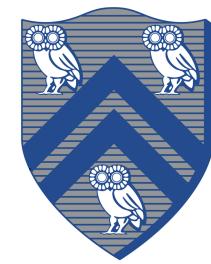
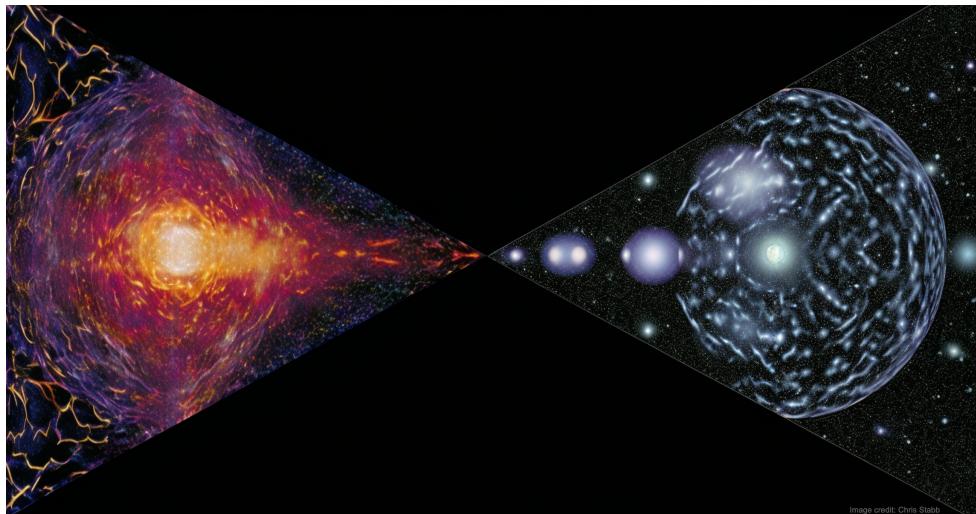


Making massive spin-2 particles from gravity during inflation & reheating



RICE

Andrew Long
Rice University
@ U of Florida IFT workshop
Oct 20, 2023

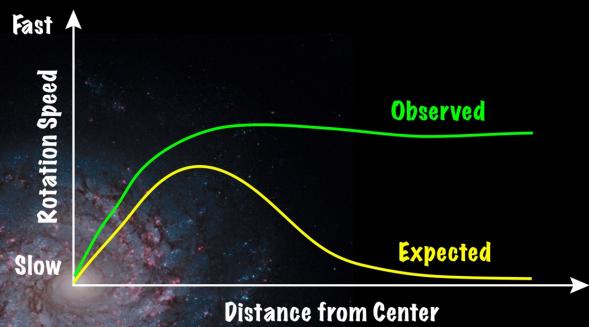
read along: 2302.04390

motivation
making dark matter
from gravity

dark matter pulls on things

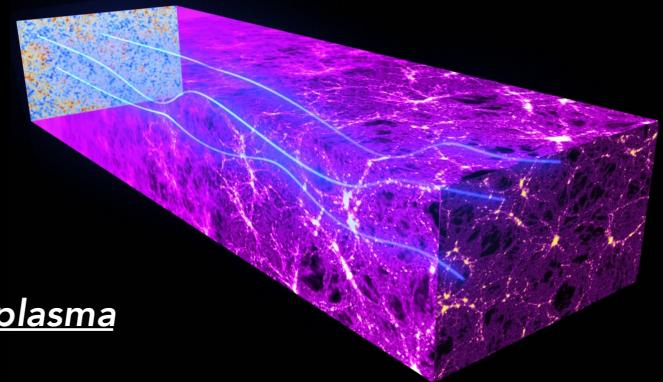
Dark matter pulls on stars in galaxies

(galactic rotation curves)



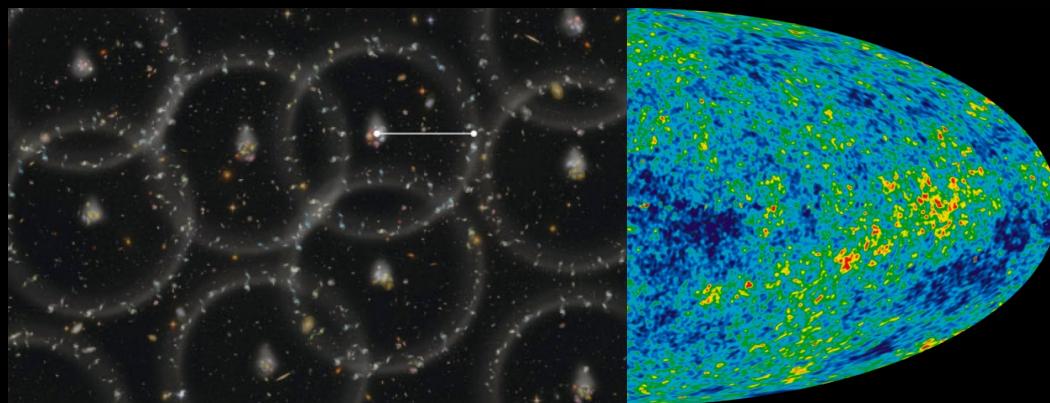
Dark matter pulls on light

(gravitational lensing)



Dark matter pulled on e^-p^+ plasma

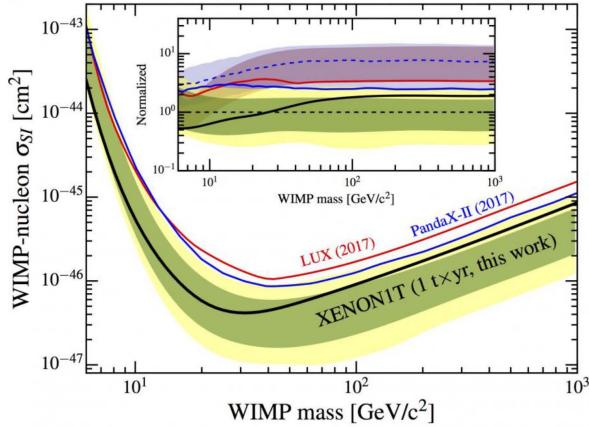
(CMB & large scale structure)



no evidence (yet) of dark matter bumping into things

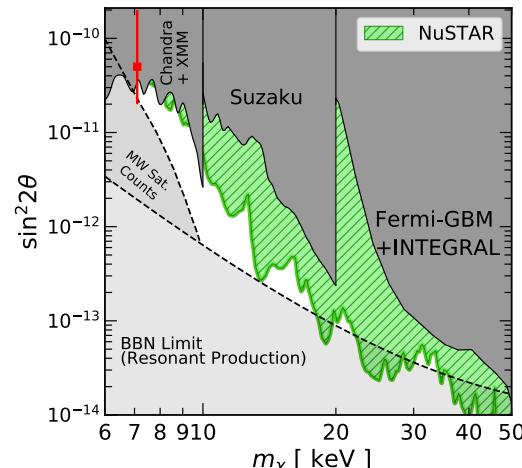
No dark matter bumping into things

(direct detection; 1805.12562)



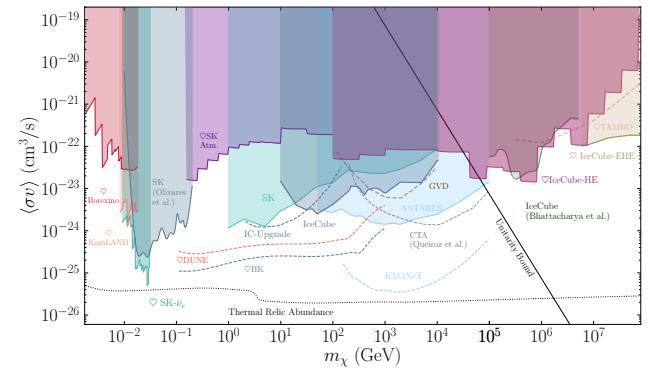
No dark matter decaying into things

(X-ray emission; 1908.09037)



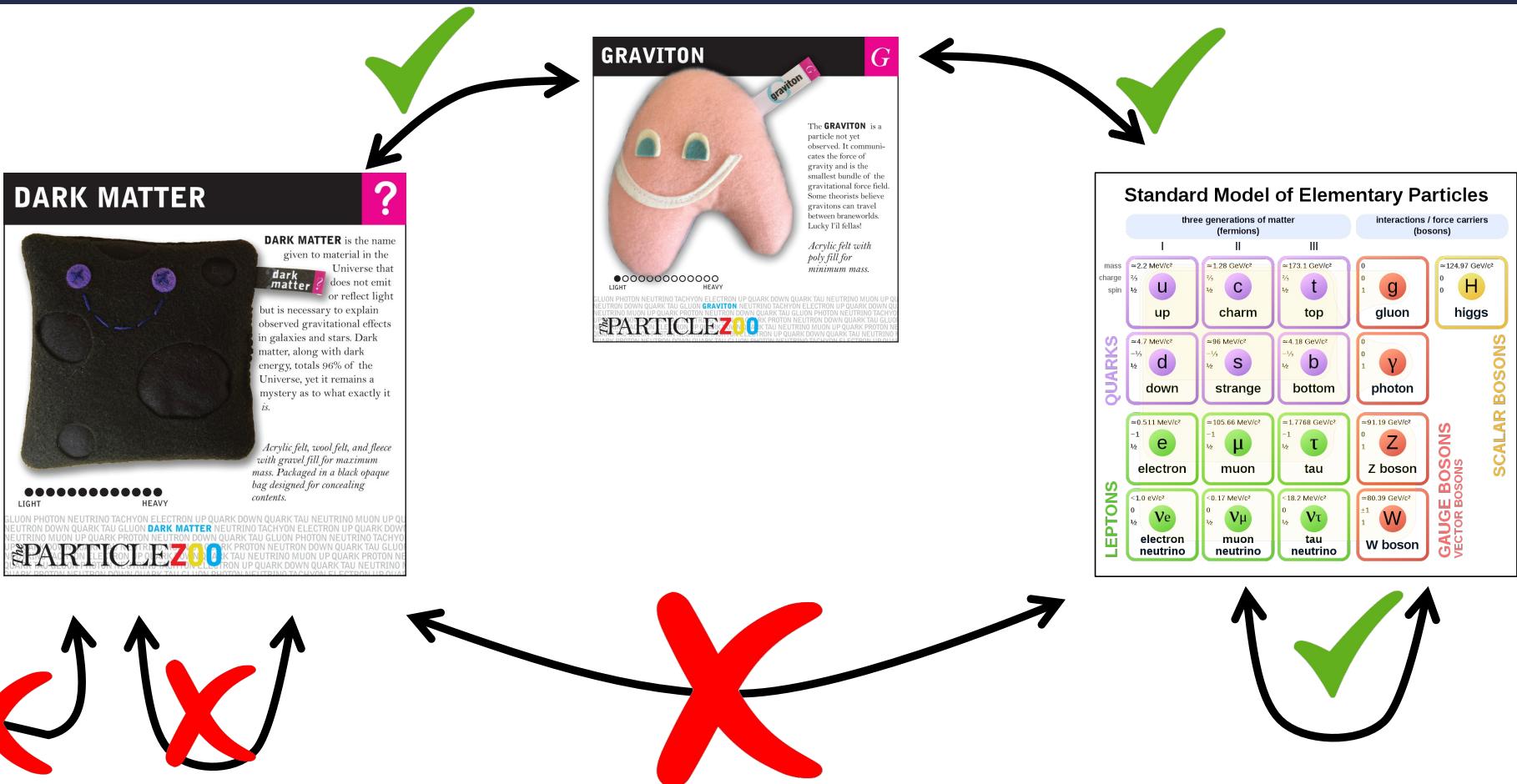
No dark matter bumping into itself

(annihilation to v's; 1912.09486)



(notwithstanding hints of new physics, there's no overwhelming evidence)

the hypothesis:



what to call it?

[Submitted on 14 Oct 1998]

WIMPZILLAS!

Edward W. Kolb, Daniel J. H. Chung, Antonio Riotto

Despicable dark relics: generated by gravity with unconstrained masses

Malcolm Fairbairn¹, Kimmo Kainulainen^{2,4}, Tommi Markkanen³ and Sami Nurmi^{2,4}

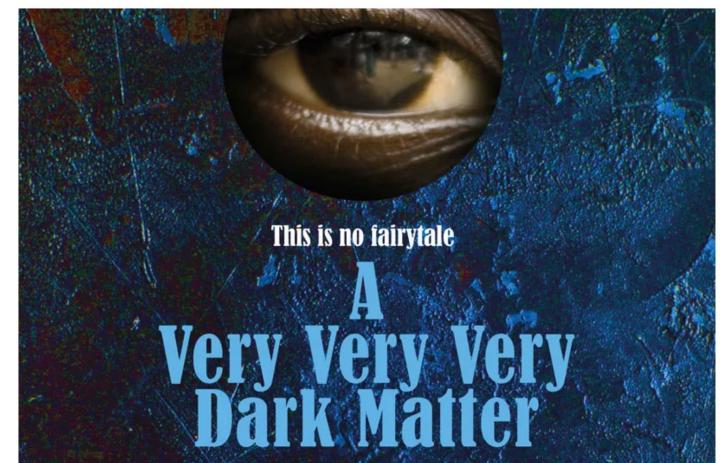
Published 3 April 2019 • © 2019 IOP Publishing Ltd and Sissa Medialab

Production of purely gravitational dark matter: the case of fermion and vector boson

Yohei Ema,^{a,b} Kazunori Nakayama^{c,d} and Yong Tang^c

Completely dark matter from rapid-turn multifield inflation

Edward W. Kolb,^a Andrew J. Long,^b Evan McDonough^c and Guillaume Payeur^{c,d}



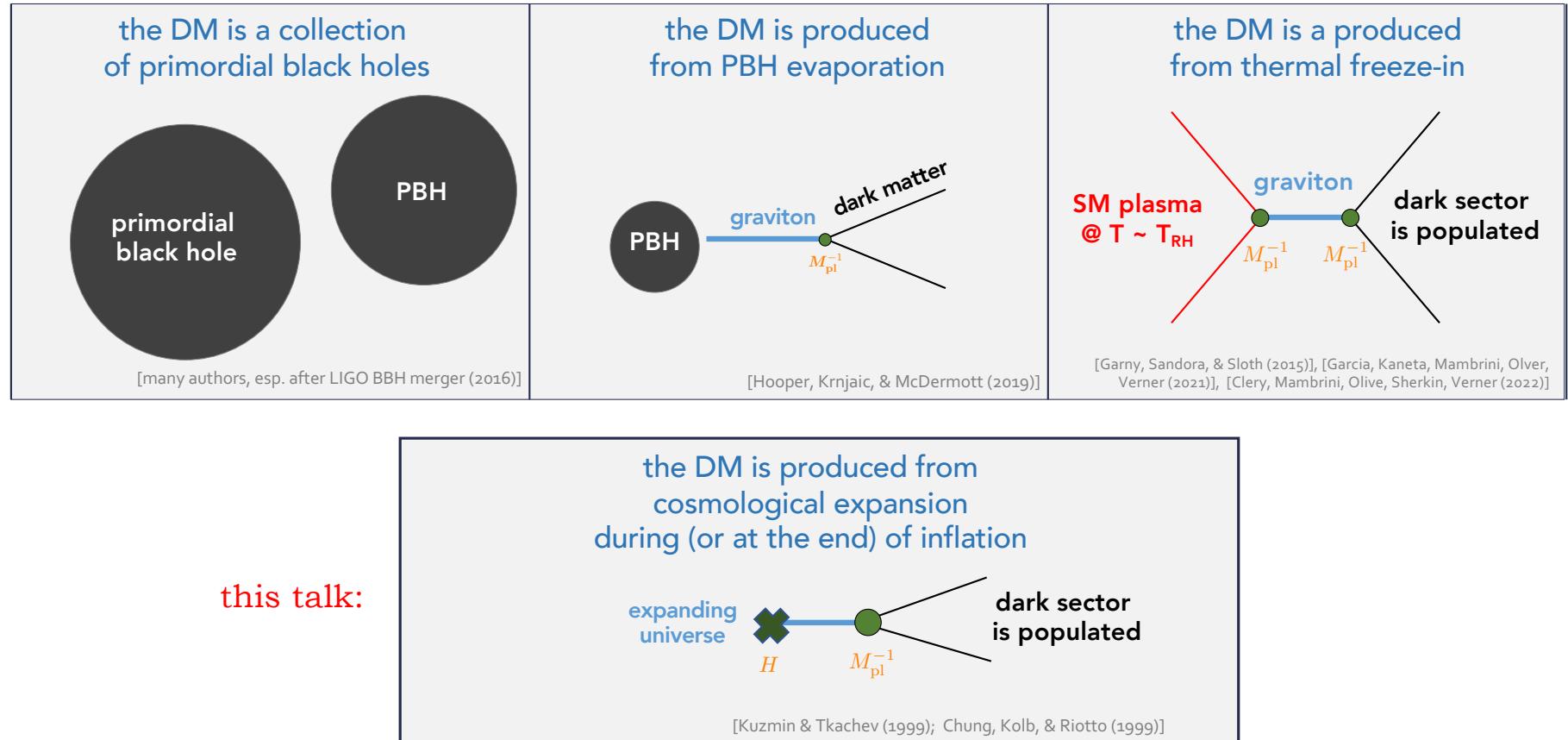
a 2018 play by Martin McDonagh

the problem:

where did all the
dark matter come from?

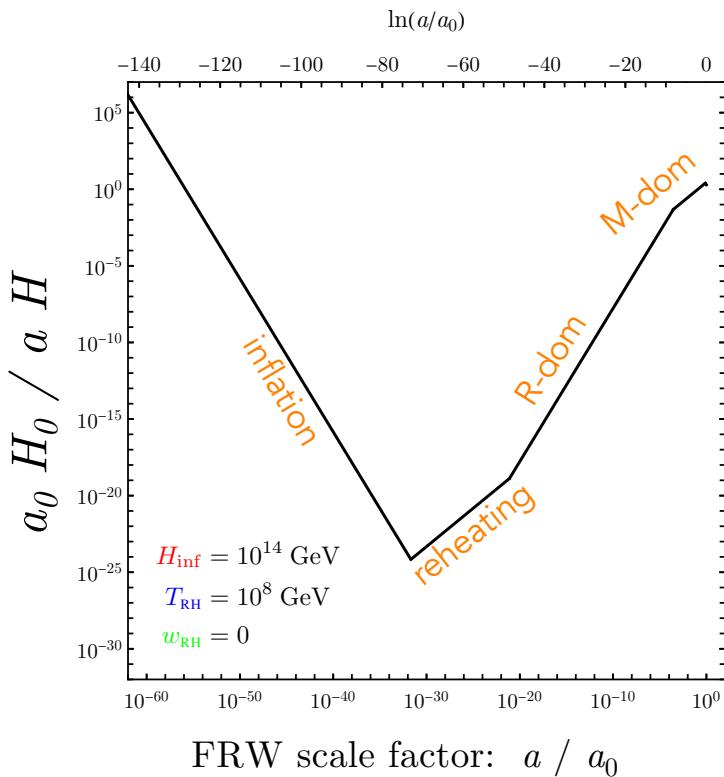
(how do we use gravity
to make dark matter?)

ideas ...



scalar spectator
inflationary quantum fluctuations

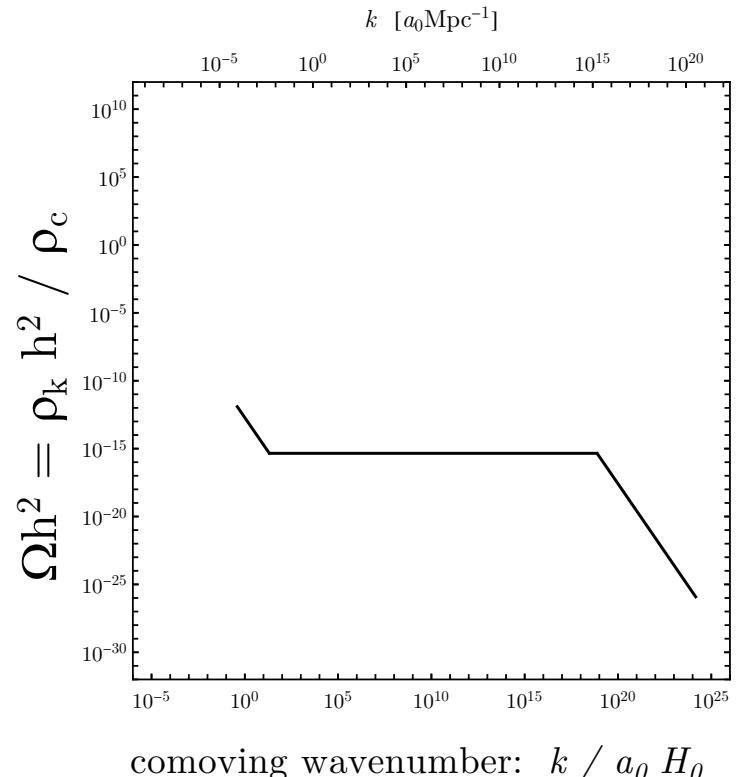
Energy spectrum of inflationary fluctuations



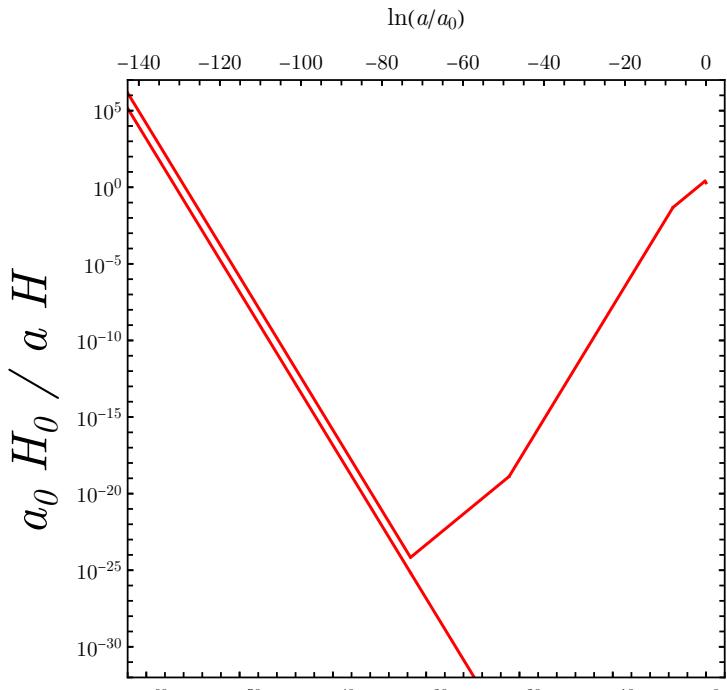
$$S[\chi] = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right]$$

$$\chi_k \approx H_{\text{inf}} / 2\pi$$

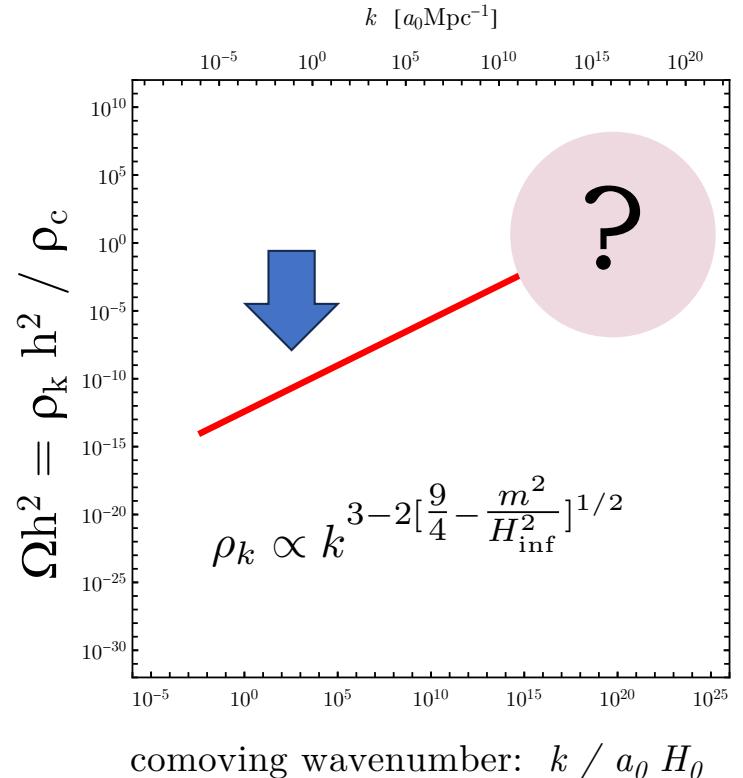
$$\rho_k \propto \begin{cases} a^{-2} & (\text{outside horizon}) \\ a^{-4} & (\text{inside horizon}) \end{cases}$$



Let's turn up the mass ...



FRW scale factor: a / a_0



Takeaway

For models with $m \sim H_{\text{inf}}$, most of the energy is carried by modes that are [on the Hubble scale](#) at the end of inflation.

Getting accurate predictions for the total particle number, requires a more careful modeling of:

1. The end of inflation & transition into the reheating epoch
2. The evolution of the spectator field & its energy

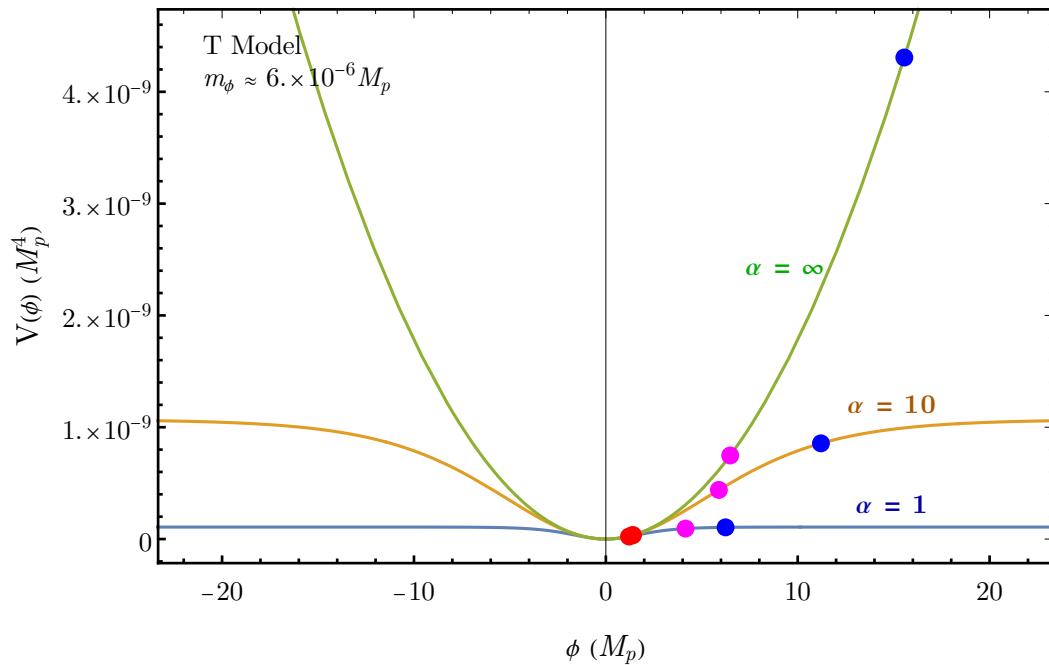
spin-0 particles
example: alpha attractor

Example: alpha attractor

[Ling & AL (2101.11621)]

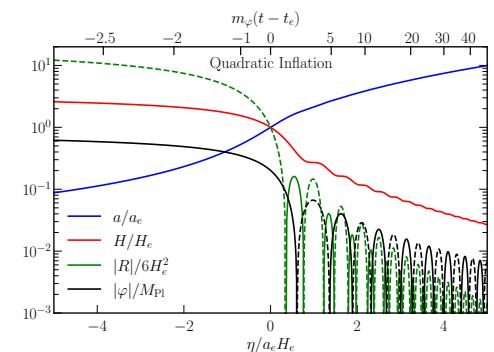
*T-model
alpha attractor*

$$V_T(\phi) = \alpha \mu^2 M_p^2 \tanh^2 \frac{\phi}{\sqrt{6\alpha} M_p}$$



$$\Rightarrow \begin{cases} \phi(t) \\ a(t) \end{cases}$$

FRW background



Scalar spectator (dark matter)

$$\text{FRW: } (ds)^2 = a(\eta)^2 [(d\eta)^2 - |dx|^2]$$

covariant action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 \right]$$

Fourier decomposition

$$\chi(\eta, \mathbf{x}) = \frac{1}{a(\eta)} \int \frac{d^3k}{(2\pi)^3} a_k \chi_k(\eta) e^{ik \cdot x} + \text{c.c.}$$

equations of motion

$$\chi''_k(\eta) + \omega_k^2(\eta) \chi_k(\eta) = 0$$

$$\omega_k^2(\eta) = k^2 + a(\eta)^2 m_\chi^2 + \frac{1}{6} a(\eta)^2 R(\eta)$$

a harmonic oscillator with
time-dependent frequency

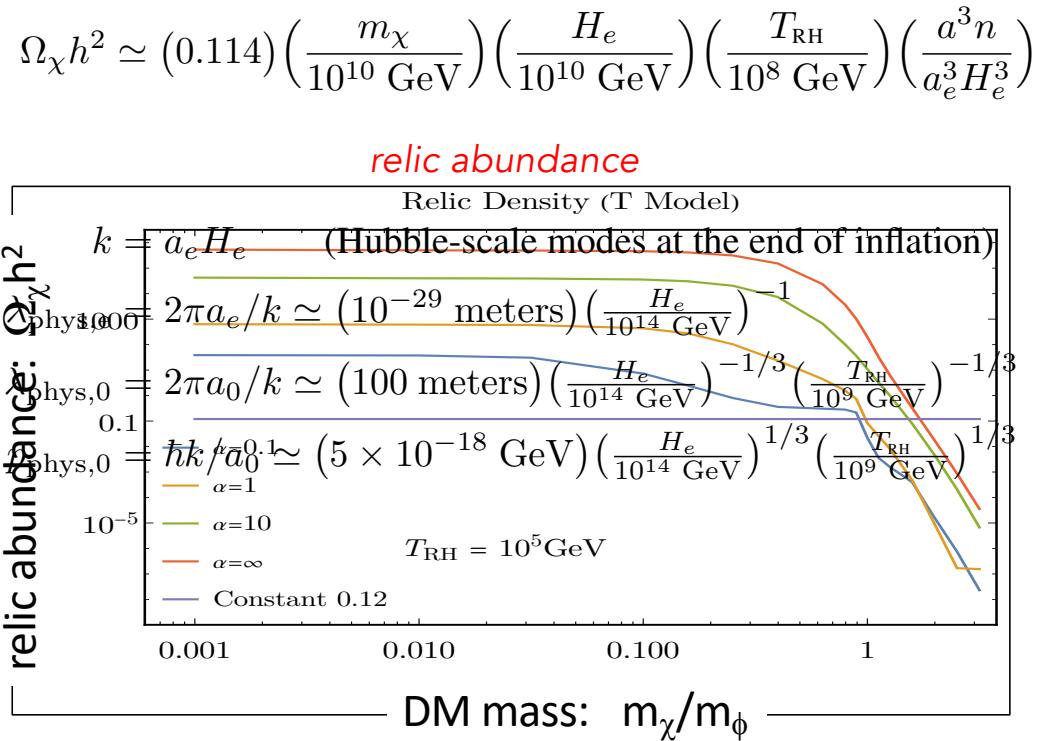
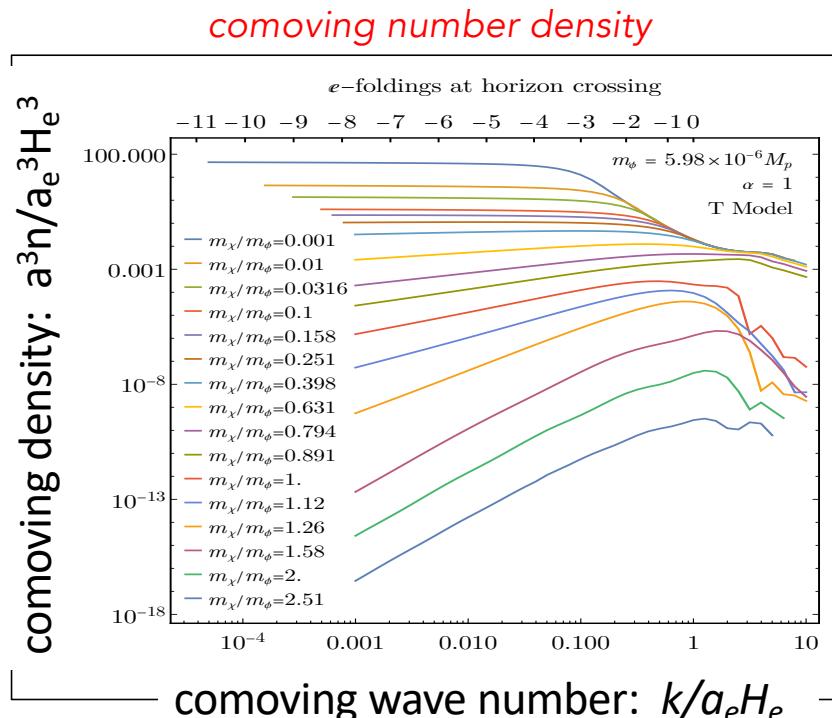
comoving number density

$$|\beta_k|^2 = \frac{\omega_k}{2} |\chi_k|^2 + \frac{1}{2\omega_k} |\partial_\eta \chi_k|^2 - \frac{1}{2}$$

$$a^3 n = \int d \ln k \frac{k^3}{2\pi^2} |\beta_k|^2$$

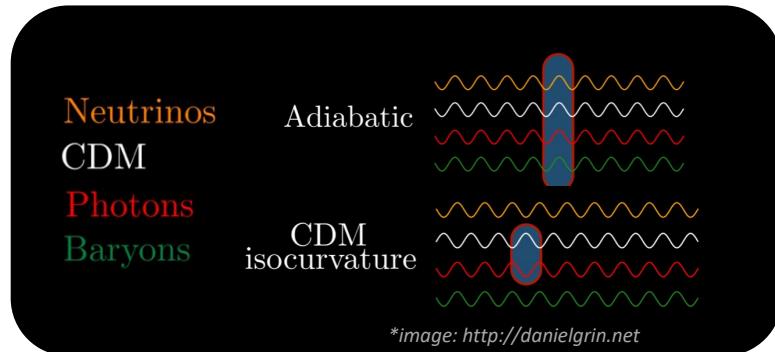
Numerical results

[Ling & AL (2101.11621)]



CMB isocurvature

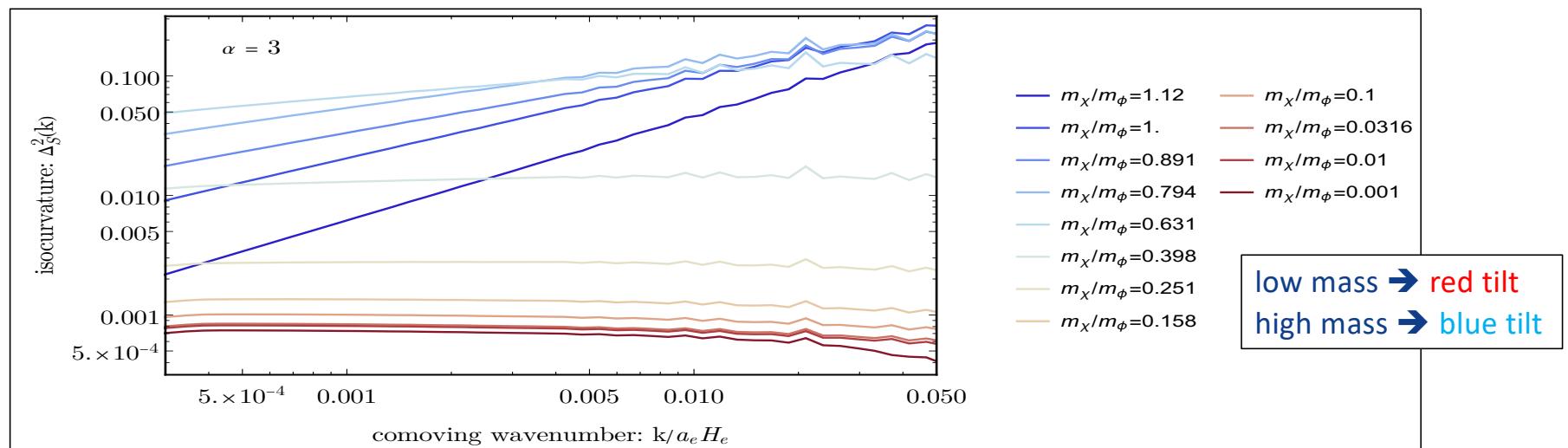
[Ling & AL (2101.11621)]



$$\Delta_S^2(k_{\text{cmb}}) < 7.3 \times 10^{-11}$$

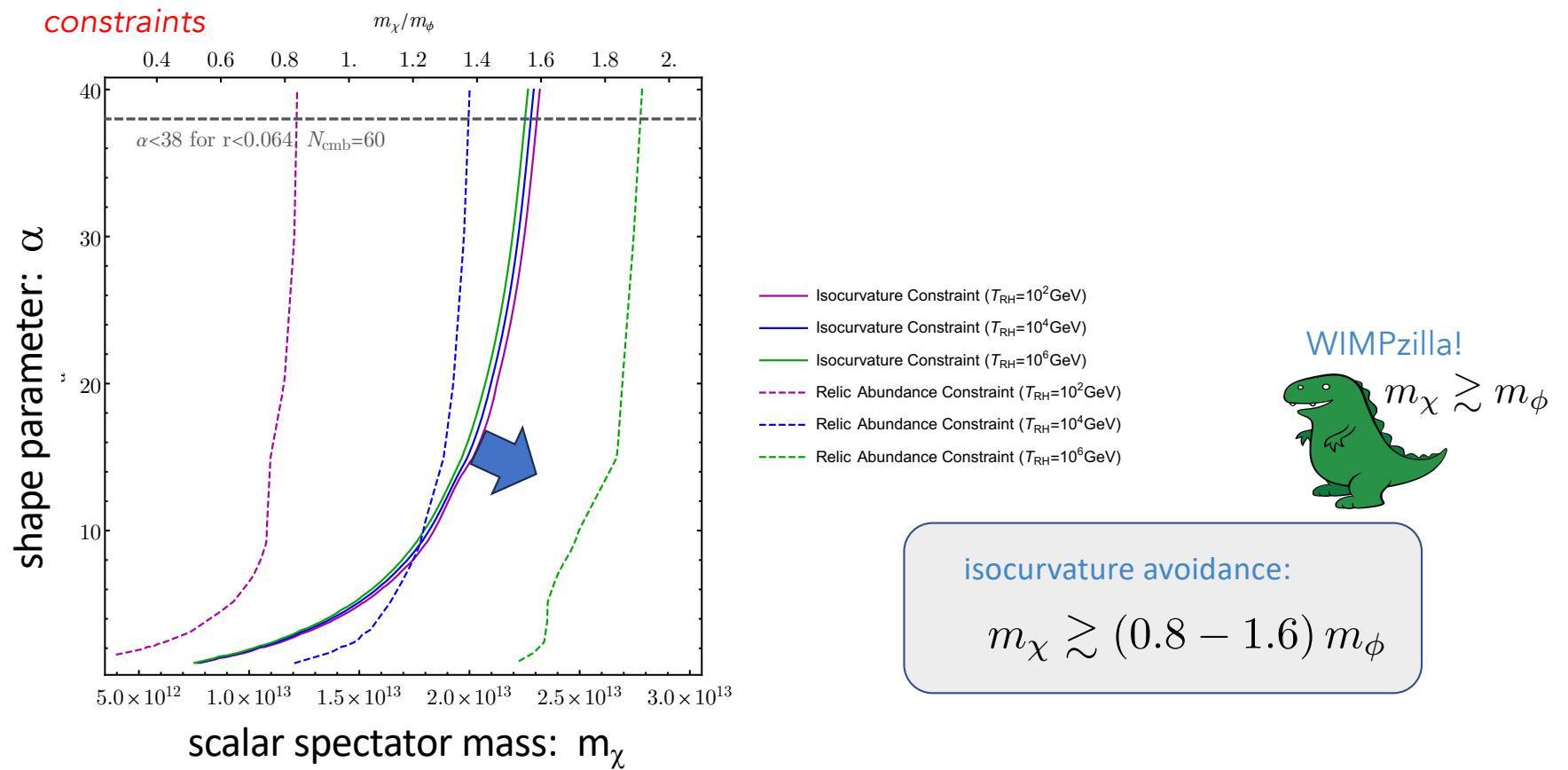
$$k_{\text{cmb}} = 0.002 \text{ Mpc}^{-1} a_0$$

$$k_{\text{cmb}}/a_e H_e \approx e^{-50} \simeq 2 \times 10^{-22}$$



Parameter space

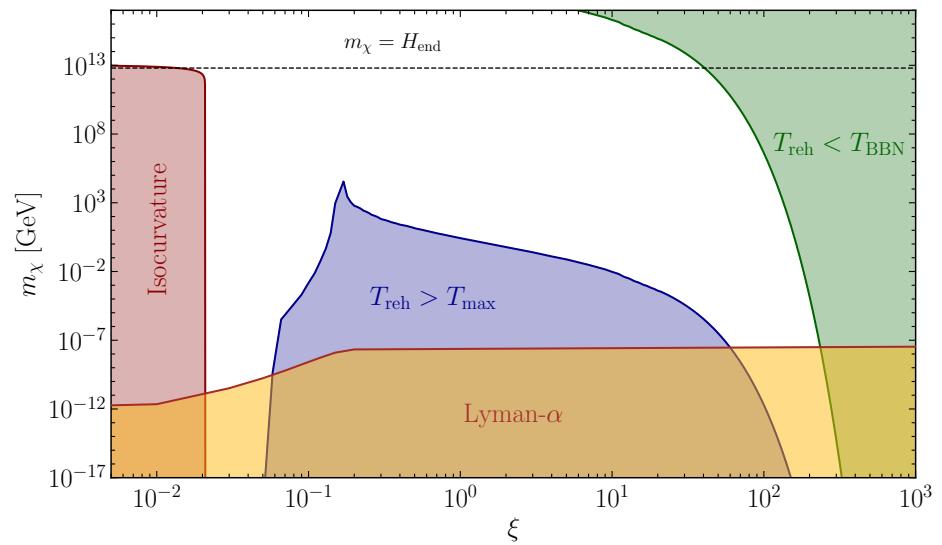
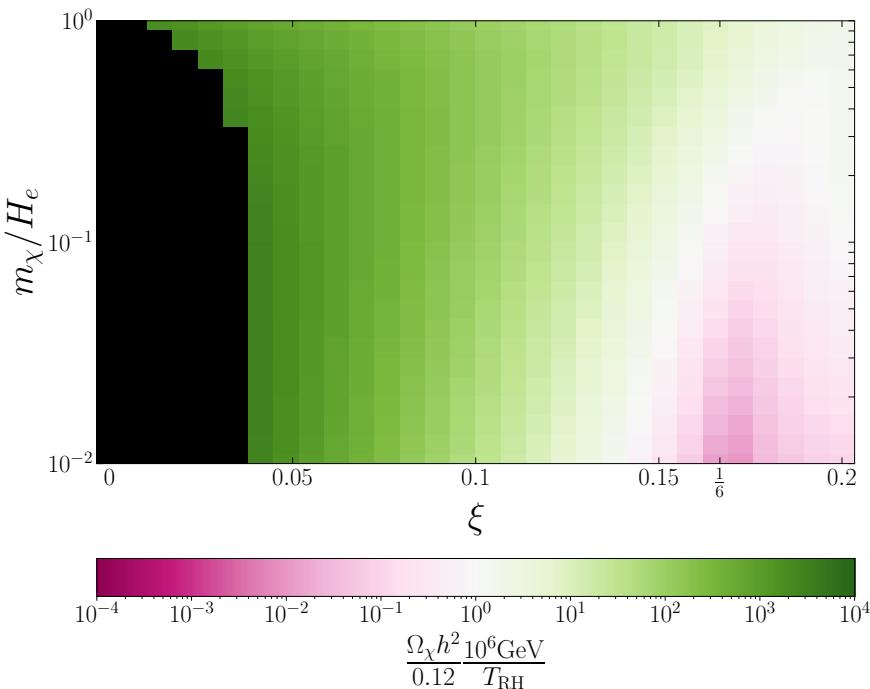
[Ling & AL (2101.11621)]



going non-minimal

[Kolb, AL, McDonough, & Payeur (2022)], [Garcia, Pierre, & Verner (2023)]
 see also: [Markkanen, Rajantie, & Tenkanen (2018); Tenkanen (2019)]

$$\mathcal{L} \supset -\frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2$$



isocurvature constraints on ultra-light scalar GPP
 can be avoided by introducing
 a “small” non-minimal coupling to gravity

CGPP for particles w/ spin

spin-0 (scalar field)

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{2}\xi\varphi^2R$$

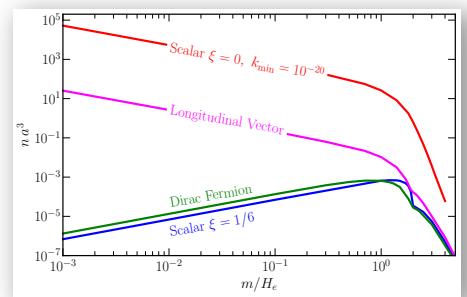
Chung, Kolb, & Riotto (1998)
Kuzmin & Tkachev (1998)
Herring, Boyanovsky, & Zentner (2020)
Brandenberger, Kamali, & Ramos (2023)

$$S = \int d^4x \sqrt{-g} \mathcal{L}$$

spin-1/2 (spinor field)

$$\mathcal{L} = \frac{i}{2}\bar{\Psi}\gamma^\mu(\nabla_\mu\Psi) - \frac{1}{2}m\bar{\Psi}\Psi + h.c.$$

Kuzmin & Tkachev (1998)
Chung, Everett, Yoo, & Zhou (2011)
Hashiba, Ling, & AL (2206.14204)



spin-1 (vector field)

Dimopoulos (2006) – not for DM; Graham, Mardon, & Rajendran (2016);
Ahmed, Grzadkowski, & Socha (2020); Kolb & AL (2009.03828)

$$\mathcal{L} = -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + \frac{1}{2}m^2g^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\xi_1 R g^{\mu\nu}A_\mu A_\nu - \frac{1}{2}\xi_2 R^{\mu\nu}A_\mu A_\nu$$

spin-3/2 (vector-spinor field)

Kallosh, Kofman, Linde, & Van Proeyen (1999); Giudice, Riotto, & Tkachev (1999); Lemoine (1999); Kolb, AL, & McDonough (2102.10113); Kaneta, Ke, Mambrini, Olive, Verner (2023)

$$\mathcal{L} = \frac{i}{4}\bar{\Psi}_\mu(\gamma^\mu\gamma^\rho\gamma^\sigma - \gamma^\sigma\gamma^\rho\gamma^\mu)(\nabla_\rho\Psi_\sigma) + \frac{1}{2}m\bar{\Psi}_\mu\gamma^\mu\gamma^\sigma\Psi_\sigma + h.c.$$

spin-2 (tensor field)

Alexander, Jenks, McDonough (2020)
Kolb, Ling, AL, & Rosen (2302.04390)

$$\mathcal{L} = \frac{1}{2}\nabla h_{\mu\nu}\nabla h^{\mu\nu} - \frac{1}{2}m^2h_{\mu\nu}h^{\mu\nu} + \dots$$

larger reps (Kalb-Ramond)

Capanelli, Jenks, Kolb, McDonough (2023)

massive spin-2 particles
on the hunt for a theory

General relativity

Covariant action for metric field $g_{\mu\nu}$

$$S[g_{\mu\nu}] = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R[g] \right]$$

Linearize around Minkowski spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}$$

$$S[h_{\mu\nu}] = \int d^4x \left[-\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda} \partial_\nu h^\mu_\lambda - \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{2} \partial_\mu h \partial^\mu h + O(h^3) \right]$$

($h = \eta^{\mu\nu} h_{\mu\nu}$)

Counting degrees of freedom

$$h_{\mu\nu} \sim 16_{\text{components}} - 6_{\text{symmetric}} - 4_{\text{gauge}} - 4_{\text{constraint}} = 2_{\text{dof}}$$

($\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$) (transverse & traceless)

→ these are the two polarization modes of the massless graviton ($x & +$ or $h = +2, -2$)

Adding a mass

Try to add mass terms

$$\delta S[h_{\mu\nu}] = \int d^4x \left[-\frac{1}{2}m_1^2 h_{\mu\nu} h^{\mu\nu} - \frac{1}{2}m_2^2 h_\mu^\mu h_\nu^\nu \right]$$

A poor choice of these mass parameters leads to a theory with a ghost (in addition to a massive spin-2)

$$m_{\text{ghost}}^2 = -\frac{1}{2} \frac{m_1^2 + 4m_2^2}{m_1^2 + m_2^2}$$

(Boulware-Deser ghost) 

A clever choice of parameters avoids the ghost and yields a healthy theory of massive spin-2 field

$$S_{\text{FP}}[h_{\mu\nu}] = \int d^4x \left[-\frac{1}{2}\partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda} \partial_\nu h^\mu_\lambda - \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$

(Fierz-Pauli action)

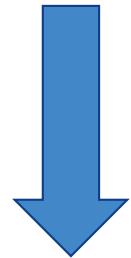
$$h_{\mu\nu} \sim 16_{\text{components}} - 6_{\text{symmetric}} - 1_{\text{gauge}} - 4_{\text{constraint}} = 5_{\text{dof}}$$

→ the five polarization modes of a massive graviton (helicity = -2, -1, 0, +1, +2)

Going to FRW background – failed attempt

(Fierz-Pauli action)

$$S_{\text{FP}}[h_{\mu\nu}] = \int d^4x \left[-\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda} \partial_\nu h^\mu_\lambda - \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$



try promoting Minkowski derivatives
to FRW covariant derivatives

$$\nabla_\lambda h_{\mu\nu} = \partial_\lambda h_{\mu\nu} - \Gamma_{\lambda\mu}^\rho h_{\rho\nu} - \Gamma_{\lambda\nu}^\rho h_{\mu\rho}$$

$$S[h_{\mu\nu}] = \int d^4x \left[-\frac{1}{2} \nabla_\lambda h_{\mu\nu} \nabla^\lambda h^{\mu\nu} + \nabla_\mu h^{\nu\lambda} \nabla_\nu h^\mu_\lambda - \nabla_\mu h^{\mu\nu} \nabla_\nu h + \frac{1}{2} \nabla_\mu h \nabla^\mu h - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$

This procedure would re-introduce the Boulware-Deser ghost. Going to an FRW bkg without also introducing the matter sector is a violation of gauge symmetry. ($\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$)

Successful attempt

Let's add a matter sector

$$S[g_{\mu\nu}] = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R[g] - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right]$$

Linearize around an FRW background

$$g_{\mu\nu} = \bar{g}_{\mu\nu}^{(\text{FRW})} + \frac{2}{M_P} u_{\mu\nu} \quad \text{and} \quad \phi = \bar{\phi}^{(\text{FRW})} + \varphi_u$$

Resulting quadratic action

$$\begin{aligned} \mathcal{L}_{\text{massless}}^{(2)} &= \mathcal{L}_{uu}^{(2)} + \mathcal{L}_{u\varphi_u}^{(2)} + \mathcal{L}_{\varphi_u\varphi_u}^{(2)} \\ \mathcal{L}_{uu}^{(2)} &= -\frac{1}{2} \nabla_\lambda u_{\mu\nu} \nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda} \nabla_\nu u^\mu{}_\lambda - \nabla_\mu u^{\mu\nu} \nabla_\nu u + \frac{1}{2} \nabla_\mu u \nabla^\mu u \\ &\quad + \left(\bar{R}_{\mu\nu} - \frac{1}{M_P^2} \nabla_\mu \bar{\phi} \nabla_\nu \bar{\phi} \right) \left(u^{\mu\lambda} u^\nu{}_\lambda - \frac{1}{2} u^{\mu\nu} u \right), \\ \mathcal{L}_{u\varphi_u}^{(2)} &= \frac{1}{M_P} \left[(\nabla_\mu \bar{\phi} \nabla_\nu \varphi_u + \nabla_\nu \bar{\phi} \nabla_\mu \varphi_u) (u^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} u) - V'(\bar{\phi}) \varphi_u u \right], \\ \mathcal{L}_{\varphi_u\varphi_u}^{(2)} &= -\frac{1}{2} \nabla_\mu \varphi_u \nabla^\mu \varphi_u - \frac{1}{2} V''(\bar{\phi}) \varphi_u^2. \end{aligned}$$

(massless spin-2 graviton + inflaton perturbation)

$$\begin{aligned} \mathcal{L}_{\text{massive}}^{(2)} &= \mathcal{L}_{vv}^{(2)} + \mathcal{L}_{v\varphi_v}^{(2)} + \mathcal{L}_{\varphi_v\varphi_v}^{(2)} \\ \mathcal{L}_{vv}^{(2)} &= -\frac{1}{2} \nabla_\lambda v_{\mu\nu} \nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda} \nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu} \nabla_\nu v + \frac{1}{2} \nabla_\mu v \nabla^\mu v \\ &\quad + \left(\bar{R}_{\mu\nu} - \frac{1}{M_P^2} \nabla_\mu \bar{\phi} \nabla_\nu \bar{\phi} \right) \left(v^{\mu\lambda} v^\nu{}_\lambda - \frac{1}{2} v^{\mu\nu} v \right) \\ &\quad - \frac{1}{2} m^2 (v^{\mu\nu} v_{\mu\nu} - v^2), \\ \mathcal{L}_{v\varphi_v}^{(2)} &= \frac{1}{M_P} \left[(\nabla_\mu \bar{\phi} \nabla_\nu \varphi_v + \nabla_\nu \bar{\phi} \nabla_\mu \varphi_v) (v^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} v) - V'(\bar{\phi}) \varphi_v v \right], \\ \mathcal{L}_{\varphi_v\varphi_v}^{(2)} &= -\frac{1}{2} \nabla_\mu \varphi_v \nabla^\mu \varphi_v - \frac{1}{2} V''(\bar{\phi}) \varphi_v^2. \end{aligned}$$

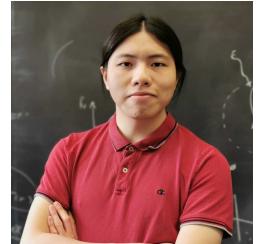
(massive spin-2 + inflaton perturbation)

Another approach: ghost-free bigravity

Field content: two metrics & two scalars

$$g_{\mu\nu}, \quad f_{\mu\nu}, \quad \phi_g, \quad \phi_f$$

talk tomorrow by
Siyang Ling
(Rice U grad student)



A theory of bigravity with a minimal coupling to matter

$$\begin{aligned} S = \int d^4x & \left[\frac{1}{2} M_g^2 \sqrt{-g} R[g] + \frac{1}{2} M_f^2 \sqrt{-f} R[f] \right. && \text{(metric kinetic terms)} \\ & - m^2 M_*^2 \sqrt{-g} V(\mathbb{X}; \beta_n) && \text{(metric interactions)} \\ & \left. + \sqrt{-g} \mathcal{L}_g(g, \phi_g) + \sqrt{-f} \mathcal{L}_f(f, \phi_f) \right] && \text{(coupling to matter)} \end{aligned}$$

Matter-sector Lagrangians

$$\mathcal{L}_g(g, \phi_g) = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi_g \nabla_\nu \phi_g - V_g(\phi_g)$$

$$\mathcal{L}_f(f, \phi_f) = -\frac{1}{2} f^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - V_f(\phi_f)$$

$$\left(\begin{array}{l} M_*^{-2} = M_g^{-2} + M_f^{-2} \\ M_P^2 = M_g^2 + M_f^2 \end{array} \right)$$

massive spin-2 particles
CGPP & dark matter

Separate out the 5 different polarization modes

Perform a scalar-vector-tensor (SVT) decomposition

$$v_{\mu\nu}(\eta, x) \sim \text{massive spin-2}$$
$$\sim (\text{helicity } \lambda = \pm 2) \oplus (\text{helicity } \lambda = \pm 1) \oplus (\text{helicity } \lambda = 0)$$

Tensor sector

$$\chi''_{k,\lambda}(\eta) + \omega_k^2(\eta) \chi_{k,\lambda}(\eta) = 0 \quad \text{for } \lambda = \pm 2$$


$$\omega_k^2(\eta) = k^2 + a^2 m^2 + \frac{1}{6} a^2 R$$

Vector sector

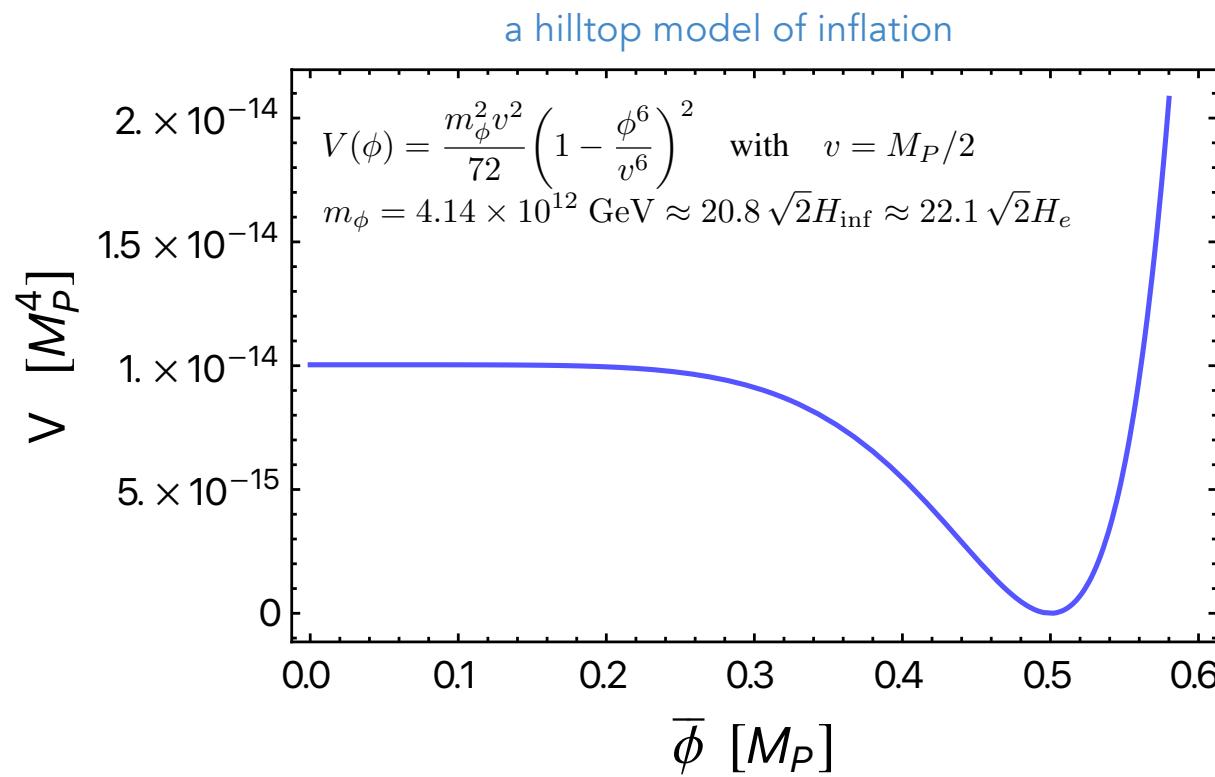
$$\chi''_{k,\lambda}(\eta) + \omega_k^2(\eta) \chi_{k,\lambda}(\eta) = 0 \quad \text{for } \lambda = \pm 1$$


$$\omega_k^2(\eta) = k^2 + a^2 m^2 - f''/f \quad \text{where } f = a^2/\sqrt{k^2 + a^2 m^2}$$

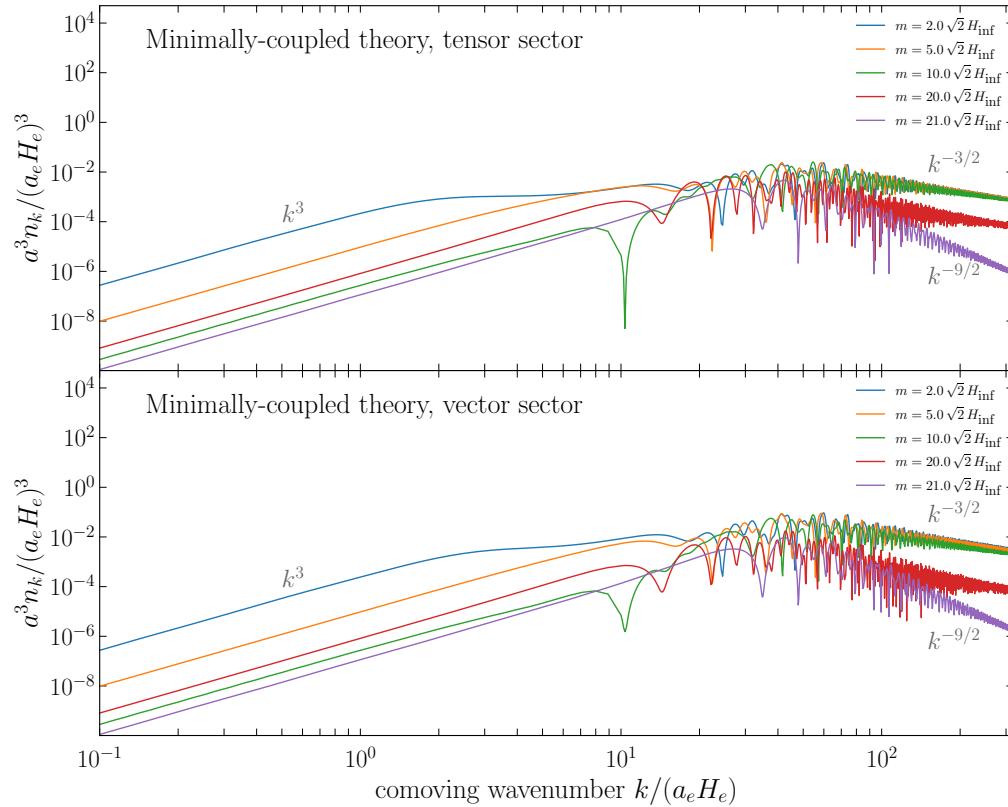
Scalar sector – it's complicated!

$$L_{S,k} = K_\Pi |\tilde{\Pi}'|^2 + M_\Pi |\tilde{\Pi}|^2 + K_{\mathcal{B}} |\tilde{\mathcal{B}}'|^2 + M_{\mathcal{B}} |\tilde{\mathcal{B}}|^2 + \lambda_1 \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0 \tilde{\Pi}^* \tilde{\mathcal{B}}$$

A numerical example



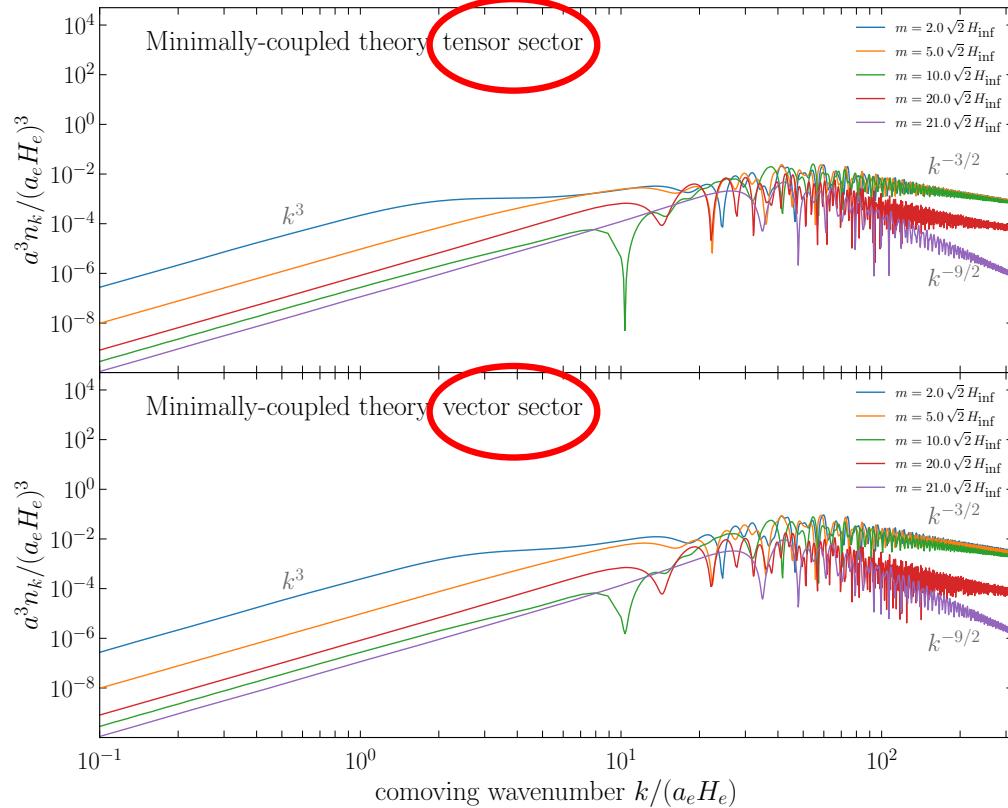
CGPP for tensor & vector sectors



Notable features:

1. Similar results for tensors & vectors
2. Low-k power law $\sim k^3$
3. High-k power law $\sim k^{-3/2}$ or $k^{-9/2}$
4. Wiggles!

CGPP for tensor & vector sectors



Notable features:

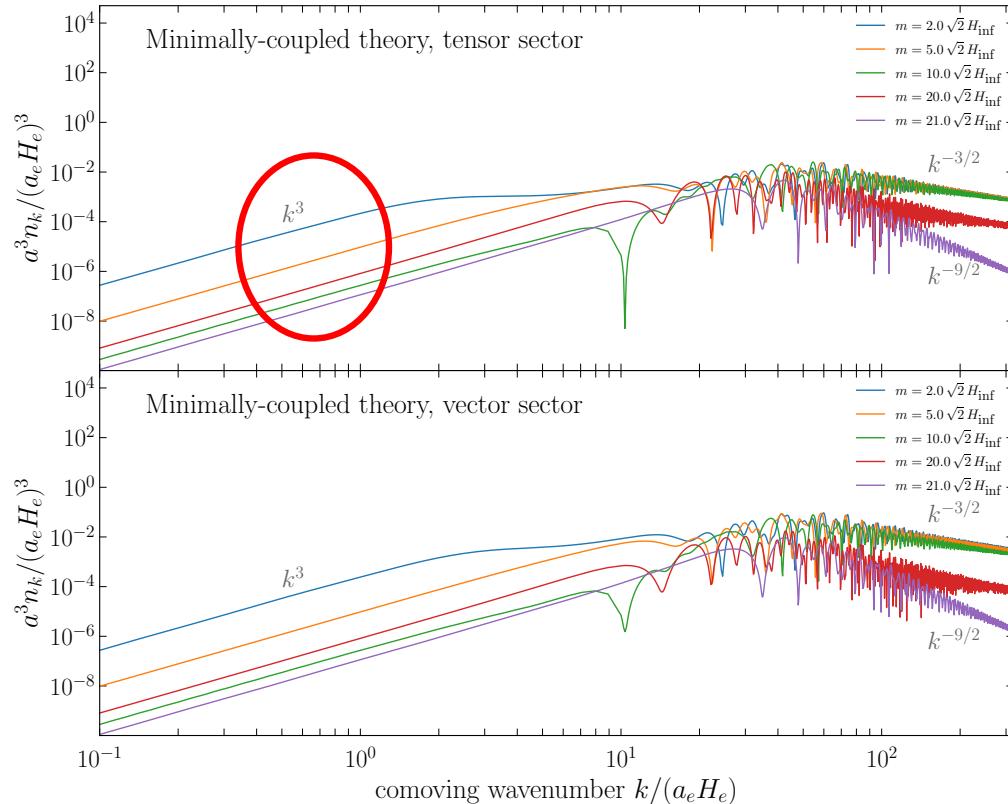
1. Similar results for tensors & vectors
2. Low-k power law $\sim k^3$
3. High-k power law $\sim k^{-3/2}$ or $k^{-9/2}$
4. Wiggles!

tensor sector: $\omega_k^2(\eta) = k^2 + a^2 m^2 + \frac{1}{6} a^2 R$

vector sector: $\omega_k^2(\eta) = k^2 + a^2 m^2 - f''/f$

equal for nonrelativistic modes

CGPP for tensor & vector sectors



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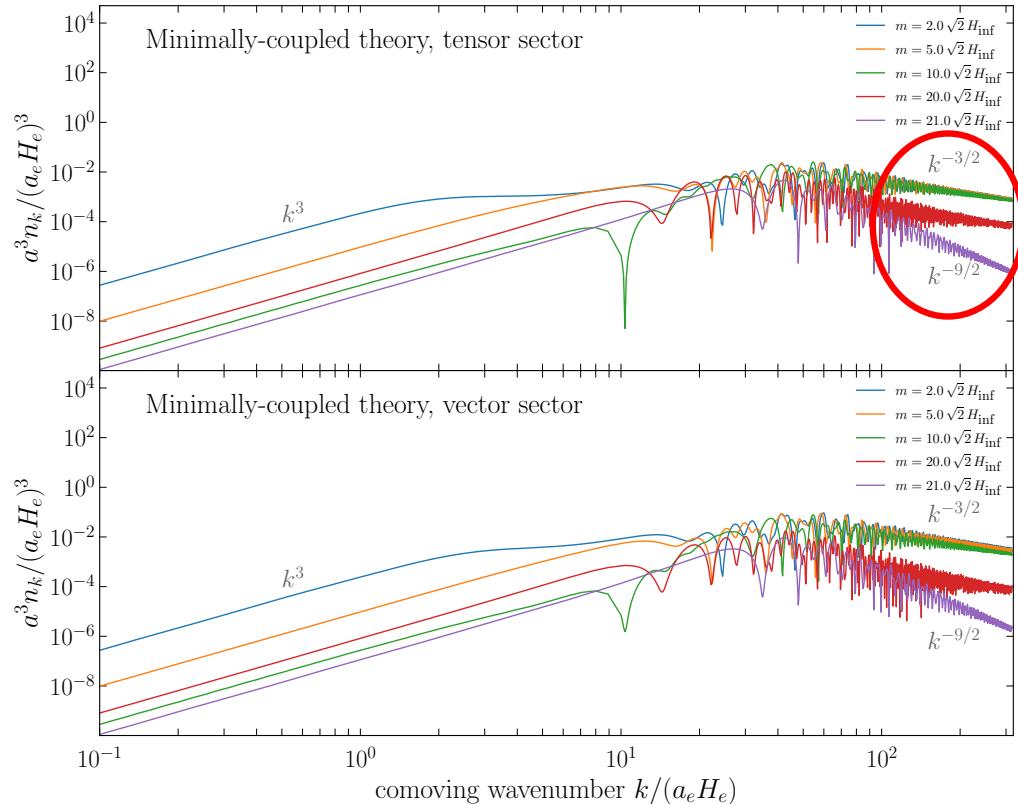
$$n_k \propto k^\nu \quad \text{with} \quad \nu = 3 - 2 \left[\frac{9}{4} - \frac{m^2}{H_{\text{inf}}^2} \right]^{1/2}$$

$$\text{Re}[\nu] = 3 \quad \text{for } m > \frac{3}{2} H_{\text{inf}}$$

low-k modes have familiar dS solution

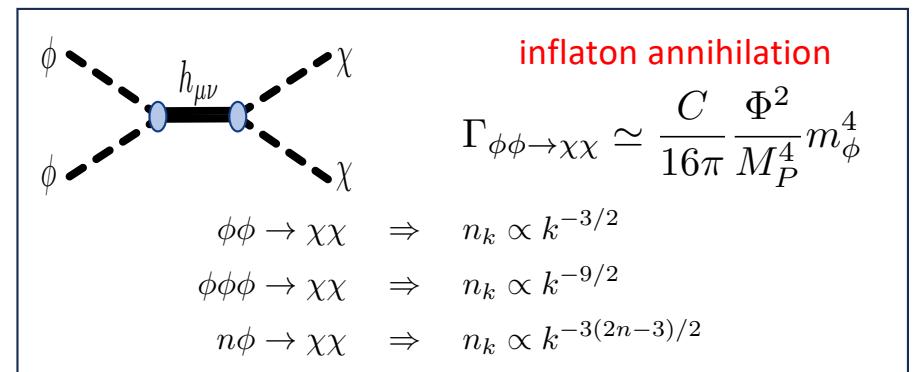
CGPP for tensor & vector sectors

[Ema, Nakayama, & Tang (2018)]
 [Chung, Kolb, AL (2018)]



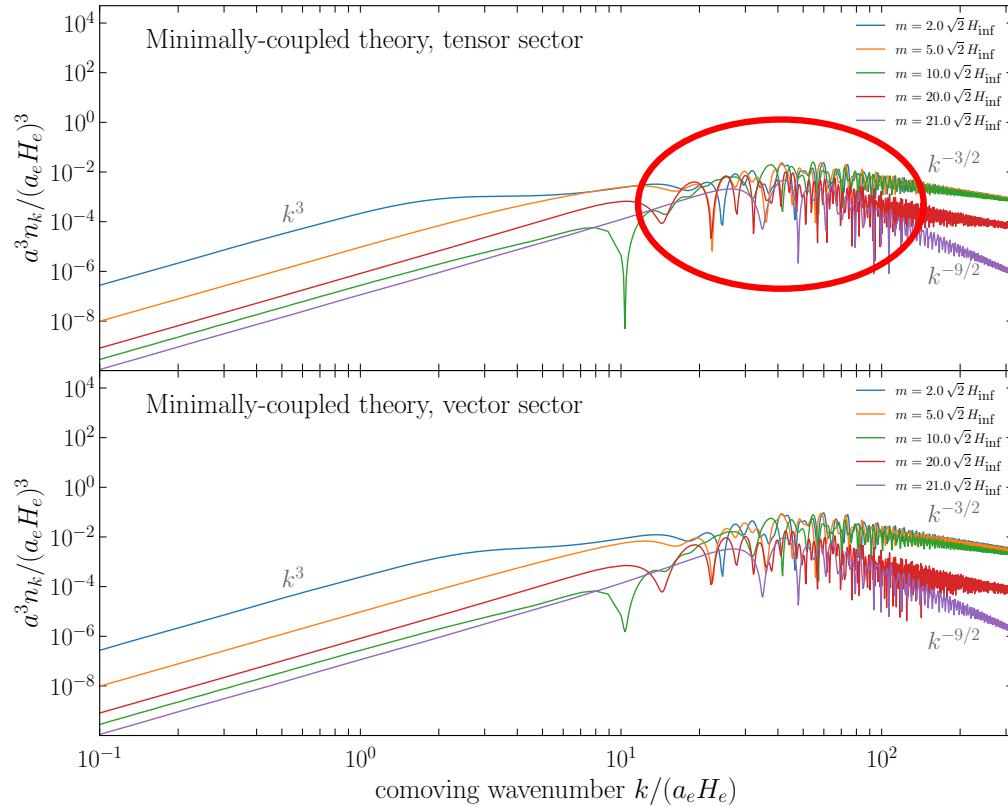
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CGPP for tensor & vector sectors

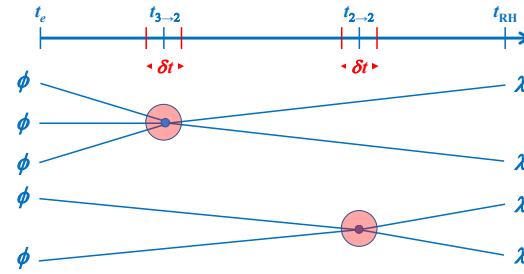
[Basso, Chung, Kolb, AL (2022)]



Notable features:

1. Similar results for tensors & vectors
2. Low-k power law $\sim k^3$
3. High-k power law $\sim k^{-3/2}$ or $k^{-9/2}$
4. Wiggles!

interference between annihilation channels



Quantum interference fringes

Basso, Chung, Kolb, AL [2209.01713]

The basic idea:

The inflaton field is in a state of indefinite particle number (coherent state).

Many-to-two scatterings will interfere with one another.

This explains the wiggly features in spectra – they are interference fringes.

What's getting calculated?

number density spectrum: $n_k \propto |\beta_k|^2$

Bogolubov coefficient: $\beta_k \approx \int_{-\infty}^{\infty} dt \frac{\dot{\omega}_k}{2\omega_k} e^{-2i \int^t dt' \omega_k/a}$

stationary phase approx.: $\beta_k \approx \beta_k^{(1 \rightarrow 2)} + \beta_k^{(2 \rightarrow 2)} + \beta_k^{(3 \rightarrow 2)} + \dots$

Quantum interference fringes

Basso, Chung, Kolb, AL [2209.01713]

Analytic results:

$$\beta_k^{(n \rightarrow 2)} = \mathcal{A}_k^{(n \rightarrow 2)} e^{i\Phi_k^{(n \rightarrow 2)}}$$

$$\Delta\Phi_k^{(n \rightarrow 2)} = \Phi_k^{(n \rightarrow 2)} - \Phi_{k, \text{leading}}^{(n \rightarrow 2)}$$

$$\mathcal{A}_k^{(1 \rightarrow 2)} = -\kappa_1^{-15/4} 3\alpha_3 \sqrt{\frac{-\frac{i}{2}\pi}{\frac{1}{4} - r_\chi^2}} r_\chi^2 (1 + \mathcal{O}(\kappa_1^{-3})) , \quad (4.3a)$$

$$\mathcal{A}_k^{(2 \rightarrow 2)} = \kappa_2^{-9/4} \frac{3}{16} \sqrt{\frac{-i\pi}{1 - r_\chi^2}} r_\chi^2 \left(1 + \frac{x_0 + x_1 r_\chi^2 + x_2 r_\chi^4 - 416r_\chi^6 + 384r_\chi^8}{1024(1 - r_\chi^2)^2} \kappa_2^{-3} + \mathcal{O}(\kappa_2^{-6}) \right) , \quad (4.3b)$$

$$\mathcal{A}_k^{(3 \rightarrow 2)} = \kappa_3^{-15/4} \frac{\alpha_3}{9} \sqrt{\frac{-\frac{3}{2}i\pi}{\frac{9}{4} - r_\chi^2}} r_\chi^2 (1 + \mathcal{O}(\kappa_3^{-3})) , \quad (4.3c)$$

$$\mathcal{A}_k^{(4 \rightarrow 2)} = \kappa_4^{-21/4} \frac{3(-21 + 68\alpha_3^2 + 24\alpha_4 + 12r_\chi^2)}{4096} \sqrt{\frac{-2i\pi}{4 - r_\chi^2}} r_\chi^2 (1 + \mathcal{O}(\kappa_4^{-3})) , \quad (4.3d)$$

$$\Delta\Phi_k^{(1 \rightarrow 2)} = \kappa_1^{-3/2} \left(\frac{y_0^{(1)} + y_1^{(1)} r_\chi^2 - 1280r_\chi^4}{480(1 - 4r_\chi^2)} + z^{(1)} + \mathcal{O}(\kappa_1^{-3}) \right) , \quad (4.4a)$$

$$\Delta\Phi_k^{(2 \rightarrow 2)} = \kappa_2^{-3/2} \left(\frac{y_0^{(2)} + y_1^{(2)} r_\chi^2 - 80r_\chi^4}{960(1 - r_\chi^2)} + z^{(2)} + \mathcal{O}(\kappa_2^{-3}) \right) , \quad (4.4b)$$

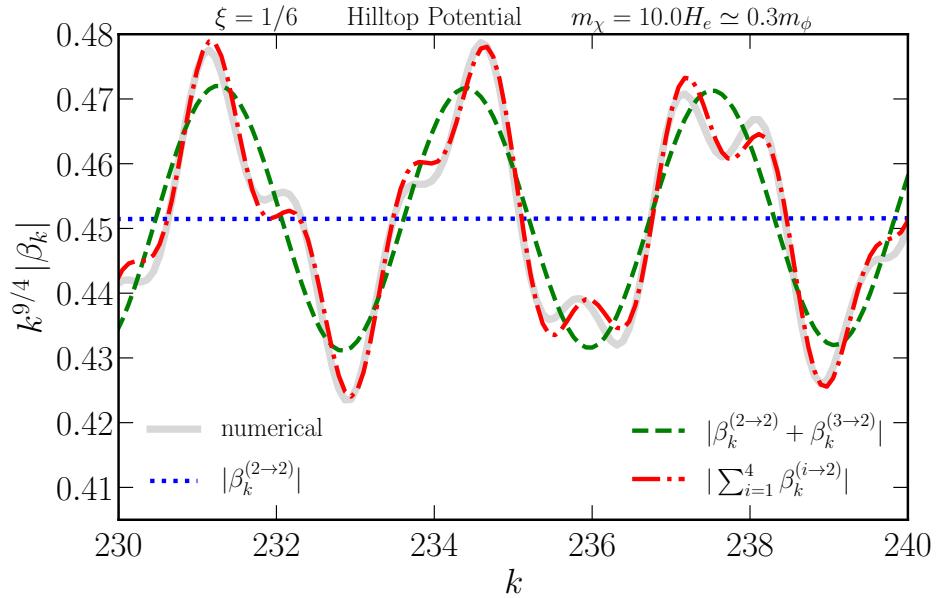
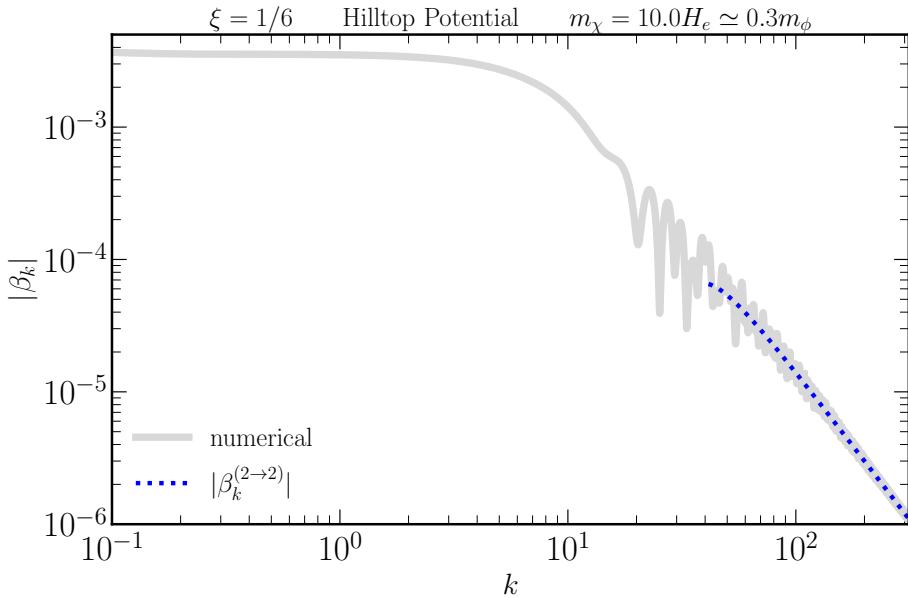
$$\Delta\Phi_k^{(3 \rightarrow 2)} = \kappa_3^{-3/2} \left(\frac{y_0^{(3)} + y_1^{(3)} r_\chi^2 - 1280r_\chi^4}{12960(9 - 4r_\chi^2)} + z^{(3)} + \mathcal{O}(\kappa_3^{-3}) \right) , \quad (4.4c)$$

$$\Delta\Phi_k^{(4 \rightarrow 2)} = \kappa_4^{-3/2} \left(\frac{y_0^{(4)} + y_1^{(4)} r_\chi^2 + y_2^{(4)} r_\chi^4 + 2588r_\chi^6}{960(4 - r_\chi^2)(-21 + 68\alpha_3^2 + 24\alpha_4 + 12r_\chi^2)} + z^{(4)} + \mathcal{O}(\kappa_4^{-3}) \right) , \quad (4.4d)$$

Quantum interference fringes

Basso, Chung, Kolb, AL [2209.01713]

Numerical validation:



What about the scalar sector?

(longitudinal polarization: $\lambda = 0$)

Scalar sector

Scalar metric perturbations mix with scalar inflaton perturbation

$L_S =$ a messy function of A, B, E, F , and φ_v

After imposing constraints (and a LOT of algebra) there are only two propagating degrees of freedom

$$L_{S,\mathbf{k}} = K_\Pi |\tilde{\Pi}'|^2 + M_\Pi |\tilde{\Pi}|^2 + K_{\mathcal{B}} |\tilde{\mathcal{B}}'|^2 + M_{\mathcal{B}} |\tilde{\mathcal{B}}|^2 + \lambda_1 \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0 \tilde{\Pi}^* \tilde{\mathcal{B}}$$

$$K_\varphi = \frac{a^2}{2} \frac{H^2 k^4 + 3a^2(m^2 - m_H^2)H^2 k^2 + \frac{3}{4}a^4 m^2(m^2 - m_H^2)H^2}{H^2 k^4 + 3a^2(m^2 - m_H^2)H^2 k^2 + \frac{3}{4}a^4 m^2(6m^2 H^2 - 4H^2 m_H^2 - m_H^4)} \quad (3.17a)$$

$$M_\varphi = \frac{a^2}{2} \frac{c_{10} k^{10} + c_8 k^8 + c_6 k^6 + c_4 k^4 + c_2 k^2 + c_0}{[H^2 k^4 + 3a^2(m^2 - m_H^2)H^2 k^2 + \frac{3}{4}a^4 m^2(6m^2 H^2 - 4H^2 m_H^2 - m_H^4)]^2} \quad (3.17b)$$

$$c_{10} = H^4$$

$$c_8 = \frac{1}{2}a^2 H^2 [(12m^2 H^2 + 8H^4 - 14H^2 m_H^2 - m_H^4) + 4 \frac{HV'(\hat{\phi})\bar{\phi}'}{aM_P^2} + 2H^2 V''(\bar{\phi})]$$

$$c_6 = \frac{3}{8}a^4 H^2 [(36m^4 H^2 + 72m^2 H^4 - 82m^2 H^2 m_H^2 - 64H^4 m_H^2$$

$$- 7m^2 m_H^4 + 40H^2 m_H^4 + 8m_H^6)$$

$$+ 8(3m^2 - 4m_H^2) \frac{HV'(\hat{\phi})\bar{\phi}'}{aM_P^2}$$

$$+ 16(m^2 - m_H^2)H^2 V''(\bar{\phi})]$$

$$c_4 = \frac{3}{8}a^6 H^2 [(16H^2 (3m^4 H^2 + 36m^2 H^4 + 16m^2 H^6 - 30m^4 H^2 m_H^2 - 76m^2 H^4 m_H^2$$

$$- 3m^2 m_H^4 + 31m^2 H^2 m_H^2 + 24H^4 m_H^4 + 6m^2 m_H^6 - 6H^2 m_H^8 - 3m^2 m_H^8)$$

$$- 4m^2 H^2 (H^2 - m_H^2) \frac{V'(\hat{\phi})^2}{M_P^2}]$$

$$+ (36m^4 H^2 + 8m^2 H^4 - 94m^2 H^2 m_H^2 + 24H^2 m_H^4 + 48H^2 m_H^6) \frac{HV'(\hat{\phi})\bar{\phi}'}{aM_P^2}$$

$$+ (36m^4 H^2 - 58m^2 H^2 m_H^2 - m^2 m_H^4 + 2H^2 m_H^6) H^2 V''(\bar{\phi})]$$

$$c_2 = \frac{9}{32}a^8 m^2 [H^2 (18m^6 H^2 + 120m^4 H^4 + 228m^2 H^6 - 78m^2 H^2 m_H^2 - 384m^2 H^4 m_H^2$$

$$- 9m^2 m_H^4 + 132m^2 H^2 m_H^2 + 128H^4 m_H^4 + 23m^2 m_H^6 - 32H^2 m_H^8 - 16m_H^8)$$

$$- 8H^2 (2m^2 H^2 - 2m^2 m_H^2 + m_H^4) \frac{V'(\hat{\phi})^2}{M_P^2}]$$

$$+ 4(6m^4 H^2 - 22m^2 H^2 m_H^2 + m^2 m_H^4 + 14H^2 m_H^4) \frac{HV'(\hat{\phi})\bar{\phi}'}{aM_P^2}$$

$$+ 4(m^2 - m_H^2)(12m^2 H^2 - 10H^2 m_H^2 - m_H^4) H^2 V''(\bar{\phi})]$$

$$c_0 = \frac{27}{32}a^{10} m^4 [-2H^2 (2m^2 H^2 - 2m^2 m_H^2 + m_H^4) \frac{V'(\hat{\phi})\bar{\phi}'}{M_P^2}$$

$$- m^2 (2H^2 - m_H^2) (4H^2 + m_H^4) \frac{HV'(\hat{\phi})\bar{\phi}'}{aM_P^2}$$

$$+ (m^2 - m_H^2) (6m^2 H^2 - 4H^2 m_H^2 - m_H^4) H^2 V''(\bar{\phi})]$$

$$L_2 = \frac{a^3 m^2 \bar{\phi}'}{2 M_P H} \frac{H^2 k^4 + \frac{3}{2}a^2(m^2 - m_H^2)H^2 k^2}{H^2 k^4 + 3a^2(m^2 - m_H^2)H^2 k^2 + \frac{3}{8}a^4 m^2(6m^2 H^2 - 4H^2 m_H^2 - m_H^4)} \quad (3.17e)$$

$$L_1 = - \frac{a^4 m^2 \bar{\phi}'}{M_P} \frac{(H^2 - \frac{1}{4}m_H^2 - \frac{1}{2}\frac{aHV'(\hat{\phi})}{\bar{\phi}'})k^4 - \frac{3}{2}a^2(m^2 - m_H^2)(H^2 + \frac{1}{4}m_H^2 + \frac{1}{2}\frac{aHV'(\hat{\phi})}{\bar{\phi}'})k^2}{H^2 k^4 + 3a^2(m^2 - m_H^2)H^2 k^2 + \frac{3}{8}a^4 m^2(6m^2 H^2 - 4H^2 m_H^2 - m_H^4)} \quad (3.17f)$$

$$L_0 = \frac{a^3 m^2 \bar{\phi}'}{2 M_P H} \frac{c_{10} k^{10} + c_8 k^8 + c_6 k^6 + c_4 k^4 + c_2 k^2}{[H^2 k^4 + 3a^2(m^2 - m_H^2)H^2 k^2 + \frac{3}{8}a^4 m^2(6m^2 H^2 - 4H^2 m_H^2 - m_H^4)]^2} \quad (3.17g)$$

$$c_{10} = H^4 \quad (3.17h)$$

$$c_8 = \frac{1}{2}a^2 H^4 [(9m^2 + 12H^2 - 13m_H^2) - 4 \frac{aHV'(\hat{\phi})}{\bar{\phi}'}]$$

$$c_6 = \frac{3}{8}a^4 H^2 [(18m^4 H^2 + 32m^2 H^4 + 64H^6 - 48m^2 H^2 m_H^2 - 64H^4 m_H^2 + m^2 m_H^4 + 28H^2 m_H^4)]$$

$$+ 8(-4m^2 H^2 + 4H^4 + m^2 m_H^2) \frac{aHV'(\hat{\phi})}{\bar{\phi}'}]$$

$$c_4 = \frac{3}{16}a^6 m^2 H^2 [(18m^4 H^2 - 24m^2 H^4 + 256H^6 - 54m^2 H^2 m_H^2 - 160H^4 m_H^2 + 9m^2 m_H^4 + 60H^2 m_H^4 - 7m_H^6)]$$

$$+ 4(-30m^2 H^2 + 32H^4 + 12m^2 m_H^2 + 4H^2 m_H^2 - 7m_H^4) \frac{aHV'(\hat{\phi})}{\bar{\phi}'}]$$

$$c_2 = \frac{9}{16}a^8 m^4 H^2 (2H^2 - m_H^2) [-(4H^2 + m_H^2)(3m^2 - 4H^2 - m_H^2)$$

$$+ 4(-3m^2 + 2H^2 + 2m_H^2) \frac{aHV'(\hat{\phi})}{\bar{\phi}'}]$$

Scalar sector

Scalar metric perturbations mix with scalar inflaton perturbation

L_S = a messy function of A, B, E, F , and φ_v

After imposing constraints (and a LOT of algebra) there are only two propagating degrees of freedom

$$L_{S,\mathbf{k}} = K_\Pi |\tilde{\Pi}'|^2 + M_\Pi |\tilde{\Pi}|^2 + \textcolor{red}{K_B} |\tilde{\mathcal{B}}'|^2 + M_{\mathcal{B}} |\tilde{\mathcal{B}}|^2 + \lambda_1 \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0 \tilde{\Pi}'^* \tilde{\mathcal{B}}$$

The second kinetic term coefficient is

$$\textcolor{red}{K_B} = \frac{3a^6m^2(m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4m^2(m^2 - m_H^2)}$$

and where we've defined: $m_H^2(\eta) = 2H(\eta)^2[1 - \epsilon(\eta)]$ where $\epsilon(\eta) = -\dot{H}/H^2$

Beware of ghosts

A wrong-sign kinetic term leads to dangerous ghosts!

For massive spin-2 particles in FRW spacetime, ghost avoidance requires:

$$m^2 > m_H^2(\eta) = 2H(\eta)^2[1 - \epsilon(\eta)] \quad \text{where} \quad \epsilon(\eta) = -\dot{H}/H^2$$

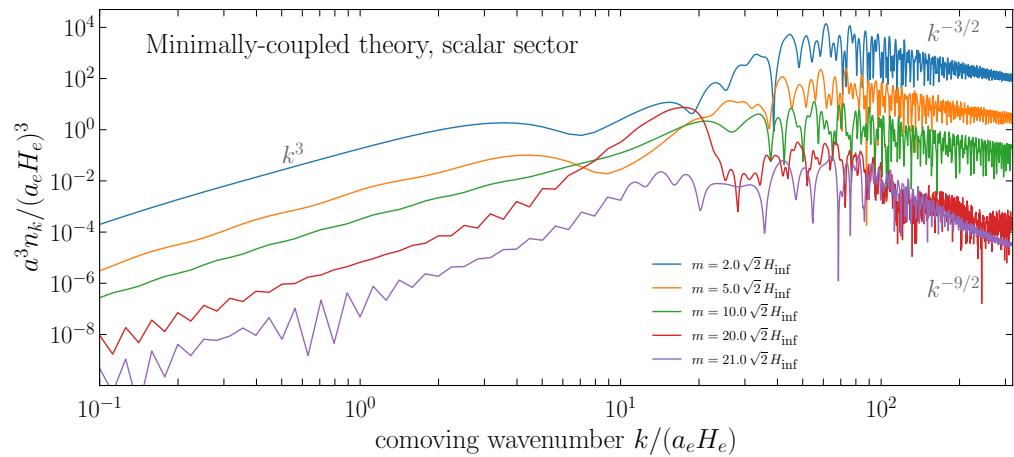
- Generalizes the Higuchi bound (for dS) to FRW spacetime
- After inflation $\epsilon > 1$ and any positive m^2 is ghost-free
- Implications for ultra-light spin-2 dark matter (e.g., time-dep mass)
- Implications for Kaluza-Klein (compact extra dimensions)
- Our numerical analysis focuses on $m^2 > 2 H_{\text{inf}}^2$ to avoid the ghost

Higuchi (1986)
see also: Fasiello & Tolley (2013)



Scalar sector - spectra

- The same low- k and high- k power law behaviors are observed in the S-sector that we noted previously in the T- and V-sectors.



Notable features:

1. Larger amplitude than T- or V-sectors
2. Lowering mass raises amplitude

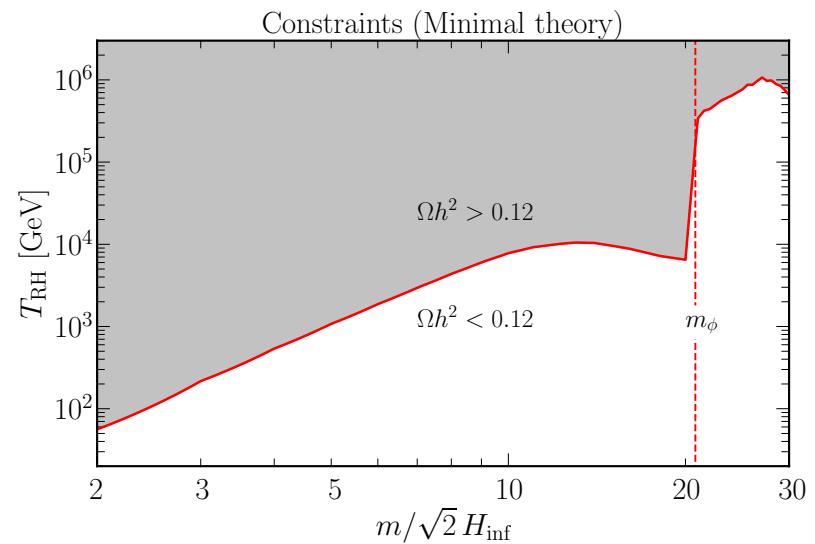
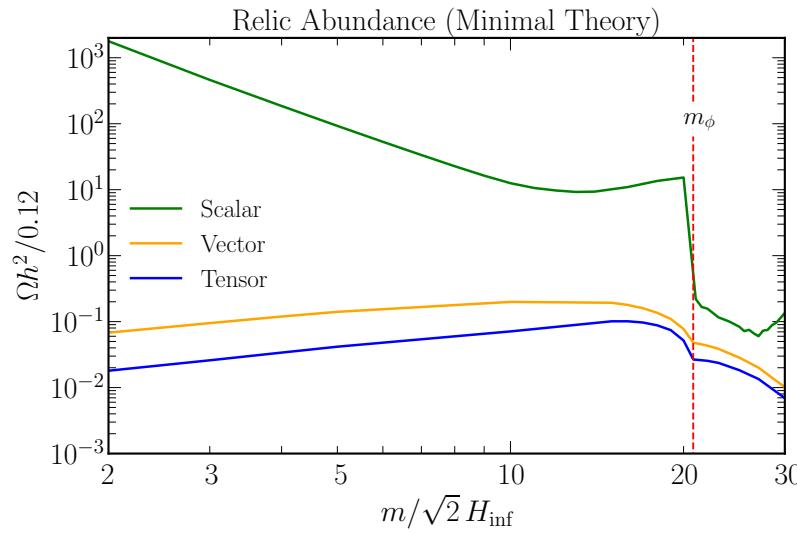
Implications for spin-2 dark matter

see also: Babichev et. al. (2016)

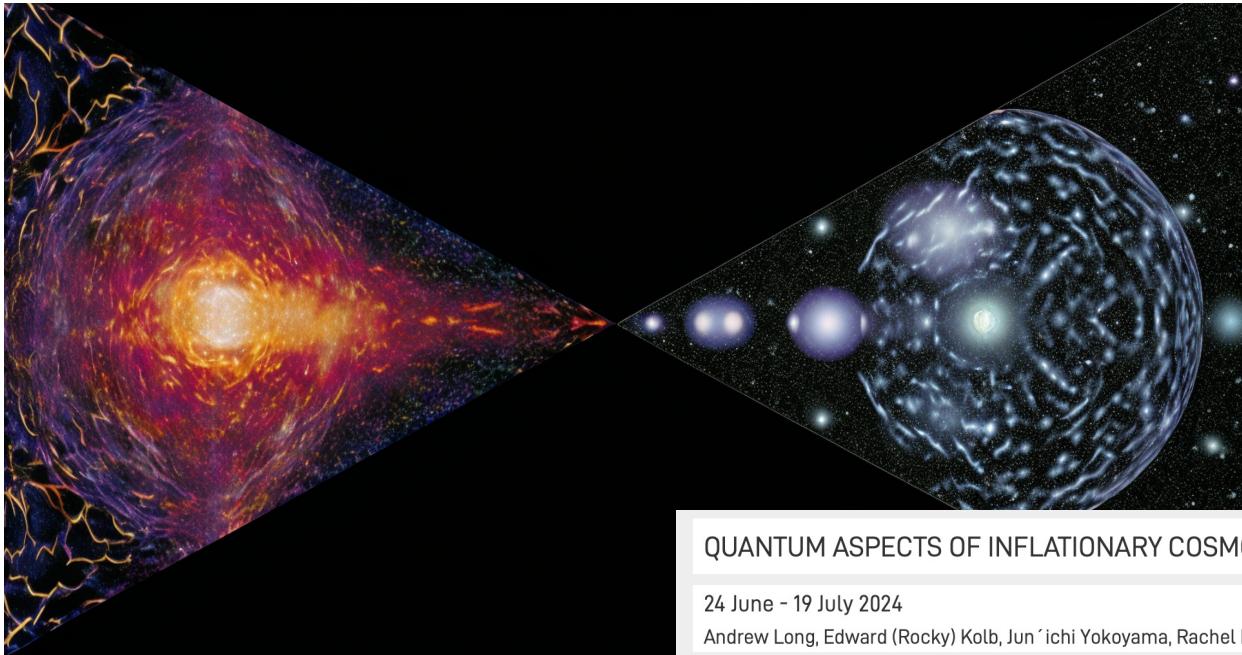
Assume: massive spin-2 particles are cosmologically long-lived

Relic abundance

$$\Omega h^2 \approx (0.114) \left(\frac{m}{10^{10} \text{ GeV}} \right) \left(\frac{H_e}{10^{10} \text{ GeV}} \right) \left(\frac{T_{\text{RH}}}{10^8 \text{ GeV}} \right) \left(\frac{a^3 n}{a_e^3 H_e^3} \right)$$



conclusion



MIAPbP
Munich Institute for
Astro-, Particle and BioPhysics

MIAPbP Workshop

Munich
June 24 – July 19, 2024

register now:
deadline is
this Sunday, Oct 22!

QUANTUM ASPECTS OF INFLATIONARY COSMOLOGY

24 June - 19 July 2024

Andrew Long, Edward (Rocky) Kolb, Jun'ichi Yokoyama, Rachel Rosen, Viatcheslav (Slava) Mukhanov

QUANTUM ASPECTS OF INFLATIONARY COSMOLOGY

① Overview

Participants

Schedule

Room reservation

Registration open

(Deadline 22 October 2023)

Contact at MIAPbP

Astrophysical and cosmological observations have revealed a wealth of information about the structure, composition, and evolution of the Universe. Although we can classify the ingredients that compose the Universe today, we don't yet know their origin. Their genesis must have been the early stages of the big bang and involved particle physics beyond the standard model. This MIAPbP program is centered around topics that sit at the connection between particle physics and cosmology:

- 1) cosmological inflation,
- 2) the end of inflation,
- 3) cosmological relics, and
- 4) gravitational particle production.

How did quantum fluctuations of the inflaton field provide the seeds for structure on cosmological scales? Did other fields play a role during inflation? How did their quantum fluctuations imprint on cosmological observables (e.g., non-Gaussianity) or survive as cosmological relics today (e.g., dark matter, matter/antimatter asymmetry)? How can these degrees of freedom be embedded into a compelling UV theory? By bringing together experts on particle physics and cosmology, we hope to develop a deeper understanding of these tough questions over the course of this 4-week MIAPbP meeting.

Summary

Question: if dark matter only interacts gravitationally, how was it produced?

Quantum fluctuations during or at the end of inflation (i.e., CGPP) can do it!

Things I talked about:

- Constraining CGPP dark matter with **CMB isocurvature** (spin-0, minimally-coupled to gravity)
- A theory of **massive spin-2** particles on an FRW background (connections to bigravity)
- CGPP interpretation as **inflaton annihilations** during reheating
- Effect of **quantum interference** leading to “fringes” in the energy spectrum
- Predicted **relic abundance** of massive spin-2 particles
- FRW-generalization of the **Higuchi bound**

Things I'd like to talk about:

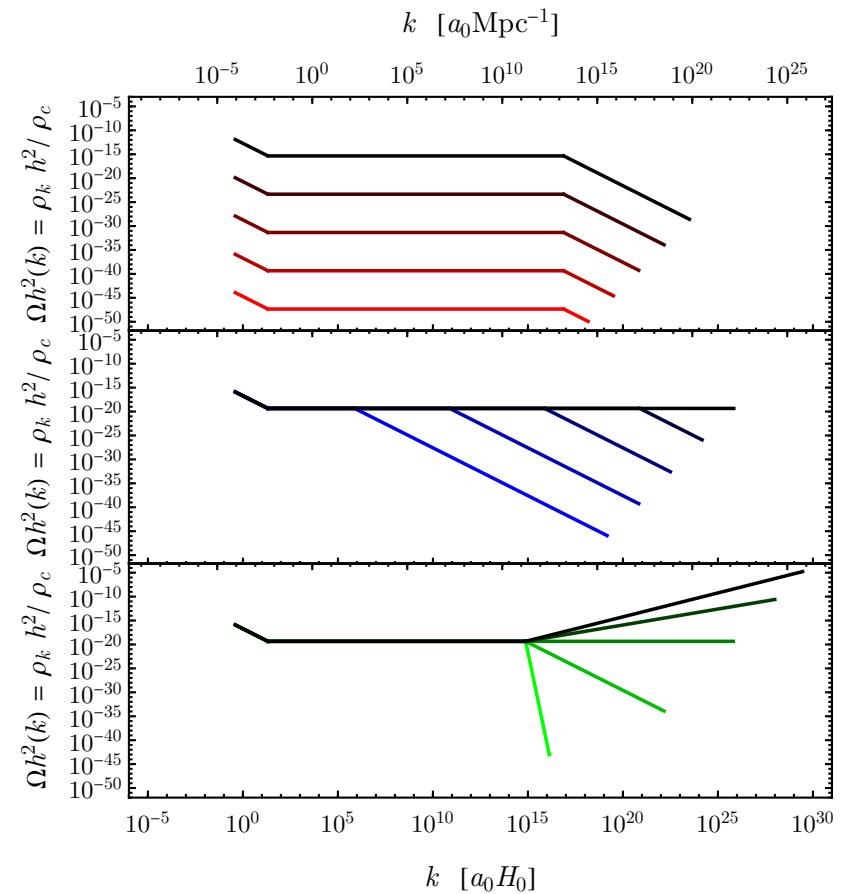
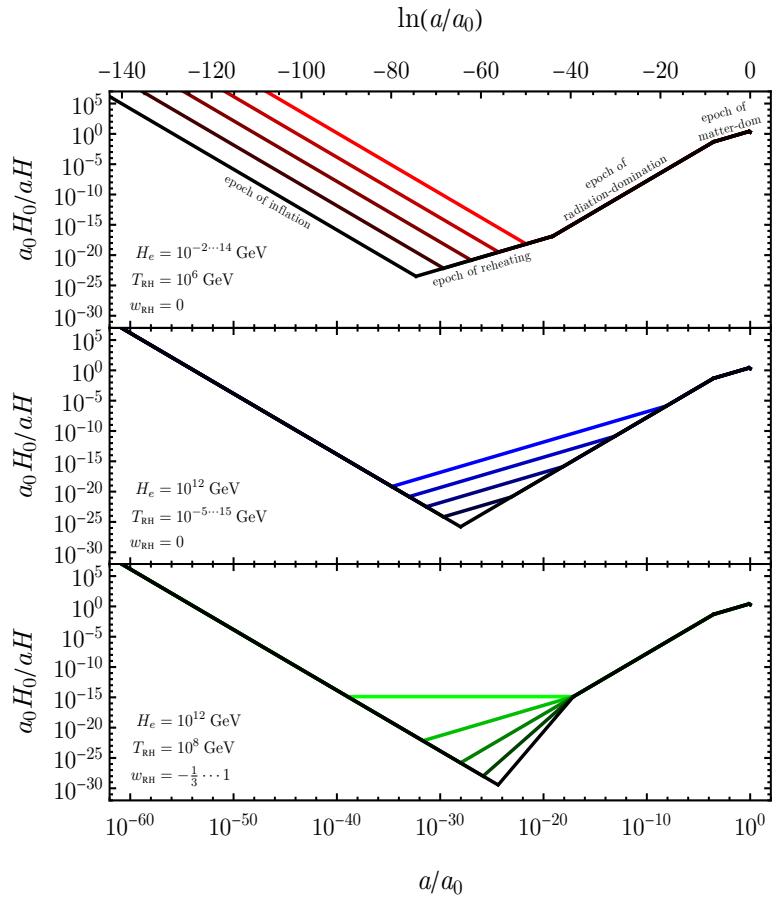
- Other theories of massive spin-2 (Kaluza Klein, string Regge trajectories)
- Observational signatures of CGPP (isocurvature, non-Gaussianity, lab tests)
- Implications for other relics (baryogenesis, dark radiation)

backup slides



massless scalar spectator spectrum

Massless scalar spectator



spin-1 particles & ultra-light vectors

A massive Proca field in FRW

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right]$$

transverse polarization modes

$$(\partial_\eta^2 + \omega_T^2) \chi_{T,k} = 0$$

$$\omega_T^2 = k^2 + a^2 m^2$$

identical to a conformally-coupled scalar

longitudinal polarization modes

$$(\partial_\eta^2 + \omega_L^2) \chi_{L,k} = 0$$

$$\omega_L^2 = k^2 + a^2 m^2 + \frac{1}{6} \frac{k^2 a^2 R}{k^2 + a^2 m^2} + 3 \frac{k^2 a^4 H^2 m^2}{(k^2 + a^2 m^2)^2}$$

extra terms from integrating out time-like component

Differentiating vectors & scalars

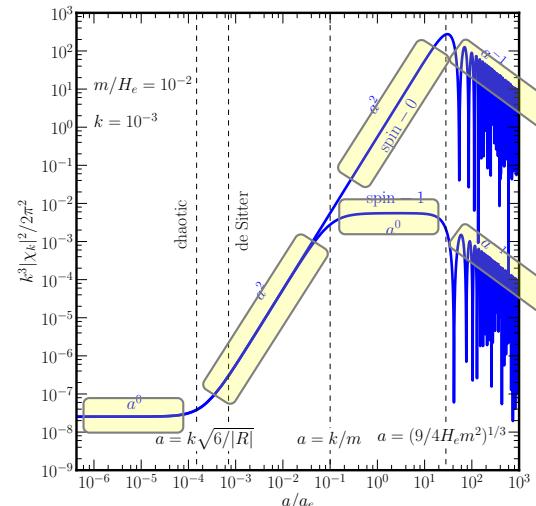
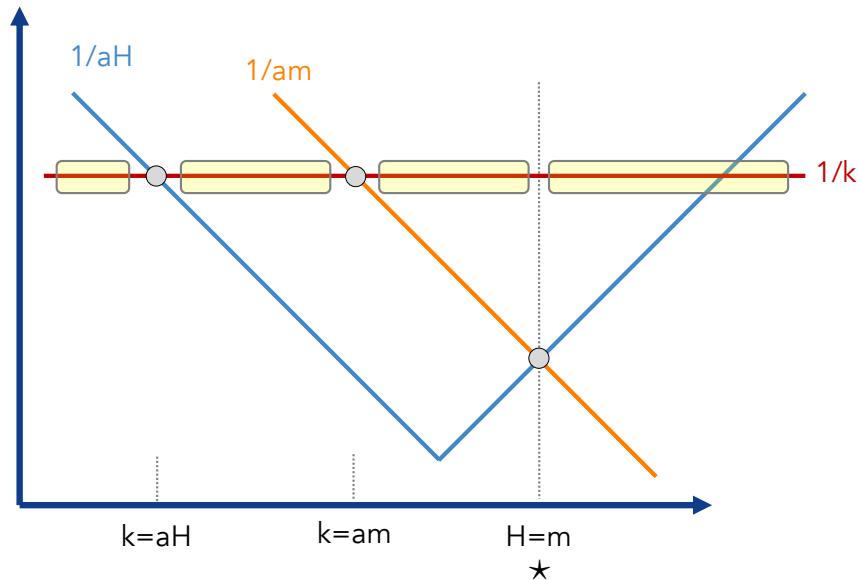
[Graham, Mardon, & Rajendran (2015)]

$$\partial_\eta^2 \chi_k + \omega^2 \chi_k = 0$$

$$\chi_k(\eta) \sim \frac{1}{\sqrt{2k}} e^{-ik\eta} \text{ (early)}$$

$$\omega_{\text{scalar}}^2 = k^2 + a^2 m^2 + \frac{1}{6} a^2 R$$

$$\omega_{L\text{-vector}}^2 = k^2 + a^2 m^2 + \frac{1}{6} \frac{k^2 a^2 R}{k^2 + a^2 m^2} + 3 \frac{k^2 a^4 H^2 m^2}{(k^2 + a^2 m^2)^2}$$



Vectors evolve differently while nonrelativistic & outside the horizon

Differentiating vectors & scalars

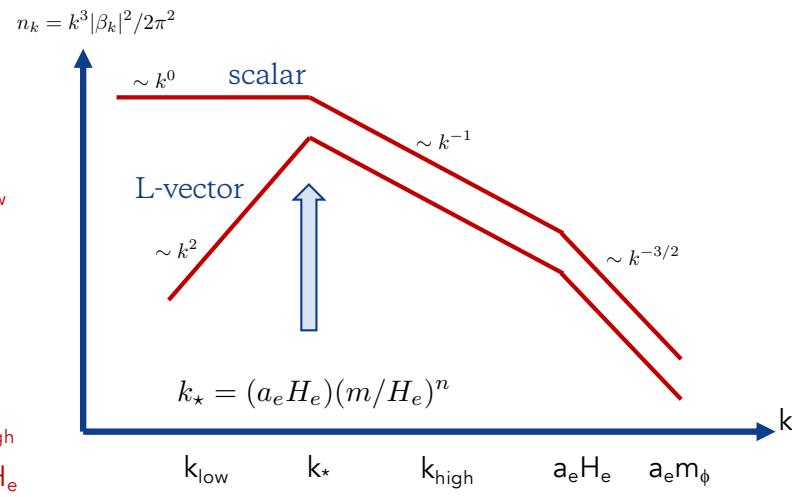
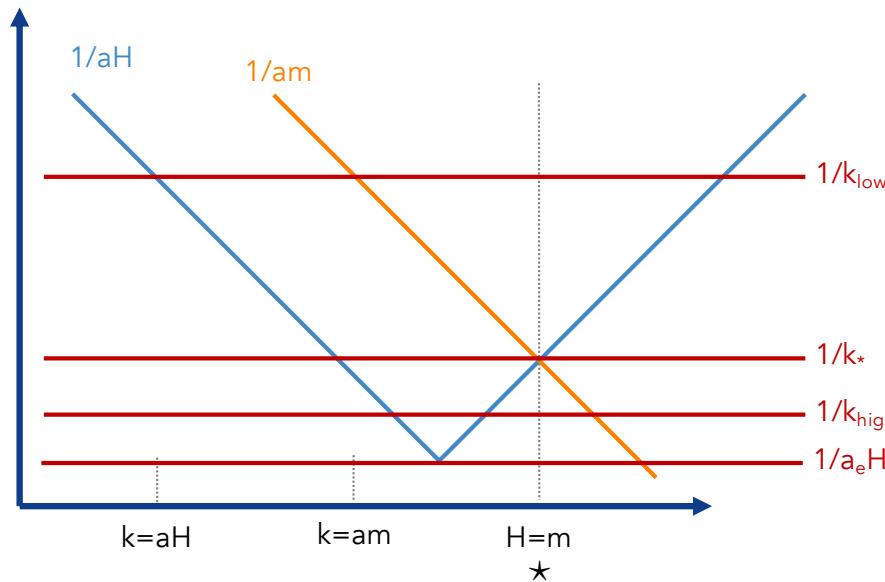
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$$\omega_{\text{scalar}}^2 = k^2 + a^2 m^2 + \frac{1}{6} a^2 R$$

$$\omega_{L\text{-vector}}^2 = k^2 + a^2 m^2 + \frac{1}{6} \frac{k^2 R}{k^2 + a^2 m^2} + 3 \frac{k^2 a^4 H^2 m^2}{(k^2 + a^2 m^2)^2}$$

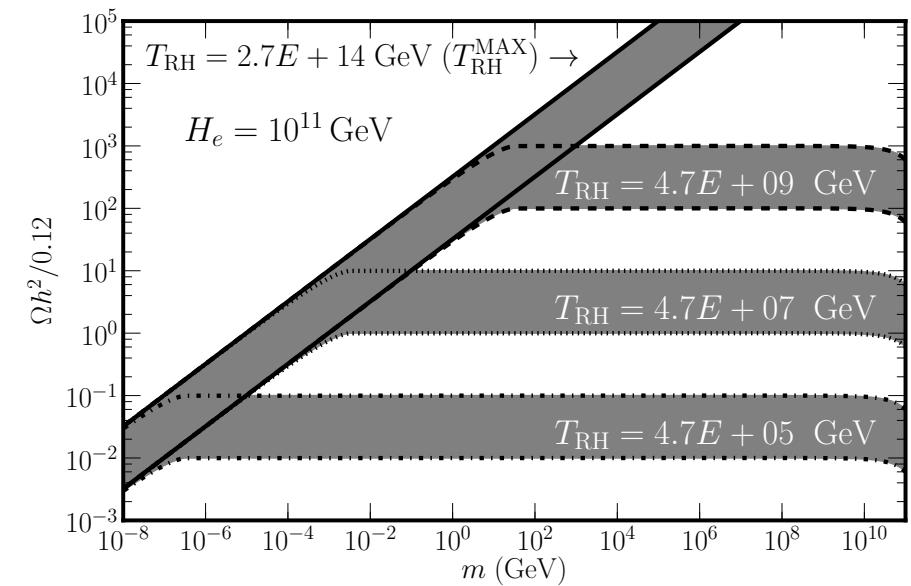


Vectors are suppressed toward low k .
Most power carried at k_*

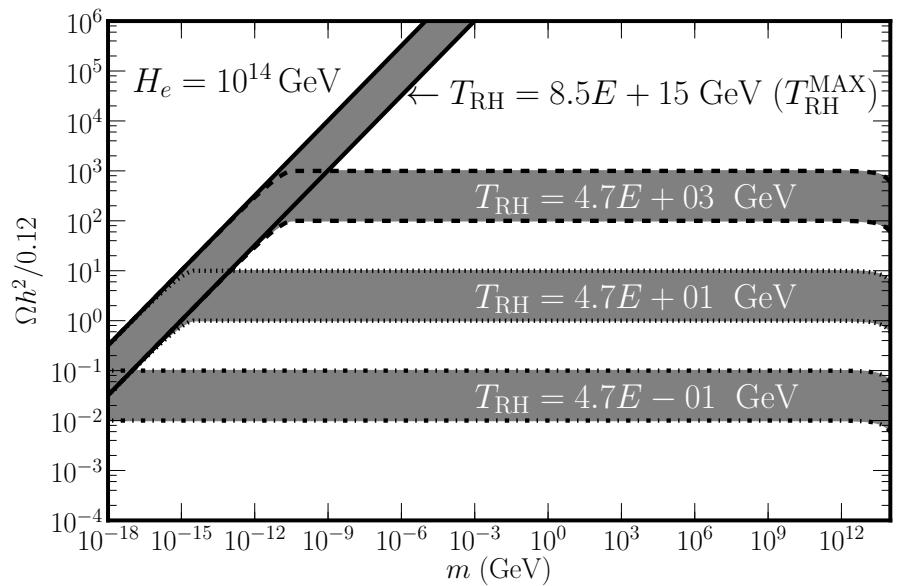
Relic abundance

[Kolb & Long (2020)], see also: [Ahmed, Grzadkowski, & Socha (2020)]

All the spin-1 dark matter is produced gravitationally for $H_{\text{inf}} < 10^{14} \text{ GeV}$ & $m > \mu\text{eV}$

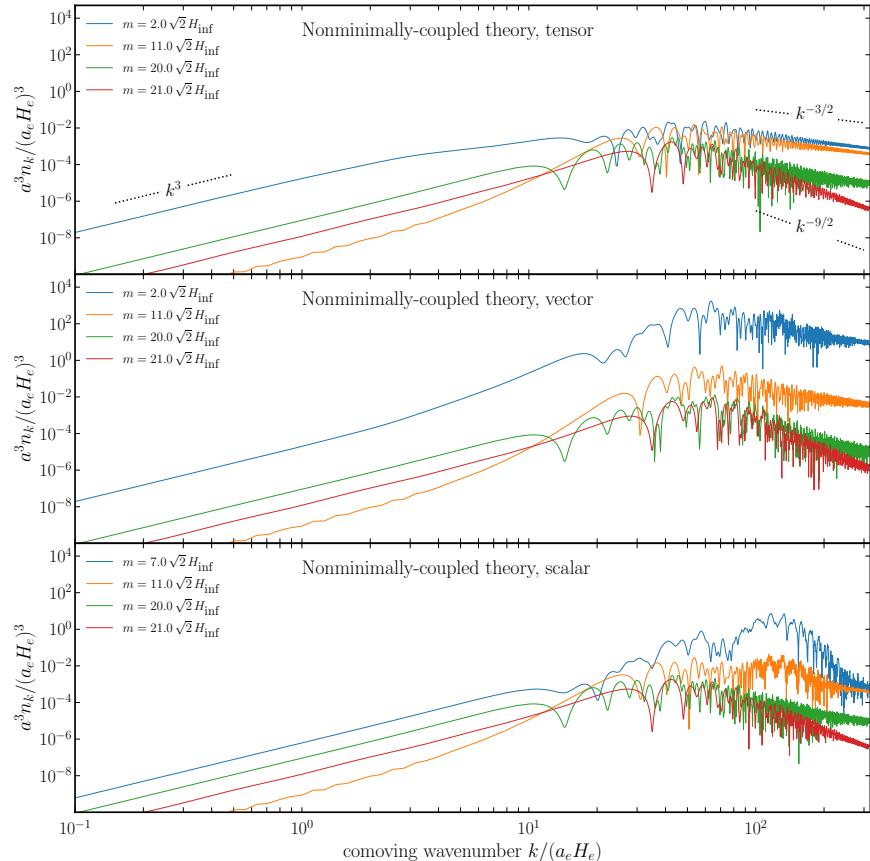
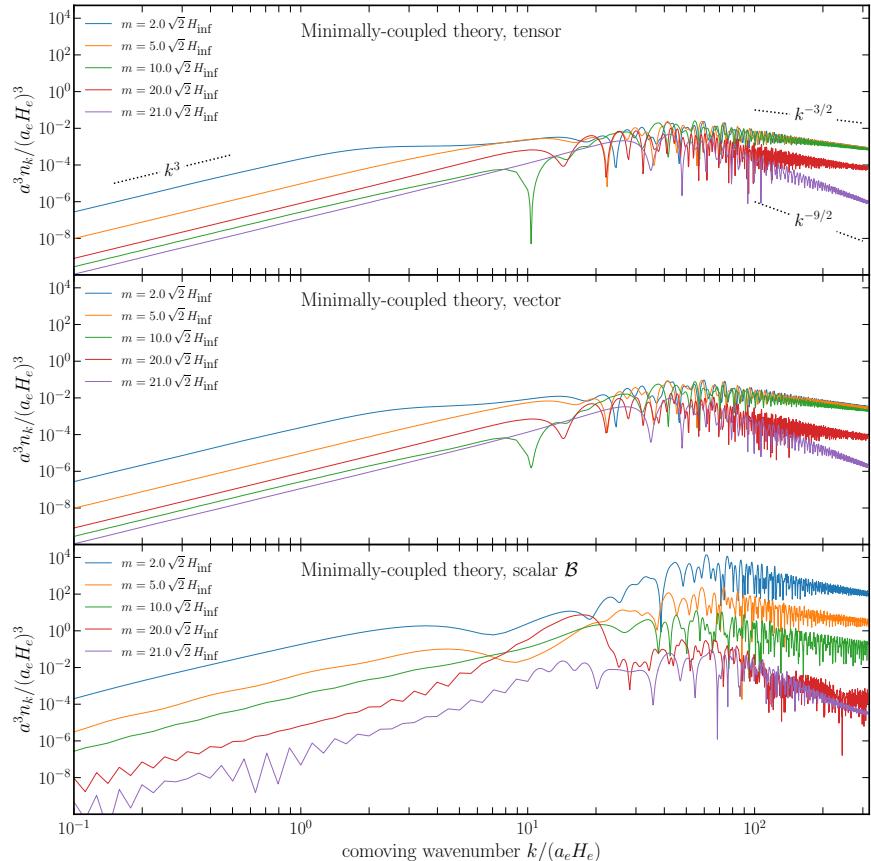


$$\Omega_\chi h^2 \simeq \begin{cases} (0.130) \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^2 \left(\frac{T_{\text{RH}}}{10 \text{ GeV}} \right) & , \quad t_e \leq t_\star < t_{\text{RH}} \\ (0.201) \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^2 \left(\frac{m}{\mu\text{eV}} \right)^{1/2} & , \quad t_e \leq t_{\text{RH}} < t_\star \end{cases}$$

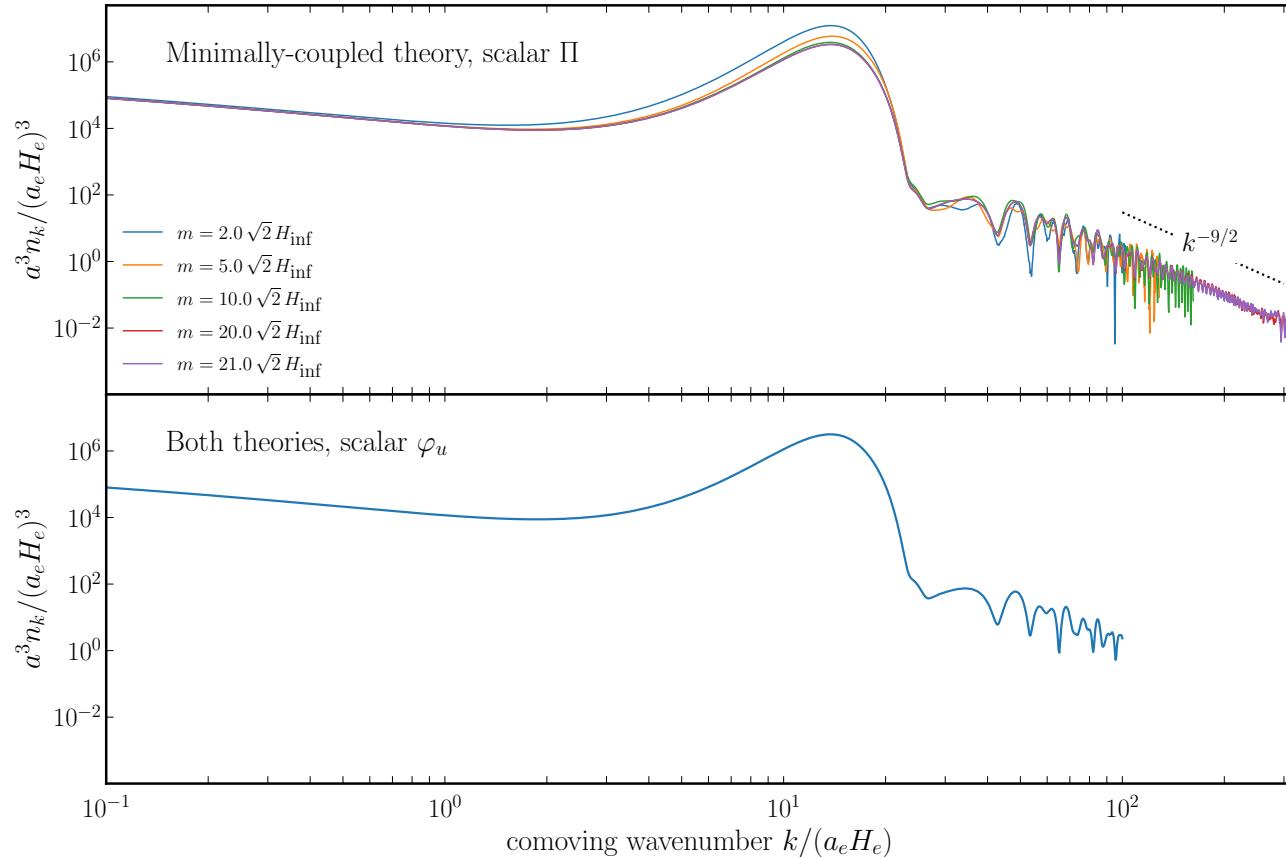


spin-2 CGPP extra plots

Numerical results – spectra



Scalar sector - spectra



- We also calculate spectra for the inflaton-like scalar perturbations. This is just the usual quasi-scale invariant spectrum of curvature perturbations.

nightmare scenario

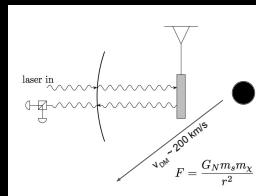
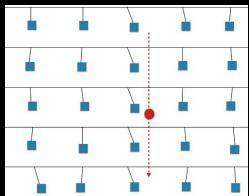
A “nightmare” scenario?

*NO SIGNATURES AT
DIRECT D, INDIRECT D, COLLIDERS
... SCARY !!??*



DIRECT DETECTION

The Windchime Project
(snowmass w.p.: 2203.07242)



COSMOLOGICAL SIGNATURES

CDM-Photon Isocurvature

Neutrinos
CDM
Photons
Baryons

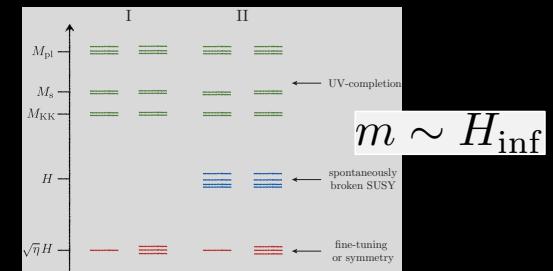
Adiabatic

CDM
isocurvature

*image: <http://danielgrin.net>

THEORETICALLY COMPELLING

You can't turn off gravity!



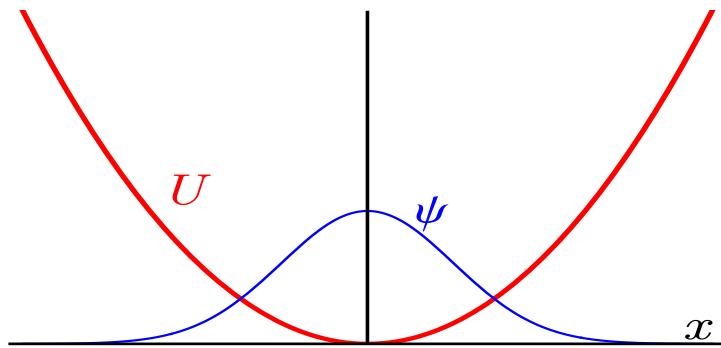
*Baumann & McAllister 1404.2601

intuition for CGPP

An analogy with 1D quantum mechanics



Spring constant is varied
slowly (adiabatically)



Spring constant is varied
abruptly (non-adiabatically)

