Future e+e- electroweak measurements : W-pair threshold lineshape

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ECFA WG1-PREC MiniWorkshop on Cross Section Lineshapes

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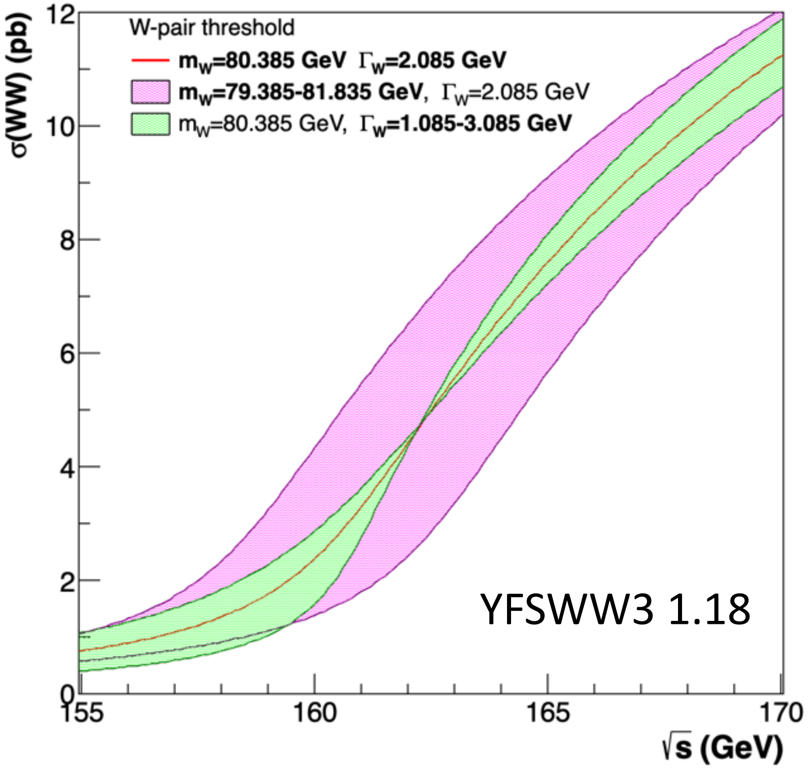
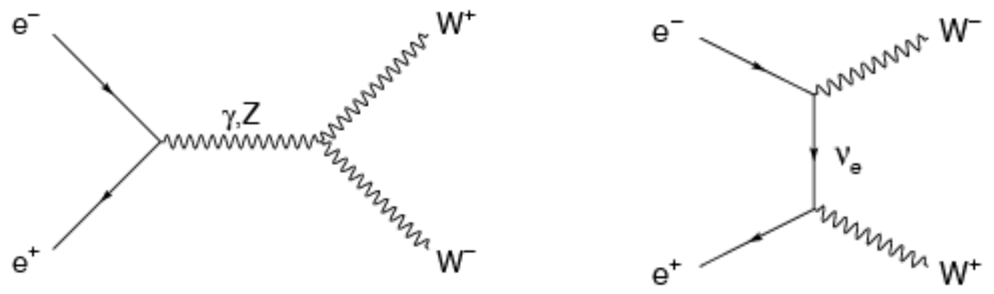
The WW threshold lineshape and the W mass



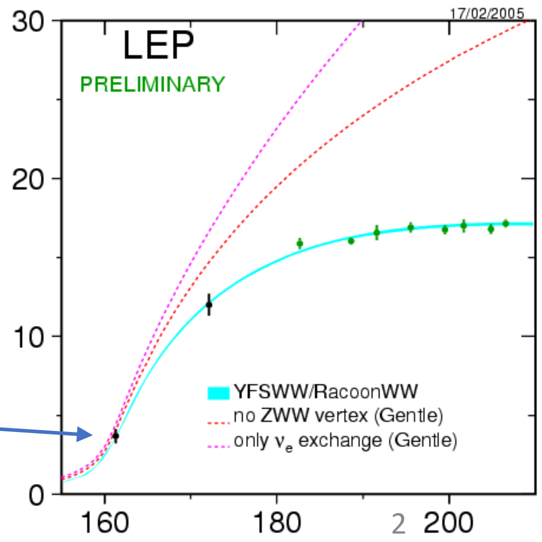


WW cross section rise $\beta = \sqrt{1 - 4m_W^2/s}$ driven by t-channel production

Extract the W mass inverting the m_W dependence

$$\sigma(m_W, E) \qquad\qquad m_W = \sigma^{-1}(E)$$

$$\Delta m_W = \left(\frac{d\sigma}{dm_W}\right)^{-1} \Delta\sigma$$



ALEPH Phys.Lett.B 401 (1997) 347 with 10/pb $m_W = 80.14 \pm 0.34$ GeV

stat extrapolation to 10/ab $\Longrightarrow$ $\Delta m_W = 0.34$ MeV

The WW threshold : W mass uncertainties

$$\sigma = \left(\frac{N}{L} - \sigma_B\right)\frac{1}{\varepsilon}$$

$$\Delta m_W(stat) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{L}} \frac{1}{\sqrt{\varepsilon p}}$$

Statistical

$$\Delta\sigma_{WW} = \frac{\Delta\sigma_B}{\varepsilon}$$

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\sigma_B}{\varepsilon} \oplus \Delta\sigma_{TH}\right)$$

Background and Theory

$$\Delta\sigma_{WW} = \sigma\left(\frac{\Delta\varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L}\right)$$

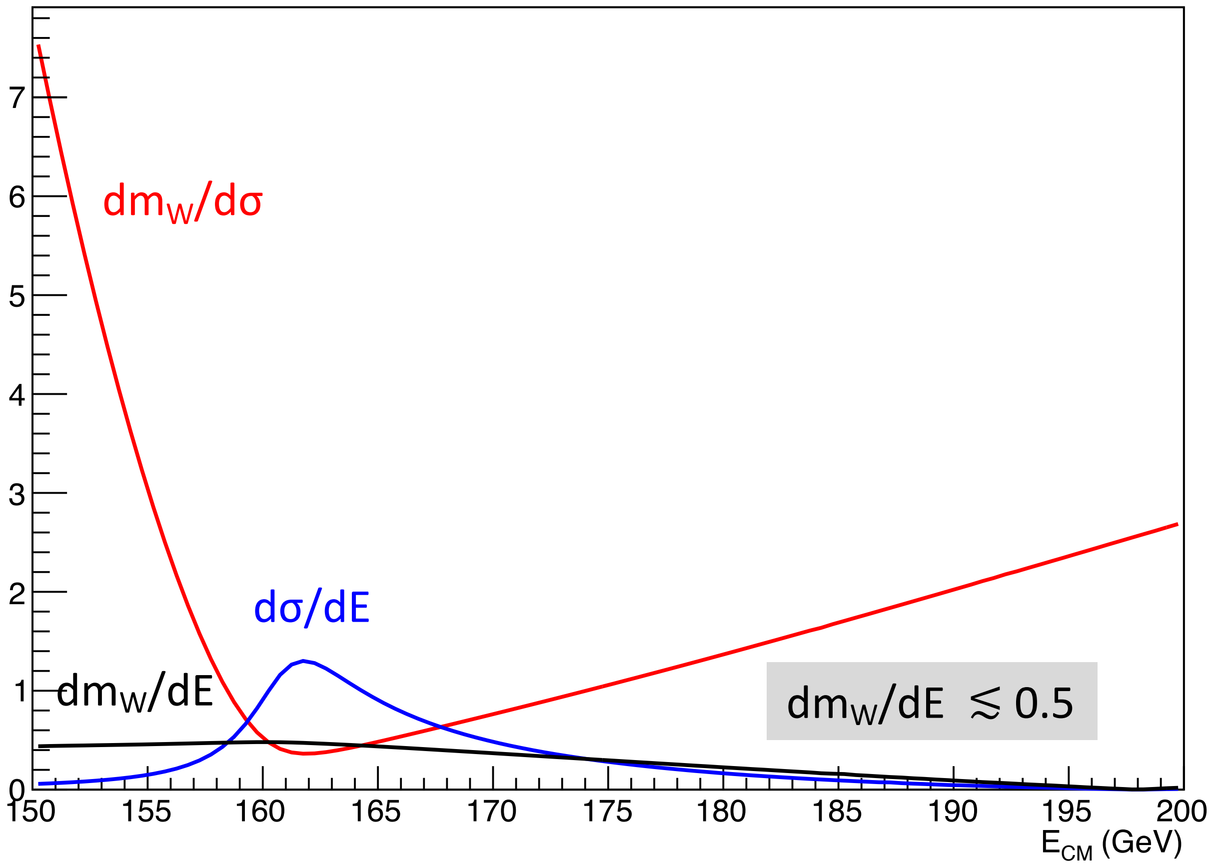
$$\Delta m_W(\varepsilon) = \sigma\left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\varepsilon}{\varepsilon} + \frac{\Delta L}{L}\right)$$

Acceptance and Luminosity

$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{d\sigma}{dE}\right)\Delta E \leq \frac{1}{2}\Delta E$$

Collision energy

The WW threshold W mass : beam energy



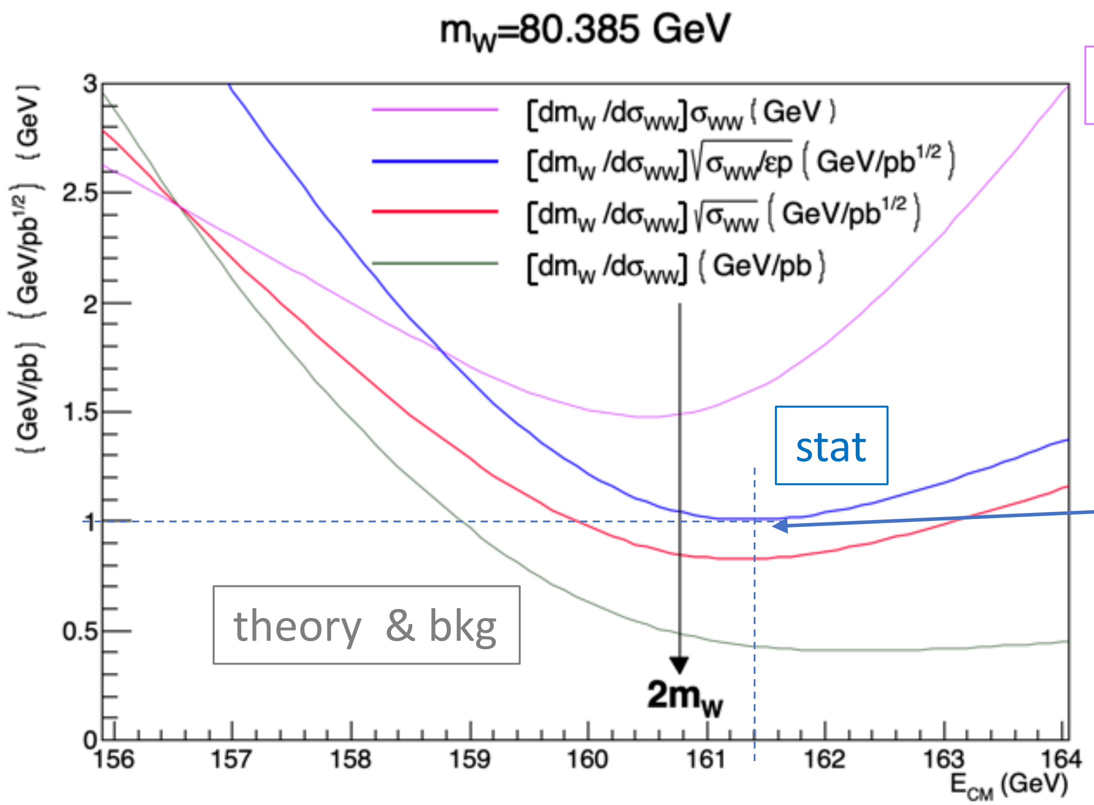
$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W}\right)^{-1}\left(\frac{d\sigma}{dE}\right)\Delta E \le \frac{1}{2}\Delta E$$

Uncertainty on beam energy $\Delta E_b = \frac{1}{2}\Delta E$ translates directly to m_W

$$\Delta E_b \cong \Delta m_W$$

Very limited variations of the dm_W/dE coefficient with E_{CM} in the threshold region

The WW threshold : W mass optimal E_{CM}



stat uncertainty assuming event selection quality
$Q=\sqrt{\epsilon p}$ with fixed ϵ=0.75 and σ_B=0.3pb

Max stat sensitivity at E_{CM}~$2m_W$ + 0.6 GeV

$$\left[\left(\frac{d\sigma}{dm_W}\right)^{-1}\frac{\sqrt{\sigma}}{\sqrt{\epsilon p}}\right]_{min} \cong 1\ \frac{GeV}{pb^{1/2}} = 1\ \frac{MeV}{ab^{1/2}}$$

With L=12/ab $\Longrightarrow$ Δm_W(stat) = 0.3 MeV

WW threshold : W mass precision requirements

Conditions to achieve Δm_W(syst)< Δm_W(stat) = 0.3 MeV
with a single point WW threshold measurement

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\sigma_B}{\varepsilon} \oplus \Delta\sigma_{TH}\right)$$

Background and Theory

$\Delta\sigma_{TH} < \mathbf{1fb}$ $(\Delta\sigma_{TH}/\ \sigma_{TH} < 2\cdot 10^{-4})$
$\Delta\sigma_B/\varepsilon <$ **1fb** $(\Delta\sigma_B/\ \sigma_B < 4\cdot 10^{-3})$

$$\Delta m_W(\varepsilon) = \sigma \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\varepsilon}{\varepsilon} + \frac{\Delta L}{L}\right)$$

Acceptance and Luminosity

$\left(\frac{\Delta\varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L}\right) < \mathbf{2\cdot 10^{-4}}$

$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{d\sigma}{dE}\right)\Delta E \leq \frac{1}{2}\Delta E$$

Collision energy

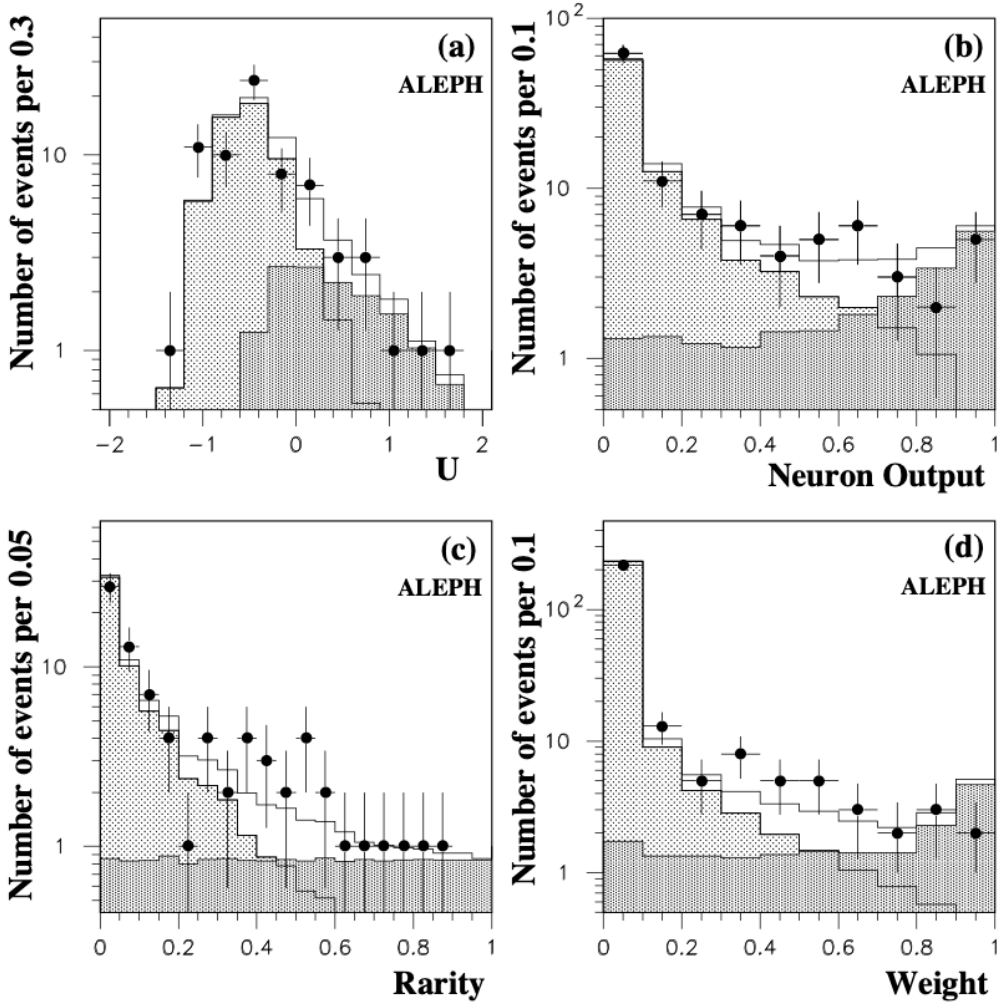
$\Delta E_b < 0.3\ MeV$ $(\Delta E_b/\ E_b < 4\cdot 10^{-6})$

The WW threshold : background

almost all bkg in the 4q channel

Selection	Expected signal	Expected background	Observed
$W^+W^- \to q\bar{q}q\bar{q}$	9.6 ± 1.0	3.44 ± 0.39	14
$W^+W^- \to q\bar{q}e\bar{\nu}_e$	3.89 ± 0.44	0.18 ± 0.27	3
$W^+W^- \to q\bar{q}\mu\bar{\nu}_\mu$	4.19 ± 0.46	0.27 ± 0.15	2
$W^+W^- \to q\bar{q}\tau\bar{\nu}_\tau$	2.32 ± 0.28	0.96 ± 0.34	7
$W^+W^- \to \ell^+\nu_\ell\ell'^-\bar{\nu}_{\ell'}$	2.58 ± 0.28	$0.19^{+0.12}_{-0.04}$	2
Combined	22.6 ± 2.4	5.0 ± 0.6	28

OPAL Phys. Lett. B 389 (1996) 416.



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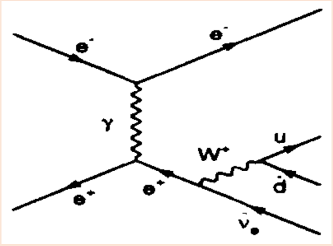
purity ~95% achieved in the last bins

4-fermion-CC03 interference effects



positive & negative effects (10-50 fb) reported in the various channels, within the LEP analyses acceptance

WW threshold : acceptance syst

Syst unc at higher E_{CM} (207 GeV) on σ_{WW} (~16pb)

Source	uncertainty (fb)			
	$\ell\nu\ell\nu$	$\ell\nu$qq	qqqq	total
Tracking	4	19	31	5[illegible]
Simulation of calorimeters	-	9	26	31
Hadronization models	-	27	8	35
Z peak q$\bar{\text{q}}$ fragmentation	-	-	20	20
Inter-W final state interaction	-	-	28	28
Background contamination	9	5	31	35
Lepton identification	1	2	-	3
Beam-related background	10	17	37	22
$\mathcal{O}(\alpha)$ corrections DPA	2	9	12	6
Luminosity	8	35	44	87
Simulation statistics	6	20	14	25
Total	17	57	87	126

ALEPH *Eur.Phys.J.C* 38 (2004) 147

Source	$\sigma_{WW}^{q\bar{q}q\bar{q}}$ (pb)	$\sigma_{WW}^{q\bar{q}l\nu}$ (pb)	$\sigma_{WW}^{l\nu l\nu}$ (pb)
Four-jet modelling	±0.051	±0.014	-
Background cross-sections	±0.009	±0.016	±0.006
Fragmentation	±0.045	±0.038	-
Final state interactions	±0.025	-	-
Radiative corrections	±0.008	±0.008	±0.002
Luminosity (theor)	±0.011	±0.010	±0.002
Luminosity (exp)	±0.045	±0.043	±0.011
Detector effects	±0.045	±0.053	±0.033
Monte Carlo statistics	±0.005	±0.014	±0.033

DELPHI *Eur.Phys.J.C* 34 (2004) 127

NP QCD effects have important impacts on both qqqq and qq$\ell\nu$

need improvements in fragmentation and hadronization modeling plus constraints from control data (Z→qq)

less worrisome than using jet properties for kin reco

can roughly scale/4 for equivalent ε effects at threshold σ_{WW} (~4pb)

target : bring table items below 4fb(/4=1fb)

WW threshold @ ILC

arXiv:1603.06016 & arXiv:1908.11299

ILC polarised collisions : enhance (x4) t-channel WW production or suppress it to control background

Channel	Efficiency (%)	$\sigma^U_{\rm bkgd}$ (fb)	$A^B_{\rm LR}$	Eff. syst. (%)	Bkgd syst.	$A^B_{\rm LR}$ syst.
lvlv	87.5	10	0.15	0.1	free	0.025
qqlv	87.5	40	0.30	0.1	free	0.012
qqqq	83.5	200	0.48	0.1	free	0.005

Table 3: Experimental assumptions for the WW event selection near threshold using a polarized scan

$\sqrt{s}$ (GeV)	L (fb^{-1})	f	$\lambda_{e^-}\lambda_{e^+}$	N_{ll}	N_{lh}	N_{hh}	N_{RR}
160.6	4.348	0.7789	$-+$	2752	11279	12321	926968
		0.1704	$+-$	20	67	158	139932
		0.0254	$++$	2	19	27	6661
		0.0254	$--$	21	100	102	8455
161.2	21.739	0.7789	$-+$	16096	67610	73538	4635245
		0.1704	$+-$	98	354	820	697141
		0.0254	$++$	37	134	130	33202
		0.0254	$--$	145	574	622	42832
161.4	21.739	0.7789	$-+$	17334	72012	77991	4639495
		0.1704	$+-$	100	376	770	697459
		0.0254	$++$	28	104	133	33556
		0.0254	$--$	135	553	661	42979
161.6	21.739	0.7789	$-+$	18364	76393	82169	4636591
		0.1704	$+-$	81	369	803	697851
		0.0254	$++$	43	135	174	33271
		0.0254	$--$	146	618	681	42689
162.2	4.348	0.7789	$-+$	4159	17814	19145	927793
		0.1704	$+-$	16	62	173	138837
		0.0254	$++$	10	28	43	6633
		0.0254	$--$	46	135	141	8463
170.0	26.087	0.7789	$-+$	63621	264869	270577	5560286
		0.1704	$+-$	244	957	1447	838233
		0.0254	$++$	106	451	466	40196
		0.0254	$--$	508	2215	2282	50979

Table 1: Illustrative example of the numbers of events in each channel for the standard 100 fb^{-1} 6-point ILC scan with 4 helicity configurations. Columns give the center-of-mass energy, $\sqrt{s}$, the apportioned integrated luminosity, the fraction for each helicity configuration, $\lambda_{e^-}\lambda_{e^+}$, and the numbers of events observed in each channel.

with 100 fb−1

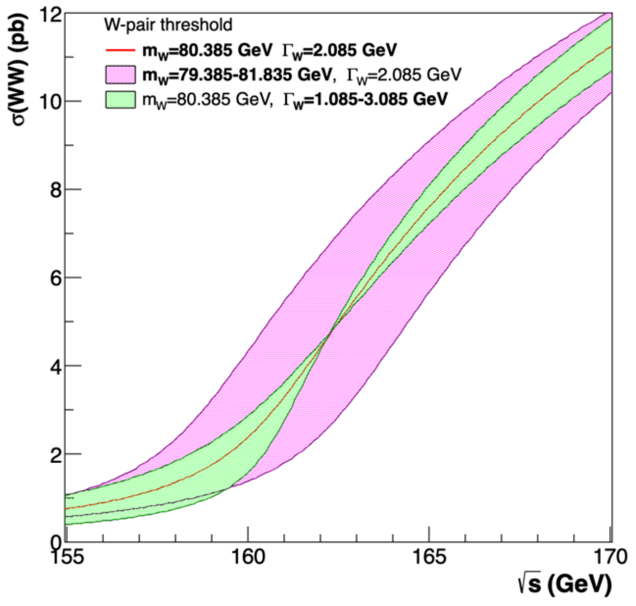
Fit type	Uncertainty source	ΔM_W [MeV]	ΔM_W (syst.) [MeV]
fixbkg	Background	3.20	2.30
fixpol	Polarization	3.73	1.27
fixeff	Efficiency	3.86	1.18
fixlum	Luminosity	3.76	0.78
fixALRB	$A^B_{\rm LR}$	3.86	0.80
fixall	Statistical	2.43	
	Systematic		3.10
standard	Total Error	3.94	

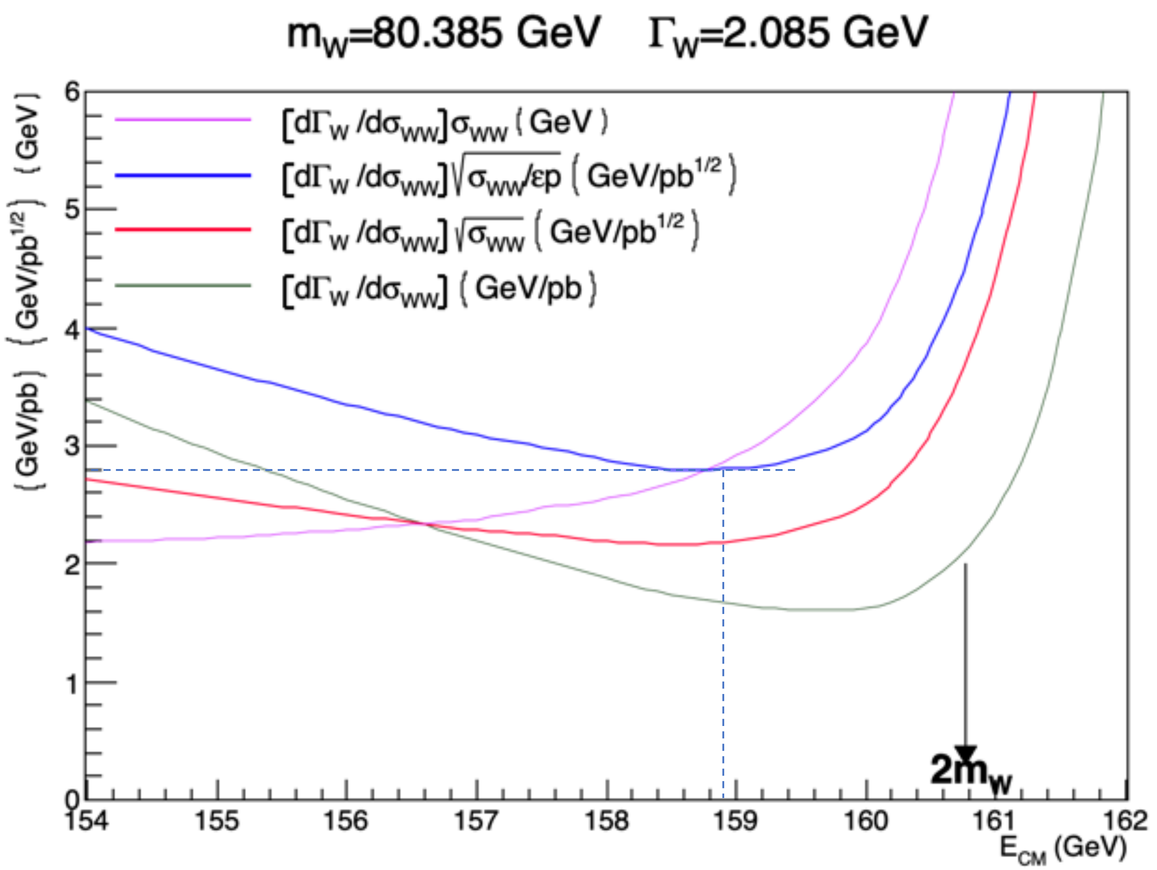
$$\Delta m_W(\text{MeV}) = 2.4\ (\text{stat}) \oplus 3.1\ (\text{syst}) \oplus 0.8\ (\sqrt{s}) \oplus \text{theory}$$

fitted $\Delta\varepsilon\sim 10^{-3}$ and $\Delta\sigma_B$ ~6 fb
additional impact of pol uncertainty

WW threshold : W mass and width

arXiv:1703.01626
arXiv:2107.04444





Max stat sensitivity at E_{CM}~ $2m_W$ - Γ_W

$$\left[\left(\frac{d\sigma}{d\Gamma_W}\right)^{-1}\frac{\sqrt{\sigma}}{\sqrt{\varepsilon p}}\right]_{min} \cong 2.8\ \frac{GeV}{pb^{1/2}} = 2.8\ \frac{MeV}{ab^{1/2}}$$

WW threshold : W mass and width

With cross section $\sigma_1\sigma_2$ measurements at two energies E_1 E_2 : uncertainty propagation

$$\begin{cases}\sigma_1 = \sigma_{WW}(E_1, m_W, \Gamma_W) \\ \sigma_2 = \sigma_{WW}(E_2, m_W, \Gamma_W)\end{cases}$$

$$\begin{cases}\Delta\sigma_1 = a_1\Delta m + b_1\Delta\Gamma \\ \Delta\sigma_2 = a_2\Delta m + b_2\Delta\Gamma\end{cases}$$

$$a_1 = \frac{d\sigma_1}{dm} \qquad b_1 = \frac{d\sigma_1}{d\Gamma}$$

$$a_2 = \frac{d\sigma_2}{dm} \qquad b_2 = \frac{d\sigma_2}{d\Gamma}$$

$$\Delta m = -\frac{b_2\Delta\sigma_1 - b_1\Delta\sigma_2}{a_2 b_1 - a_1 b_2}$$

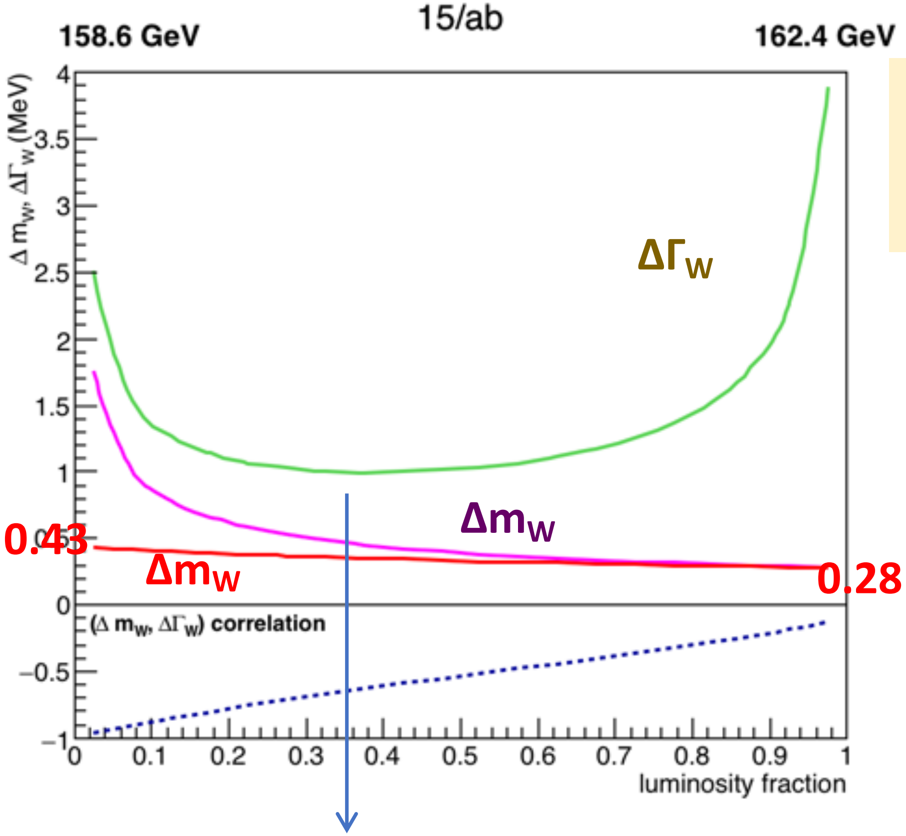
$$\Delta\Gamma = \frac{a_2\Delta\sigma_1 - a_1\Delta\sigma_2}{a_2 b_1 - a_1 b_2}$$

$\Delta m, \Delta\Gamma$ linear correlation with uncorrelated $\Delta\sigma_1, \Delta\sigma_2$

$$r = -\frac{1}{\Delta m \Delta\Gamma}\frac{a_2 b_2 \Delta\sigma_1^2 + a_1 b_1 \Delta\sigma_2^2}{\left(a_2 b_1 - a_1 b_2\right)^2}$$

WW threshold : W mass and width

Scans of possible E_1 E_2 data taking energies and luminosity fractions f (at the E_2 point)



A -minimum of **$\Delta\Gamma_W$=0.91 MeV** with **Δm_W=0.55 MeV**
taking data at E_1=**156.6** GeV E_2=**162.4** GeV f=**0.25**
yields **Δm_W=0.47 MeV** (as single par)

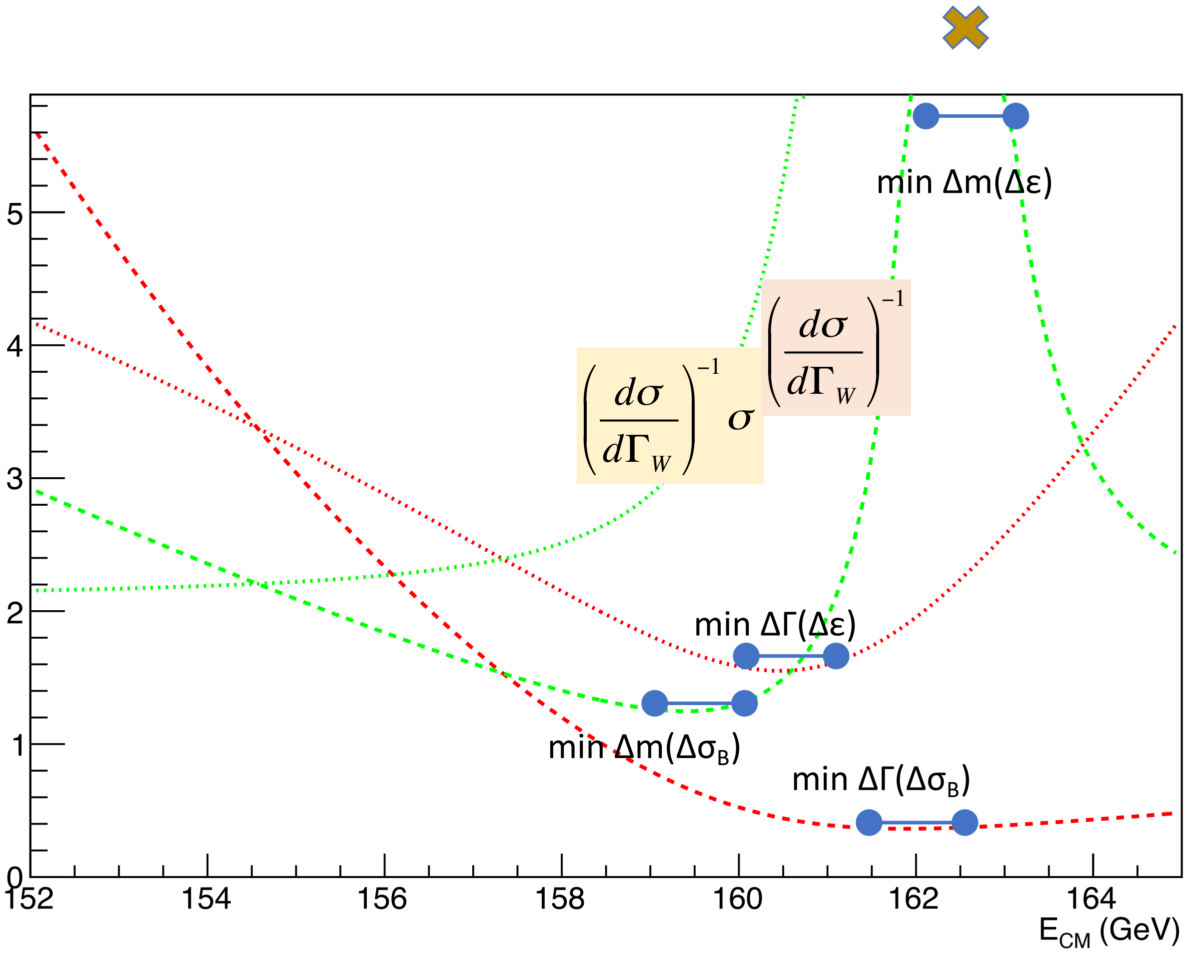
B- minimum of **Δm_W=0.28 MeV** **$\Delta\Gamma_W$=3.3 MeV** with
E_1=**155.5** GeV E_2=**162.4** GeV f=**0.95**
yields **Δm_W=0.28 MeV** (as single par)

C- minimum of **$\Delta\Gamma_W$=0.96 MeV** **+Δm_W=0.41 MeV** with
E_1=**157.5** GeV E_2=**162.4** GeV f=**0.45**
yields and **Δm_W=0.37 MeV** (as single par)

Δm_W =0.45 MeV , $\Delta\Gamma_W$=1 MeV (r=-0.6)
Δm_W=0.35 MeV

Δm_W , $\Delta\Gamma_W$: error on W mass and width from fitting both
Δm_W : error on W mass from fitting only m_W

WW threshold : W mass and width



optimal E points with limiting correlated systs

Scans of (E_1, E_2, f) data taking **assuming limiting syst uncertainties**, either Δε+ΔL or $\Delta\sigma_B + \Delta\sigma_{TH}$

$\left(\frac{d\sigma}{dm_W}\right)^{-1}\sigma$

More complex situation, depends very much on the correlation of uncertainties between the energy points (that can be quite large)

Correlated syst can cancel taking data at different E_{CM} points where the relevant differential factors are equal (around their minima)

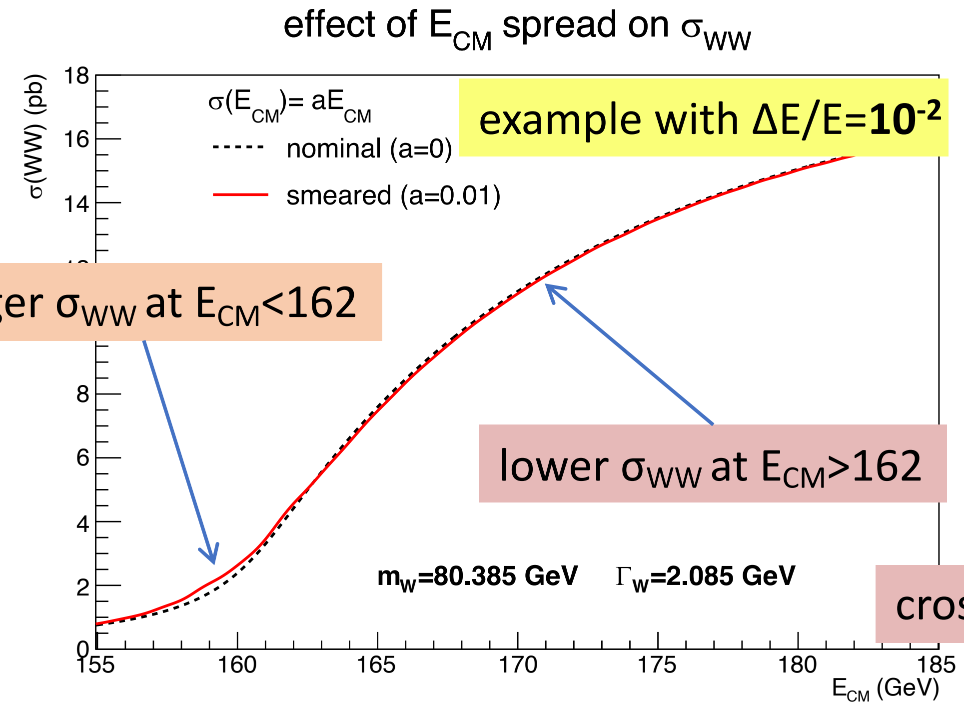
$\left(\frac{d\sigma}{dm_W}\right)^{-1}$

>2 energy points will be beneficial to reduce the impact of (correlated) systematic uncertainties careful choice of additional points recommended

partially explored in Eur. Phys. J. C 80 no. 1, (2020) 66

WW threshold : energy spread effects

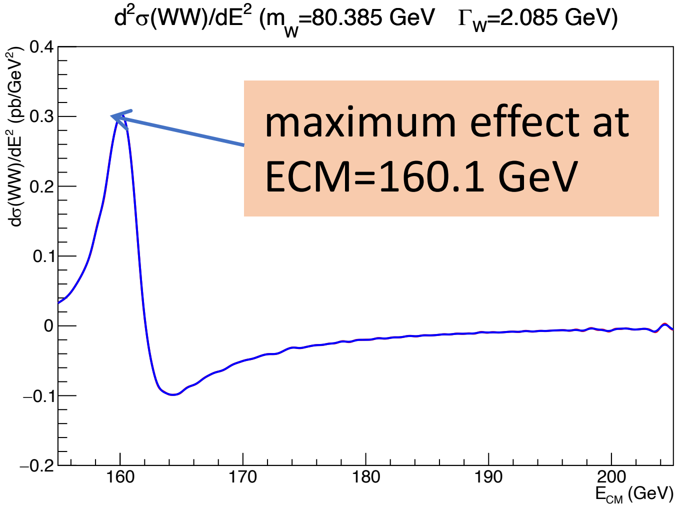
arXiv:1909.12245



example with $\Delta E/E$=**10^{-2}**

larger σ_{WW} at E_{CM}<162

lower σ_{WW} at E_{CM}>162



maximum effect at ECM=160.1 GeV

crossing point near the $d\sigma_{WW}/\Gamma_W$=0 point

$\sigma(E_{CM})$=**(0.47-1.10) 10^{-3}** E_{CM}

Optimal m_W & Γ_W points @E_{CM}=**157.3 & 162.6** GeV

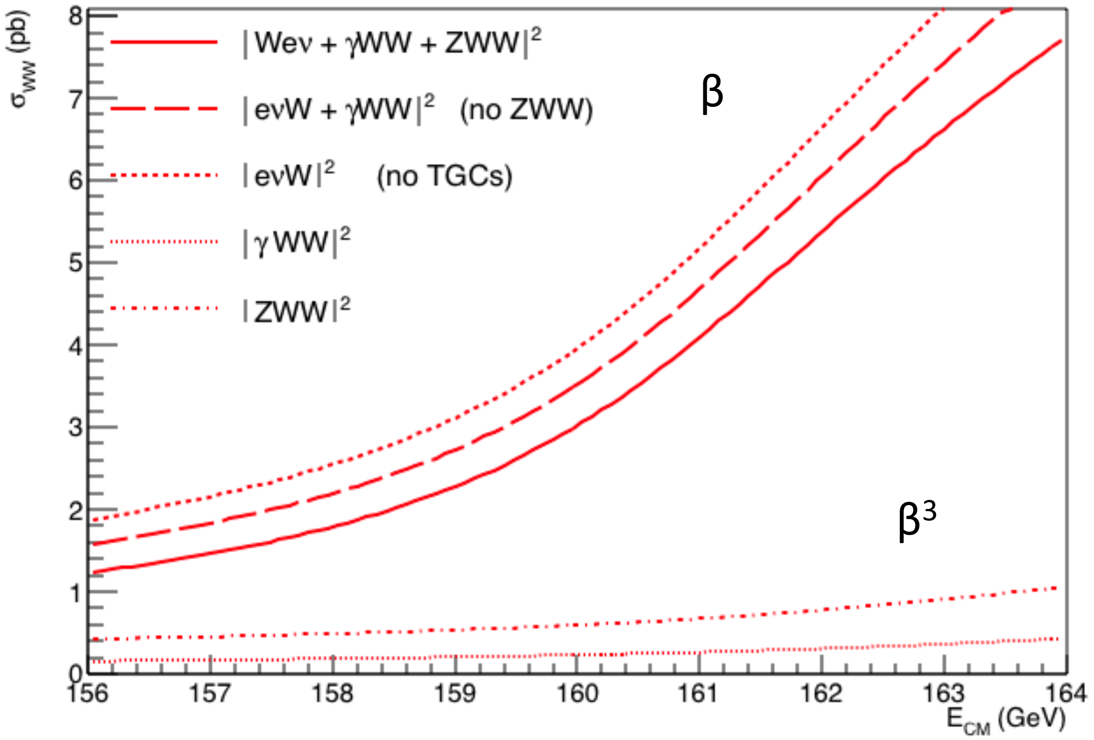
➔ $\Delta\sigma_{WW}$ = +(0.24-1.3) fb & = -(0.18-1.0) fb

➔ **Δm_W = -(0.09-0.48) MeV**

➔ **$\Delta\Gamma_W$ = +(0.6-3.3) MeV**

Maximum effects are at the level of Δm_W(stat) and 2x $\Delta\Gamma_W$ (stat) so that control on the beam energy **RMS <50%** is required to avoid additional syst contributions from this source

TGCs at threshold

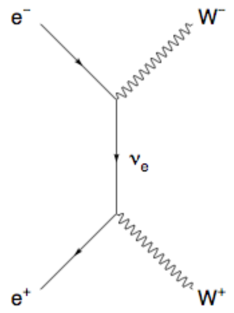


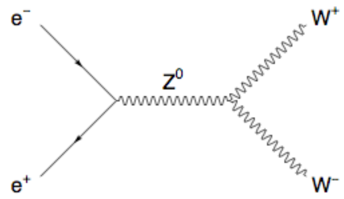
SU(2)⊗U(1) Gauge Cancellations

without TGCs

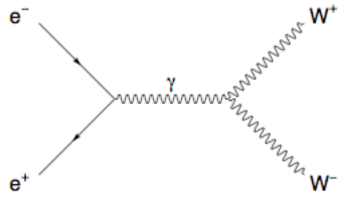
σ_{WW} +40% @157GeV +25%@162GeV

Weν





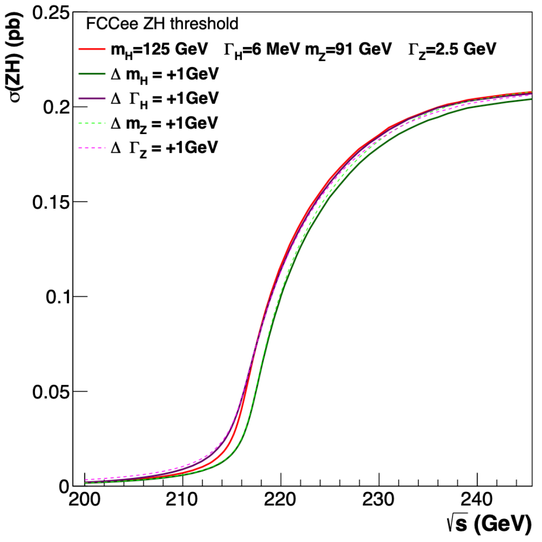
WWZ

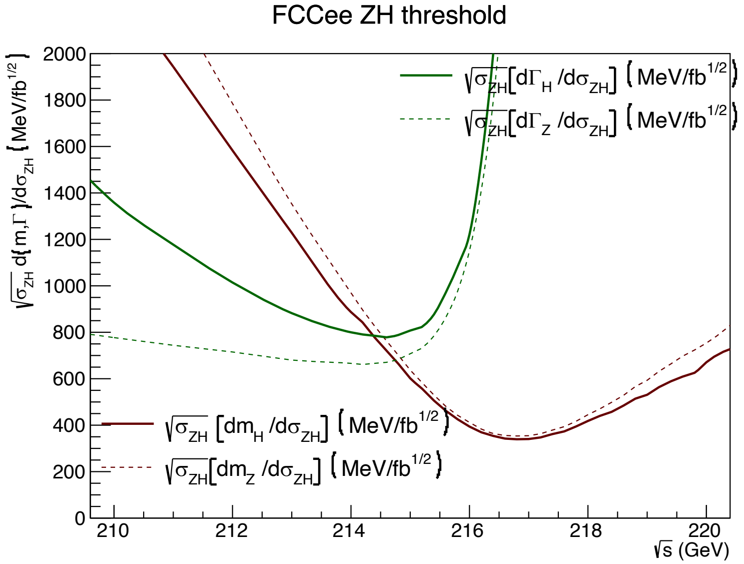


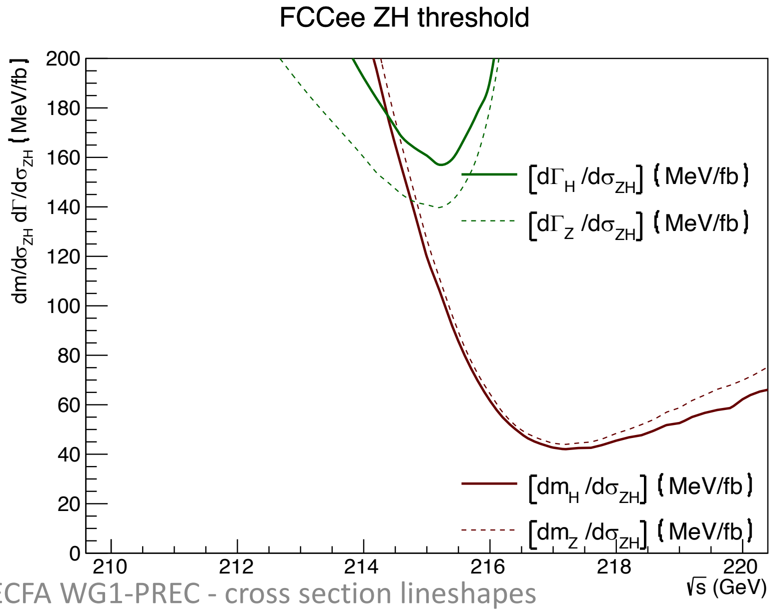
WWγ

interlude : the ZH threshold

On the way to the electron-Yukawa (with ee→H)







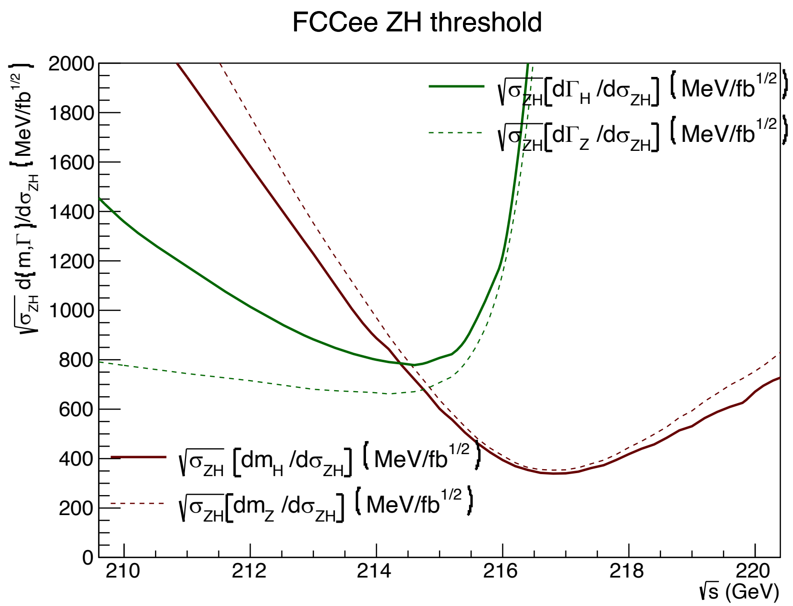
Optimal data-taking point for min $\boldsymbol{\Delta}m_H$(stat)
Is $E_{CM} \simeq m_Z+m_H+0.6\sim$ **217 GeV**

$\sqrt{\sigma_{ZH}}(dm_H/d\sigma_{ZH})_{min}$=350 MeV/√fb

With **5/ab ⇒ $\boldsymbol{\Delta m_H}$(stat) = 5 MeV**
Not including $Q=\sqrt{\sum \varepsilon_i p_i}$ (over all channels)

$(dm_H/d\sigma_{ZH})$=40 MeV/fb

interlude : the ZH threshold

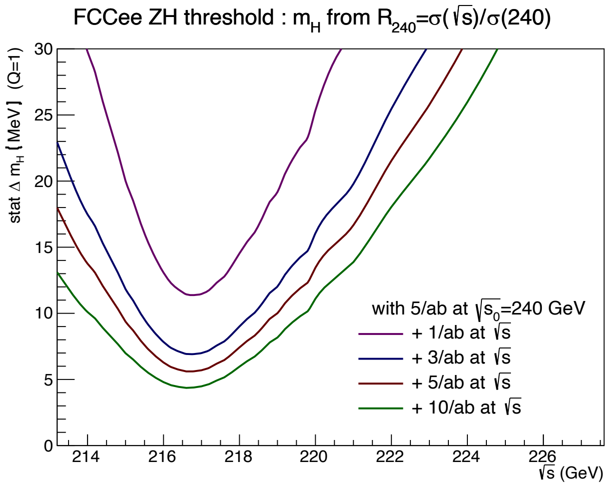


need syst control on :

- ΔE(beam)<5 MeV (**5x10^{-5}**)
- Δε/ε, ΔL/L < **10^{-3}**
- Δσ_B<0.1 fb (~**10^{-3}**)

Taking some /ab at E_{CM}≃214-215GeV (off shell) would allow **Δ*Γ***$_H$≃40 MeV

⟹ *not very interesting*



arXiv:2106.15438

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work ahead

- Explore in more detail the **systematic uncertainties (cancellation) effects with multi-point (n≥3) cross section measurements**. Evaluate benefits of additional model independence.
  - reduction / cancellation of **acceptance & luminosity systs** is of particular interest
- Design a realistic a modern analysis with event classifiers, evaluate performances and the corresponding **impact of systematic uncertainties.** Feedback to theory and detector design.
- Explore BSM/EFT interest and utility of multi-point precision $\boldsymbol{\sigma}_{\mathrm{WW}}$ measurements at threshold, also with other 4f productions (We$\boldsymbol{\nu}$, Zee, ..)
- Sensitivity to $\sin^2\boldsymbol{\theta}_{\mathrm{W}}$ with total $\boldsymbol{\sigma}_{\mathrm{WW}}$ at higher energies (>200 GeV)

Summary

- WW lineshape data can provide both m_W and Γ_W with unprecedented precision
  - optimal data taking at E_{CM}= $2m_w$+1.5 GeV (**Γ_W-insensitive**) and E_{CM} =$2m_w$- Γ_W (**off shell**) yileds with 12/ab stat precision Δm_W =0.5 MeV and $\Delta\Gamma_W$=1.2 MeV, some challenges from syst uncertainties (acceptance control at few 10^{-4} level)
  - interest of additional E_{CM} points for syst control and investigate other lineshape properties
  - threshold data can be used for other measurements as direct N_ν from radiative Z, single V ($We\nu$, Zee), …