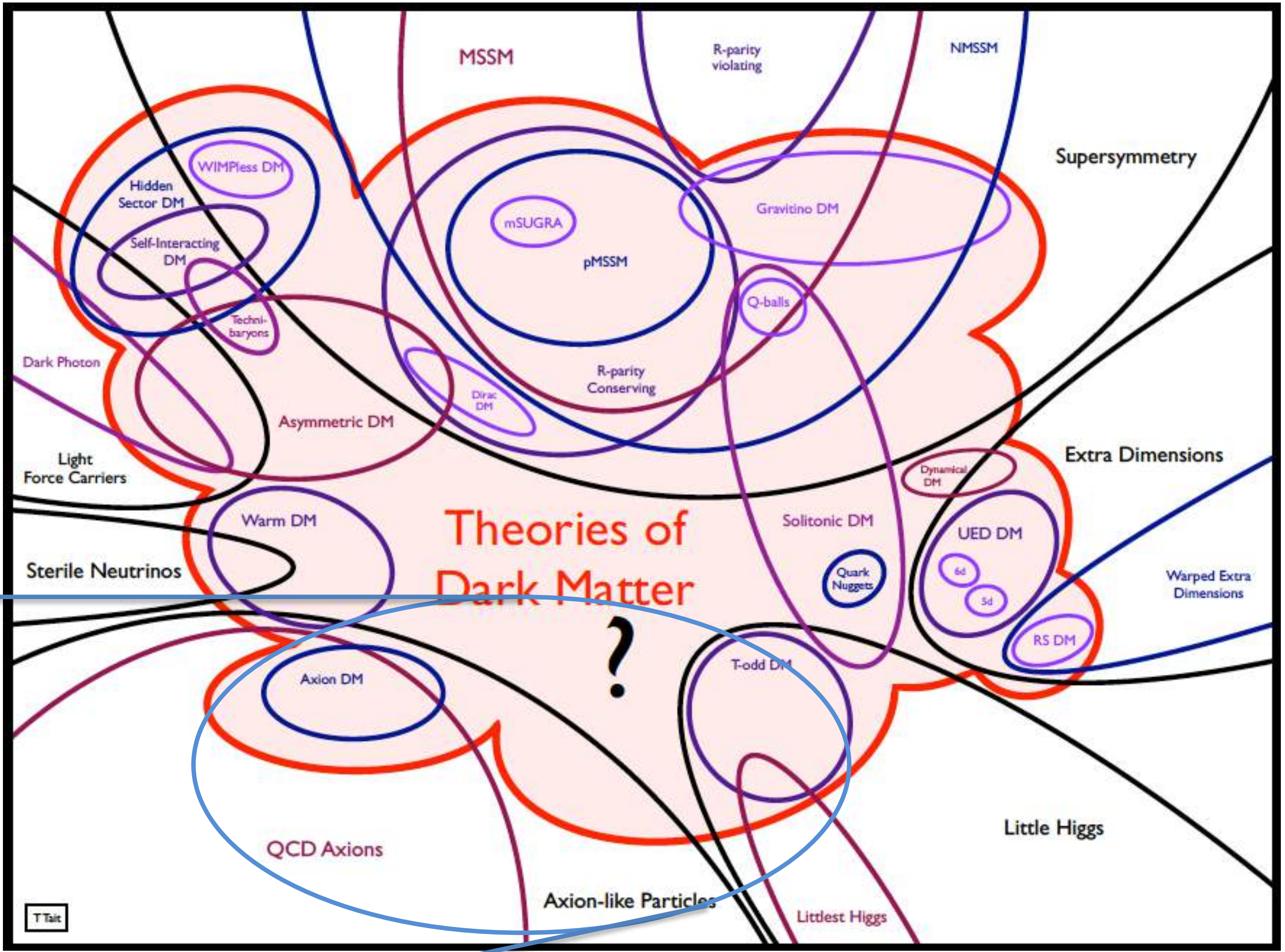
The cosmological axion domain wall problem

Subir Sarkar

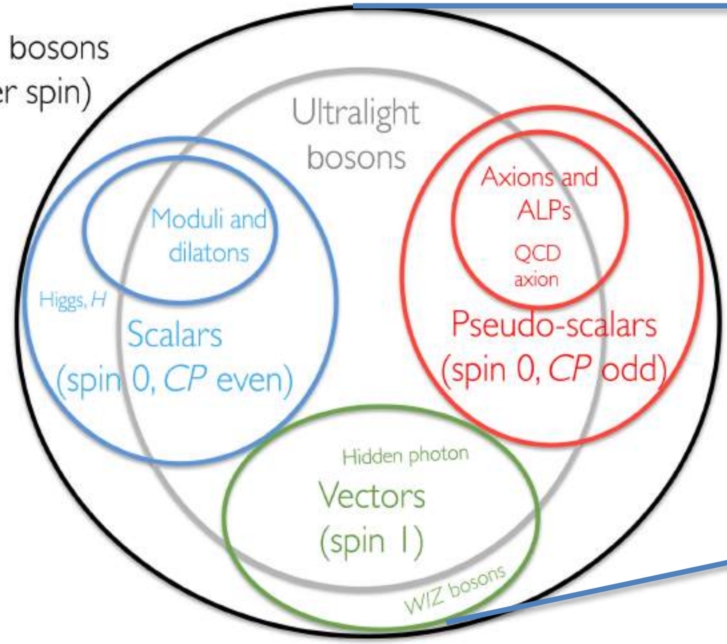
We revisit the domain wall problem for QCD axion models with more than one quark charged under the Peccei-Quinn symmetry. Symmetry breaking during or after inflation results in the formation of a domain wall network which would cause cosmic catastrophe if it comes to dominate the Universe. The network may be made unstable by invoking a 'tilt' in the axion potential due to Planck scale suppressed non-renormalisable operators. (Alternatively the random walk of the axion field during inflation can generate a 'bias' favouring one of the degenerate vacuua, but we find that this mechanism is in practice irrelevant.) Consideration of the axion abundance generated by the decay of the wall network then requires the Peccei-Quinn scale to be rather low – thus **ruling out the DFSZ axion with mass below ~11 meV**, where most experimental searches are in fact focussed.

(with Konstantin Beyer, *SciPost Physics* **15**:003, 2023)

As WIMPs prove elusive, so has interest grown in axions as dark matter candidates ...



Courtesey: Tim Tait



Chadha-Day *et al*, *Sci.Adv.***8**:abj361,2022

Axion dark matter

$$\mathcal{L}_{\text{eff}} = F^2 + \bar{\Psi}\,\not{D}\Psi + \bar{\Psi}\Psi\Phi + (D\Phi)^2 + \Phi^2 \quad \boxed{+\theta_{\text{QCD}} F\tilde{F}}$$

The SM admits a term which would lead to *CP* violation in strong interactions, hence an (unobserved) electric dipole moment for neutrons ⇒ requires $\theta_{\text{QCD}} < 10^{-10}$

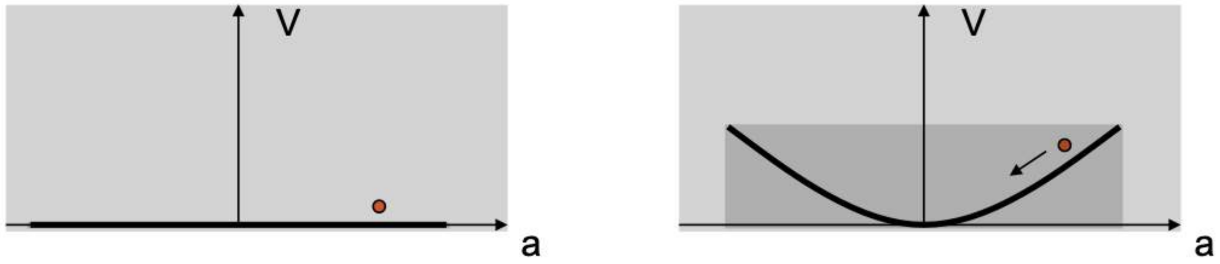
To achieve this without fine-tuning, θ_{QCD} must be made a dynamical parameter, through the introduction of a new $U(1)_{\text{Peccei-Quinn}}$ symmetry which must be broken … the resulting (pseudo) Nambu-Goldstone boson is the QCD **axion** which subsequently gets a mass via mixing with the pion: $m_a = m_\pi\,(f_\pi/f_a) \sim 5.7\ \text{eV}\ (10^6\ \text{GeV}/\,f_a)$

Courtesey: Javier Redondo

When the temperature drops to Λ_{QCD} the axion potential turns on and the coherent oscillations of relic axions contain energy density that behaves like *cold* dark matter with $\Omega_a h^2 \sim 10^{11}\ \text{GeV}/f_a$, so this requires $f_a \sim 10^{10\text{-}12}\ \text{GeV} \Rightarrow m_a \sim 10^{-6}$–$10^{-4}\ \text{eV}$

Experimental searches for axions are therefore focussed on this mass range, however the above argument has not accounted for the contribution from topological defects that form in the axion field

Axion production by vacuum realignment



$T \geq 1\,\mathrm{GeV}$ $\qquad$ $T \leq 1\,\mathrm{GeV}$

Preskill, Wise & Wilczek, *Phys.Lett.* **120B**:127,1983;
Abbott & Sikivie, *ibid*:133;
Dine & Fischler, *ibid*:137

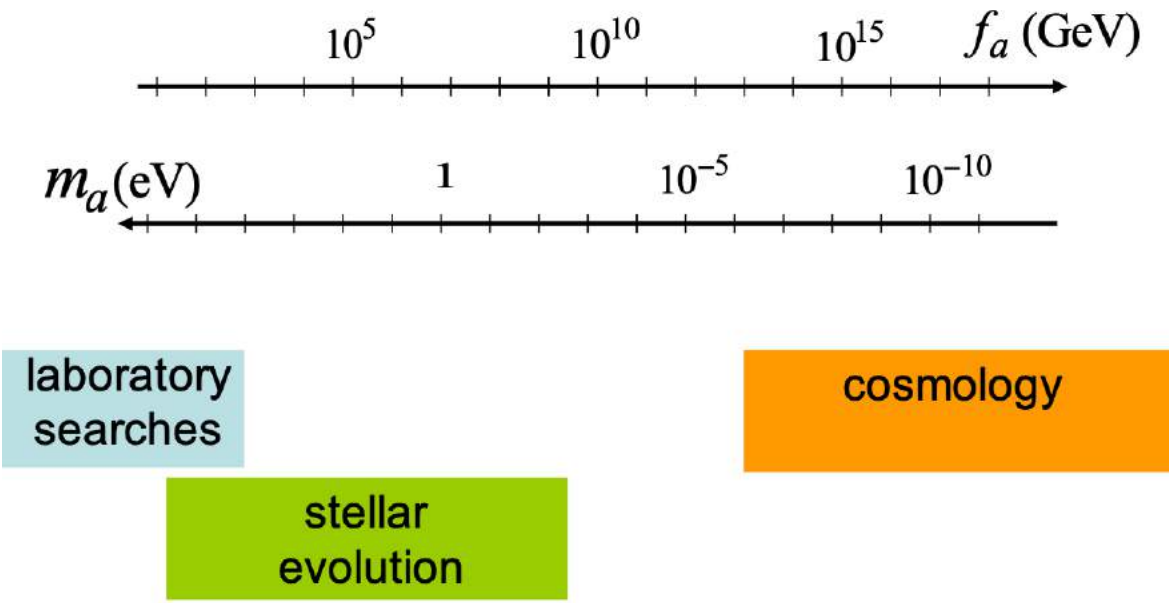
$$n_a(t_1) \simeq \frac{1}{2} m_a(t_1) a(t_1)^2 \simeq \frac{1}{2t_1} f_a^2 \alpha(t_1)^2$$

$$\rho_a(t_1) \simeq m_a n_a(t_1) \left(\frac{R_1}{R_0}\right)^3 \propto m_a^{-\frac{7}{6}}$$

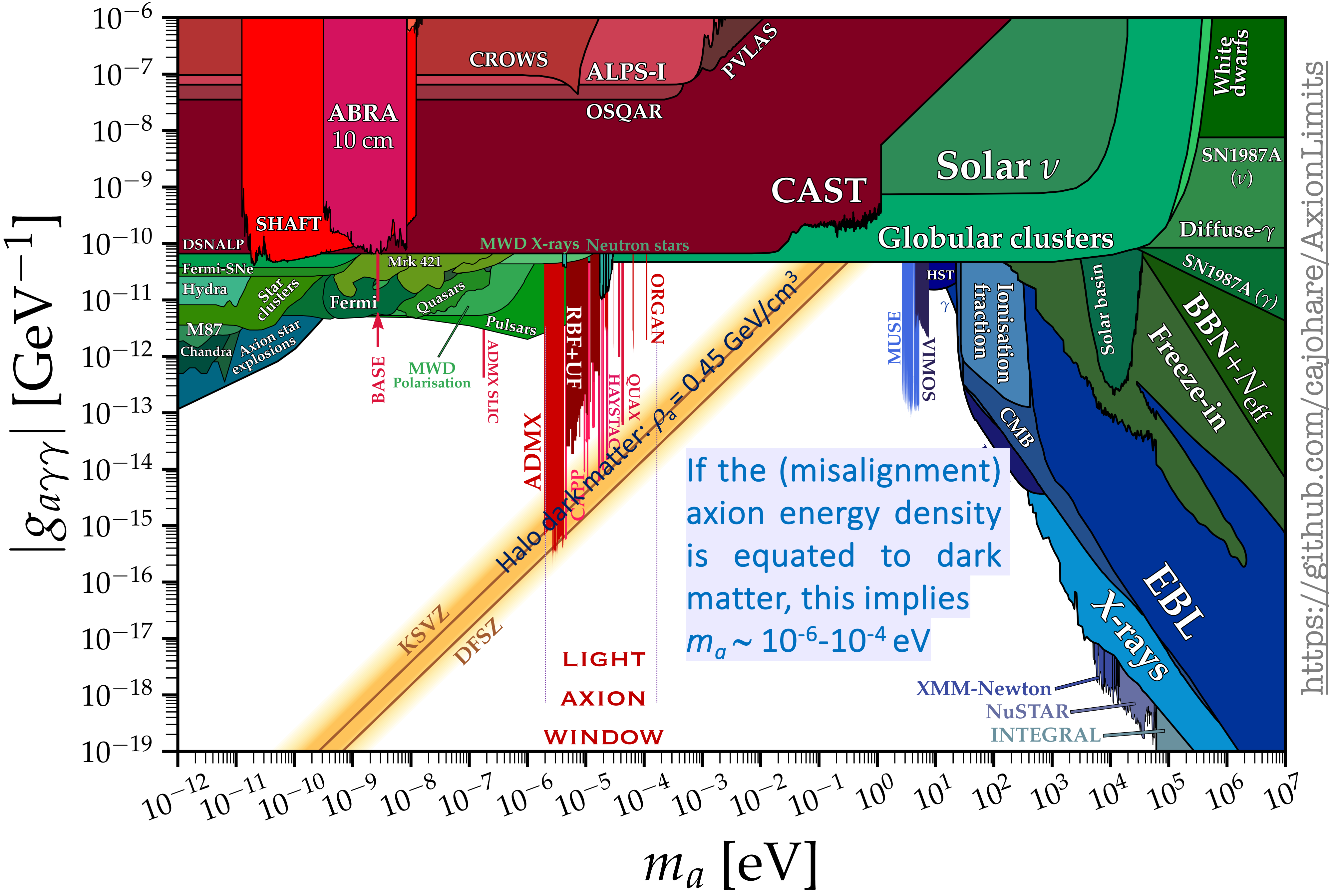
initial misalignment angle

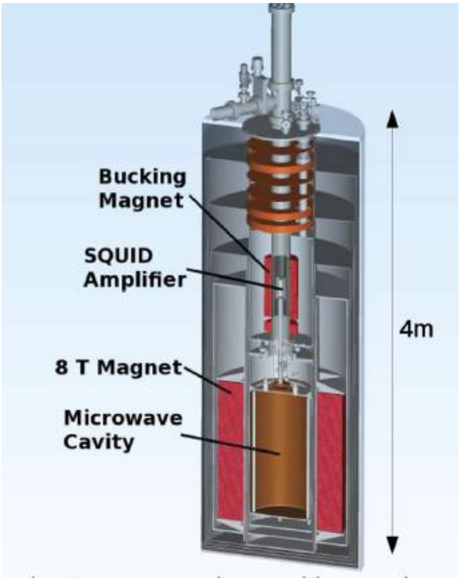
Axions produced by vacuum realignment are cold dark matter

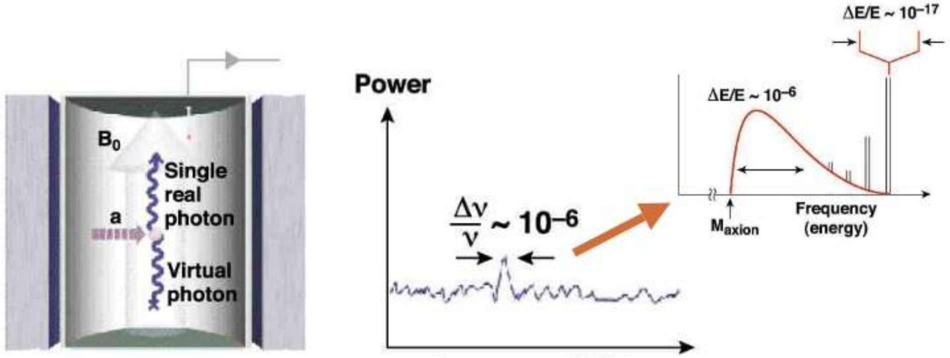
Axion constraints

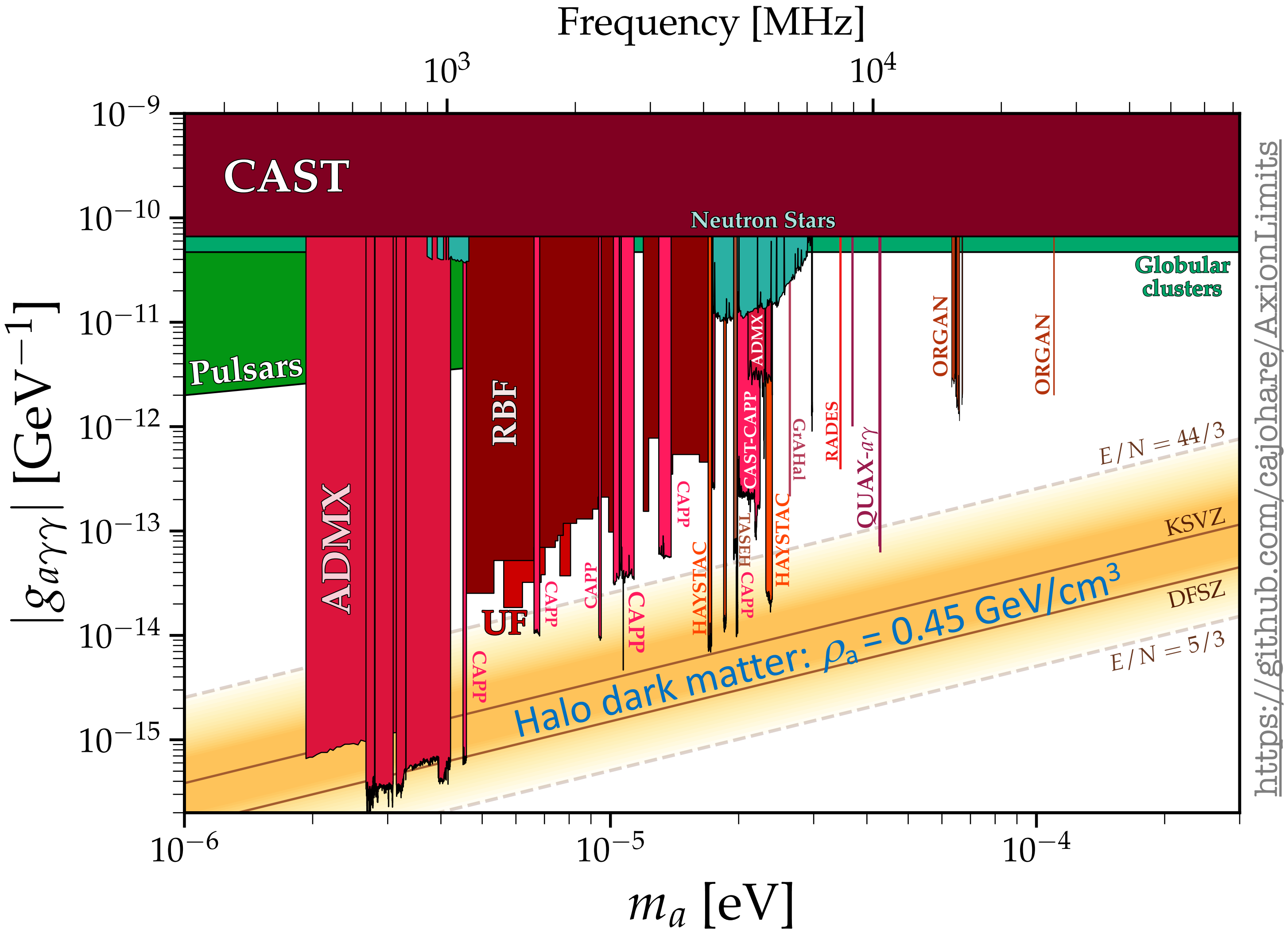


CONSTRAINTS ON QCD AXIONS (AND AXION-LIKE PARTICLES)





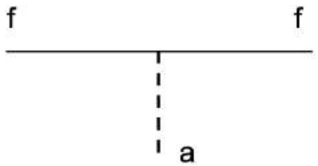




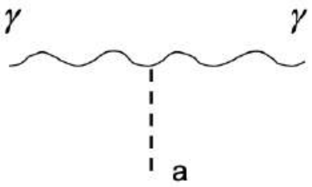
Most searches are focussed therefore on this QCD 'light axion window'

But are they in fact looking in the right mass range?

PARTICLE PHYSICS MODELS FOR QCD AXIONS



$$\mathcal{L}_{a\bar{f}f} = i g_f \frac{a}{f_a} \bar{f} \gamma_5 f$$



$$\mathcal{L}_{a\gamma\gamma} = g_\gamma \frac{a}{f_a} \vec{E} \cdot \vec{B}$$

g_γ = 0.97 in KSVZ model
0.36 in DFSZ model

The axion *always* couples to two photons (through the anomaly) but may or may not couple to fermions depending on how it arises in the UV-complete theory.

1) **KSVZ**: Only a scalar field σ with $f_{\rm PQ} = \langle\sigma\rangle \gg v_{\rm F}$, and a superheavy quark Q with $M_Q \sim f_{\rm PQ}$ carry PQ charge i.e. $N_{\rm DW} = 1$

The KSVZ axion does *not* interact with leptons – only with light quarks

J.E. Kim M. Shifman A. Vainshtein V.I. Zakharov

2) **DFSZ**: Adds a scalar field σ which carries PQ charge, with $f_{\rm PQ} = \langle\sigma\rangle \gg v_{\rm F}$ i.e. $N_{\rm DW} > 1$

The DFSZ axion interacts with *both* leptons and quarks and is more generic

M. Dine W. Fischler M. Srednicki A. Zhitnitsky

In the early universe after P-Q symmetry breaking the vacuum manifold is not simply connected ⇒ cosmic strings
… and at $T < \Lambda_{\rm QCD}$ the symmetry is further broken to $\mathbb{Z}(N_{\rm DW})$ ⇒ domain walls

DISCRETE SYMMETRY BREAKING IN THE EARLY UNIVERSE IS INCONSISTENT WITH COSMOLOGY

Cosmological consequences of a spontaneous breakdown of a discrete symmetry

Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun'

Institute for Applied Mathematics, USSR Academy of Sciences

(Submitted January 31, 1974)

Zh. Eksp. Teor. Fiz. **67**, 3–11 (July 1974)

In theories involving spontaneous symmetry breakdown one may expect a domain structure of the vacuum. Such a structure does not exist near a cosmolgical singularity, when the temperature is above the Curie point, but this structure must appear later during the cosmological expansion and cooling down. We discuss the properties of the domain interfaces and of the space with domains in the large, the law of cosmological expansion in the presence of domains, and the influence of domains of the homogeneity of the Universe at a late stage.

We can thus draw three fundamental conclusions:

1. Spontaneous violation of ~~CP-invariance~~ P-Q symmetry leads to the formation of a domain structure of the vacuum. This occurs because during the cooling down of causally disjointed points of the Universe the signs of the condensate appear in a random fashion.

2. For $\lambda < 1$ the walls of the domains are so heavy that their existence would lead to a radical change of the cosmological evolution of the Universe.

3. If there is no mechanism that leads to the disappearance of domains at a sufficiently early stage of the evolution of the Universe, the domains would lead to conclusions which are in contradiction with experiment.

Thus, either the model of spontaneous breakdown of ~~CP~~ P-Q-symmetry discussed by us is false, or there must exist mechanisms which facilitate the disappearance of the domains.

That this is a problem for axions was recognised by Sikivie, who also suggested a solution:

"It is possible, however, to introduce a small $Z(N)$ breaking interaction into axion models without upsetting the Peccei-Quinn mechanism. In that case the domain walls disappear a certain time after their formation in the early universe"

Phys.Rev.Lett.**48**:1156,1982

BUT WHAT ABOUT THE ENERGY CONTAINED IN THE DOMAIN WALLS?

It has been argued that this is *not* a problem

- If the PQ transition occurs after inflation, there are domain walls. It has been argued that these lead to an enhanced axion dark matter density; there is a large parameter associated with this i.r. divergence..

My Claim: Provided that there are symmetry violating effects which eliminate the domain walls before they dominate the energy density of the universe, domains of higher energy collapse. Most of their energy is converted to kinetic energy of the domain walls, which is in turn converted, at the final stages of collapse, to extremely relativistic axions, which again don't contribute significantly to the dark energy budget

… this overlooks however that the axions from wall decay turn non-relativistic subsequently and thus contribute to the ***dark matter*** *budget!*

Michael Dine | The Strong CP Problem and Its Implications

Dine *et al, JCAP* **11**:041,2021
Dine, arXiv:2307.04710

To calculate this contribution, we must follow in detail the evolution and collapse of the domain wall network, and take into account the **contribution of the radiated axions to the present dark matter abundance**

BUT APPARENTLY THERE IS NO CLEAR CONCLUSION REGARDING THIS ISSUE

Snowmass 2021 White Paper
Axion Dark Matter

arXiv:2203.14923v3 [hep-ex]

4.1.2 The Post-Inflation Scenario

In the complementary scenario, the PQ symmetry breaks (for the last time) after inflation has ended. The axion field, again, takes random values $\theta_\mathrm{i} \sim \mathcal{U}(-\pi, \pi)$ in causally disconnected regions. However, the observable universe now consists of a huge number of these regions, such that the energy density of axions today can be calculated as an average over the causally disconnected regions. As a consequence, the axion DM abundance becomes independent of any one θ_i and, for QCD axions, only depends on f_a. There may also be significant local DM overdensities and thus a highly non-linear evolution, giving rise to observable consequences discussed below.

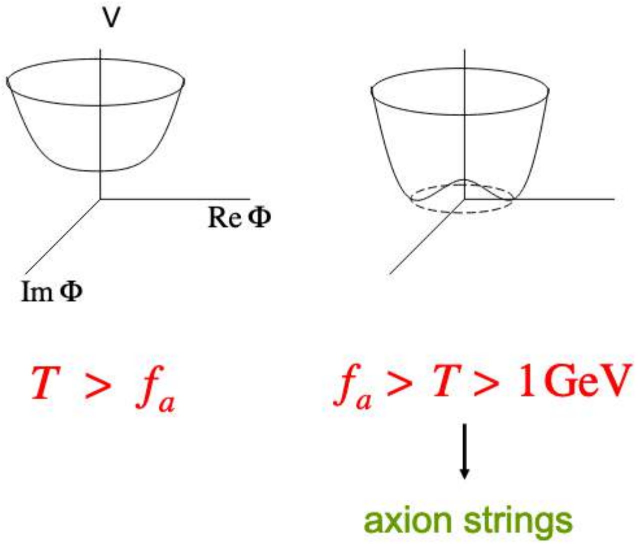
Topological defects created during PQ breaking [81] are not inflated away in this scenario. They can increase the axion DM density through their evolution and decays [83–86]. While the phenomenology of cosmic strings, domain walls, and axitons has been studied extensively over the past decades [83–86, 143–155], no definitive consensus has been reached regarding the relevance of their contribution to the axion DM density. Recently, the contribution from string decay and wall annihilation have also been considered for the case of ALPs [156, 157].

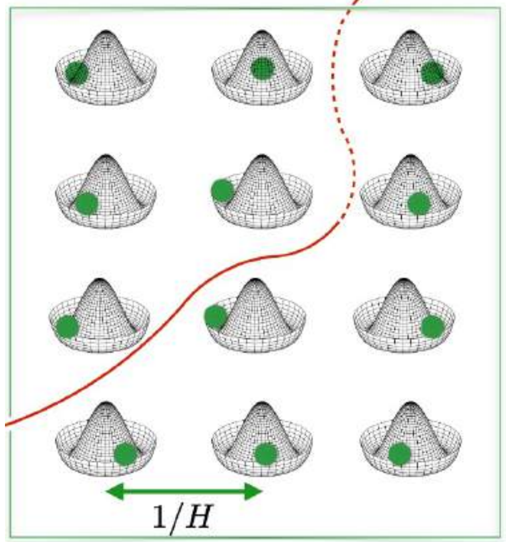
Report of the Topical Group on Wave Dark Matter for Snowmass 2021

For example in … so-called post inflationary scenario … insufficient understanding of the production that receives possibly significant contributions from topological defects [17-33] in addition to the misalignment production.

arXiv:2209.08125 [hep-ph]

Axion Strings





String interactions are complicated, understood by numerical simulations

String energy density follows a scaling law

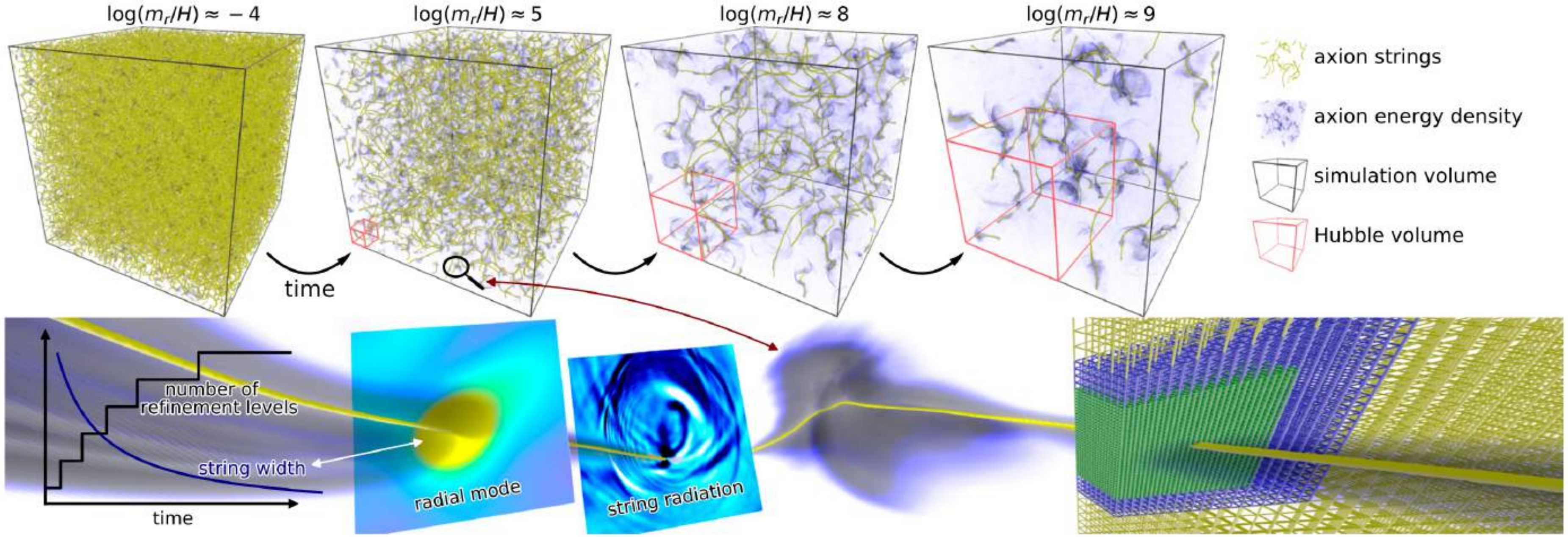
$$\rho_{\rm strings} \simeq \xi \mu H^2$$

$$10^3 > \xi > 1$$

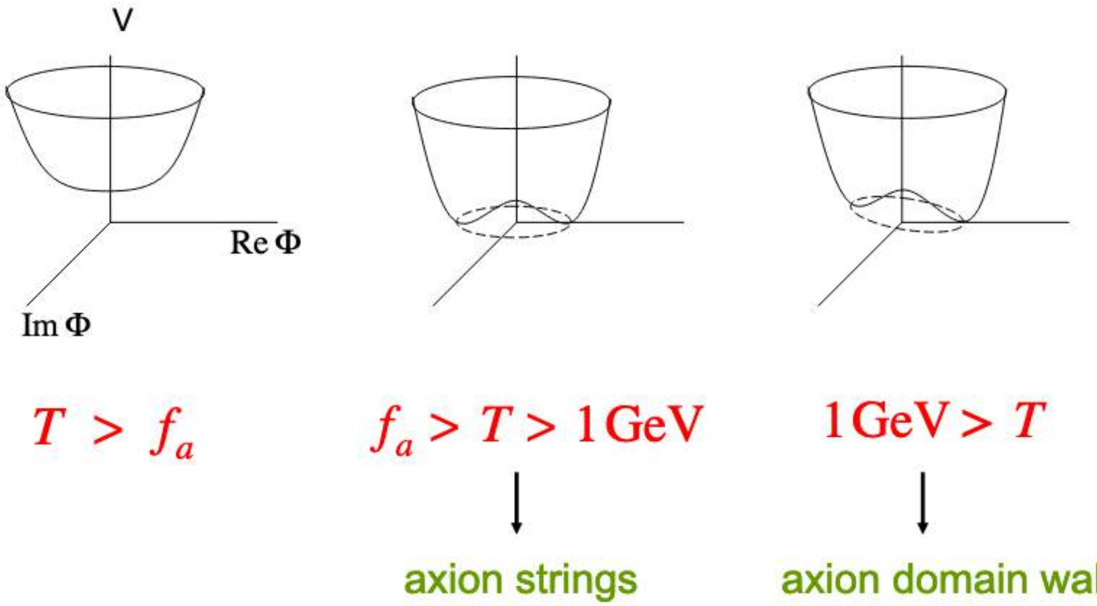
Equivalent to ξ strings per Hubble volume

Network is dominated by infinitely long strings with structure at scale 1/H

State-of-the-art simulation of cosmic string network evolution (Buschmann *et al*, *Nature Comm.* 13:1049,2022)



Axions radiated by the string network contribute ~25% more DM (O'Hare *et al*, *PR* D**105**:055025,2022)



If there are $N_{\rm DW} > 1$ quarks charged under $U(1)_{\rm PQ}$, then the Z_N symmetry breaking results in the formation of domain walls at $t_{\rm QCD} \sim 10^{-5}$ s (Sikivie, *PRL* **48**:1156,1982)

Their energy density scales as: $\rho_{\rm wall} \propto t^{-1}$ (*cf.* $\rho_{\rm m}, \rho_{\rm r} \propto t^{-2}$)

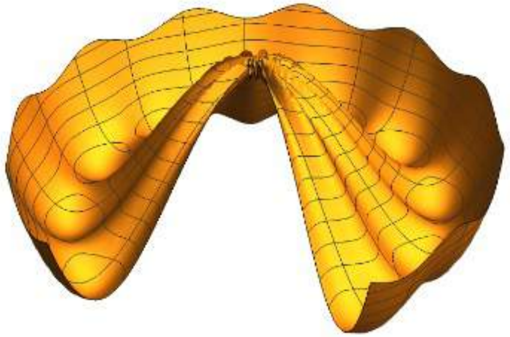
so the wall network *must* collapse before they come to dominate at:

$$t = 3/32\pi G_N \sigma_{DW} \sim 6\text{x}10^3\text{ s }(10^{10}\text{ GeV}/f_{PQ}),\text{ where } \sigma_{DW} \sim 9 m_a f_{PQ}^2$$

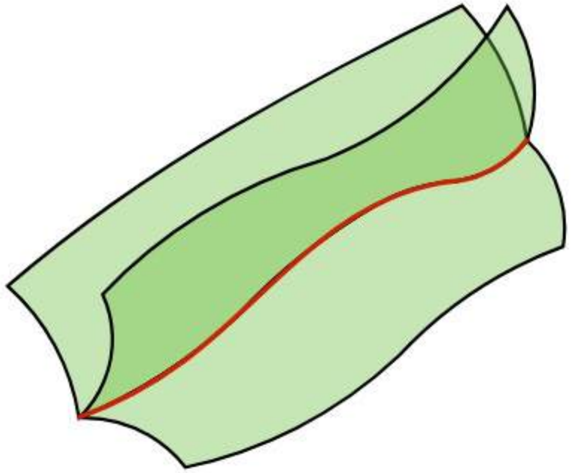
… otherwise the universe would begin to undergo (power-law) inflation without end!

A *stronger* constraint follows from accounting for the energy dumped by wall decay

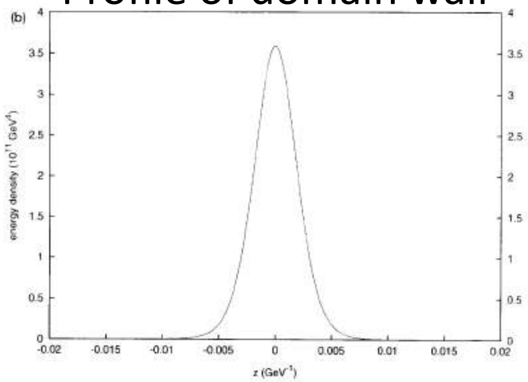
$U(1) \to \mathbb{Z}_N$



$$V(a) = m_a^2 f_{PQ}^2 [1 - \cos(N_{DW}\, a)]$$



Profile of domain wall

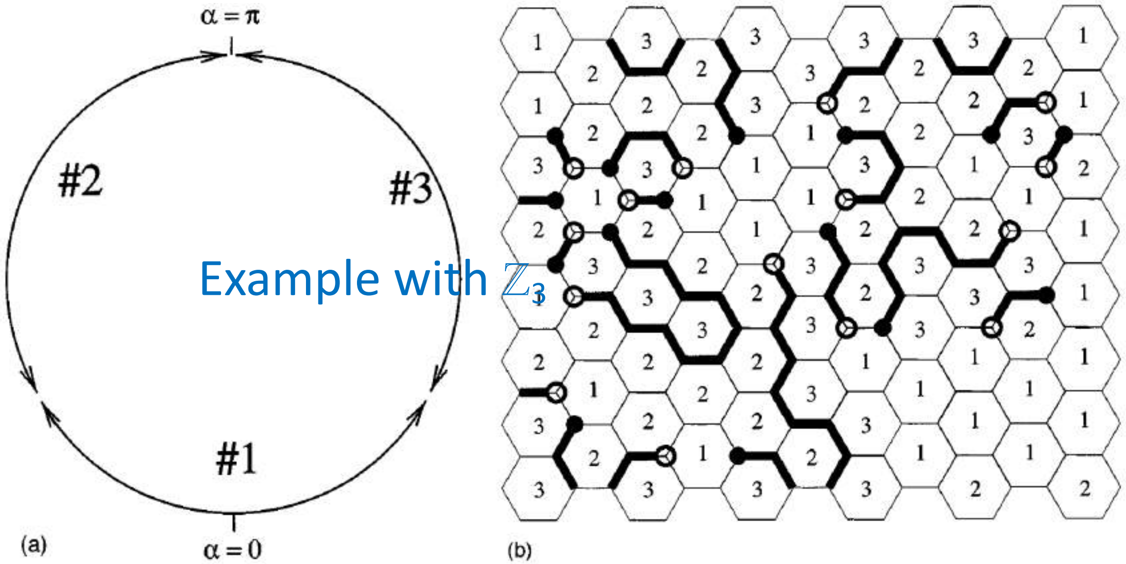


In the 'scaling' regime, there is $O(1)$ wall per Hubble volume

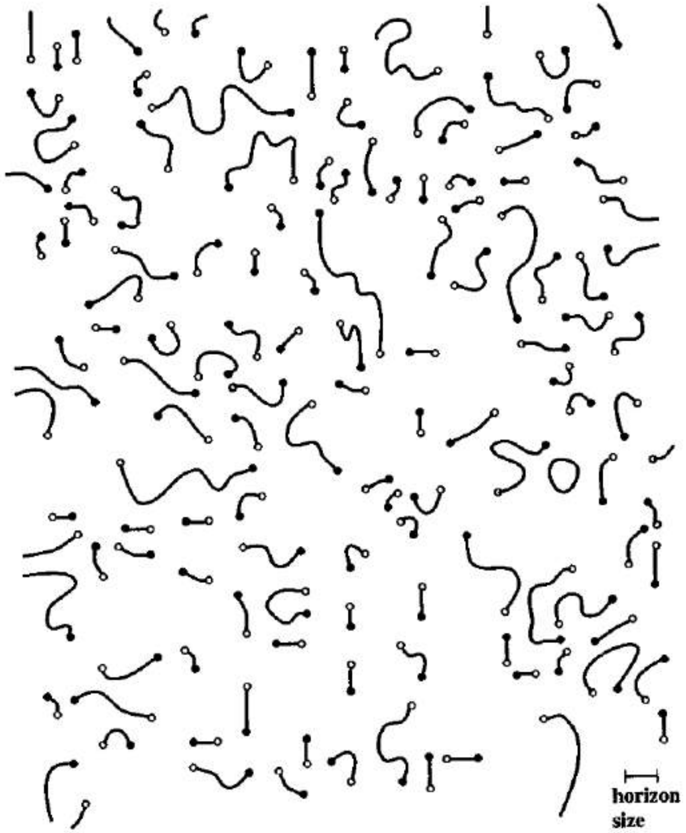
When $H < m_a$, domain walls ending on strings form

$N_{DW} = 1$ String network disappears soon after

$N_{DW} > 1$ String/domain wall network survives



Example with $\mathbb{Z}_3$



Chang *et al*, *Phys.Rev.* D**59**:023505,1998

Classical equation of motion for a scalar field in an expanding background

$$\frac{\partial^2\phi}{\partial\eta^2}+2\frac{d\ln a}{d\ln\eta}\frac{1}{\eta}\frac{\partial\phi}{\partial\eta}-\nabla^2\phi=-a^2\frac{\partial V}{\partial\phi}\ ,\quad V(\phi)=V_0\left(\frac{\phi^2}{\phi_0^2}-1\right)^2$$

$d\eta\equiv dt/a(t)$ conformal time

However the a^2 term on the right-hand side makes the potential barrier appear higher as time goes on, resulting in a wall solution which grows increasingly narrow in comoving cordinates!

The solution (Press, Spergel & Ryden, *ApJ* **347**:590,1989) is to generalize the EoM to:

$$\frac{\partial^2\phi}{\partial\eta^2}+\alpha\frac{d\ln a}{d\ln\eta}\frac{1}{\eta}\frac{\partial\phi}{\partial\eta}-\nabla^2\phi=-a^\beta\frac{\partial V}{\partial\phi}\ ,$$

with $\beta = 0$ and $\alpha = 3$ to ensure momentum conservation

(Check that simulation gives similar results, over a limited range in η, as for: $\alpha = \beta = 2$)

We find that the wall network exhibits "Kibble scaling" ⇒ correlation length $\xi = v\eta$, and the comoving area of the wall network per unit volume scales as:

$$(A/V)\simeq(A/V)_0\frac{\xi_0}{\xi}\ \propto\eta^{-\nu},$$

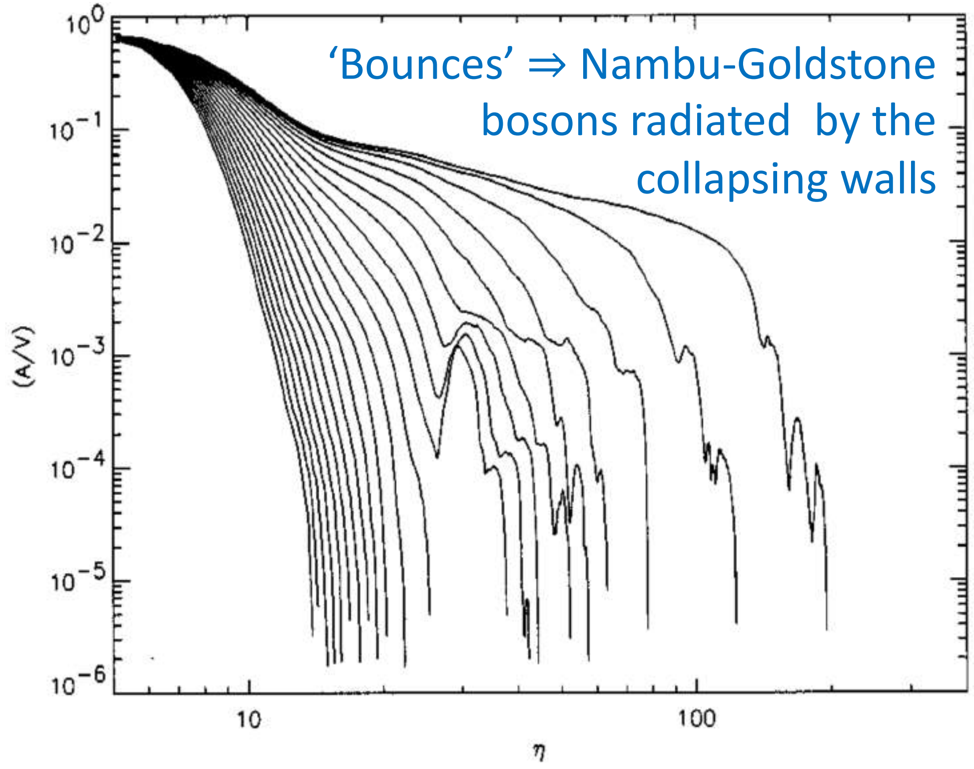
where $\nu \simeq 1$ (0.92 ± 0.08 on a 128^3 grid)

Larsson, S.S. & White, *Phys.Rev.D* **55**:5129,1997

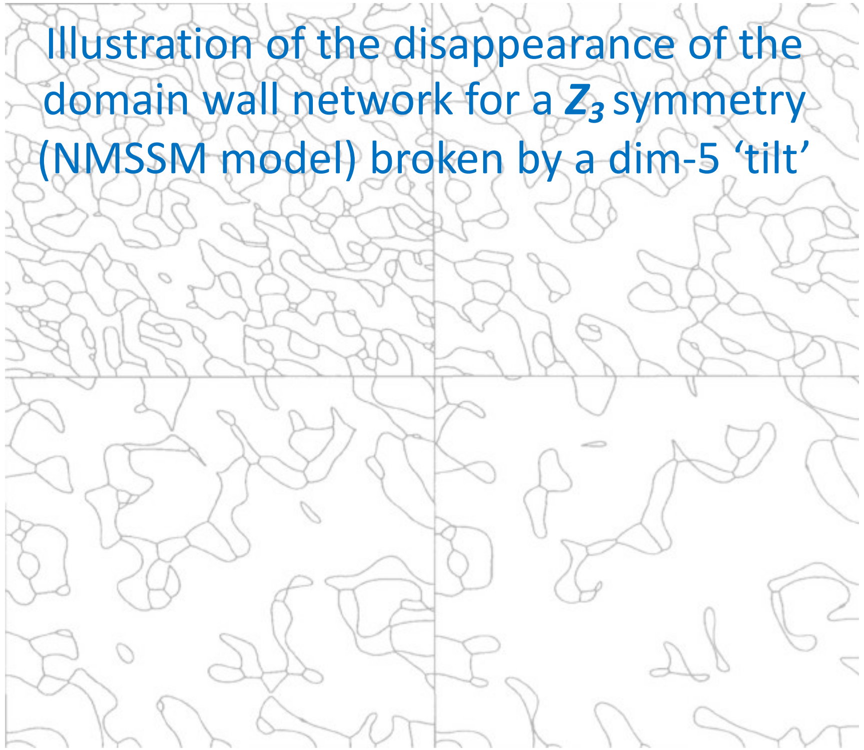
This result also follows from relativistic generalisation of an analytic model describing dynamics of topological defects in condensed matter (Hindmarsh, *PRL* **77**:4495,1996)

Our numerical simulations show that a 'tilt' μ in the potential causes an *exponential* decay of the wall network (from the scaling solution)

$$\rho_{\rm DW} \propto \frac{\sigma_{\rm DW}}{\eta} \exp\left[-\mu^3 \left(\frac{\eta}{\eta_{\rm DW}}\right)^3\right]$$



Larsson, S.S. & White, *Phys.Rev.D* **55**:5129,1997



Abel, White & S.S., *Nucl.Phys.B* **454**:663,1995

The collapse of the wall network averts cosmological disaster

… the higher the 'tilt', the sooner will this happen (e.g. before $t_{\rm BBN}$ or $t_{\rm QCD}$)

Discrete (global) symmetries are *violated* by quantum gravity effects … this naturally induces a 'tilt' in the axion potential due to Planck scale suppressed operators

$$\delta V_{M_{\rm Pl}} = \frac{|g|\, e^{i\delta}}{M_{\rm Pl}^{2m+n-4}} |\phi|^{2m} \phi^n$$

After $U(1)_{\rm PQ}$ breaks spontaneously and the complex PQ field acquires a vev $v_a = N_{\rm DW} f_{\rm PQ}$,

$$V(a) = m_a^2 f_{\rm PQ}^2 \left[(1 - \cos(N_{\rm DW} a)) + \mu \left(1 - \cos\left(na + \delta\right)\right)\right]$$

where: $\mu \equiv |g|\, (M_{\rm Pl}/m_a)^2\, (f_{\rm PQ}/\sqrt{2}M_{\rm Pl})^{2m+n-2}$

A *lower* limit on μ comes from requiring the wall network to collapse soon enough to avert cosmological disaster; e.g. requiring this to happen by $t_{\rm BBN} \sim 1$ s requires:

$$|g| \left(\frac{f_{\rm PQ}}{\sqrt{2} M_{\rm Pl}} \right)^{2m+n-1} > 1.2 \times 10^{-83}$$

But $\mu \neq 0$ reintroduces the strong-CP problem so is constrained by the neutron EDM!

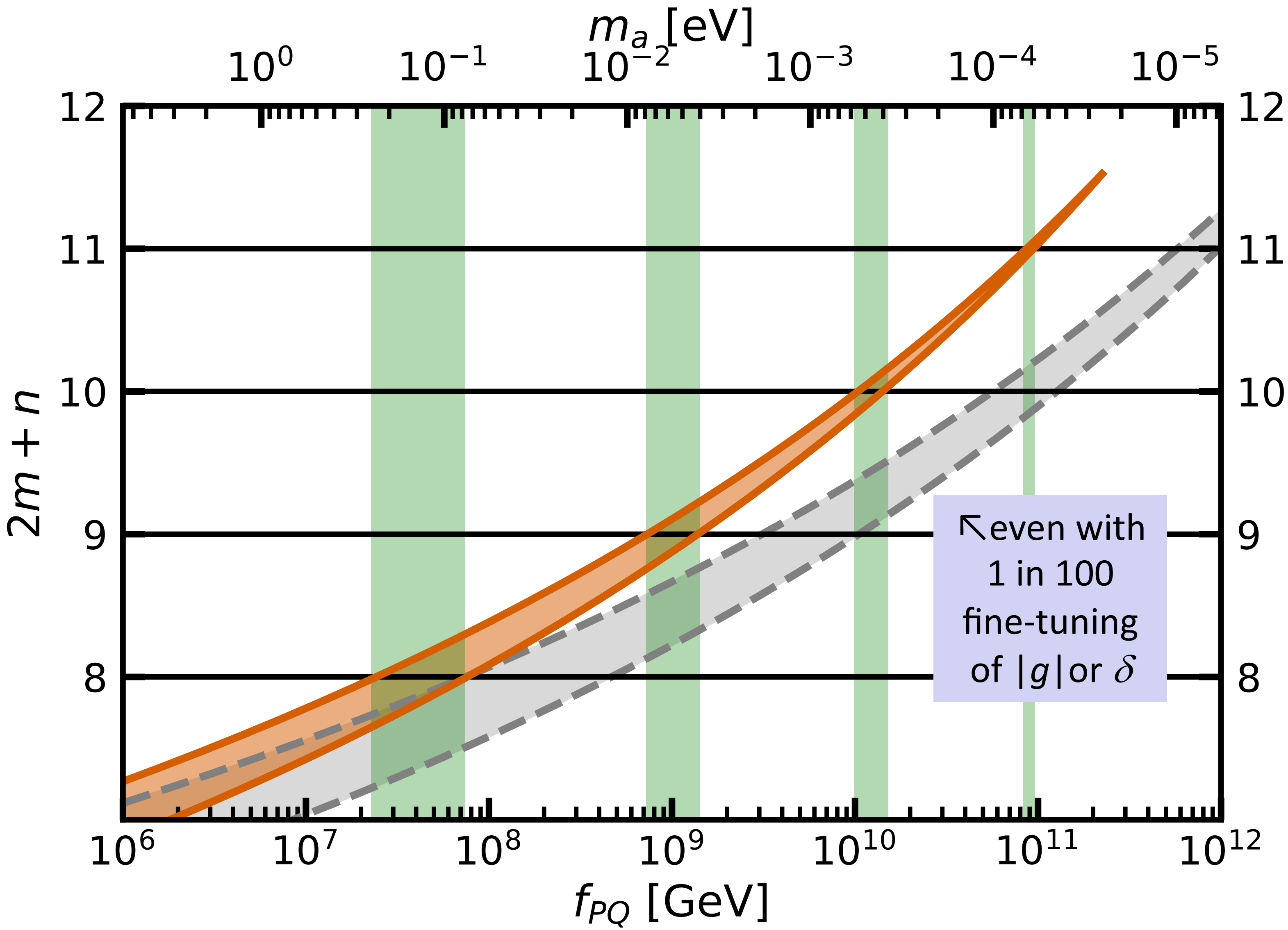
$$\langle \theta \rangle \simeq |g| \left(\frac{M_{\rm Pl}}{m_a} \right)^2 \left(\frac{f_{\rm PQ}}{\sqrt{2} M_{\rm Pl}} \right)^{2m+n-2} \frac{n}{N_{\rm DW}} \sin\delta, < 10^{-10}$$

$$|g| \left(\frac{f_{\rm PQ}}{\sqrt{2} M_{\rm Pl}} \right)^{2m+n} \frac{n}{N_{\rm DW}} < 1.6 \times 10^{-91}$$

Beyer & S.S *SciPost Physics* **15**:003, 2

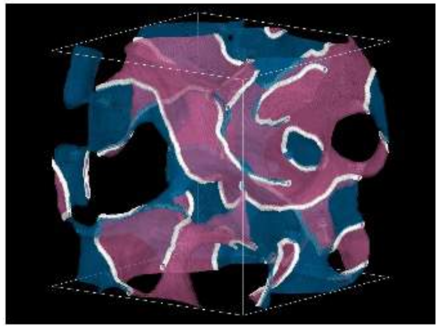
The 'tilt' solution to the axion domain wall problem is therefore severely constrained

$$\frac{\log\left(1.6\times10^{-91}N_{\rm DW}/n\right)-\log\left(|g|\right)}{\log\left(f_{\rm PQ}/\sqrt{2}M_{\rm Pl}\right)} < 2m+n < 1+\frac{\log\left(1.2\times10^{-83}\right)-\log\left(|g|\right)}{\log\left(f_{\rm PQ}/\sqrt{2}M_{\rm Pl}\right)}.$$

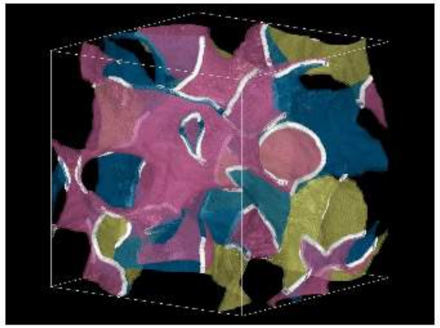


Beyer & S.S *SciPost Physics* **15**:003, 2

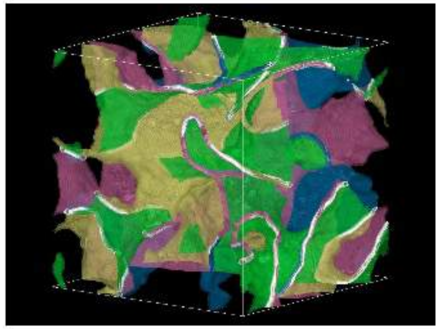
Our previous findings are confirmed by detailed simulations of the collapse of the wall network and the radiation of NGBs and (sub-dominant) gravitational waves



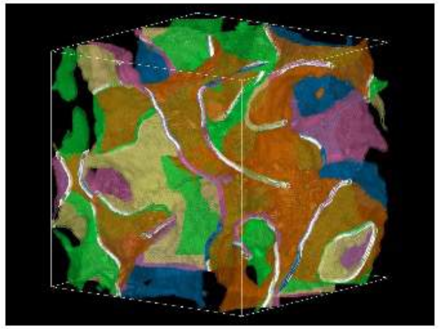
(a) $N_{\mathrm{DW}} = 2$



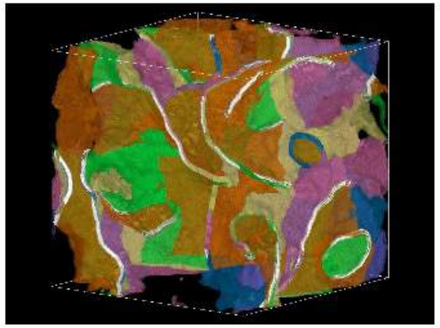
(b) $N_{\mathrm{DW}} = 3$



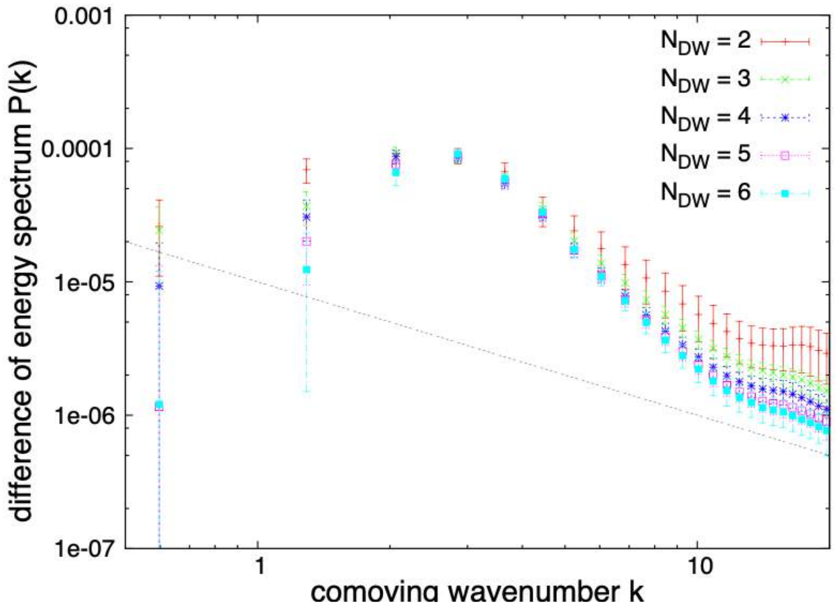
(c) $N_{\mathrm{DW}} = 4$

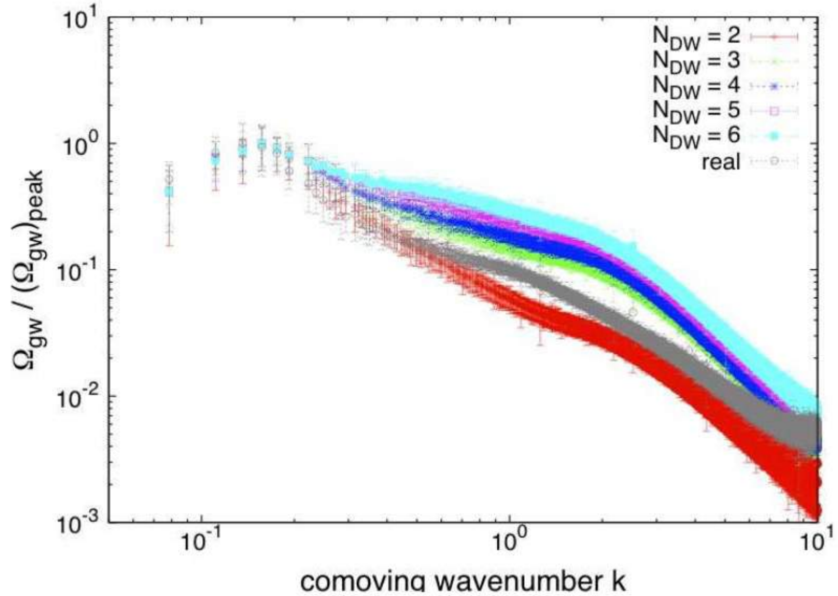


(d) $N_{\mathrm{DW}} = 5$



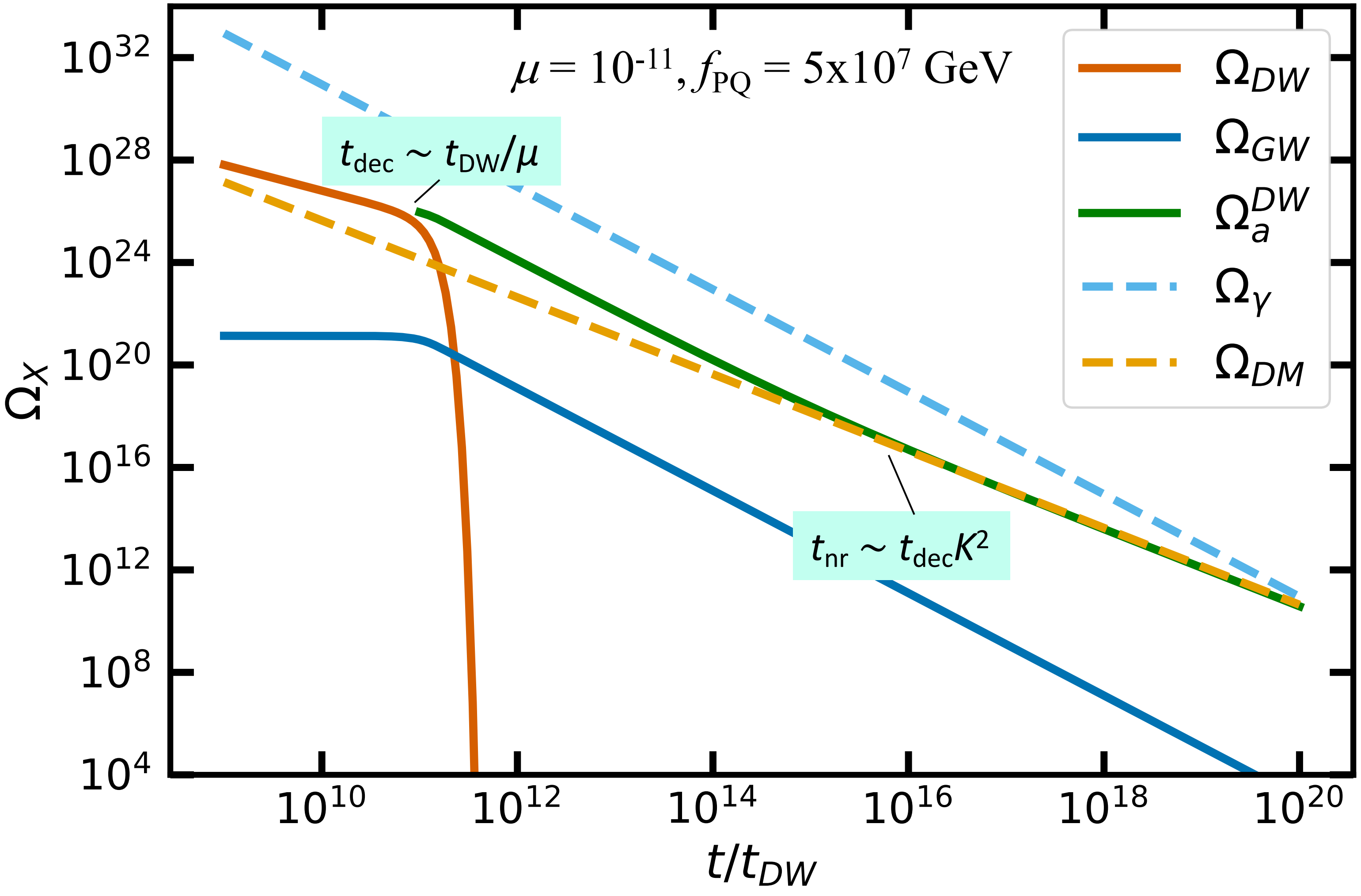
(e) $N_{\mathrm{DW}} = 6$





Hiramitsu, Kawasaki, Saikawa & Sekiguchi, *JCAP* **01**:001,2013

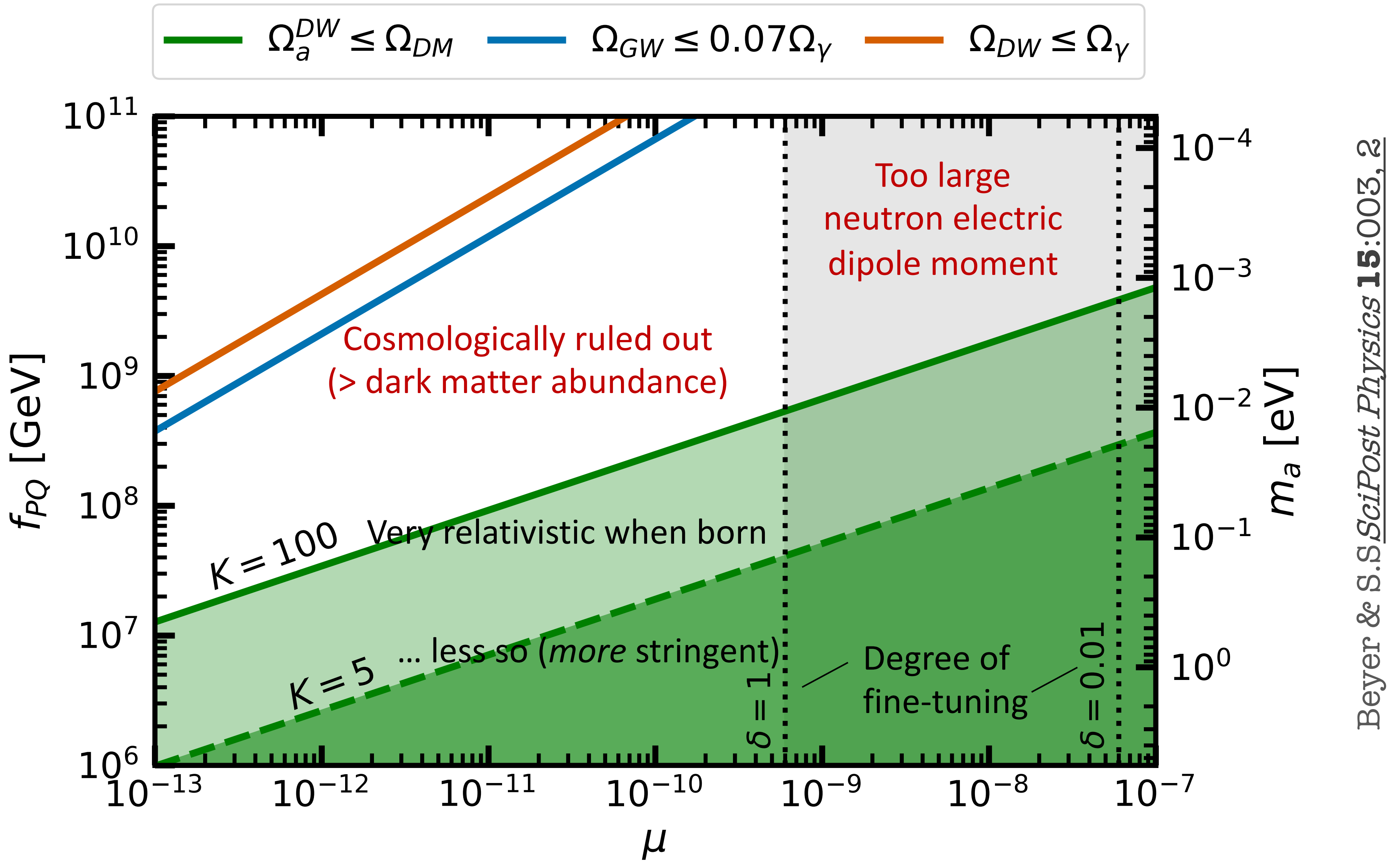
The decay of the axion wall network generates non-thermal axions which are initially relativistic ($K \equiv \mathrm{K.E.}/m_a \gg 1$), but subsequently turn non-relativistic



Beyer & S.S *SciPost Physics* **15**:003, 2

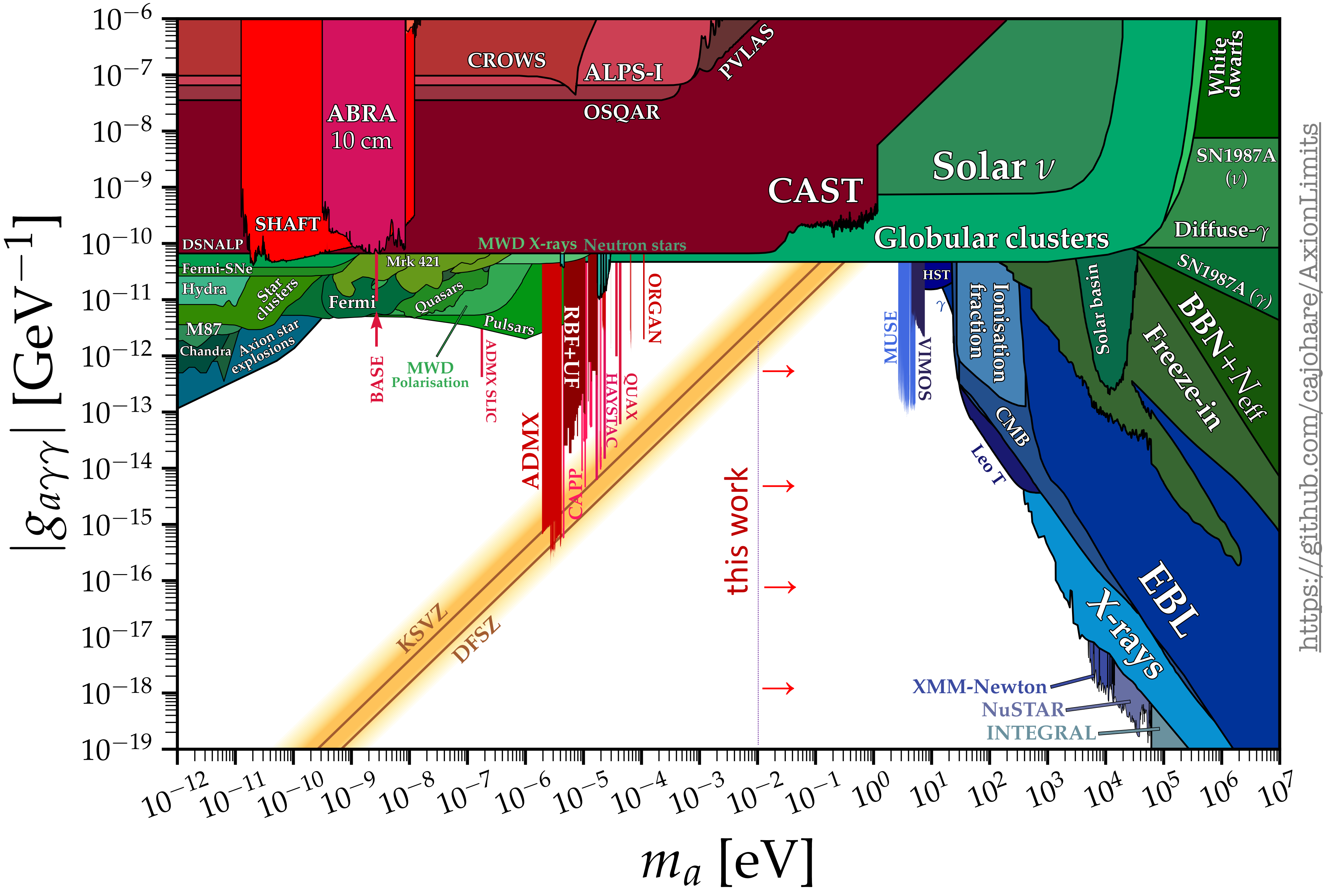
This requires the walls to decay *very* soon after they are born (at $T \sim \Lambda_{QCD}$) in order that the relic abundance of decay axions does not exceed that of DM

But if the 'tilt' is large enough to make the wall network decay cosmologically safe, then the breaking of the $\mathbb{Z}_N$ symmetry may violate the neutron EDM bound: $\langle\theta\rangle < 10^{-10}$



This implies the limit: $\boldsymbol{f_{PQ} \lesssim 5.4\times10^8}$ **GeV** $x_a^{6/7} \Rightarrow \boldsymbol{m_a \gtrsim 11}$ **meV** $x_a^{-6/7}$ (taking $N_{DW} = 6$, $x_a \Rightarrow$ fraction of DM)

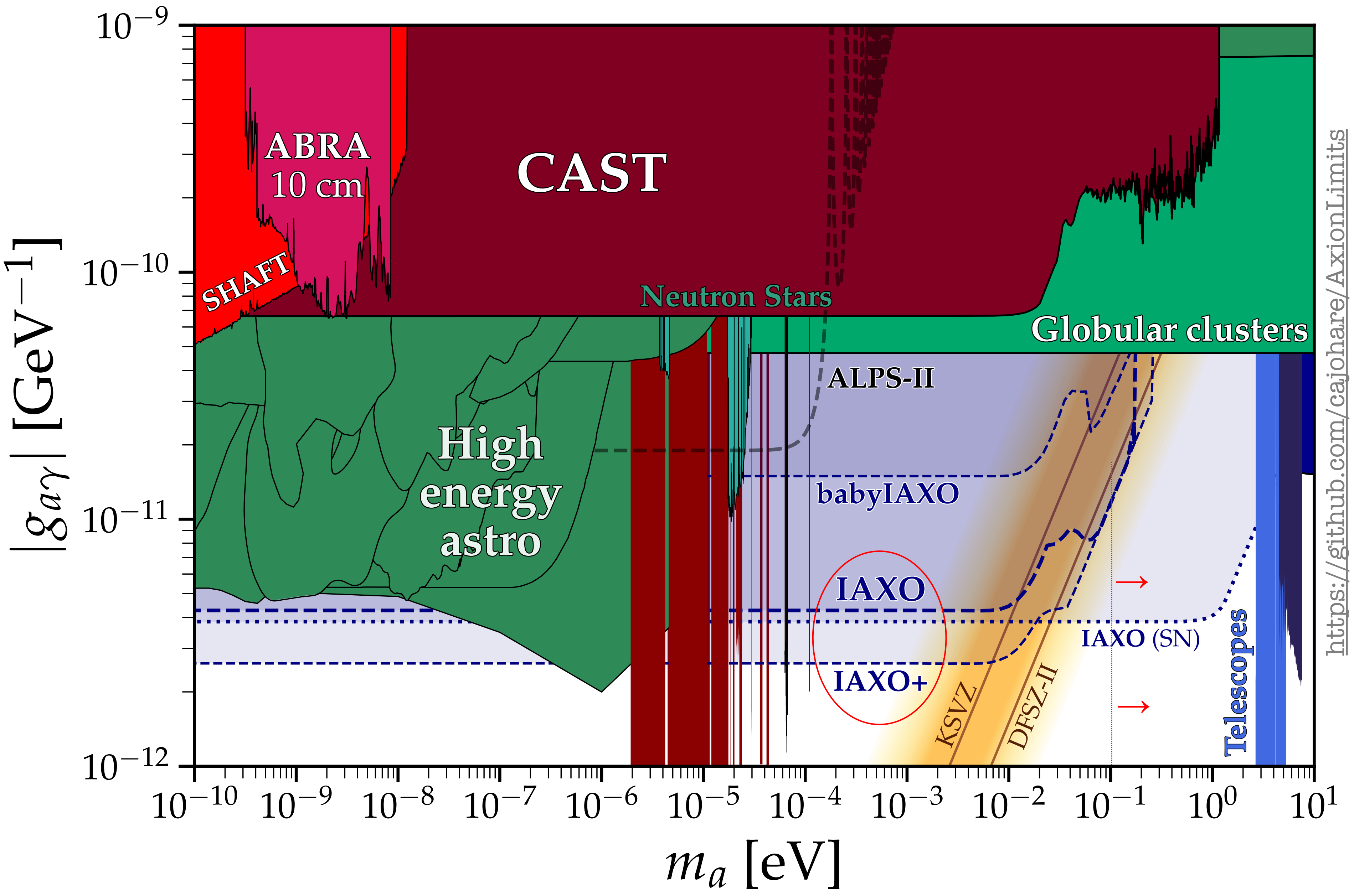
LIMITS & PROJECTIONS ON AXIONS (AND ALPs)



This strengthens the case for experiments which probe QCD axions in mass range ~ 0.01-1 eV

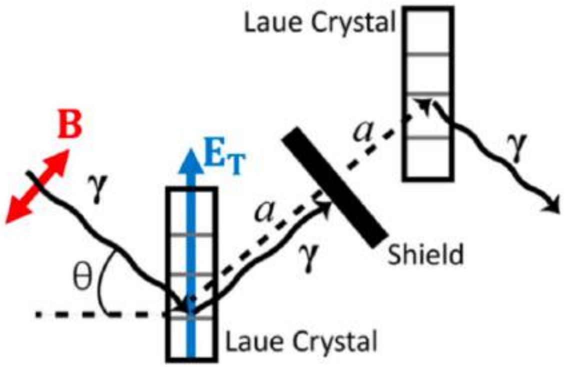
Such heavy axions are also being targeted by a laser experiment at XEFL Hamburg

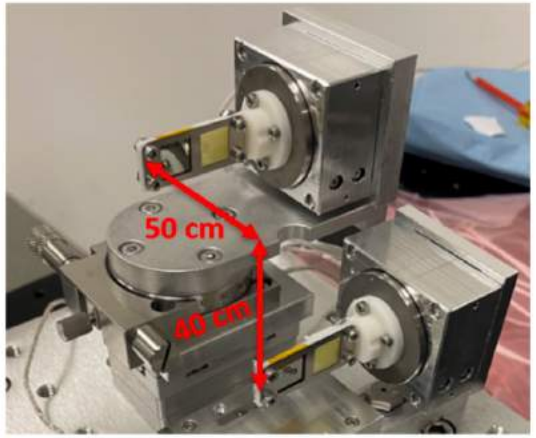
(Beyer *et al.*, *Phys.Rev. D* **101**:095018,2020, *Phys.Rev. D* **105**:035031,2022)

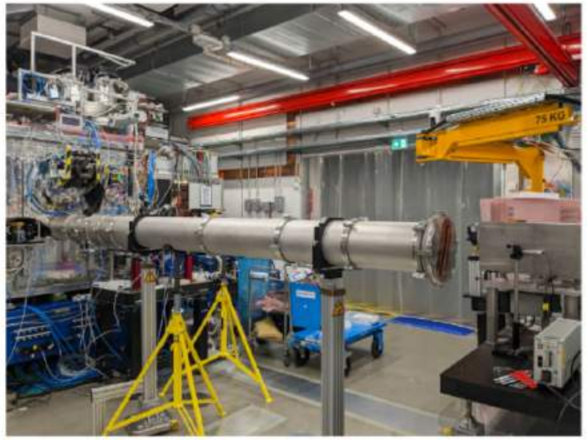


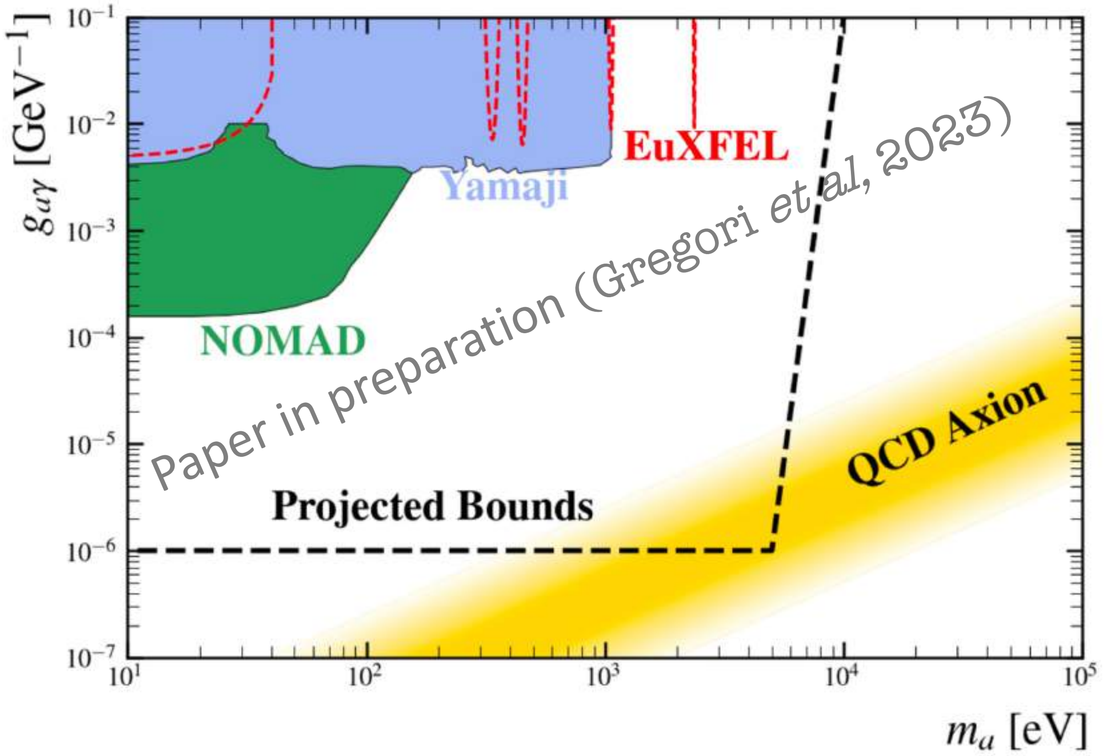
Axion Search using a free electron laser at XFEL Hamburg (HED 3326)

(motivated by Beyer *et al.*, *Phys.Rev. D* **105**:035031,2022)









Summary

- QCD axion models (DFSZ) where the Peccei-Quinn symmetry breaks after inflation have a potential problem with the formation of domain walls.

- The domain wall network will come to dominate the Universe leading to (power-law) inflation with no end … i.e. an *unacceptable* cosmology.

- The walls can be destabilized by non-renormalizable, Planck scale suppressed operators which violate the discrete global symmetry – the domain wall network will then decay away.

- However this generates an (unobserved) electric dipole moment of the neutron, hence such a 'tilt' of the potential is phenomenologically restricted.

- These *opposing* constraints require that $f_{\mathrm{PQ}} \lesssim 5\mathrm{x}10^{8}\,\mathrm{GeV} \Rightarrow m_a \gtrsim 11\,\mathrm{meV}$.

- Direct experiments targeting the axion mass range ~ 0.01-1 eV* are thus particularly relevant, e.g. IAXO and experiments using lasers at XFEL & ELI.

*Such axions are supposedly excluded by SN1987a but may in fact account for observations of anomalous stellar cooling (Gianotti *et al*, *JCAP* **10**:010,2017).