

Parity Domain Walls and the Cosmology of Nelson-Barr

based on [\[2212.03882\]](#)

Qianshu Lu

Institute for Advanced Study and New York University

With Pouya Asadi, Samuel Homiller, and Matthew Reece

The Early Universe: a window to new physics, October 21, 2023

The Strong CP Problem

The quark sector of the Standard Model has two possible sources of CP violation:

- The complex phase δ_{CKM} in the CKM matrix
- $\bar{\theta} = \theta + \arg \det(m_u) + \arg \det(m_d)$, where θ comes from

$$\frac{\theta}{16\pi^2} \int d^4x \text{tr} \left(G_{\mu\nu} G^{\mu\nu} \right)$$

The Strong CP problem: $\delta_{\text{CKM}} \sim \mathcal{O}(1)$ while $\bar{\theta} \lesssim 10^{-10}$

To CP or not CP

Two sectors of solution to the strong CP problem, depending on whether CP(or sometimes just P) is a true symmetry of nature



C/P: δ_{CKM} and $\bar{\theta}$ both $\mathcal{O}(1)$ at tree-level
axion dynamically relaxes $\bar{\theta}$ to zero

[Pecci & Quinn '77]

light degree of freedom
well studied, rich phenomenology

To CP or not CP

Two sectors of solution to the strong CP problem, depending on whether CP(or sometimes just P) is a true symmetry of nature



C/P: δ_{CKM} and $\bar{\theta}$ both $\mathcal{O}(1)$ at tree-level
axion dynamically relaxes $\bar{\theta}$ to zero

[Pecci & Quinn '77]

light degree of freedom
well studied, rich phenomenology

CP: δ_{CKM} and $\bar{\theta}$ both 0 at tree-level
 δ_{CKM} generated from spontaneous
breaking of CP

[Nelson '84, Barr '84]

Phenomenological
consequences less explored

The story/plan in one slide

[McNamara & Reece 2212.00039]

Domain walls from spontaneous breaking of parity are absolutely stable, even when parity is gauged



Scale of $\langle CP \rangle$ tied with scale of inflation

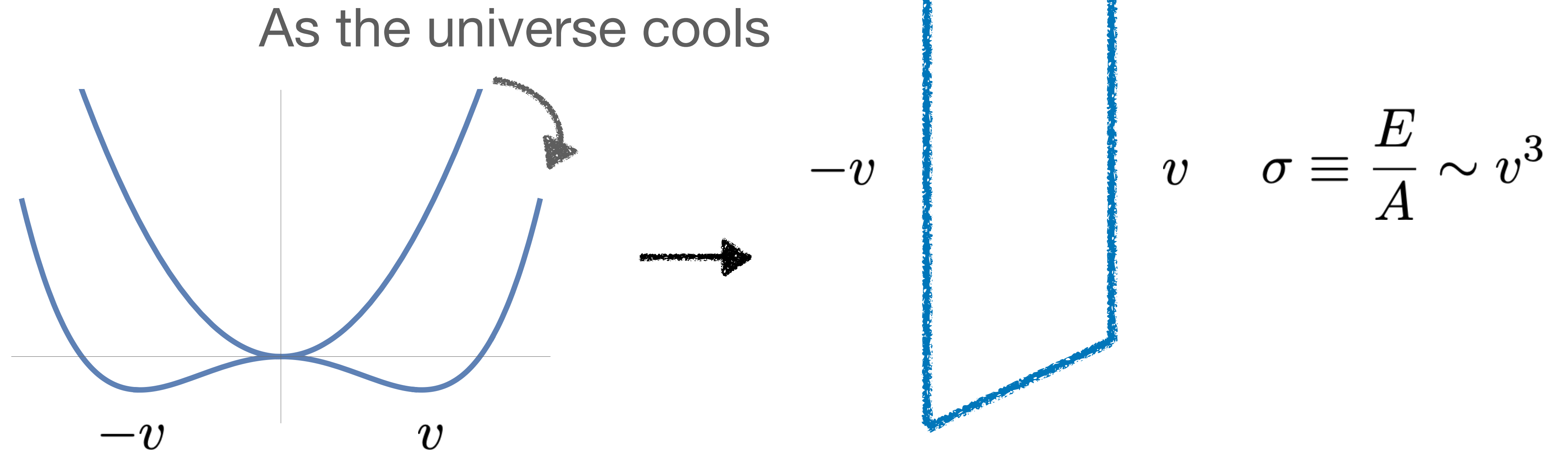


cosmological consequences associated with the “quality” of Nelson-Barr models

Part 1: Stability of Parity Domain Walls

[McNamara & Reece 2212.00039]

Domain walls are dangerous



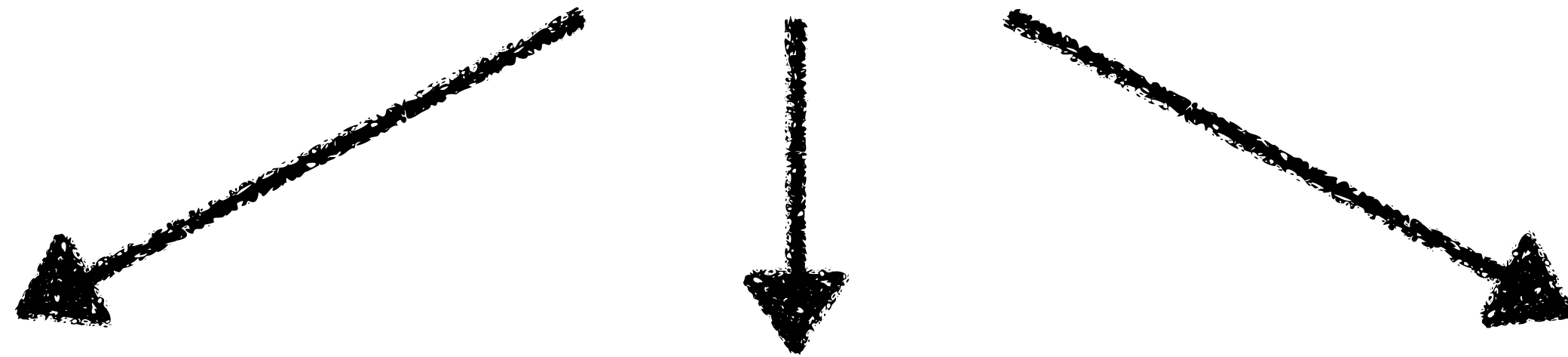
Absolutely stable domain walls will dominate the universe and contradict Standard Cosmology if

$$v \gtrsim 1\text{MeV}$$

Getting rid of domain walls: internal symmetry

Domain walls from spontaneous breaking of internal symmetry

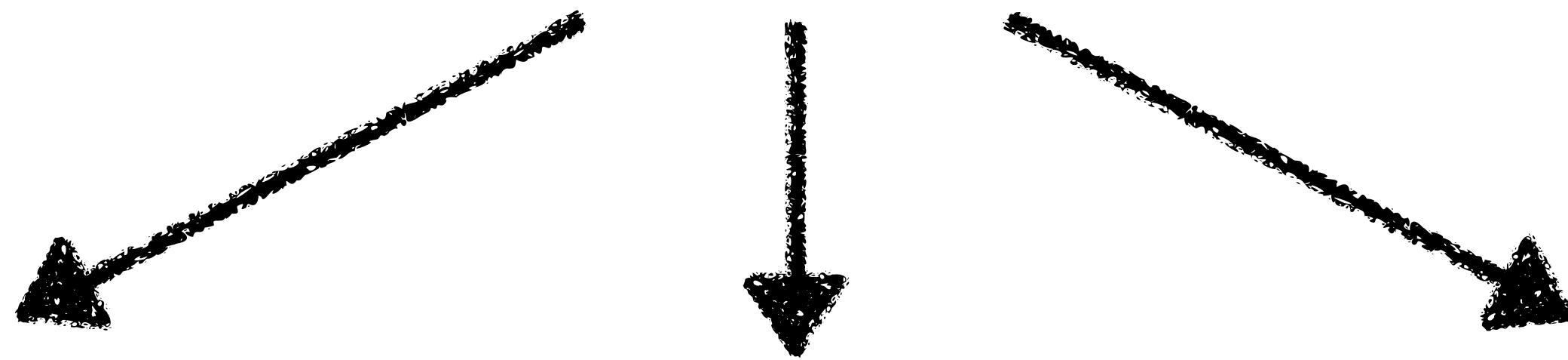
(e.g. $\mathbb{Z}_2 \rightarrow 1$)



Getting rid of domain walls: internal symmetry

Domain walls from spontaneous breaking of internal symmetry

(e.g. $\mathbb{Z}_2 \rightarrow 1$)



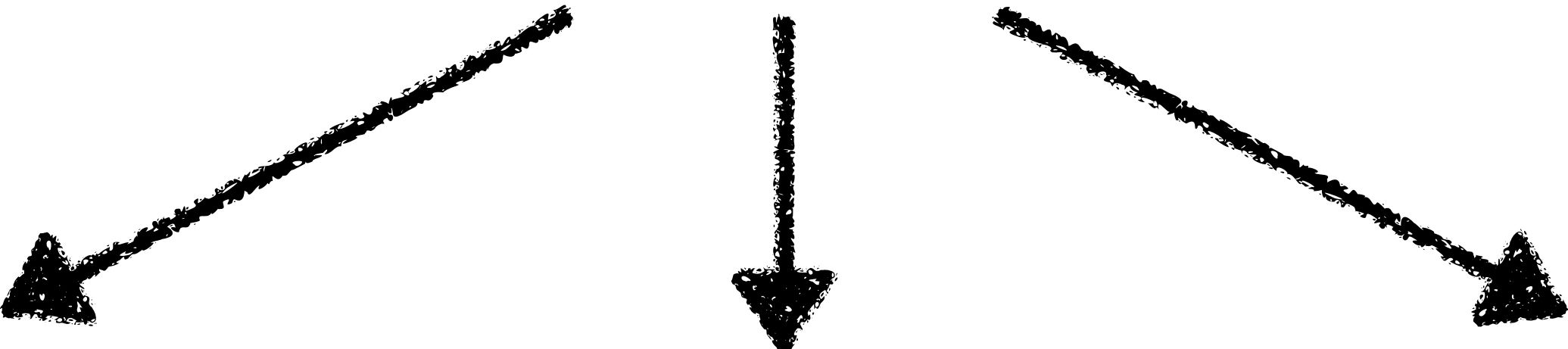
Global: DWs are stable

(if explicit breaking,
needs to be protected)

Getting rid of domain walls: internal symmetry

Domain walls from spontaneous breaking of internal symmetry

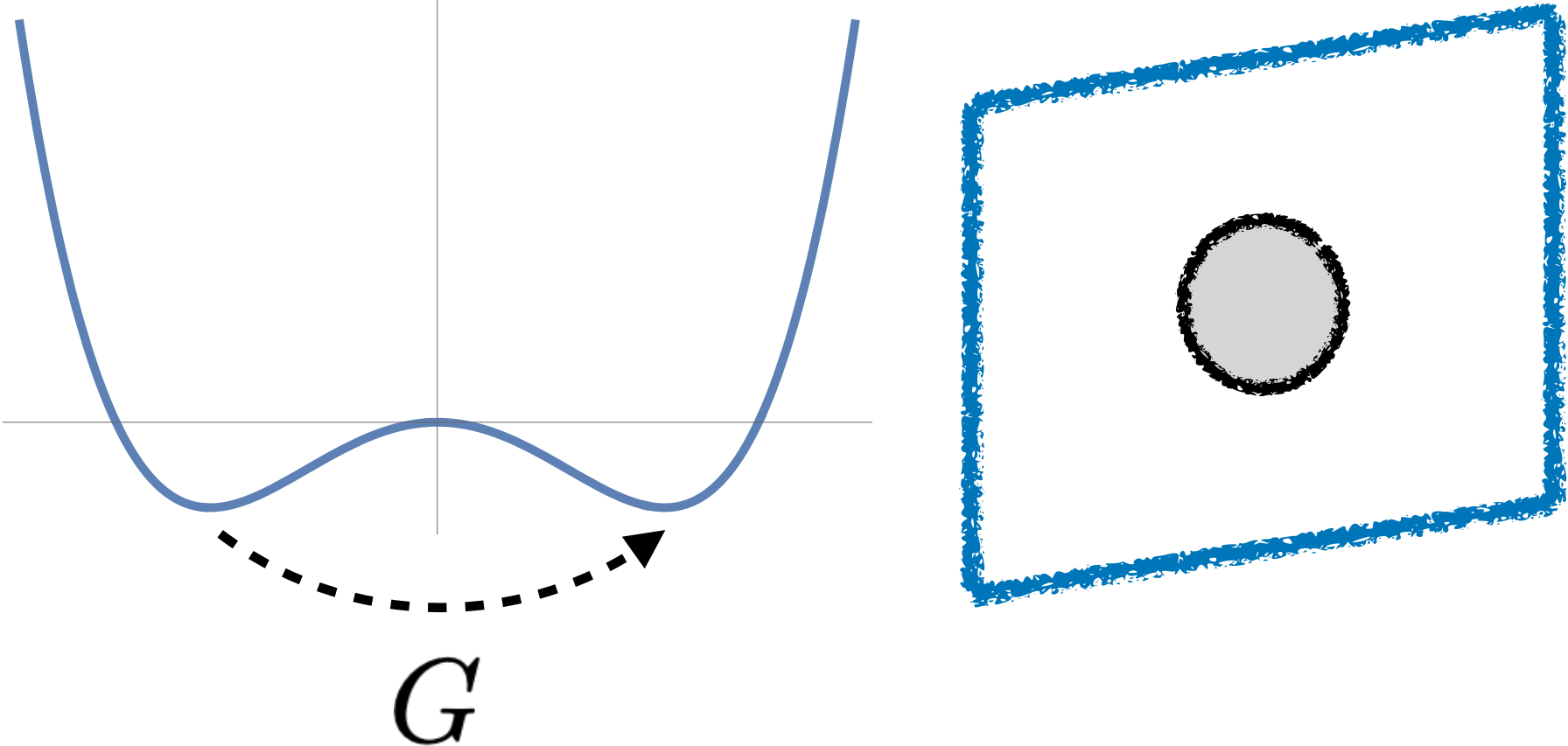
(e.g. $\mathbb{Z}_2 \rightarrow 1$)



Global: ~~DWs~~ are stable

Gauged:
DW destroyed by strings

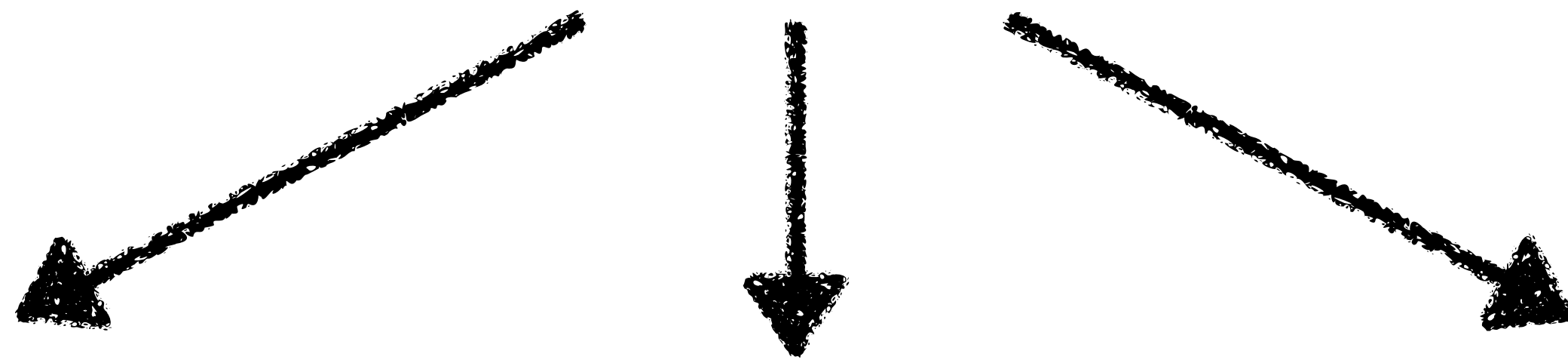
$G \rightarrow \mathbb{Z}_2 \rightarrow 1$



Getting rid of domain walls: internal symmetry

Domain walls from spontaneous breaking of internal symmetry

(e.g. $\mathbb{Z}_2 \rightarrow 1$)

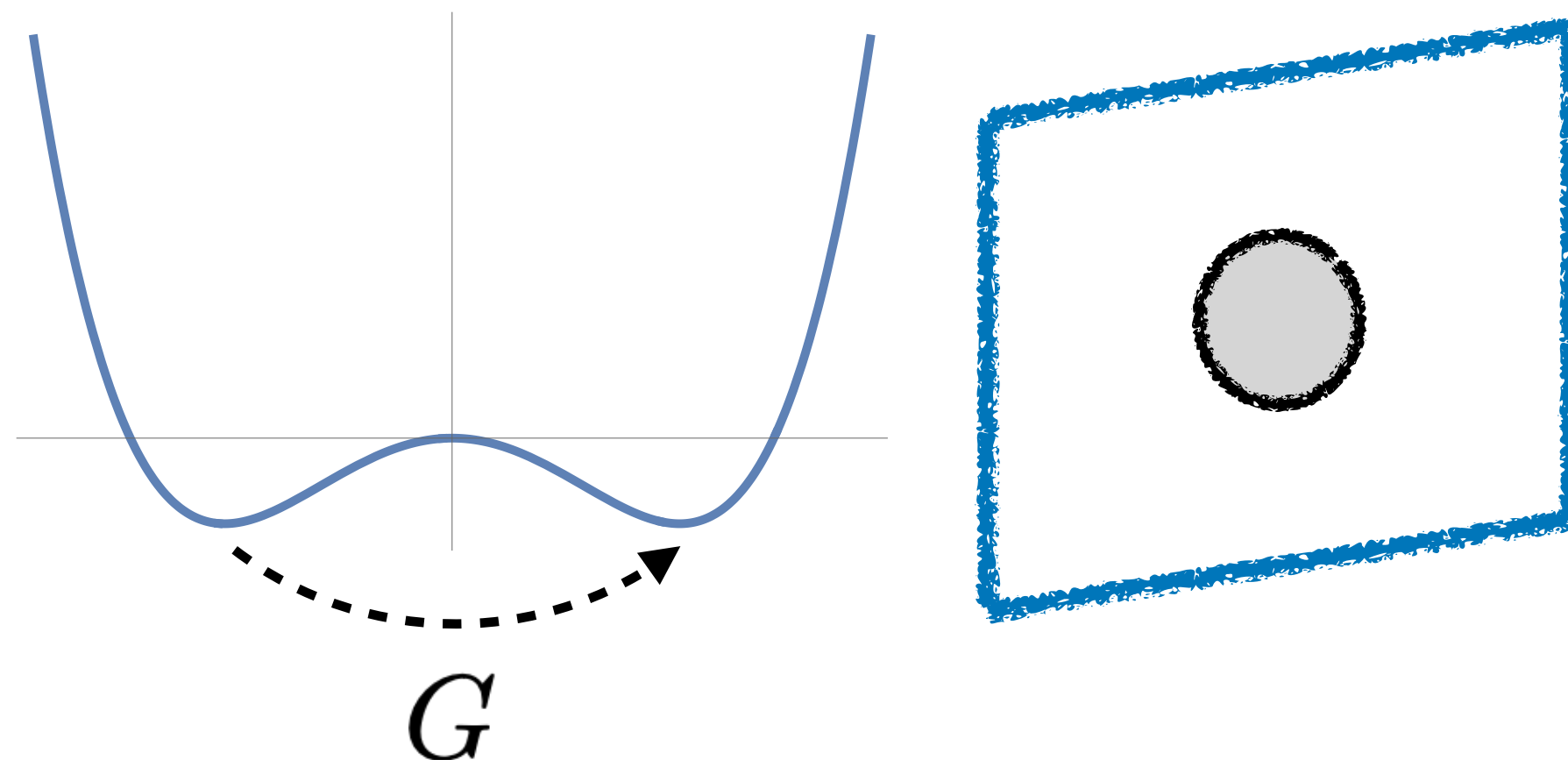


Global: ~~DWs~~ are stable

Gauged:
DW destroyed by strings

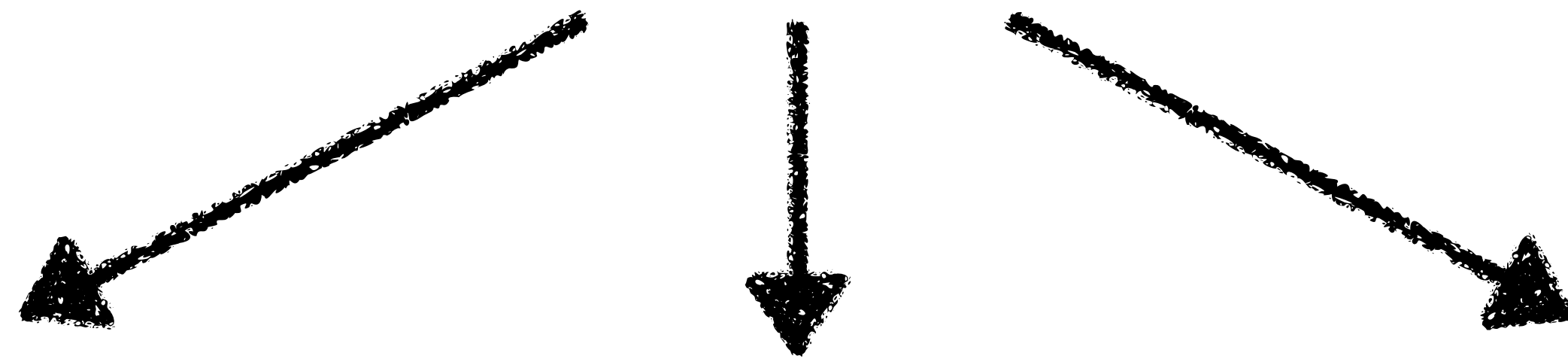
Inflate away

$G \rightarrow \mathbb{Z}_2 \rightarrow 1$



Getting rid of domain walls: Parity symmetry

Domain walls from spontaneous breaking of parity
(or parity with any internal symmetry attached to it)



Global: ~~DWs~~ are stable

Gauged:

Inflate away

[McNamara & Reece 2212.00039]

Parity string does not exist

Internal symmetry vs parity

Internal symmetry: (background) gauge field configuration can be chosen independently of the spacetime manifold

Internal symmetry vs parity

Internal symmetry: (background) gauge field configuration can be chosen independently of the spacetime manifold

Parity: the spacetime manifold **determines** the gauge field configuration

$$g_{i \rightarrow j}^P = \text{sign det} \left(\frac{\partial x_j^\nu}{\partial x_i^\mu} \right) \in \mathbb{Z}_2$$

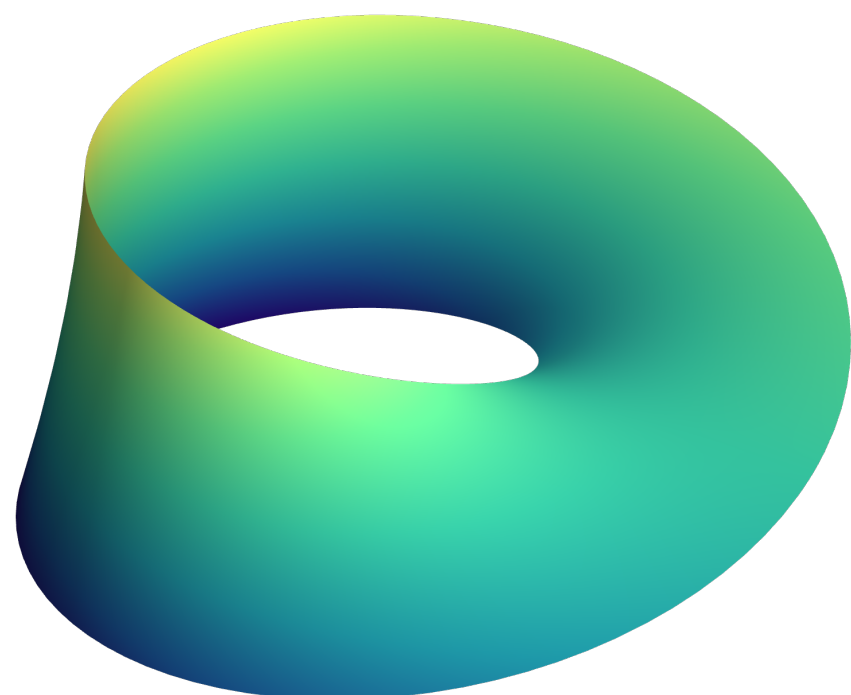
Internal symmetry vs parity

Internal symmetry: (background) gauge field configuration can be chosen independently of the spacetime manifold

Parity: the spacetime manifold **determines** the gauge field configuration

$$g_{i \rightarrow j}^P = \text{sign det} \left(\frac{\partial x_j^\nu}{\partial x_i^\mu} \right) \in \mathbb{Z}_2$$

Non-orientable: Going around a cycle forces a parity transformation



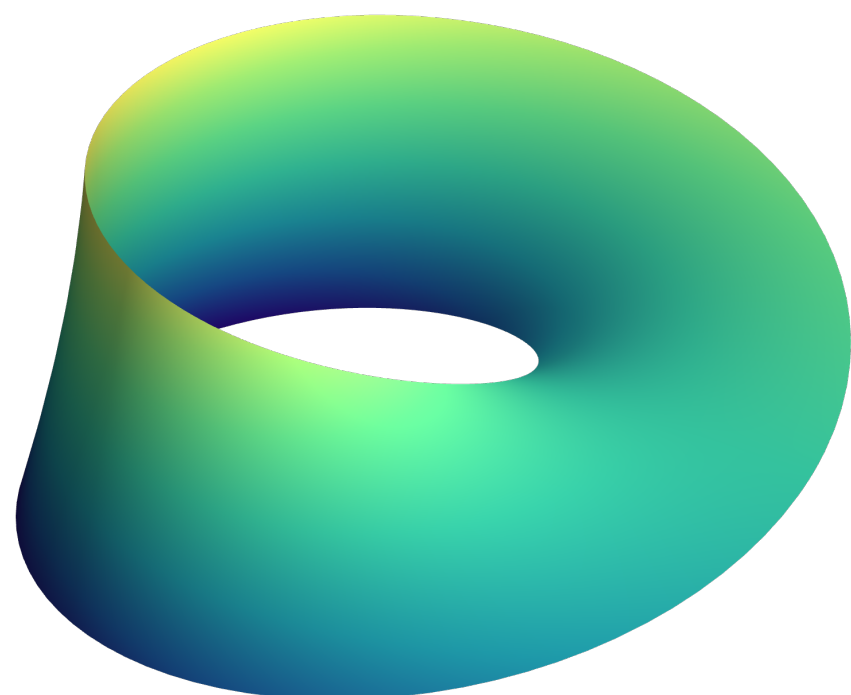
Internal symmetry vs parity

Internal symmetry: (background) gauge field configuration can be chosen independently of the spacetime manifold

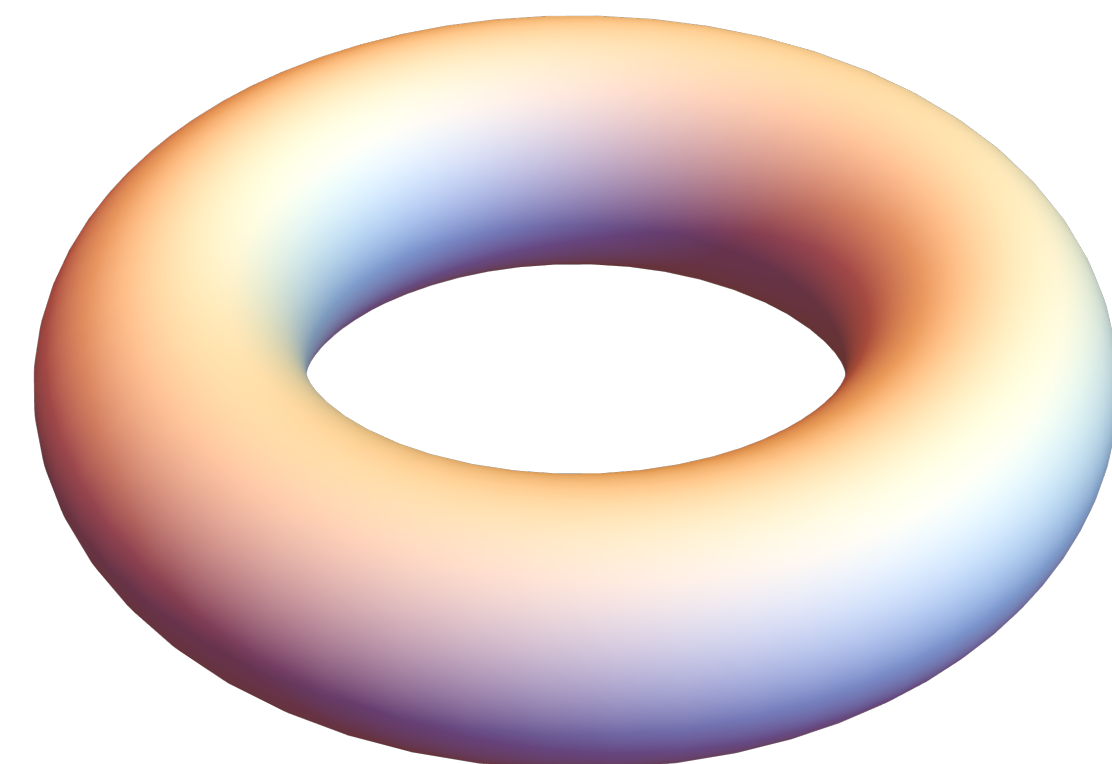
Parity: the spacetime manifold **determines** the gauge field configuration

$$g_{i \rightarrow j}^P = \text{sign det} \left(\frac{\partial x_j^\nu}{\partial x_i^\mu} \right) \in \mathbb{Z}_2$$

Non-orientable: Going around a cycle forces a parity transformation

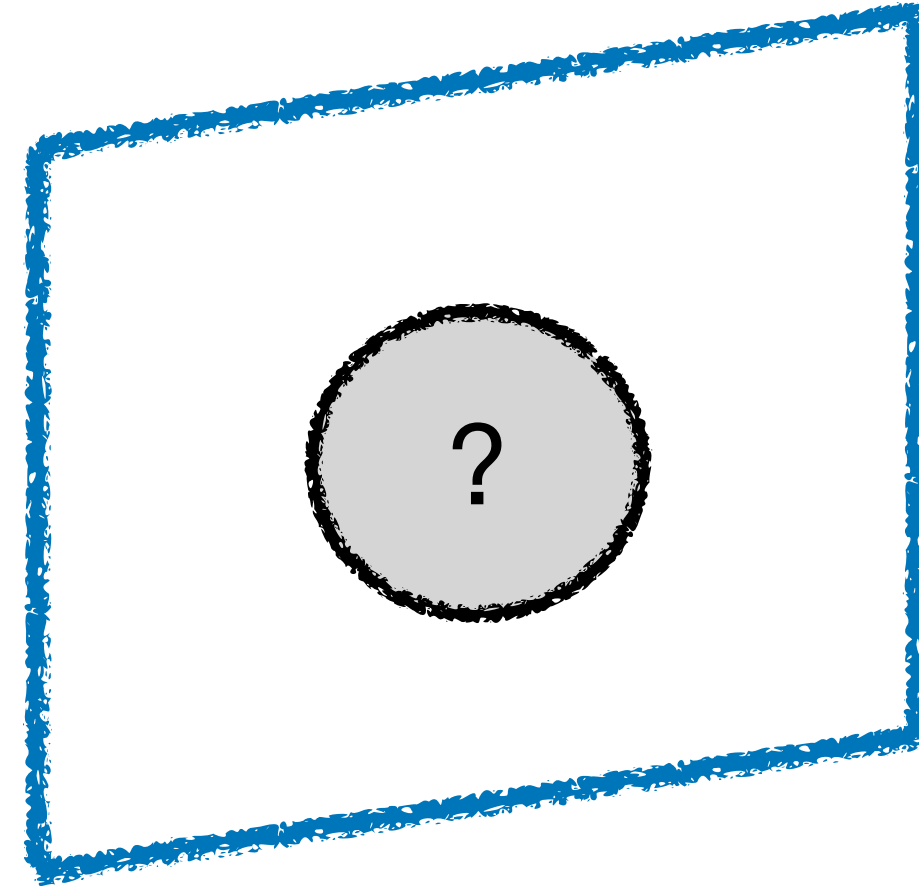


Orientable: No parity transformation along any cycle



Internal symmetry vs parity

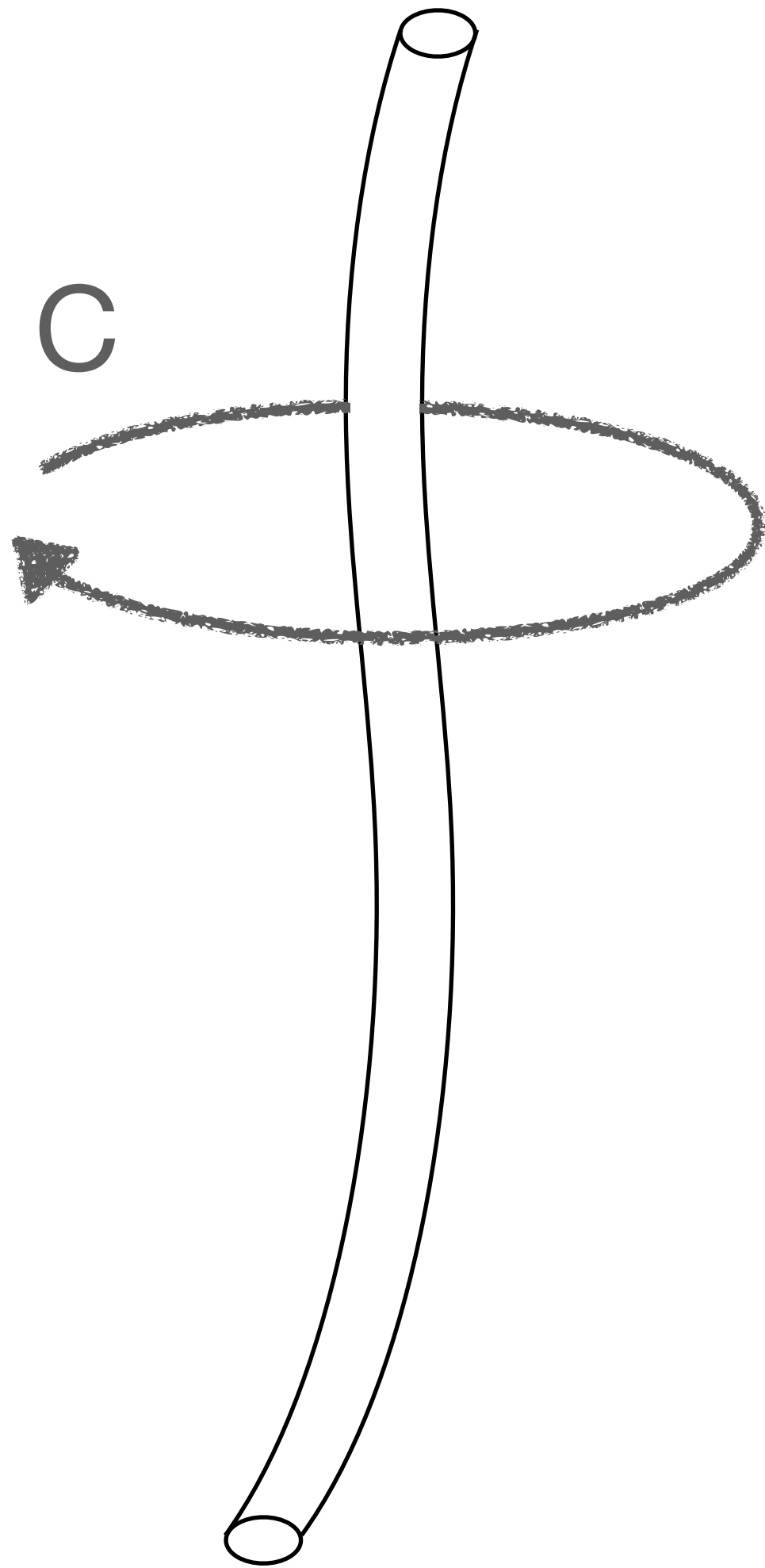
+ orientation



- orientation

To open a hole in a parity domain wall, must go through topology change of the underlying manifold

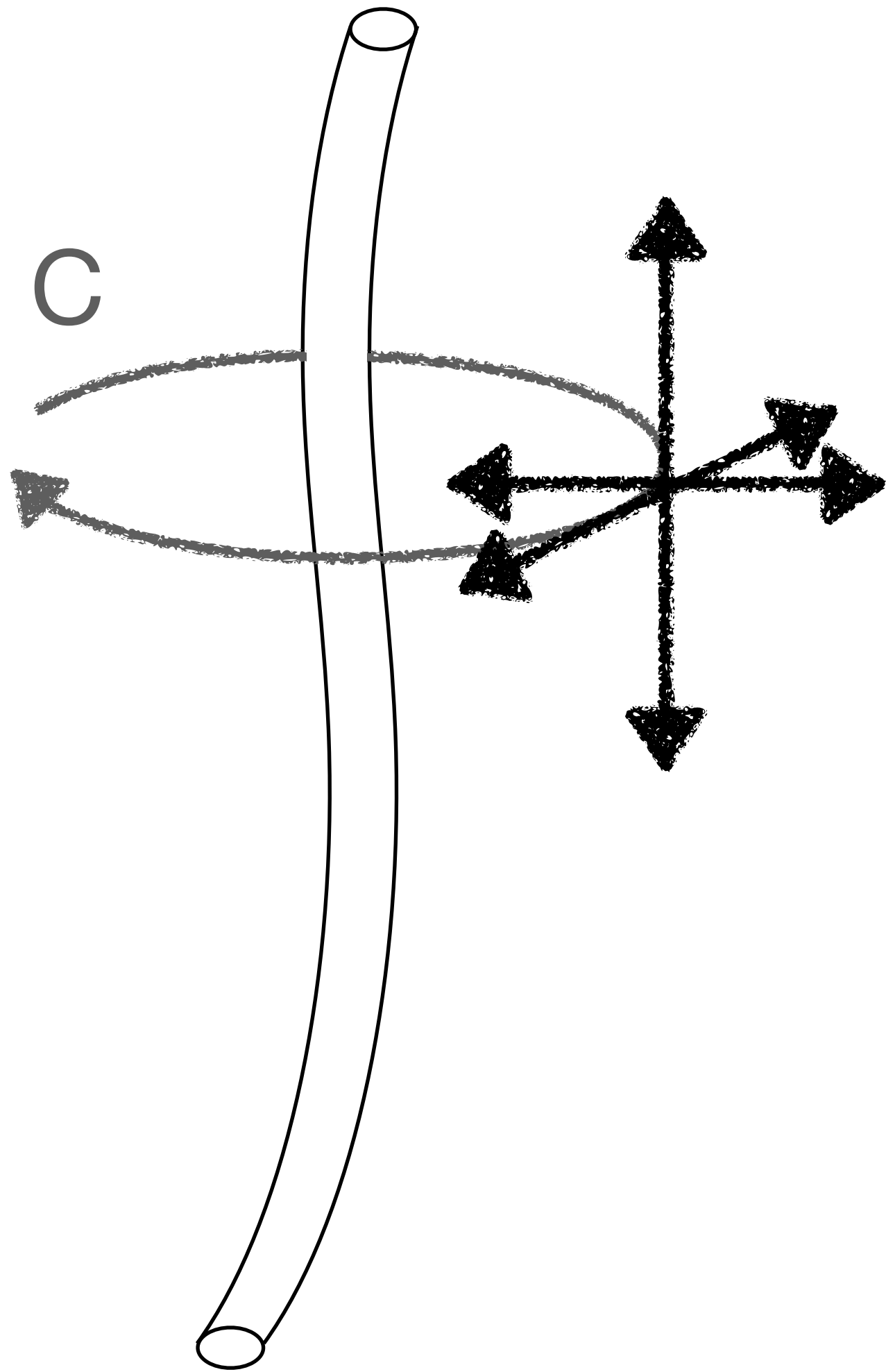
String-like object cannot cause parity flip



Let's assume a parity string exist.

We want the string to implement a parity transformation as we go around it, along trajectory C.

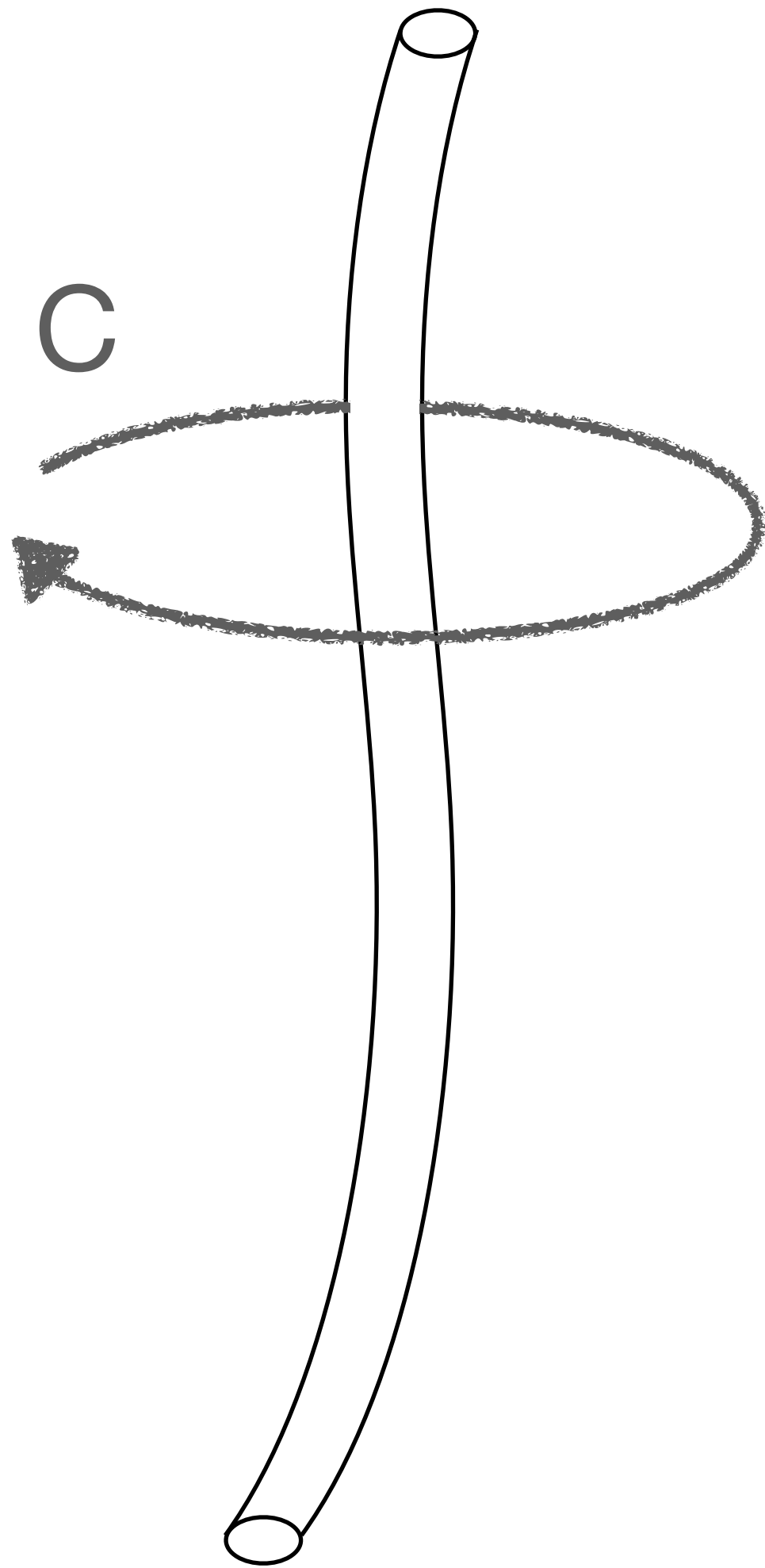
String-like object cannot cause parity flip



Parity transformation in up-down or left-right direction will destroy the string configuration

The only direction left is front-back: but there is no continuous way to do a front-back flip as you traverse a circle

String-like object cannot cause parity flip



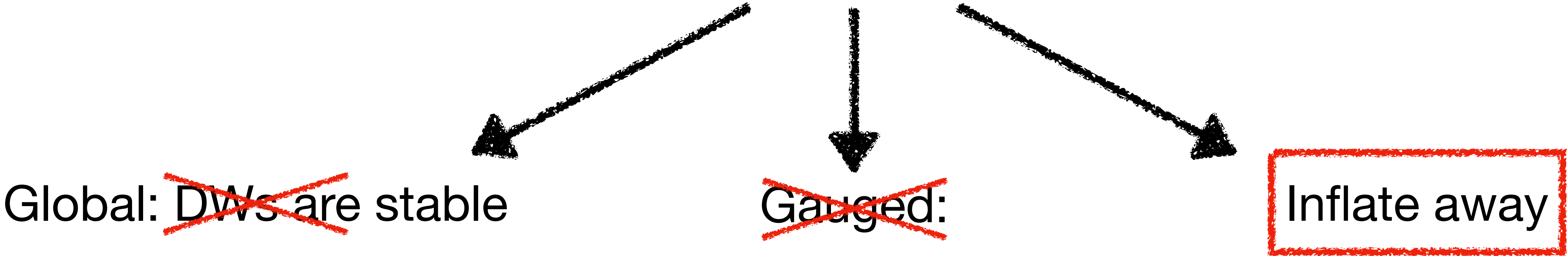
Parity transformation in up-down or left-right direction will destroy the string configuration

The only direction left is front-back: but there is no continuous way to do a front-back flip as you traverse a circle

all closed, 1-dimensional manifolds are orientable,
i.e. parity string cannot exist

Getting rid of domain walls: Parity symmetry

Domain walls from spontaneous breaking of parity
(or parity with any internal symmetry attached to it)



[McNamara & Reece 2212.00039]
Parity string does not exist

Part 2: Nelson-Barr Quality Problem and Cosmology

A Minimal Nelson-Barr construction

Bento, Branco, Parada (BBP) '91

Add to SM a pair of vector-like fields with quantum numbers of bottom-type quarks, $D \bar{D}$ and N pseudoscalars η_a

Assume CP symmetry, and a discrete symmetry \mathbb{Z}_N

$$\eta_a \rightarrow e^{2\pi i k/N} \eta_a, \quad D \rightarrow e^{-2\pi i k/N} D, \quad \bar{D} \rightarrow e^{2\pi i k/N} \bar{D}$$

The down-type quark Lagrangian is then

$$\mathcal{L} \supset \mu_D \bar{D} D + (\lambda_d)_j^i Q_i H^c \bar{d}^j - f_i^a \eta_a D \bar{d}^i + \text{h.c}$$

where all parameters are real because of CP

$Q_i H^c \bar{D}$ $\eta_a \bar{D} D$ terms forbidden because of \mathbb{Z}_N

A Minimal Nelson-Barr construction

Bento, Branco, Parada (BBP) '91

Assume that the scalar potential generates complex vevs, $\langle \eta_a \rangle$

$$\text{At tree level, } \mathcal{L} \supset (Q \quad D) \begin{pmatrix} \lambda_d v / \sqrt{2} & 0 \\ \sum_a f_i^a \langle \eta_a \rangle & \mu_D \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{D} \end{pmatrix}$$

$$\equiv (Q \quad D) \mathcal{M}_0 \begin{pmatrix} \bar{d} \\ \bar{D} \end{pmatrix}$$

$$\Rightarrow \bar{\theta} = \theta + \arg \det \mathcal{M}_0 = 0 \quad \checkmark$$

Effective mass matrix for the Standard Model quarks:

$$(m_0^2)_j^i = (m_d)_k^i \left(\delta_l^k + \frac{F^{\dagger k} F_l}{F_p F^{\dagger p} + \mu_D^2} \right) (m_d^T)_j^l, \quad F_i \equiv \sum_a f_i^a \langle \eta_a \rangle$$

$\mathcal{O}(1)$ complex phase

The Nelson-Barr “Quality” Problem

Corrections from higher-dimensional operators to $\bar{\theta}$:

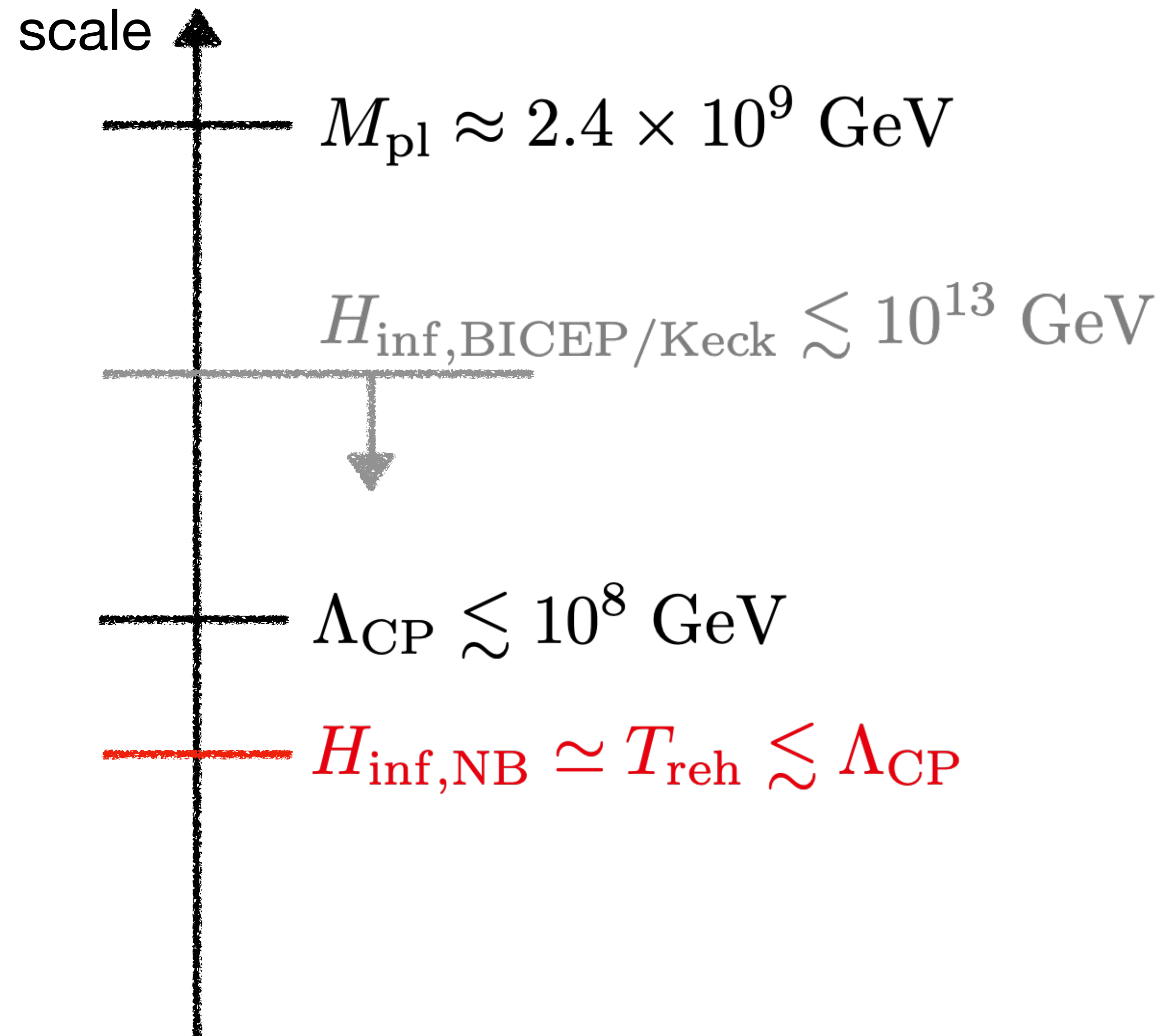
$$\frac{1}{\Lambda_{\text{EFT}}} \eta_a^\dagger \eta_b \bar{D} D, \quad \frac{1}{\Lambda_{\text{EFT}}} \eta_a^\dagger Q_i H^c \bar{D}$$

From dimensional analysis: $\Delta \bar{\theta} \sim \frac{\Lambda_{\text{CP}}}{\Lambda_{\text{EFT}}}$

CP breaking scale cannot be too large:

$$\Lambda_{\text{EFT}} = M_{\text{pl}} \Rightarrow \Lambda_{\text{CP}} \lesssim 10^8 \text{ GeV}$$

Inflation scale must be lower



Cosmological implications of $H_{\text{inf}} \lesssim 10^8 \text{ GeV}$

Inflation model building

Tensor-to-scalar ratio directly measure the scale of inflation:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \approx 10^8 \frac{H_{\text{inf}}^2}{M_{\text{pl}}^2}$$

Current bound from BICEP/Keck:

$$r < 0.036$$

From the constraint that inflation happens after spontaneous CP breaking:

$$r \leq 1.7 \times 10^{-13} \left(\frac{\Lambda_{\text{CP}}}{10^8 \text{ GeV}} \right)^2$$

Cosmological implications of $H_{\text{inf}} \lesssim 10^8 \text{ GeV}$

Gravitational waves from cosmic string

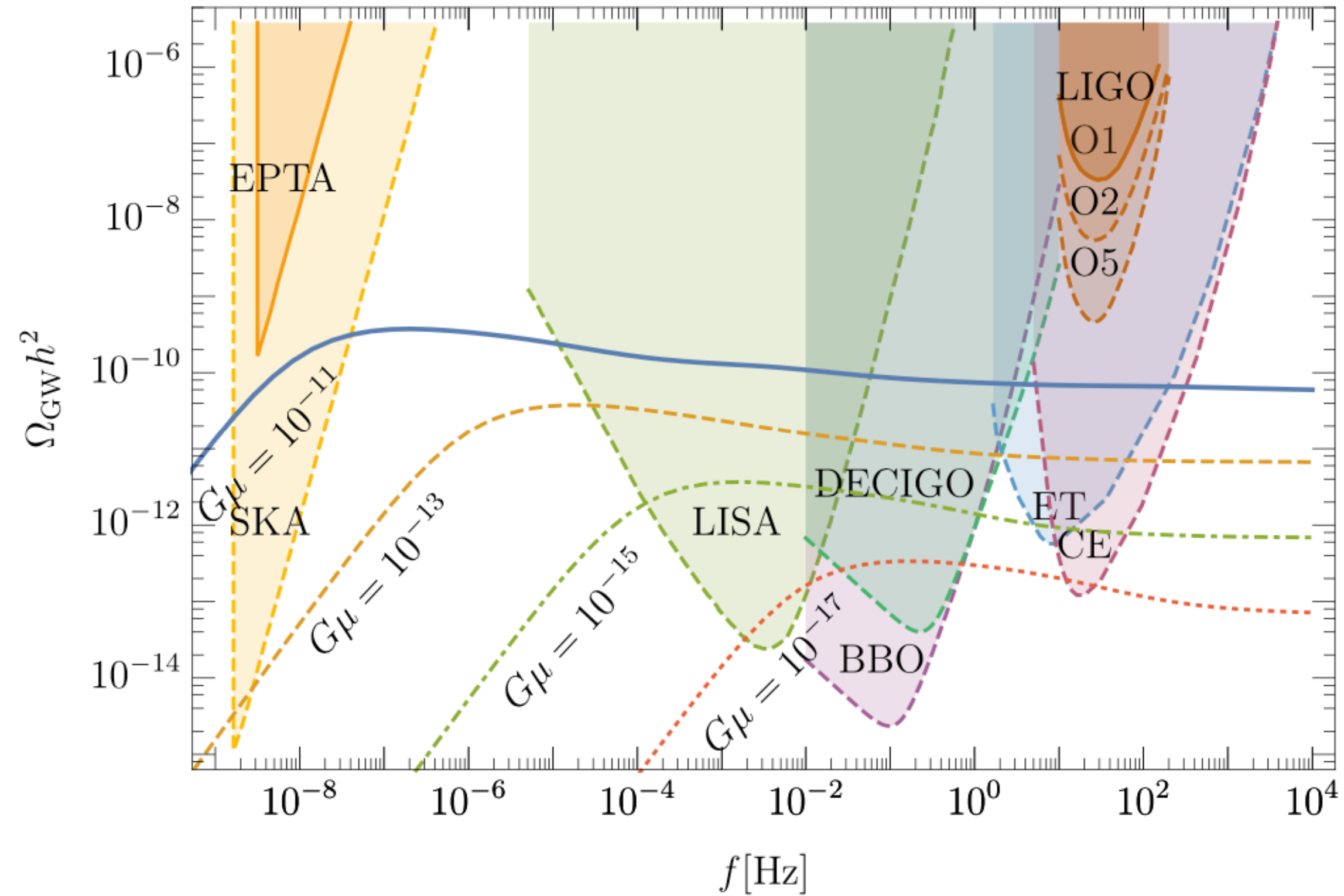
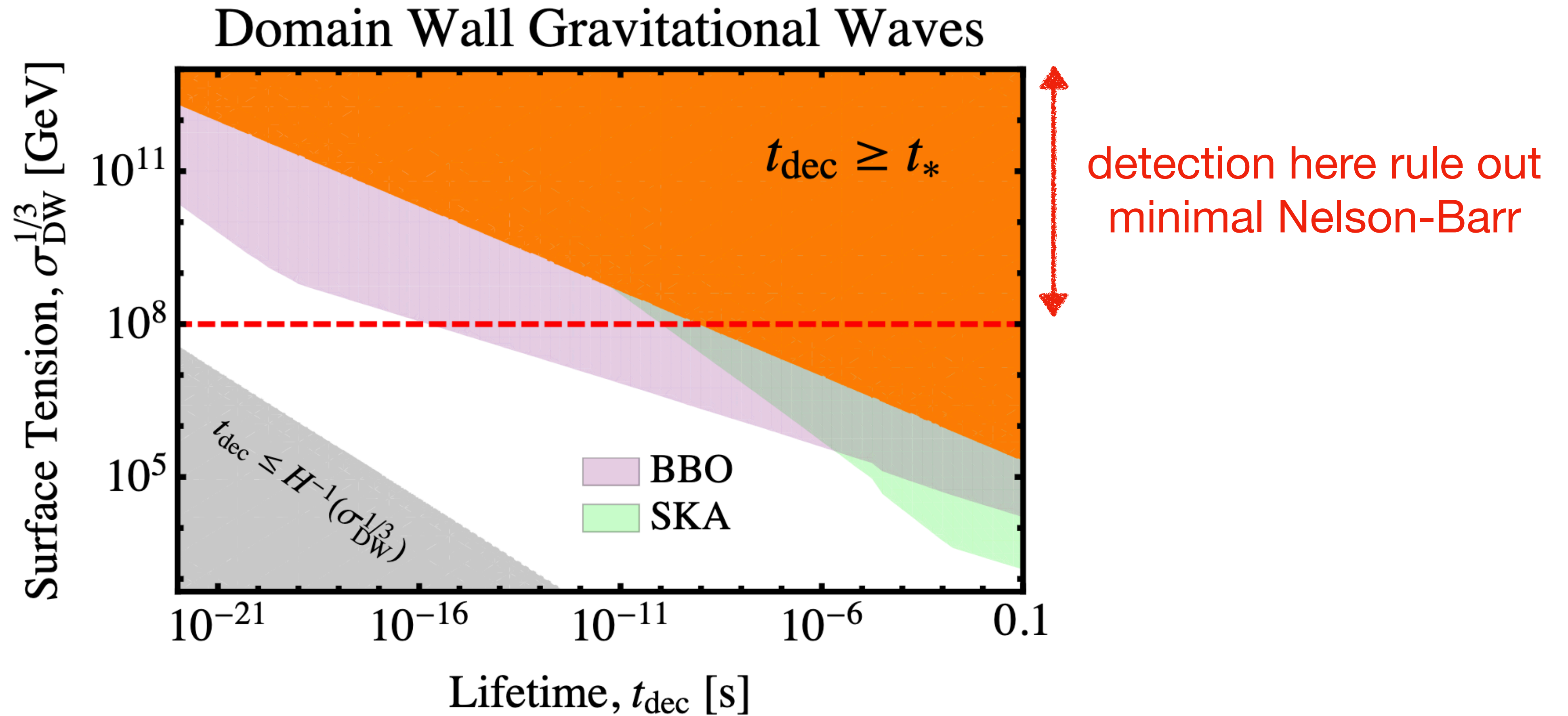


Image from [Cui, Lewicki, Morrissey, Wells '19]

Detection of gravitational wave from cosmic string
rules out minimal Nelson-Barr

Cosmological implications of $H_{\text{inf}} \lesssim 10^8 \text{ GeV}$

Gravitational waves from OTHER domain walls

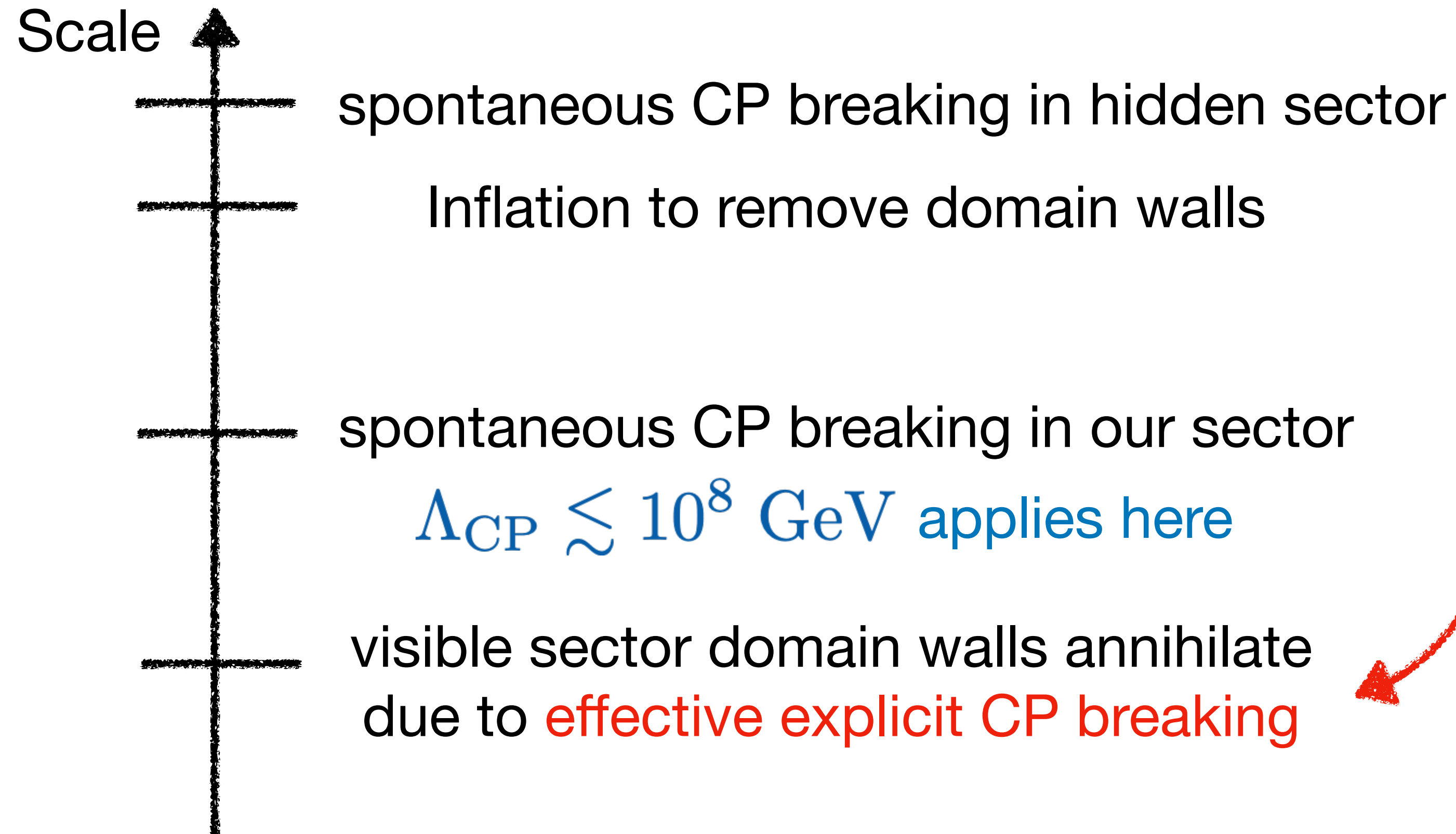


Calculation method from [Hiramatsu, Kawasaki, Saikawa '13]

To make inflation high scale again

Two possible solutions:

1. sequester CP breaking in a hidden sector



Challenges: keeping $\bar{\theta}$ small enough, fast annihilation of DWs, etc

To make inflation high scale again

Two possible solutions:

2. Alleviate quality problem by forbidding higher-dimensional operators:
chiral Nelson-Barr models

Chirally charging D , \bar{D} under a new symmetry

$U(1)_X$ will forbid the dimension-5 operators

$$\frac{1}{\Lambda_{\text{EFT}}} \eta_a^\dagger \eta_b \bar{D} D, \quad \frac{1}{\Lambda_{\text{EFT}}} \eta_a^\dagger Q_i H^c \bar{D}$$

Need a new scalar ρ to give masses to D , \bar{D}

$$\mathcal{L} \supset -y_D \rho D \bar{D}, \quad \mu_D = y_D \langle \rho \rangle$$

The rest of the model is the same as the minimal Nelson-Barr construction

To make inflation high scale again

Chiral Nelson-Barr

To cancel anomalies, need to add another set of fermions, B, \bar{B}

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
D	3	—	$-1/3$	-1
\bar{D}	$\bar{3}$	—	$+1/3$	-5
B	3	—	$+1/3$	$+1$
\bar{B}	$\bar{3}$	—	$-1/3$	$+5$
ρ	—	—	0	$+6$
η_a	—	—	0	$+2$

$U(1)_X$ is taken to be a linear combination of hyper charge and B-L, which is always anomaly free for SM

To make inflation high scale again

Chiral Nelson-Barr

All possible dimension-5 operators are forbidden.
Quality problem arise again at dimension-6:

$$\eta_a^\dagger \eta_b \rho D \bar{D}, \quad \eta_a^\dagger \rho Q_i H^c \bar{D}, \quad \eta_a \eta_b \eta_c^\dagger D d_j$$

Correction to $\bar{\theta}$ is now

$$\Delta \bar{\theta} \simeq \frac{1}{y_D} \frac{\Lambda_{\text{CP}}^2}{\Lambda_{\text{EFT}}^2} \Rightarrow \Lambda_{\text{CP}} \lesssim 10^{13} \text{ GeV}$$

Now the scale of CP breaking is high enough to recover most of the cosmology we are familiar with

Conclusion

- Despite the long history of the Nelson-Barr mechanism, consequences of spontaneous parity breaking has only been clarified recently
- Spontaneous breaking of CP leads to **exactly stable** domain walls [**McNamara & Reece 2212.00039**], which must be inflated away
- Nelson-Barr quality problem: the scale of spontaneous CP breaking cannot be too high
- These facts lead to phenomenological consequences of otherwise unconstrained Nelson-Barr models
- Chiral Nelson-Barr models is one set of solution to the domain wall-quality problem. Other possibilities and their phenomenological consequences remain to be explored.

Cosmological implications of $H_{\text{inf}} \lesssim 10^8 \text{ GeV}$

Inflation model building

Such a low-scale inflation require extremely flat potential,

$$\epsilon \equiv \frac{M_{\text{pl}}^2}{2} \left(\frac{V'}{V} \right)^2 = \frac{r}{16}$$

And extremely small field inflation,

$$\frac{\Delta\phi}{M_{\text{pl}}} \lesssim 10^{-6}.$$

Cosmological implications of $H_{\text{inf}} \lesssim 10^8 \text{ GeV}$

thermal leptogenesis

With some mild assumptions about reheating, we also have

$$T_{\text{reh}} \leq H_{\text{inf}} \lesssim 10^8 \text{ GeV}$$

Constrains baryogenesis scenario where baryon asymmetry comes from asymmetric decay of thermal particles.

e.g. leptogenesis: $\delta \equiv \frac{\Gamma_{N \rightarrow H\nu} - \Gamma_{N \rightarrow H\bar{\nu}}}{\Gamma_{N \rightarrow H\nu} + \Gamma_{N \rightarrow H\bar{\nu}}}$

[Davidson & Ibarra bound '02]

$$\delta \lesssim \frac{3}{8\pi} \frac{M_N m_\nu}{v^2} \Rightarrow T_{\text{reh}} \gtrsim 10^{8-10} \text{ GeV}$$