak symmetry is broken to the Maxwellian symmetry and particles get masses.

<u>Maxwell</u>

$$\partial_\nu A^{\mu\nu} = j^\mu$$

Currents due to charges in physical system.

<u>Electroweak</u>

$$D_\nu W^{\mu\nu a} = j^{\mu a}, \quad (a = 1, 2, 3)$$

$$\partial_\nu Y^{\mu\nu} = j^{\mu Y}$$

Currents due to charges in physical system including Higgs.

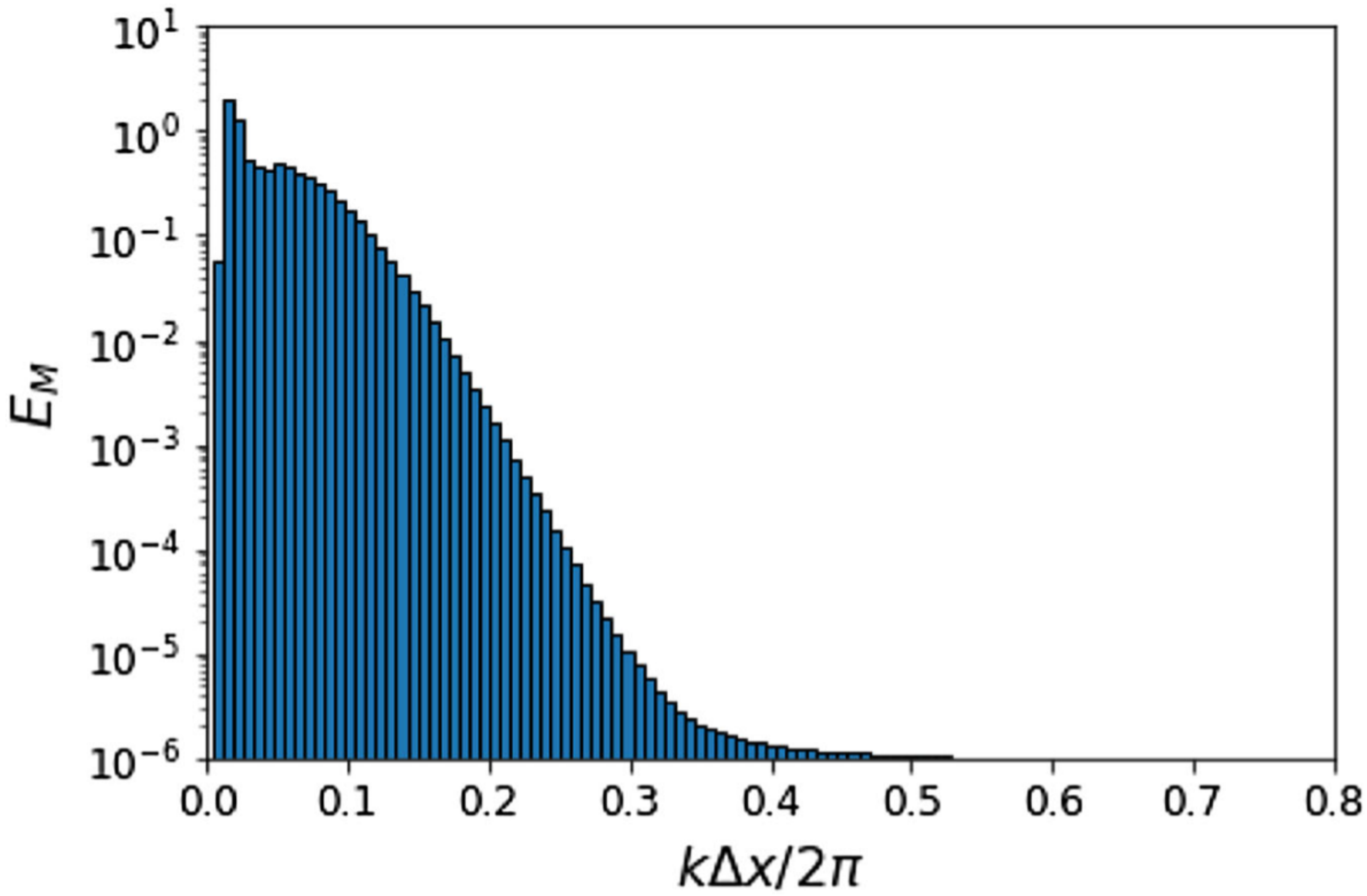
$$\Box\Phi + V'(\Phi) = 0$$

Predictions of electroweak equations are as reliable as predictions of Maxwell equations. (I am not introducing any BSM physics.)

B Spectrum contd.

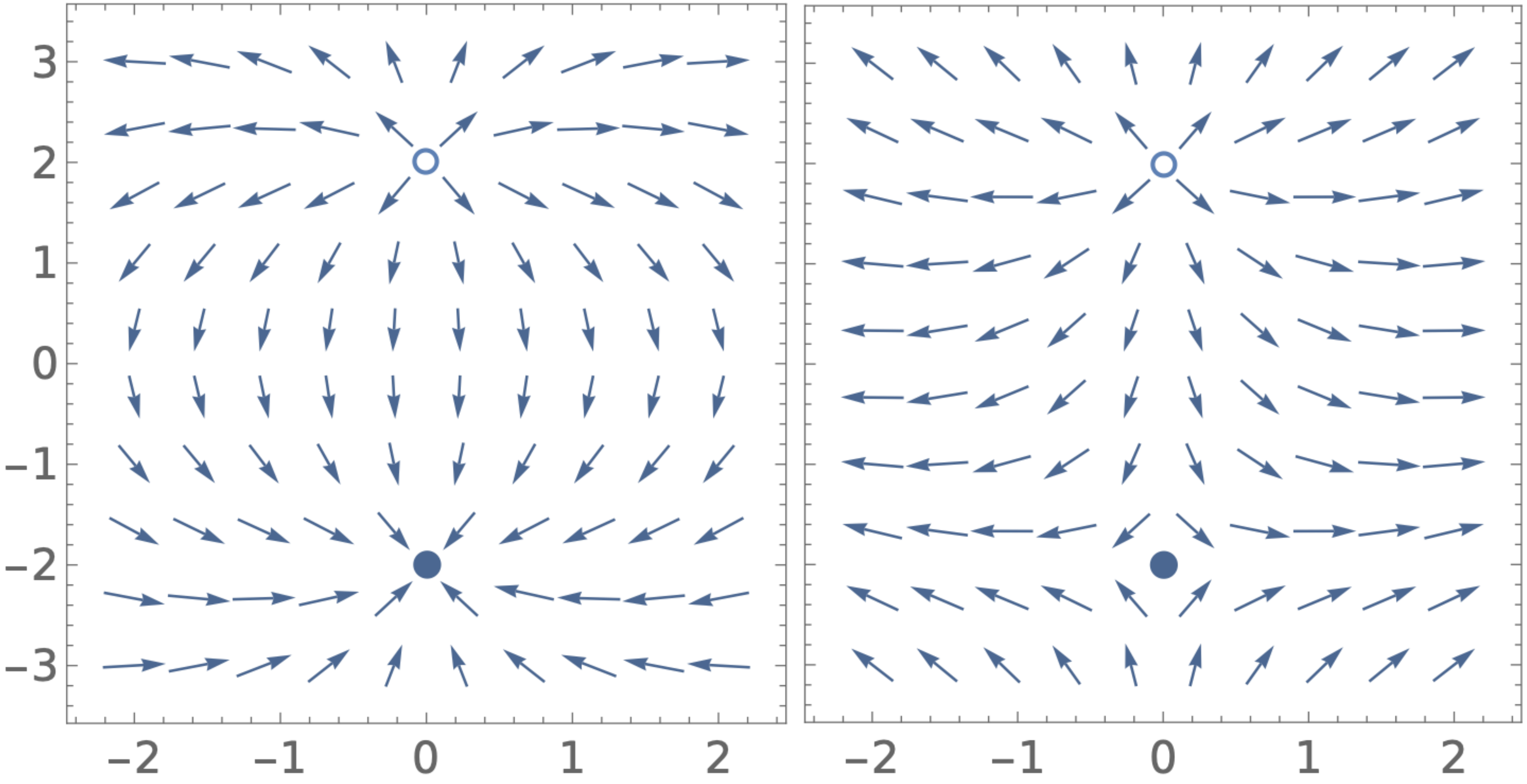
Zhang, Ferrer & TV, 2019

Another simulation.



Twisted dumbbells

Ayush Saurabh & TV, 2017



twist=pi

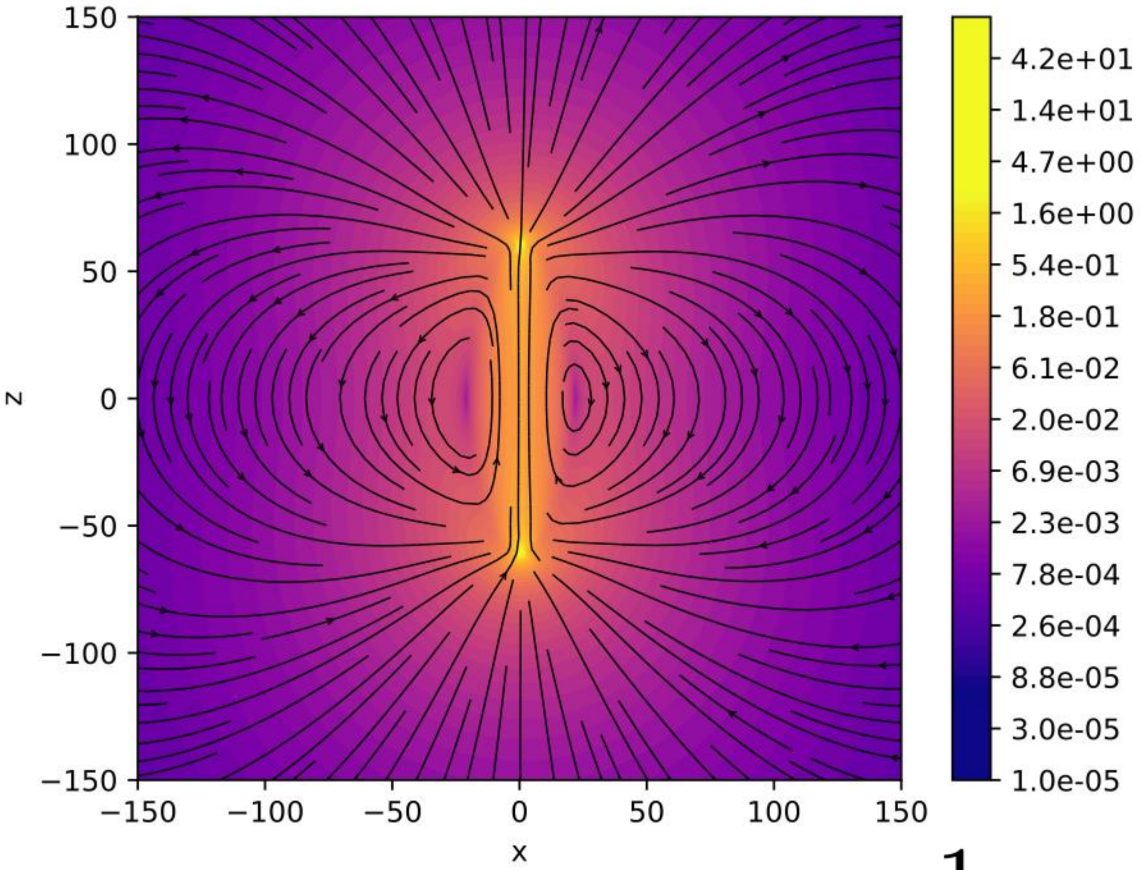
twist=0

Magnetic field of dumbbells

Teerthal Patel & TV, 2023

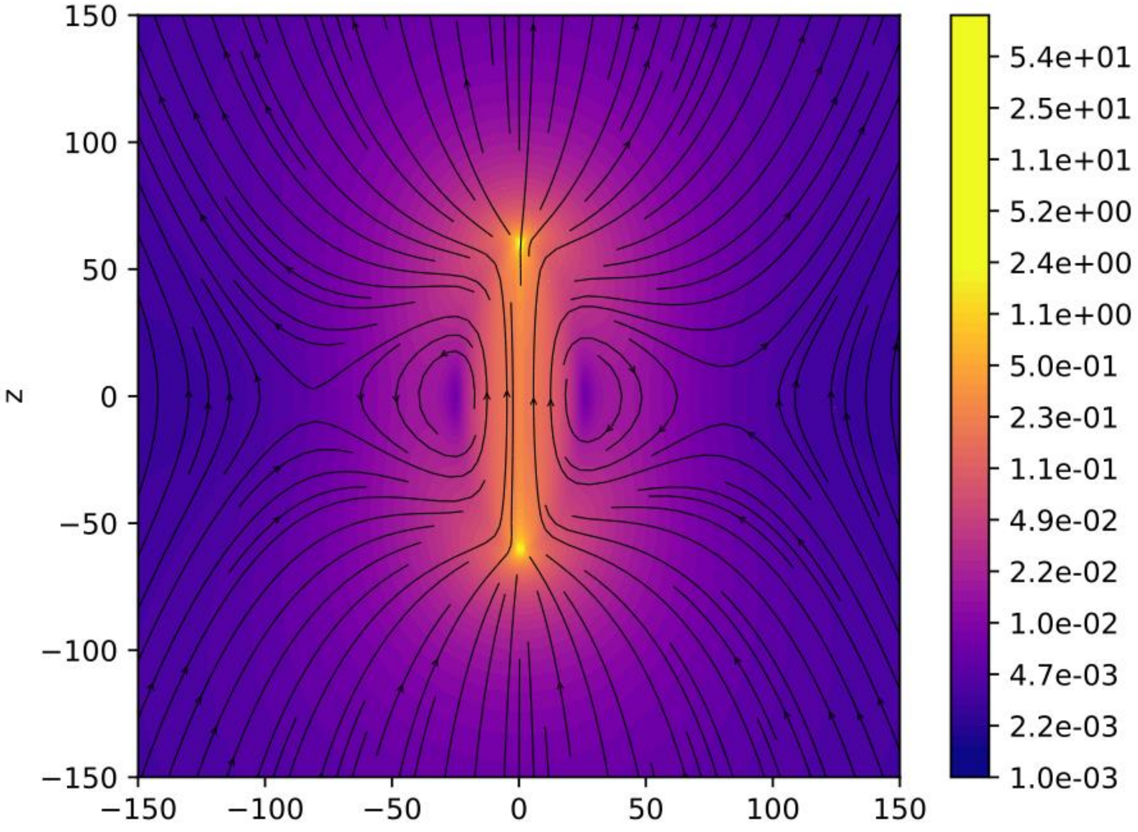
Impose topological constraint: fix monopole-antimonopole positions.

Relax field configuration for different Higgs field configurations.



twist=0

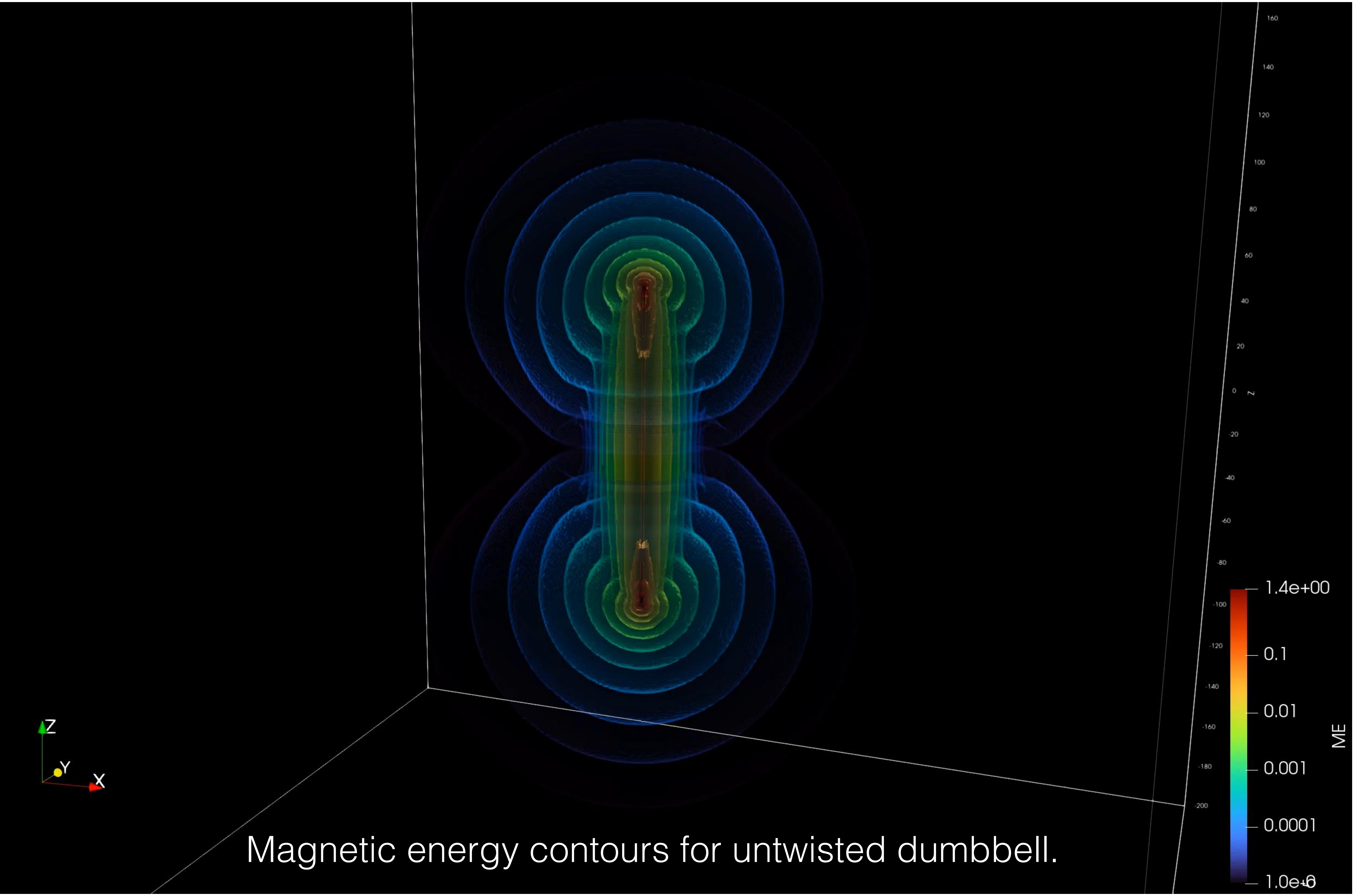
$$B \propto \frac{1}{r^3}$$



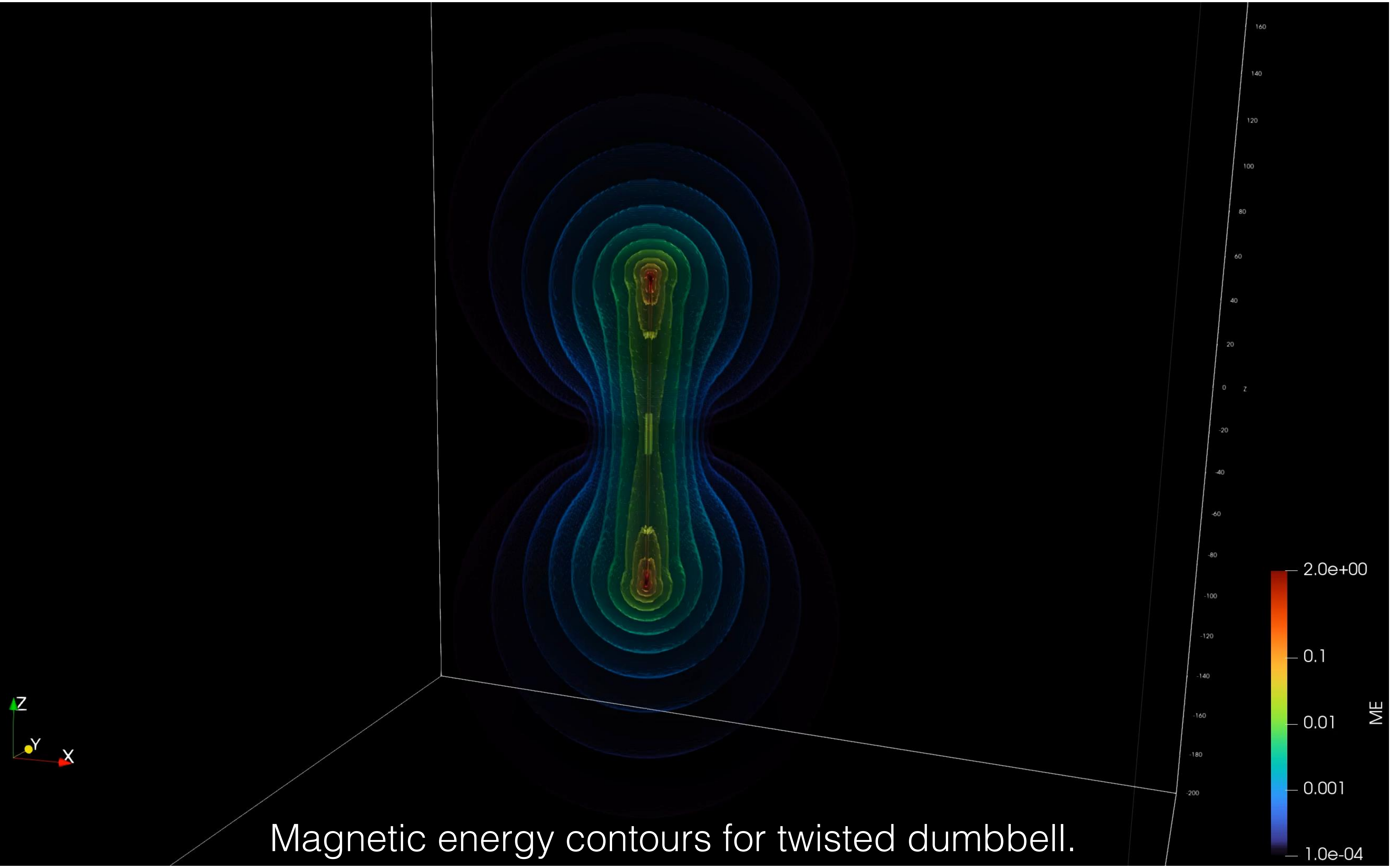
twist=pi

$$B \propto \frac{\cos\theta}{r^2}$$

Teerthal Patel & TV (ongoing)



Magnetic energy contours for untwisted dumbbell.



Magnetic energy contours for twisted dumbbell.