



# Standard Model and open problems

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# Disclaimer

Many lectures already about the Standard Model (SM) and extensions. Hard to say something “new”

Students with very different profiles: theory/experiment, master/PhD, ...

My “solution” is to focus on aspects of the SM that are rarely discussed in this kind of lectures

We will need a new language (not standard quantum field theory). Much easier to learn, no previous background needed

Feel free to ask me any questions you have, both in class and during breaks

# Standard Model of Elementary Particles

		three generations of matter (elementary fermions)			three generations of antimatter (elementary antifermions)			interactions / force carriers (elementary bosons)	
		I	II	III	I	II	III		
QUARKS	mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
	charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
	spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
		<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b><math>\bar{u}</math></b> antiup	<b><math>\bar{c}</math></b> anticharm	<b><math>\bar{t}</math></b> antitop	<b>g</b> gluon	<b>H</b> higgs
		<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\bar{d}</math></b> antidown	<b><math>\bar{s}</math></b> antistrange	<b><math>\bar{b}</math></b> antibottom	<b><math>\gamma</math></b> photon	
		<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b><math>e^+</math></b> positron	<b><math>\bar{\mu}</math></b> antimuon	<b><math>\bar{\tau}</math></b> antitau	<b>Z</b> Z <sup>0</sup> boson	
LEPTONS	mass	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$
	charge	0	0	0	0	0	0	1	-1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b><math>\bar{\nu}_e</math></b> electron antineutrino	<b><math>\bar{\nu}_\mu</math></b> muon antineutrino	<b><math>\bar{\nu}_\tau</math></b> tau antineutrino	<b><math>W^+</math></b> W <sup>+</sup> boson	<b><math>W^-</math></b> W <sup>-</sup> boson	

GAUGE BOSONS  
VECTOR BOSONS

SCALAR BOSONS

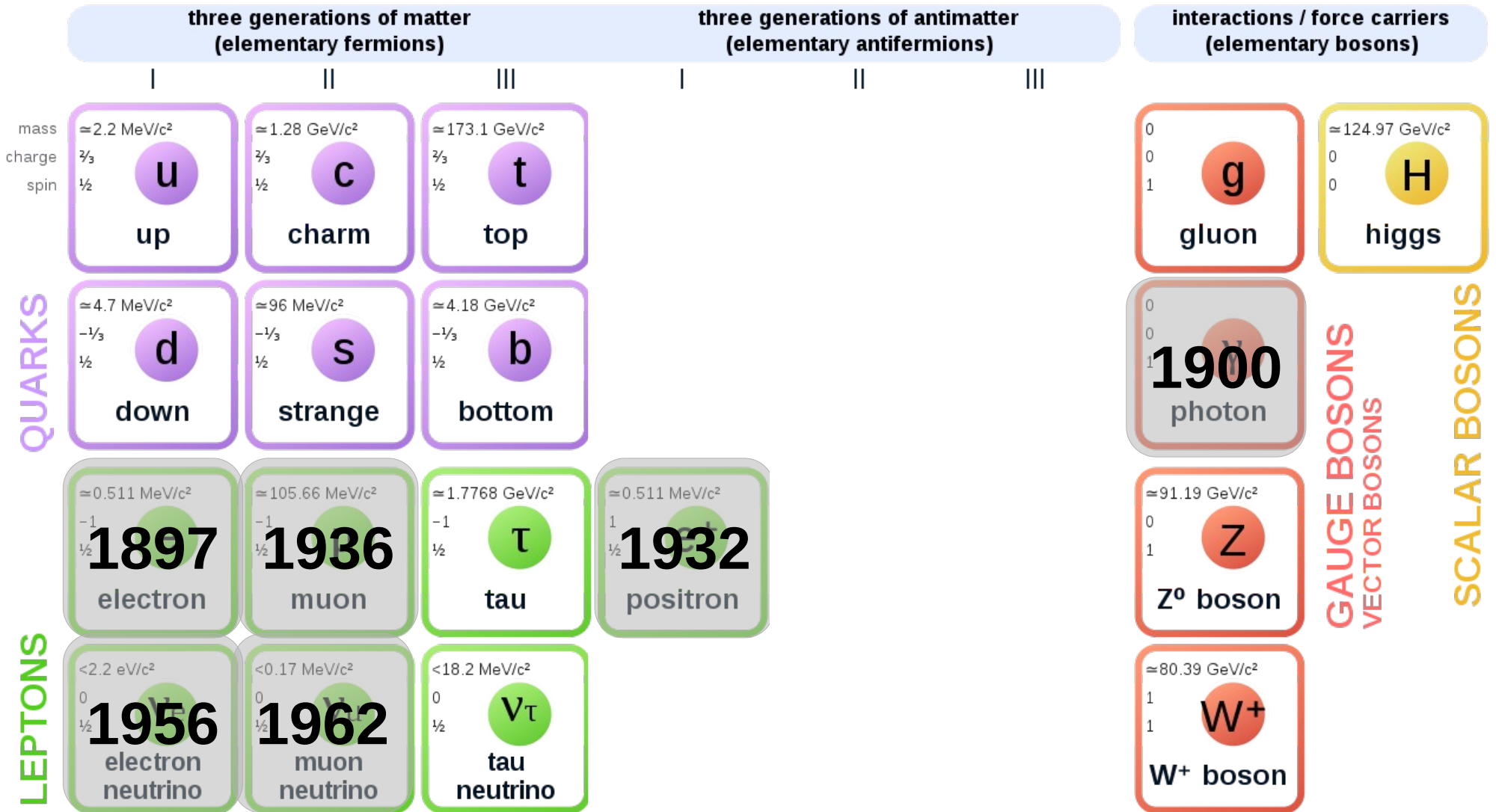
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QUARKS	$\approx 4.7 \text{ MeV}/c^2$		$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	<b>1900</b> photon
	$-\frac{1}{3}$	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\bar{d}</math></b> antidown	<b><math>\bar{s}</math></b> antistrange	<b><math>\bar{b}</math></b> antibottom	0	
LEPTONS	$\approx 0.511 \text{ MeV}/c^2$		$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	0	<b>Z</b> Z <sup>0</sup> boson
	$-\frac{1}{2}$	<b>1897</b> electron	<b>1936</b> muon	<b><math>\tau</math></b> tau	<b>1932</b> positron	<b><math>\bar{\mu}</math></b> antimuon	<b><math>\bar{\tau}</math></b> antitau	1	
	$< 2.2 \text{ eV}/c^2$		$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	1	<b>W<sup>+</sup></b> W <sup>+</sup> boson
	$\frac{1}{2}$	<b>1956</b> electron neutrino	<b>1962</b> muon neutrino	<b><math>\nu_{\tau}</math></b> tau neutrino	<b><math>\bar{\nu}_e</math></b> electron antineutrino	<b><math>\bar{\nu}_{\mu}</math></b> muon antineutrino	<b><math>\bar{\nu}_{\tau}</math></b> tau antineutrino	1	
								1	<b>W<sup>-</sup></b> W <sup>-</sup> boson

GAUGE BOSONS  
VECTOR BOSONS

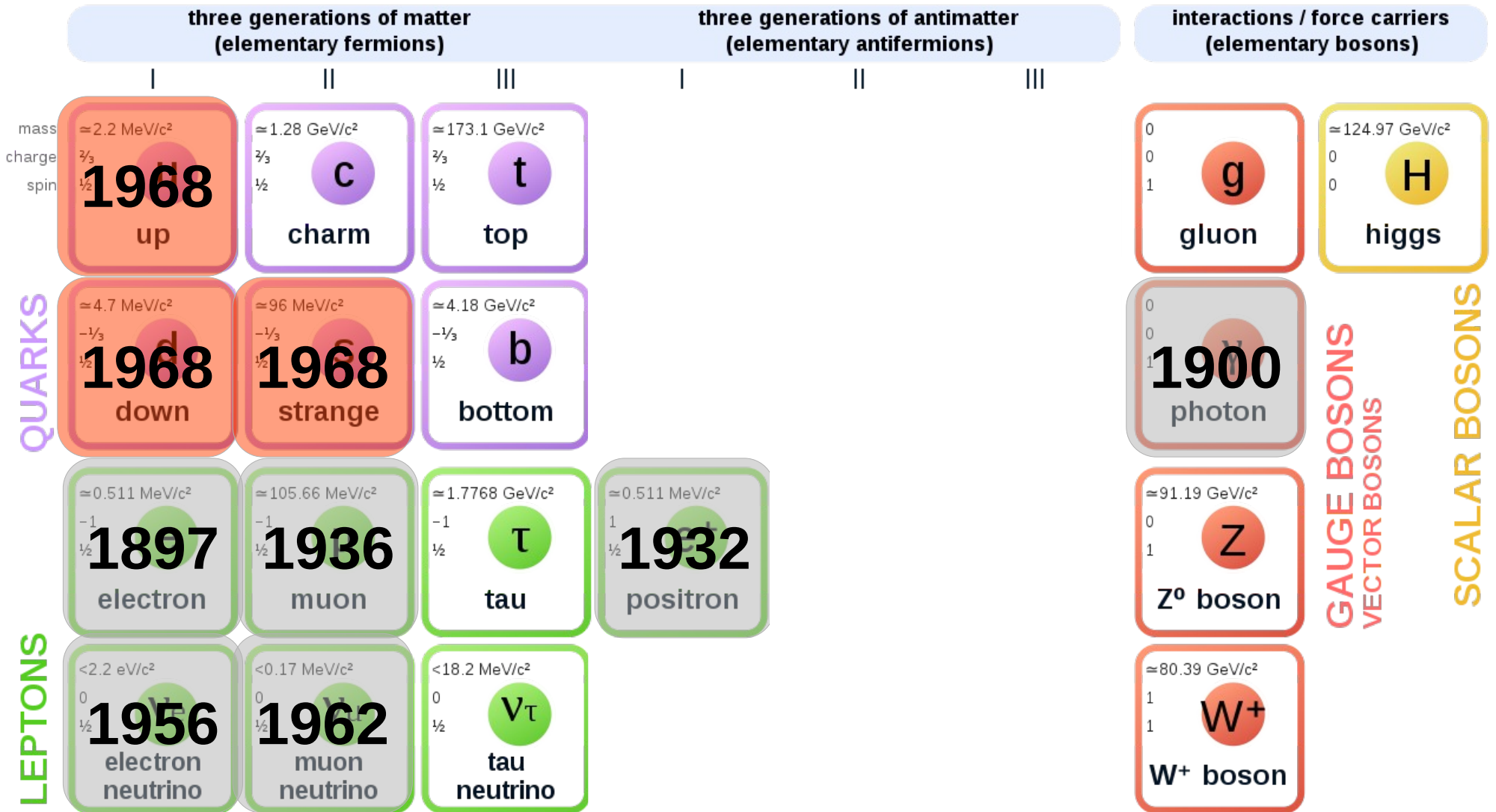
SCALAR BOSONS

# Standard Model of Elementary Particles

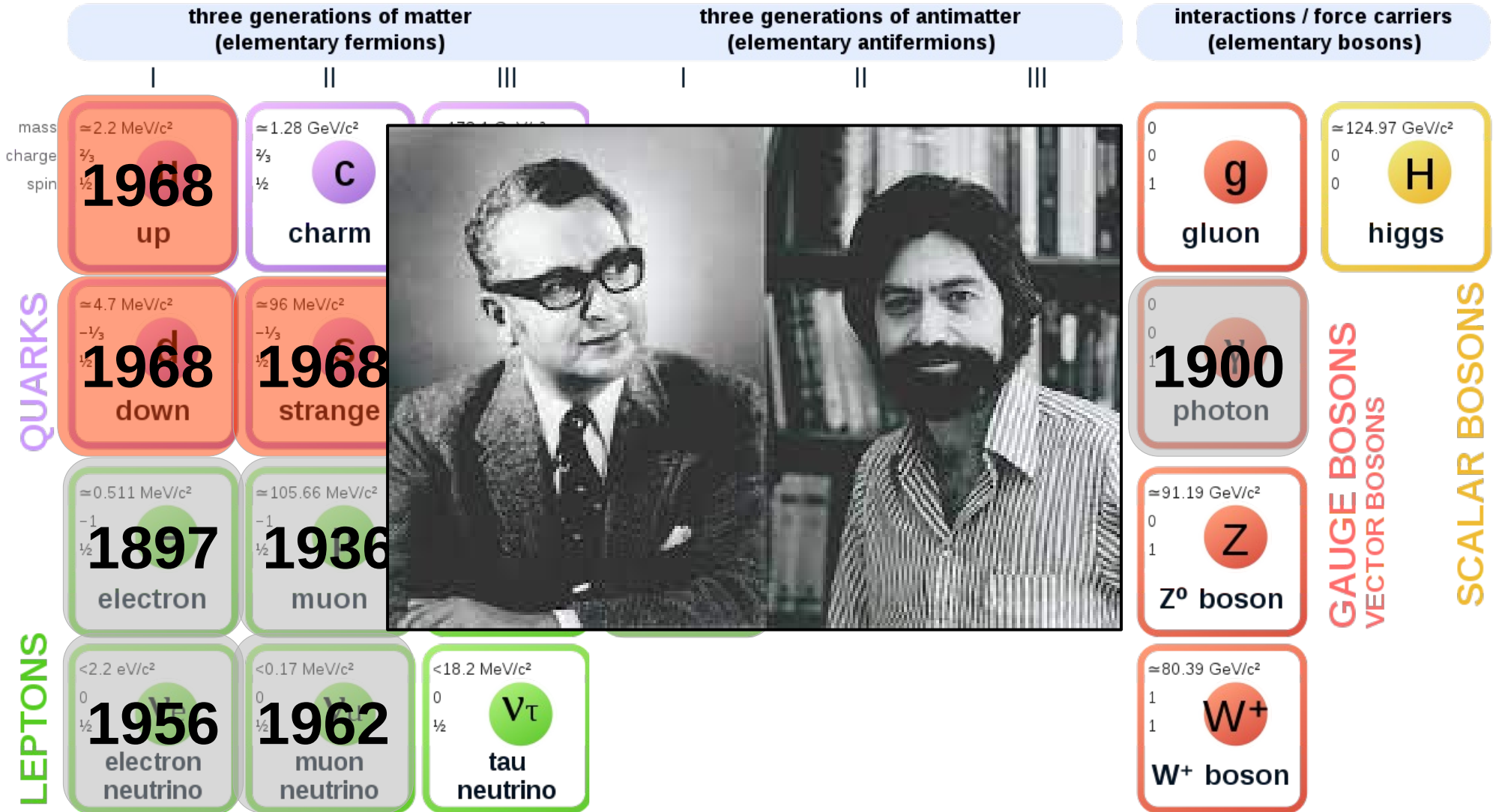




# Standard Model of Elementary Particles



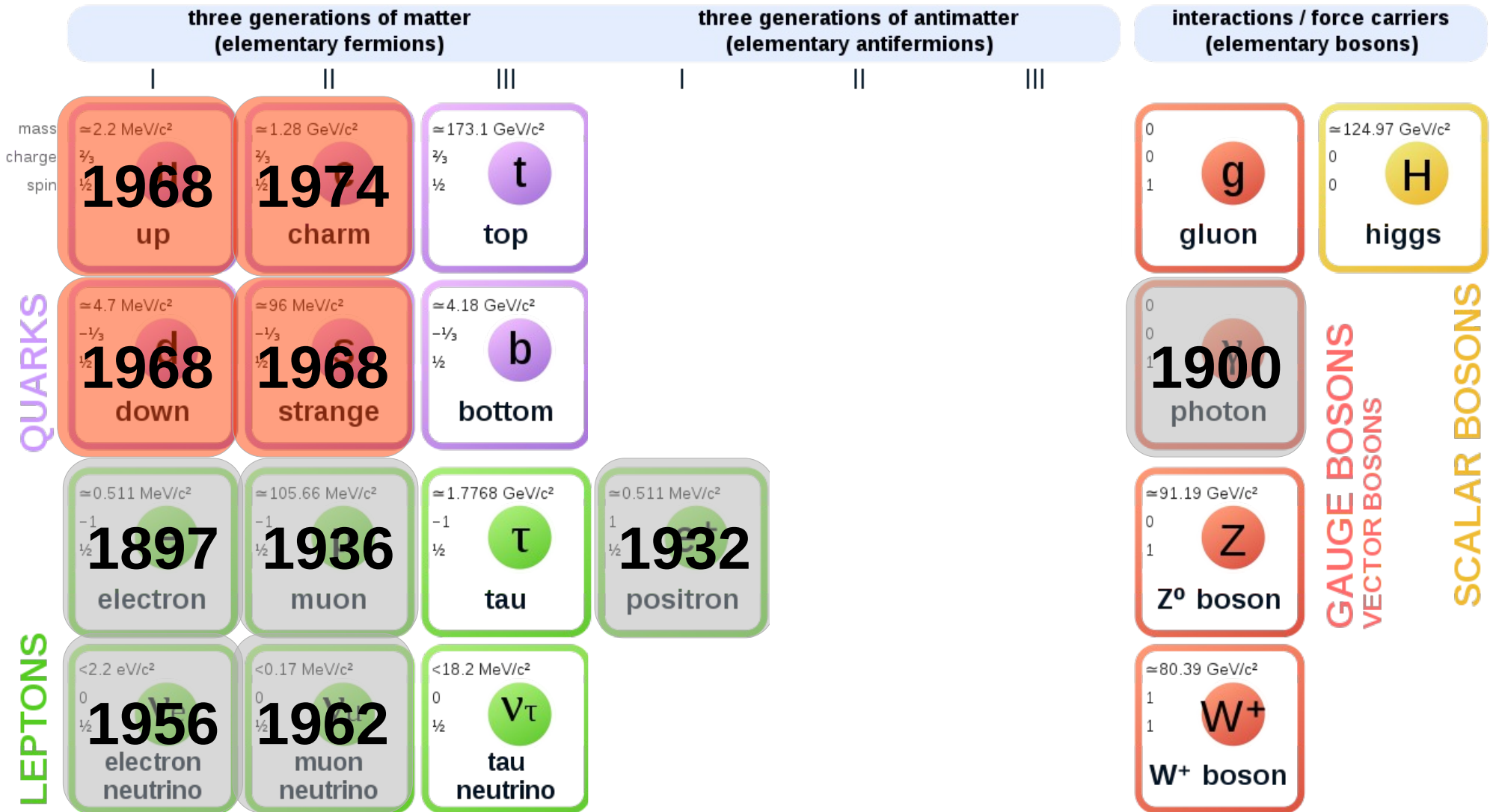
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VECTOR BOSONS

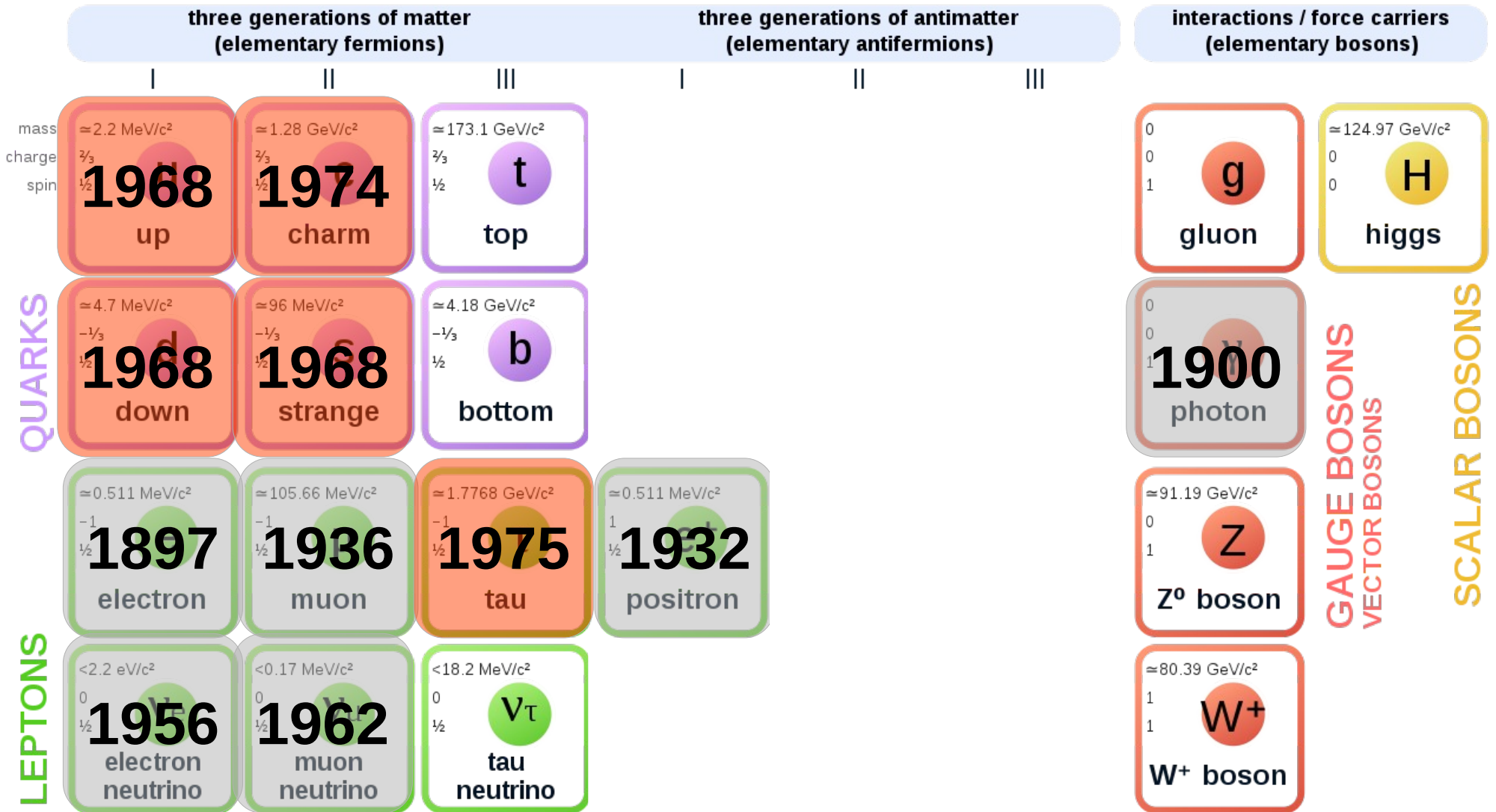
SCALAR BOSONS

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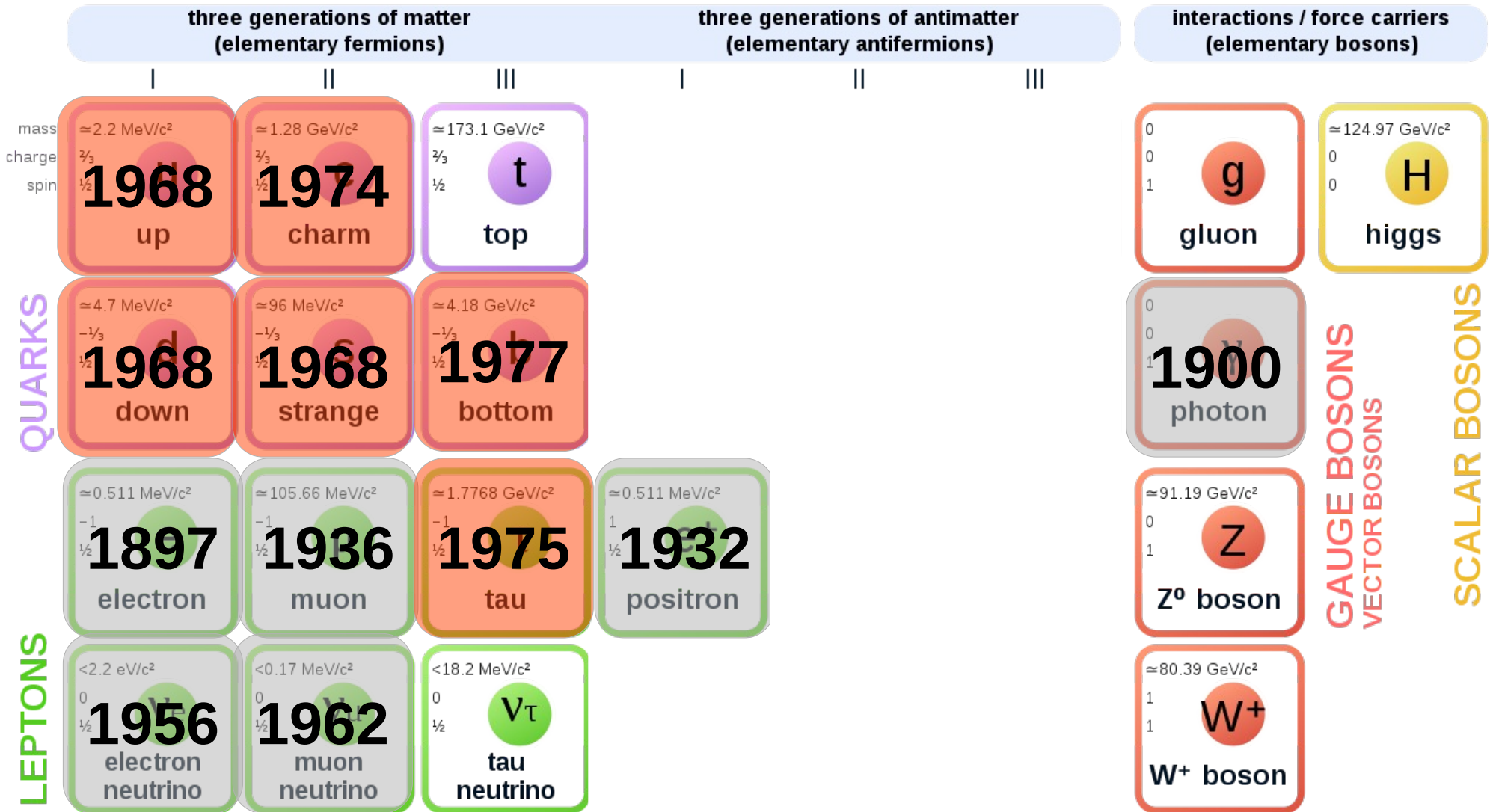




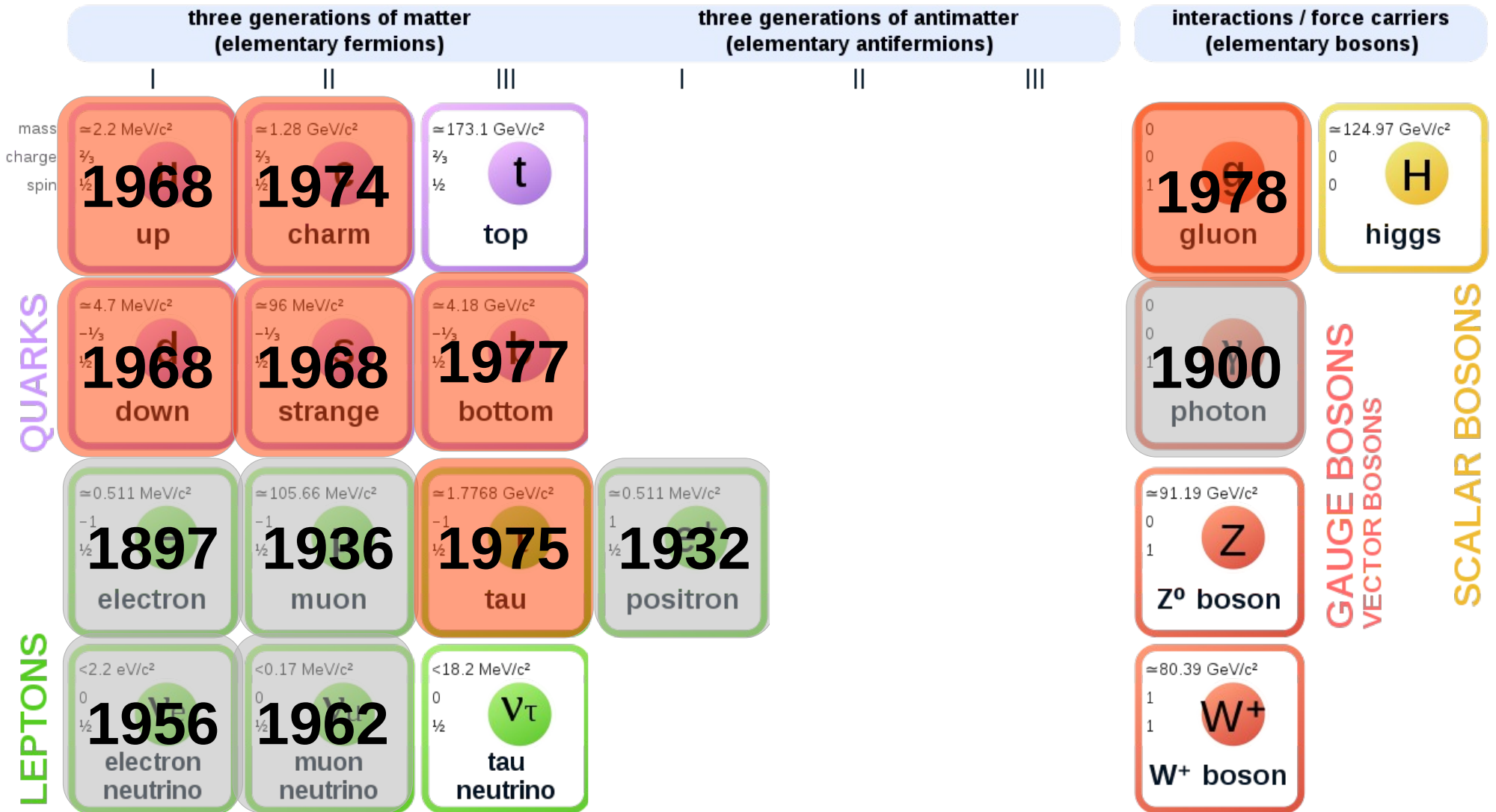
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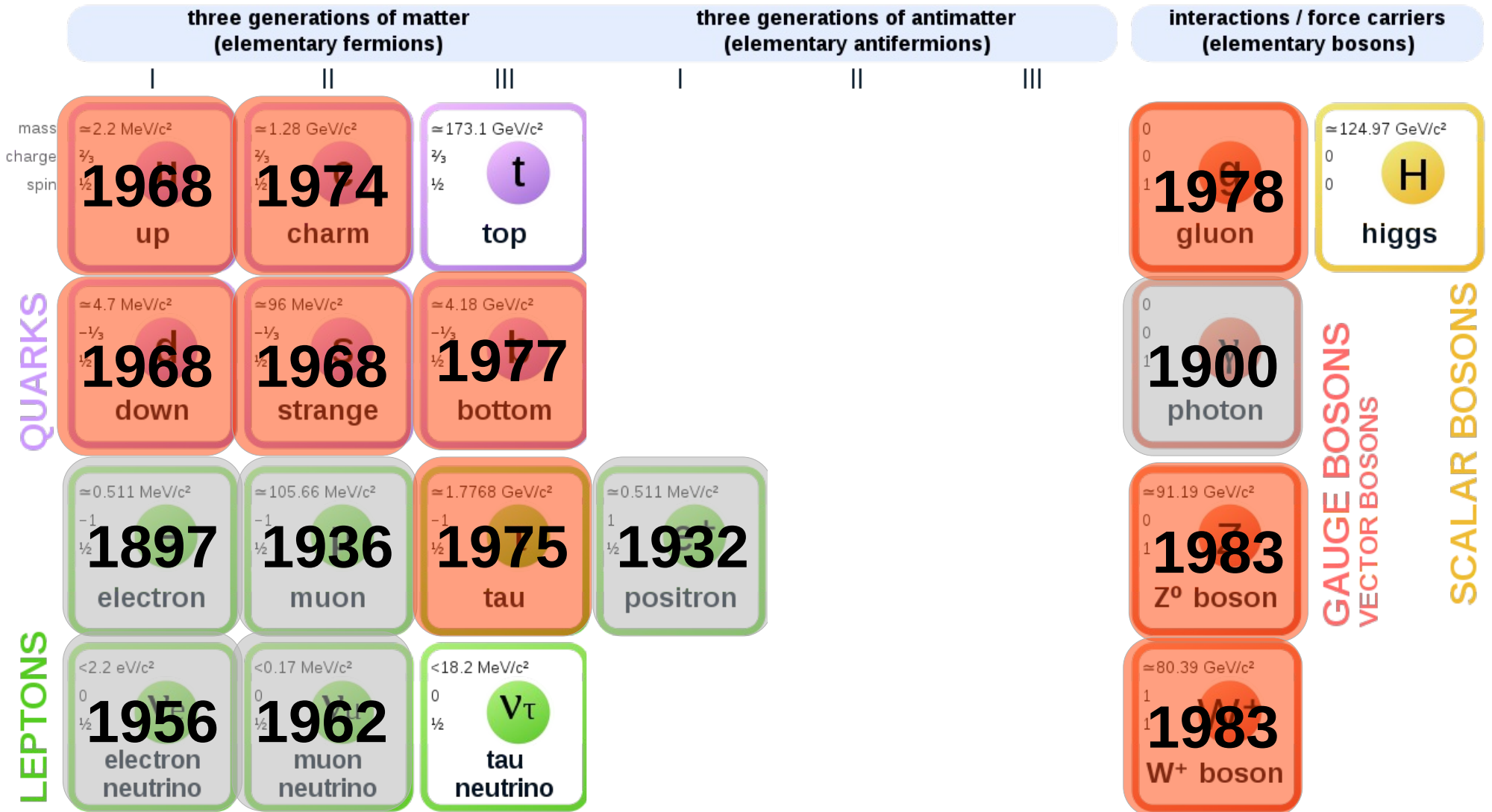
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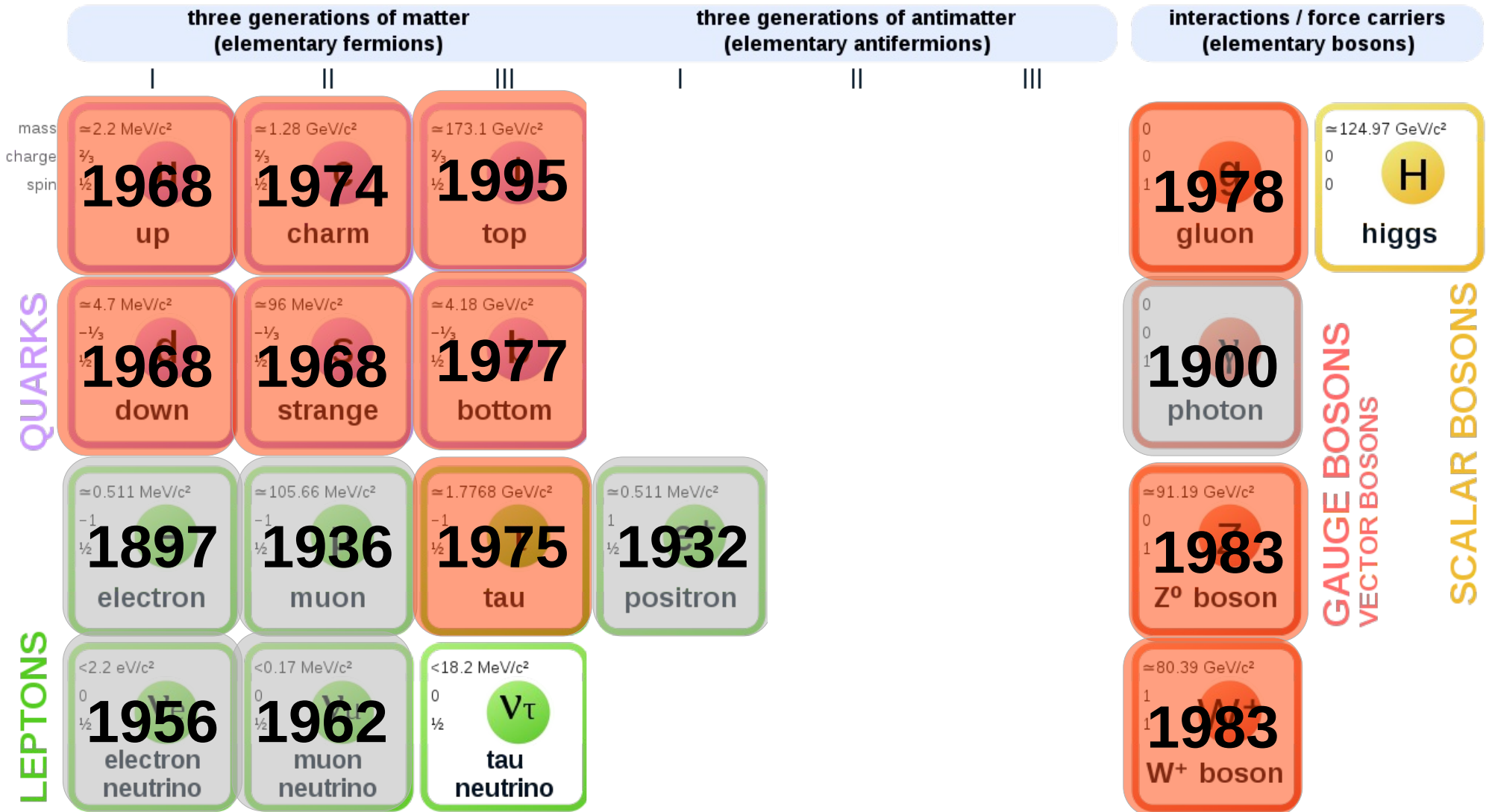
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	charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
	spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
		<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b><math>\bar{u}</math></b> antiup	<b><math>\bar{c}</math></b> anticharm	<b><math>\bar{t}</math></b> antitop	<b>g</b> gluon	<b>H</b> higgs
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		$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\bar{d}</math></b> antidown	<b><math>\bar{s}</math></b> antistrange	<b><math>\bar{b}</math></b> antibottom	<b><math>\gamma</math></b> photon	
		$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	
		-1	-1	-1	1	1	1	0	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b><math>e^+</math></b> positron	<b><math>\bar{\mu}</math></b> antimuon	<b><math>\bar{\tau}</math></b> antitau	<b>Z</b> Z <sup>0</sup> boson	
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		0	0	0	0	0	0	1	-1
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
		<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b><math>\bar{\nu}_e</math></b> electron antineutrino	<b><math>\bar{\nu}_\mu</math></b> muon antineutrino	<b><math>\bar{\nu}_\tau</math></b> tau antineutrino	<b><math>W^+</math></b> W <sup>+</sup> boson	<b><math>W^-</math></b> W <sup>-</sup> boson

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	spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
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		<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b><math>e^+</math></b> positron	<b><math>\bar{\mu}</math></b> antimuon	<b><math>\bar{\tau}</math></b> antitau	<b>Z</b> Z <sup>0</sup> boson	
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	spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
		<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b><math>\bar{\nu}_e</math></b> electron antineutrino	<b><math>\bar{\nu}_\mu</math></b> muon antineutrino	<b><math>\bar{\nu}_\tau</math></b> tau antineutrino	<b><math>W^+</math></b> W <sup>+</sup> boson	<b><math>W^-</math></b> W <sup>-</sup> boson

GAUGE B  
VECTOR BOS

SCALAR

# Observations

There is one anti-particle for each particle

All particles have spin smaller than or equal to 2

There is the same number of leptons and quarks

There are three families of fermions

Values of particle masses seemingly unrelated (neutrino masses are tiny)

...

...



# The SM Lagrangian

SM Lagrangian (gauge) invariant under symmetry group  $SU(3) \times SU(2) \times U(1)$  + Einstein-Hilbert action

Interactions better seen in terms of Feynman diagrams

$$\begin{aligned} \mathcal{L}_{\text{SM}} = & -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \bar{q} i \not{D} q + \bar{l} i \not{D} l + \bar{u} i \not{D} u + \bar{d} i \not{D} d + \bar{e} i \not{D} e \\ & + (D_\mu \phi)^\dagger (D^\mu \phi) + \mu_\phi^2 |\phi|^2 - \lambda_\phi |\phi|^4 - (\bar{q} \tilde{\phi} Y_u u + \bar{q} \phi Y_d d + \bar{l} \phi Y_e e + \text{h.c.}) \end{aligned}$$

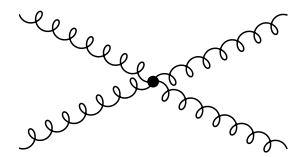
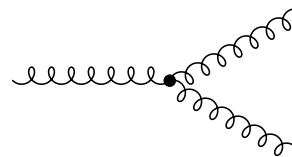
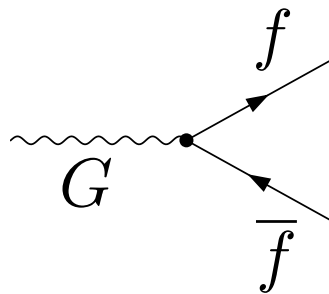
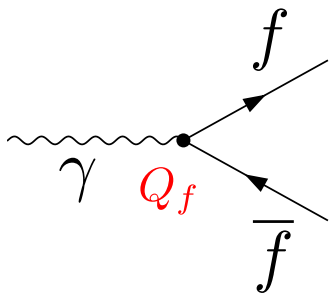
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$$D_\mu = \partial_\mu - igT_A F_\mu^A \quad F_{\mu\nu}^A = \partial_\mu F_\nu^A - \partial_\nu F_\mu^A + gf_{ABC}F_\mu^B F_\nu^C$$



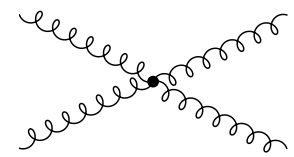
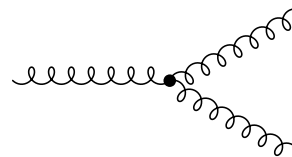
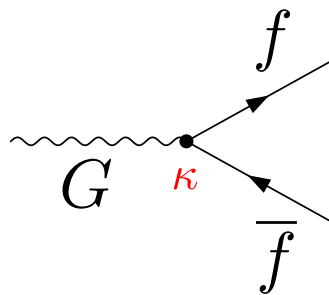
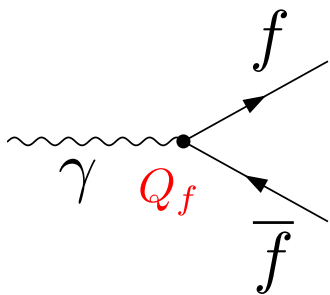
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# Observations

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Forces are described by Yang-Mills theory

Gravity couples equally to all particles (equivalence principle)

# n-gluon scattering

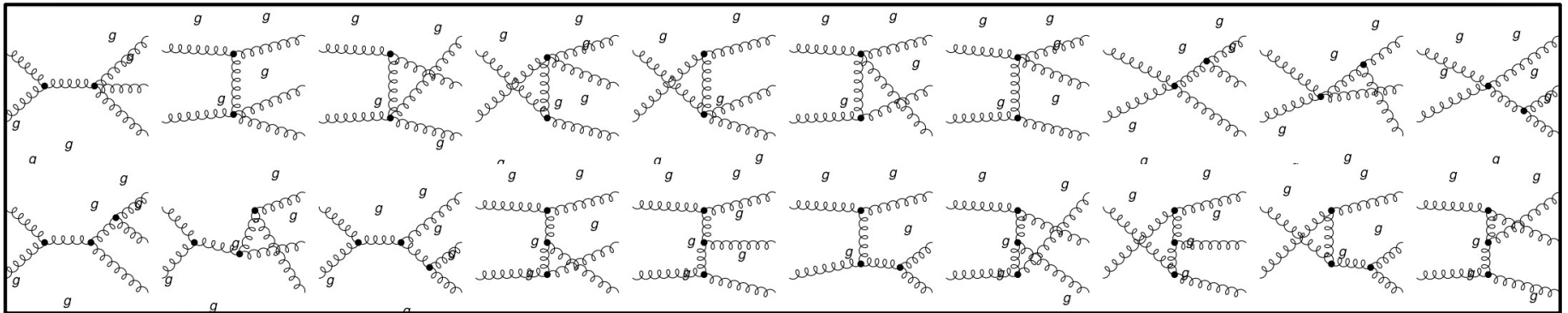
The scattering amplitude for  $n$ -gluon collision with only two-minus helicity gluons is surprisingly simple, despite the number of Feynman diagrams for  $n=4, 5, 6, \dots$  being  $4, 25, 220, \dots$



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No terms of the type  $(p_i + p_j + p_k)^2$ . They all cancel!



$$|\mathcal{A}(- - + + + \dots)|^2 \sim \frac{(p_1 \cdot p_2)^2}{(p_1 \cdot p_2)(p_2 \cdot p_3) \cdots (p_n \cdot p_1)}$$

# n-gluon scattering

This result is known as the Parke and Taylor's equation

They make some very interesting comments in their paper:

results have been available so far. Before presenting the helicity amplitude, let us make it clear that this result is an educated guess which we have compared to the existing computations and verified by a series of highly nontrivial and nonlinear consistency checks.

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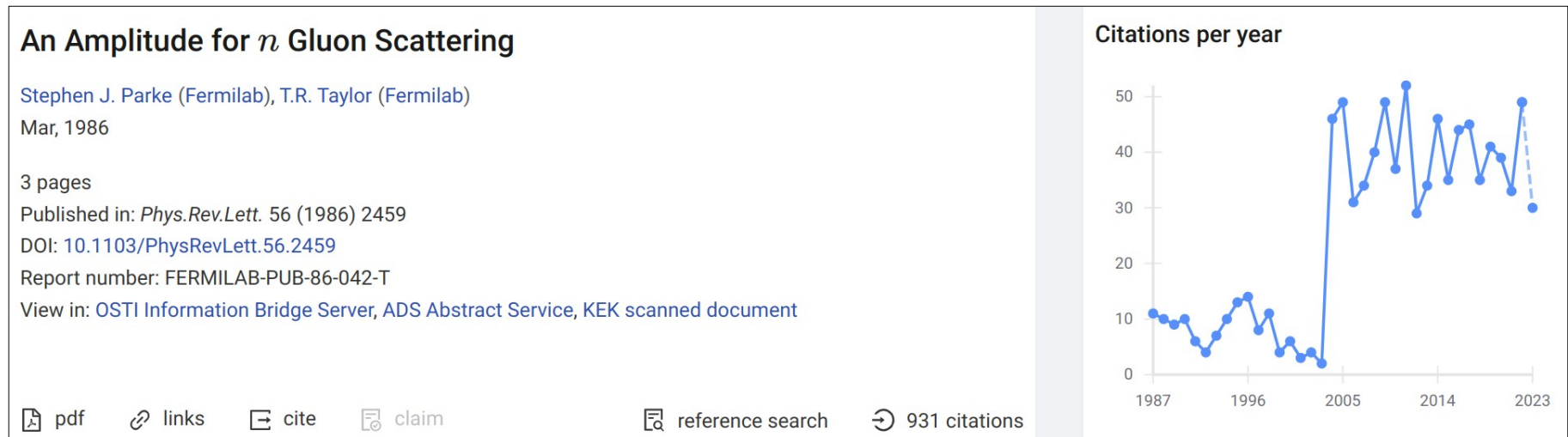
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We do not expect such a simple expression for the other helicity amplitudes. Also, we challenge the string theorists to prove more rigorously that Eq. (3) is correct.

# n-gluon scattering

String theorists (successfully) accepted the challenge



# n-gluon scattering

String theorists (successfully) accepted the challenge

## An Amplitude for $n$ Gluon Scattering

Stephen J. Parke (Fermilab), T.R. Taylor (Fermilab)  
Mar, 1986

3 pages  
Published in: *Phys.Rev.Lett.* 56 (1986) 2459  
DOI: [10.1103/PhysRevLett.56.2459](https://doi.org/10.1103/PhysRevLett.56.2459)  
Report number: FERMILAB-PUB-86-042-T  
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### Citations per year

Year	Citations
1987	10
1988	10
1989	10
1990	10
1991	10
1992	10
1993	10
1994	10
1995	10
1996	10
1997	10
1998	10
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2012	30
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2014	45
2015	35
2016	45
2017	35
2018	40
2019	35
2020	45
2021	30
2022	45
2023	30

## Perturbative gauge theory as a string theory in twistor space #2

Edward Witten (Princeton, Inst. Advanced Study) (Dec, 2003)

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# Traditional quantum field theory

Particles are embedded in (non-observable) fields; unphysical degrees of freedom (gauge redundancy), quantisation, Feynman rules, all sort of auxiliary objects (gamma matrices, polarizations, ...)

This language of quantum field theory is highly redundant

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Particles are embedded in (non-observable) fields; unphysical degrees of freedom (gauge redundancy), quantisation, Feynman rules, all sort of auxiliary objects (gamma matrices, polarizations, ...)

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Perform the following transformation on the Lagrangian of a free theory:

$$\varphi \rightarrow \varphi + \frac{\varphi^2}{\Lambda}$$

Show that the four-point amplitude still vanishes in the new theory given by

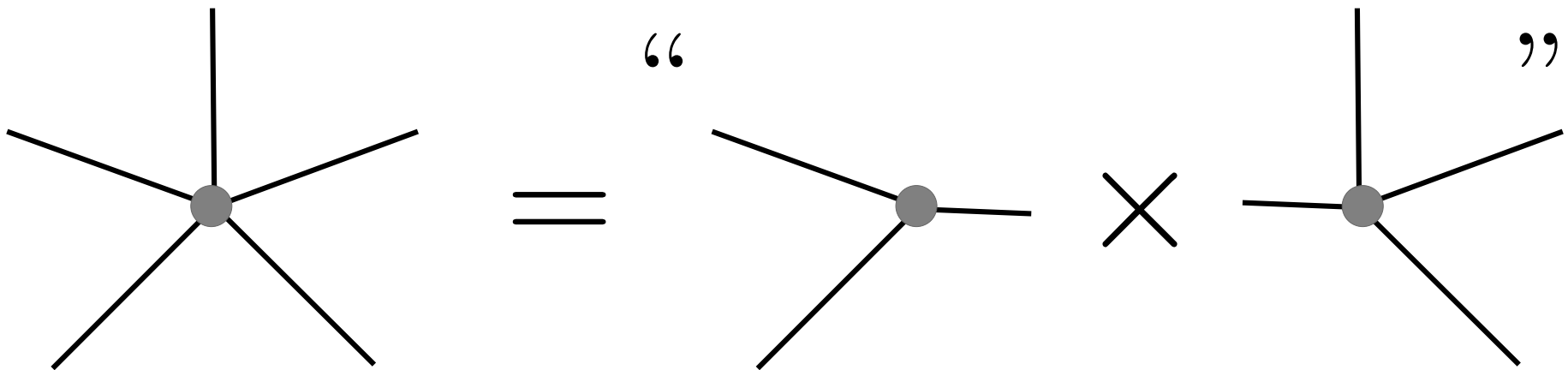
$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{2}{\Lambda} \varphi (\partial_\mu \varphi) (\partial^\mu \varphi) - \frac{m^2}{\Lambda} \varphi^3 + \frac{2}{\Lambda^2} \varphi^2 (\partial_\mu \varphi) (\partial^\mu \varphi) - \frac{m^2}{2\Lambda^2} \varphi^4$$



# A new approach

Forget fields (and all related things). Compute  $n$ -point amplitudes directly, relying on  $(n-1)$ -point amplitudes and consistency conditions with complex momenta

3-point amplitudes completely fixed by momentum conservation, little group covariance, ...



# Spinor-helicity variables

In this new approach, there are no Feynman diagrams, no polarization vectors, no gamma matrices, no four-momenta, ...

There are only angle ( $\langle \rangle$ ) and squares ( $[ ]$ )

$$\begin{aligned} p &= (p^0, \vec{p}) \\ \sigma^\mu &= (1, \sigma^i) \end{aligned} \quad P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$$

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$$\sigma^\mu = (1, \sigma^i)$$

$$\det P = 0 \Rightarrow P = \begin{pmatrix} \langle \rangle & \\ & [ ] \end{pmatrix}$$

$$p \rangle \quad [p$$

$$p \rangle = \frac{z}{\sqrt{p_0 - p_3}} \begin{pmatrix} p_0 - p_3 \\ -p_1 - ip_2 \end{pmatrix}$$

$$p] = \frac{z^{-1}}{\sqrt{p_0 - p_3}} \begin{pmatrix} p_0 - p_3 & -p_1 + ip_2 \end{pmatrix}$$

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$$\langle p_i p_j \rangle \equiv \langle ij \rangle = ( \quad ) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} ( \quad )$$

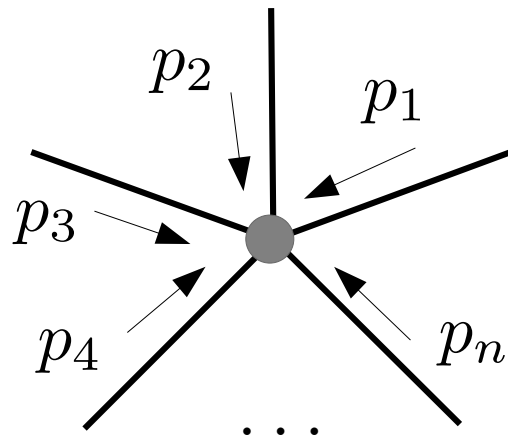
$$[p_i p_j] \equiv [ij] = ( \quad ) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} ( \quad )$$

# Spinor-helicity variables

The main properties of angles and squares are: antisymmetry, Schouten identity and (when applied to amplitudes) momentum conservation

$$\langle ij \rangle = -\langle ji \rangle \quad [ij] = -[ji]$$

$$\langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 42 \rangle + \langle 14 \rangle \langle 23 \rangle = 0$$



$$\sum_j |j\rangle [j] = 0 \Rightarrow \sum_j \langle ij \rangle [jk] = 0$$

# Spinor-helicity variables

The main properties of angles and squares are: antisymmetry, Schouten identity and (when applied to amplitudes) momentum conservation

Also, all objects that arise in Feynman diagrams can be simply written as products of angles and squares

$$s_{ij} = (p_i + p_j)^2 = \langle ij \rangle [ij]$$

$$(\overline{u}_1 \gamma^\mu P_R u_2) (\overline{u}_3 \gamma_\mu P_R u_4) = 2 \langle 13 \rangle [42]$$

# Amplitudes in spinor-helicity

Amplitudes are simply linear combinations of angles and squares, subject to constrain from little-group scaling

Amplitudes have only single poles (in Mandelstan invariants), with residues given by products of amplitudes!

$n$ -point amplitudes have energy dimension (in natural units)  
 $[A] = 4-n$

$$\mathcal{A}(1, 2, \dots, n) = \sum_i \langle 12 \rangle^{a_i} \langle 13 \rangle^{b_i} \dots [34]^{c_i} \dots$$

$$\#[i] - \#\langle i \rangle = 2h_i$$

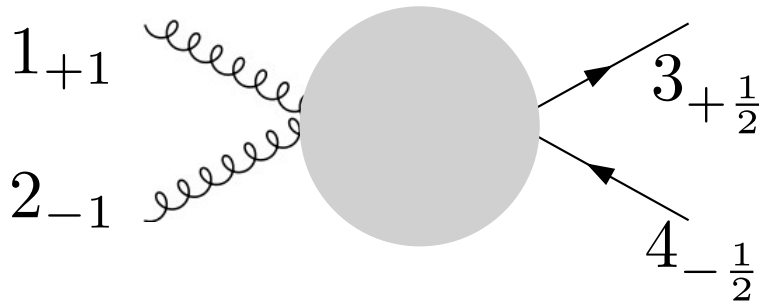
$$\text{residue}(\mathcal{A}, \text{pole} = s, t, u) = \mathcal{A}' \times \mathcal{A}''$$



# Good and bad amplitudes

Let us consider the amplitude describing the scattering of two gluons (of helicities  $+1$  and  $-1$ ) and two fermions (of helicities  $+1/2$  and  $-1/2$ ) in a scaleless theory

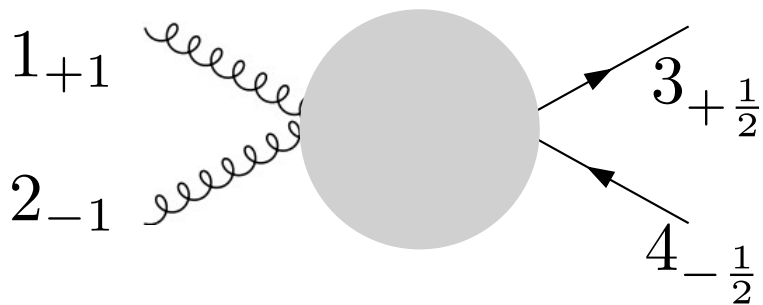
Only one of the following amplitudes is compatible with the previous principles



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$$\mathcal{A}_1 = g \frac{\langle 23 \rangle [14]}{s}$$

$$\mathcal{A}_2 = \kappa \frac{\langle 23 \rangle \langle 24 \rangle [13]^2}{s}$$

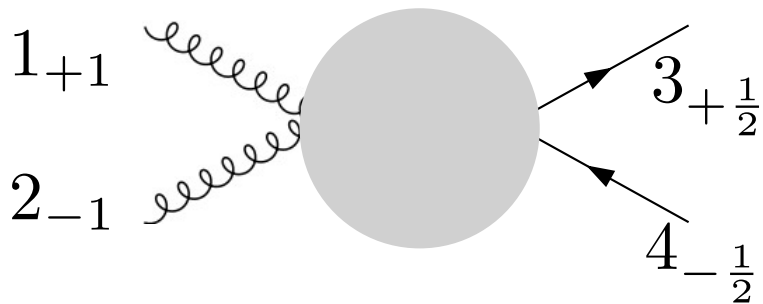
$$\mathcal{A}_3 = g \frac{\langle 23 \rangle \langle 24 \rangle [13]^2}{s^2}$$

$$\mathcal{A}_4 = g \frac{\langle 23 \rangle \langle 24 \rangle [13]^2}{st}$$

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$$\mathcal{A}_4 = g \frac{\langle 23 \rangle \langle 24 \rangle [13]^2}{st}$$

Which of the following amplitudes can not describe  $1_0 2_0 \rightarrow 3_{+1} 4_{-1}$  ?

$$\mathcal{A}_1 = g \frac{\langle 14 \rangle^2 [13]^2}{\langle 12 \rangle [12]}$$

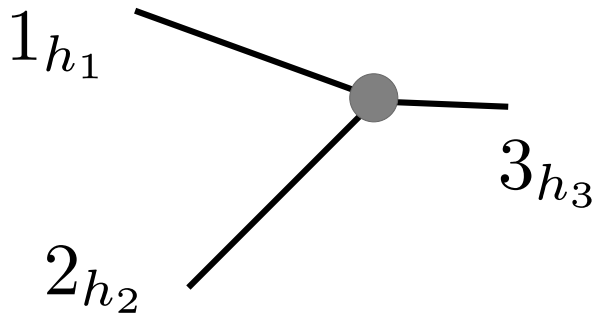
$$\mathcal{A}_1 = g \frac{\langle 14 \rangle^2 [13]^2 t}{\langle 12 \rangle [12] us}$$

$$\mathcal{A}_1 = \kappa \frac{\langle 14 \rangle^2 [13]^2 [12]}{st}$$

# 3-point amplitudes

Completely fixed by little-group covariance and momentum conservation

They are product of only angles or only squares



$$(p_1 + p_2)^2 = p_3^2 = 0 \Rightarrow \langle 12 \rangle [12] = 0$$

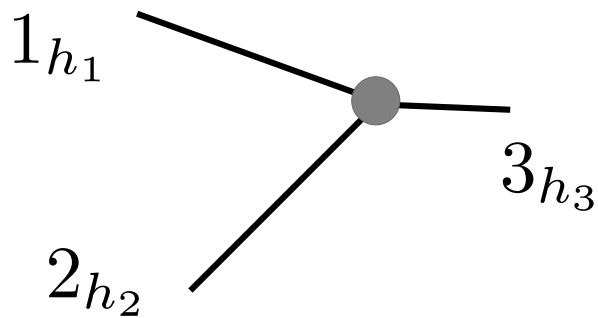
$$\dots \Rightarrow \langle 23 \rangle [23] = 0$$

$$\dots \Rightarrow \langle 31 \rangle [31] = 0$$

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$$\dots \Rightarrow \langle 23 \rangle [23] = 0$$

$$\dots \Rightarrow \langle 31 \rangle [31] = 0$$

$$\langle 12 \rangle \neq 0 \Rightarrow [12] = 0$$

$$\langle 12 \rangle [23] = -\langle 11 \rangle [13] - \langle 13 \rangle [33] = 0 \Rightarrow [23] = 0$$

$$\dots \Rightarrow [31] = 0$$

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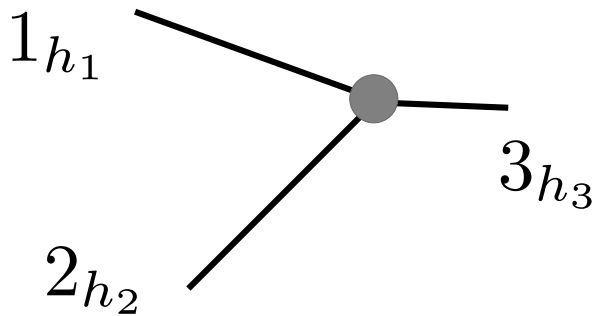
In the limit of real momenta,  $\langle ij \rangle = [ji]^*$ , so all angles and all squares vanish. This is a (convoluted) manifestation that momentum conservation implies that only trivial 3-point amplitudes are possible

$$p_i \in \mathbb{R} \implies \langle ij \rangle = [ji]^* \implies \mathcal{A}_3 = 0$$

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$$\mathcal{A} = \kappa [12]^a [23]^b [31]^c$$

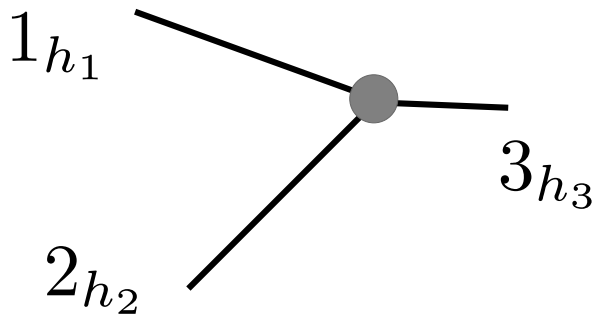
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Proof that little-group scaling provides the unique solution:

$$a = h_1 + h_2 - h_3$$

$$b = h_2 + h_3 - h_1 \quad \text{for } h = \sum_i h_i > 0$$

$$c = h_1 + h_3 - h_2$$

$$a = h_3 - h_1 - h_2$$

$$b = h_1 - h_2 - h_3 \quad \text{for } h = \sum_i h_i < 0$$

$$c = h_2 - h_3 - h_1$$

# An Universe with only photons

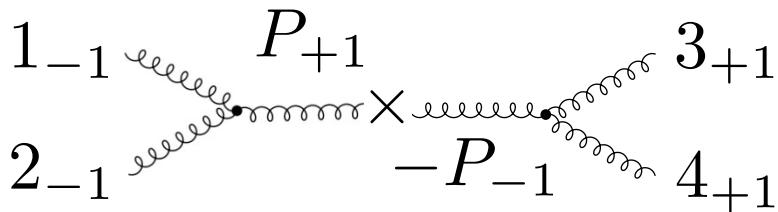
Let us assume that the Universe contains a single type of spin-1 particle (a photon)

Let us compute the amplitude for  $1_{-1} 2_{-1} \rightarrow 3_{+1} 4_{+1}$ , starting with residue in s-channel

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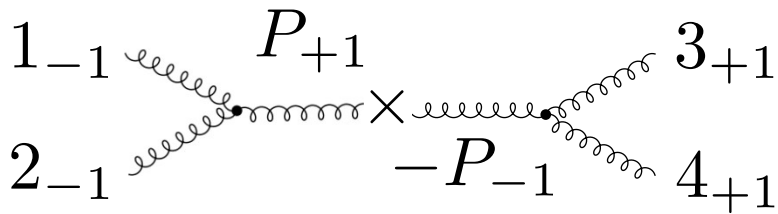
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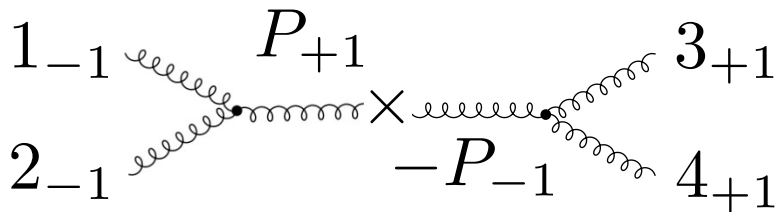
$$r_s = g \frac{\langle 12 \rangle^3}{\langle 2P \rangle \langle P1 \rangle} \times g \frac{[34]^3}{[4-P][P3]}$$

$$= g^2 \frac{\langle 12 \rangle^3 [34]^3}{\langle 2P \rangle [P4] \langle 1P \rangle [P3]} = g^2 \frac{\langle 12 \rangle^2 [34]^2}{u}$$

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Proof the result above using first that  $-P] = i P]$  (and likewise for angles), then momentum conservation ( $P > [P = -1 > [1 - 2 > [2 = -3 > [3 - 4 > [4$ ) and finally  $u = \langle 14 \rangle [41]$

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We do not include the following channel because it gives dimensionful couplings

$$1_{-1} 2_{-1} \begin{array}{c} \text{-----} P_{-1} \text{-----} \\ \text{-----} \times \text{-----} \\ \text{-----} -P_{+1} \text{-----} \end{array} 3_{+1} 4_{+1} \Rightarrow \kappa^2 \langle 12 \rangle \langle 2P \rangle \langle P1 \rangle \times [34][4P][P3]$$

$$[\kappa] = -1$$

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The rest of the residues can be computed equally easily. Little group scaling is manifest

Consistent factorisation indicates that  $g$  must vanish; photons are not self-interacting. We could have guessed this at the level of 3-point amplitudes by Bose statistics

$$r_s = g^2 \frac{\langle 12 \rangle^2 [34]^2}{u} \quad r_t = g^2 \frac{\langle 12 \rangle^2 [34]^2}{s} \quad r_u = g^2 \frac{\langle 12 \rangle^2 [34]^2}{t}$$



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$$\mathcal{A} = \langle 12 \rangle^2 [34]^2 \left( \frac{A}{st} + \frac{B}{tu} + \frac{C}{su} \right) \Rightarrow \begin{aligned} C - A &= g^2 \\ A - B &= g^2 \\ B - C &= g^2 \end{aligned} \Rightarrow g = 0$$

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Why the following ansätze do not work? Can local terms be added to the amplitude?

$$\mathcal{A} = \langle 12 \rangle^2 [34]^2 \left( \frac{A}{s} + \frac{B}{t} + \frac{C}{u} \right), \quad \mathcal{A} = \langle 12 \rangle^2 [34]^2 \frac{A}{stu}$$

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Let us assume now that the Universe contains several types of spin-1 particles (gluons). Let us compute the amplitude for

$$1_{-1}^a 2_{-1}^b \rightarrow 3_{+1}^c 4_{+1}^d$$

The residues are the same as before, but with the coupling  $g$  depending on  $a,b,c,d$ . Consistent factorisation gives rise to Jacobi identity

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$$r_s = g_{abe}g_{cde} \frac{\langle 12 \rangle^2 [34]^2}{u}$$

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Let us assume now that the Universe contains several types of spin-1 particles (gluons). For each type, we have a coupling for

$$\begin{aligned}
 \tilde{\mathcal{M}}(1^-2^-3^+4^+5^+6^+7^+) &= \frac{\langle 16 \rangle}{\langle 17 \rangle \langle 76 \rangle} \tilde{\mathcal{M}}(1^-2^-3^+4^+5^+6^+) \\
 &= \frac{\langle 16 \rangle}{\langle 17 \rangle \langle 76 \rangle} \frac{\langle 15 \rangle}{\langle 16 \rangle \langle 65 \rangle} \tilde{\mathcal{M}}(1^-2^-3^+4^+5^+) \\
 &= \frac{\langle 15 \rangle}{\langle 17 \rangle \langle 76 \rangle \langle 65 \rangle} \frac{\langle 14 \rangle}{\langle 15 \rangle \langle 54 \rangle} \tilde{\mathcal{M}}(1^-2^-3^+4^+) \\
 &= \frac{\langle 14 \rangle}{\langle 17 \rangle \langle 76 \rangle \langle 65 \rangle \langle 54 \rangle} \frac{\langle 13 \rangle}{\langle 14 \rangle \langle 43 \rangle} \tilde{\mathcal{M}}(1^-2^-3^+) \\
 &= \frac{\langle 12 \rangle^4}{\langle 71 \rangle \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 67 \rangle},
 \end{aligned}$$

The re  
depend  
Jacobi

$r_s = g_{ace}g_{bde} \frac{\langle 12 \rangle^2 [34]^2}{s} \Rightarrow g_{abe}g_{cde} + g_{ace}g_{bde} + g_{ade}g_{bce} = 0$

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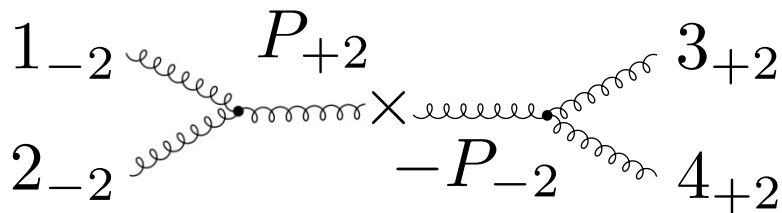


# What happens with gravity?

Let us assume that the Universe contains a single type of spin-2 particle (the graviton)

Let us compute the amplitude for  $1_{-2} 2_{-2} \rightarrow 3_{+2} 4_{+2}$

The first thing we notice is that we can not avoid dimensionful coupling. Namely, there is necessarily a new scale, the Planck mass!

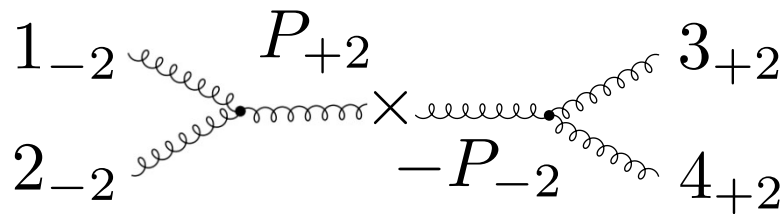


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$$r_s = \kappa^2 \frac{\langle 12 \rangle^6}{\langle 2P \rangle^2 \langle P1 \rangle^2} \times \frac{[34]^6}{[4-P]^2 [-P3]^2} = \kappa^2 \frac{\langle 12 \rangle^4 [34]^4}{u^2} = -\kappa^2 \frac{\langle 12 \rangle^4 [34]^4}{ut}$$

$$\kappa = \frac{g}{m_P^2}$$

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Compute the residue on the s-channel of the amplitude that has just been shown

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Computing all residues, we fix the amplitude for (quantum) graviton scattering!

$$r_s = -\kappa^2 \frac{\langle 12 \rangle^4 [34]^4}{ut} \quad r_t = -\kappa^2 \frac{\langle 12 \rangle^4 [34]^4}{st} \quad r_u = -\kappa^2 \frac{\langle 12 \rangle^4 [34]^4}{st}$$

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$$\Rightarrow \mathcal{A} = -\kappa^2 \frac{\langle 12 \rangle^4 [34]^4}{stu} \quad \Rightarrow \frac{d\sigma}{d\Omega} = 4G_N^2 E^2 \frac{\cos^{12} \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}}$$

# What happens with gravity?

Let us take some time to reflect on what we have just done.

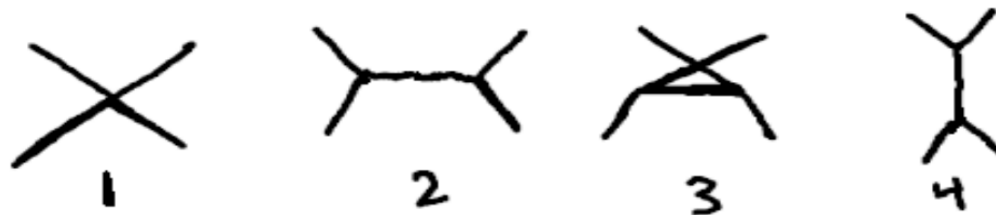
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In 1963 I gave [Walter G. Wesley] a student of mine the problem of computing the cross section for a graviton-graviton scattering in tree approximation, for his Ph.D. thesis [28]. The relevant diagrams are these:



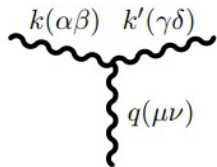
Given the fact that the vertex function in diagram 1 contains over 175 terms and that the vertex functions in the remaining diagrams each contain 11 terms, leading to over 500 terms in all, you can see that this was not a trivial calculation, in the days before computers with algebraic manipulation capacities were available. And yet the final results were ridiculously simple. The cross section for scattering in the



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$$\begin{aligned}
 &= \frac{i\kappa}{2} \left( P_{\alpha\beta,\gamma\delta} \left[ k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2} \eta^{\mu\nu} q^2 \right] \right. \\
 &+ 2q_\lambda q_\sigma \left[ I^{\lambda\sigma}_{\alpha\beta} I^{\mu\nu}_{\gamma\delta} + I^{\lambda\sigma}_{\gamma\delta} I^{\mu\nu}_{\alpha\beta} - I^{\lambda\mu}_{\alpha\beta} I^{\sigma\nu}_{\gamma\delta} - I^{\sigma\nu}_{\alpha\beta} I^{\lambda\mu}_{\gamma\delta} \right] \\
 &+ \left[ q_\lambda q^\mu (\eta_{\alpha\beta} I^{\lambda\nu}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu}_{\alpha\beta}) + q_\lambda q^\nu (\eta_{\alpha\beta} I^{\lambda\mu}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu}_{\alpha\beta}) \right. \\
 &- q^2 (\eta_{\alpha\beta} I^{\mu\nu}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu}_{\alpha\beta}) - \eta^{\mu\nu} q^\lambda q^\sigma (\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma}) \left. \right] \\
 &+ \left[ 2q^\lambda \left( I^{\sigma\nu}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\mu + I^{\sigma\mu}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\nu \right. \right. \\
 &- \left. \left. I^{\sigma\nu}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu \right) \right. \\
 &+ \left. q^2 (I^{\sigma\mu}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I_{\alpha\beta,\sigma}{}^\nu I^{\sigma\mu}_{\alpha\delta}) + \eta^{\mu\nu} q^\lambda q_\sigma (I^{\rho\sigma}_{\gamma\delta} I_{\alpha\beta,\lambda\rho} + I^{\rho\sigma}_{\alpha\beta} I_{\gamma\delta,\lambda\rho}) \right] \\
 &+ \left[ (k^2 + (k-q)^2) \left( I^{\sigma\mu}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I^{\sigma\nu}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\mu - \frac{1}{2} \eta^{\mu\nu} P_{\alpha\beta,\gamma\delta} \right) \right. \\
 &\left. - k^2 \eta_{\gamma\delta} I^{\mu\nu}_{\alpha\beta} - (k-q)^2 \eta_{\alpha\beta} I^{\mu\nu}_{\gamma\delta} \right] \left. \right).
 \end{aligned}$$

# What happens with gravity?

Let us take some time to reflect on what we have just done.

We have computed a quantum gravity process. This task is crazily complicated within standard quantum field theory

Our amplitude is valid only at energies significantly smaller than the Planck mass. At higher energies, the amplitude grows too fast in contradiction with unitarity

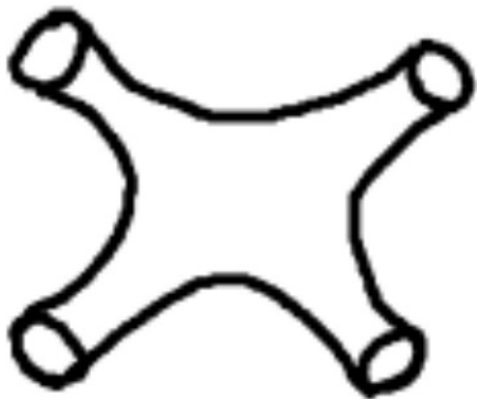
$$\mathcal{A} = -\frac{g^2}{m_P^2} \frac{\langle 12 \rangle^4 [34]^4}{stu} \longrightarrow \infty$$
$$E \longrightarrow \infty$$

# What happens with gravity?

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This has been later computed within string theory, where unitarity is preserved at all energies



$$\Rightarrow \mathcal{A} = -\frac{\langle 12 \rangle^4 [34]^4 \Gamma(-\alpha' s) \Gamma(-\alpha' t) \Gamma(-\alpha' u)}{\Gamma(1 + \alpha' s) \Gamma(1 + \alpha' t) \Gamma(1 + \alpha' u)}$$

# Higher-spin particles

Let us assume now that the Universe contains a single type of spin-3 particle

Let us compute the amplitude for  $1_{-3} 2_{-3} \rightarrow 3_{+3} 4_{+3}$

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Let us compute the amplitude for  $1_{-3} 2_{-3} \rightarrow 3_{+3} 4_{+3}$

There is no amplitude that has only single poles with residues given by product of three amplitudes. It is enough to investigate the s-channel

Therefore, particles with spin  $> 2$  must be free

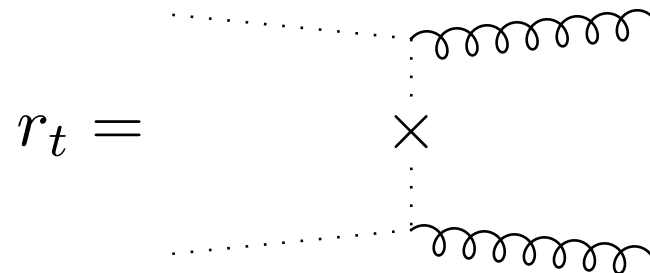
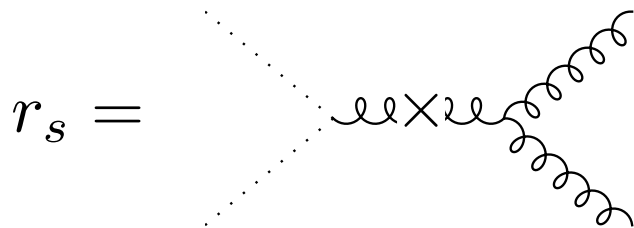
$$r_s = \kappa^2 \frac{\langle 12 \rangle^6 [34]^6}{u^2 t} \sim \kappa^2 \frac{\langle 12 \rangle^6 [34]^6}{t u^2} \sim \dots$$

# Interactions with gravitons

Let us assume now that the Universe contains a graviton and a scalar

Let us compute the amplitude for  $1_0 2_0 \rightarrow 3_{+2} 4_{-2}$

Consistent factorisation implies that the graviton couples with the scalar with the same strength as with itself!

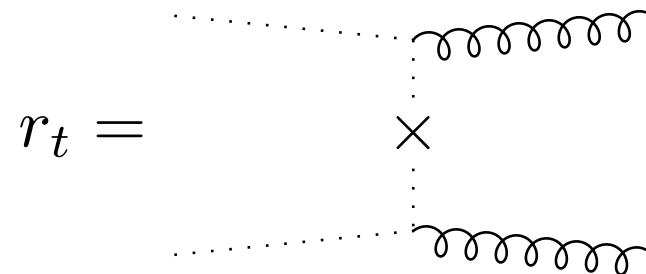
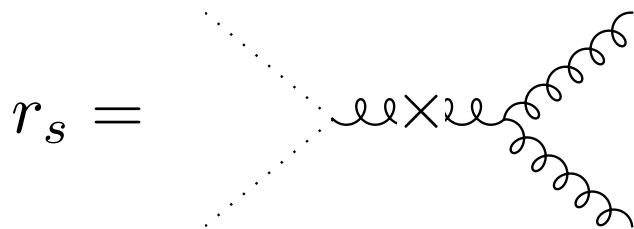


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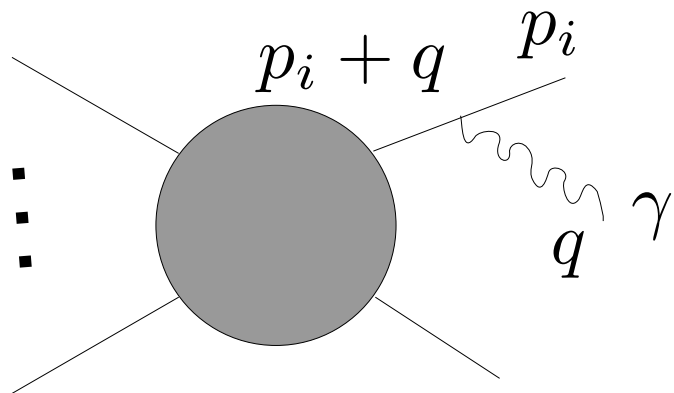


$$\mathcal{A} = -\kappa^2 \frac{\langle 14 \rangle^4 [13]^4}{stu}$$

# Equivalence principle from fields

The universal coupling of gravity can be deduced from standard quantum field theory by analysing amplitudes in the soft limit

Following Weinberg, let us first prove charge conservation



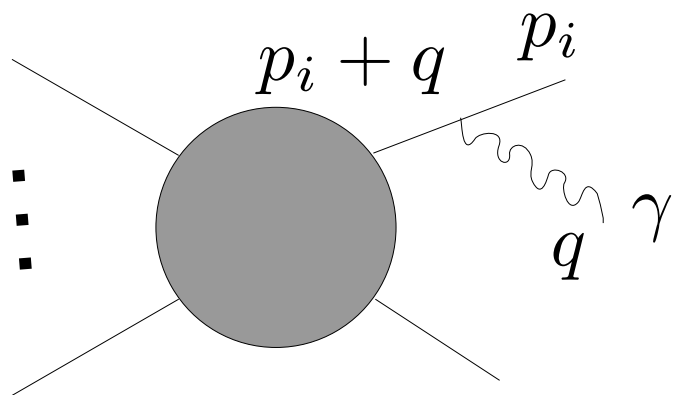
$$q \rightarrow 0$$



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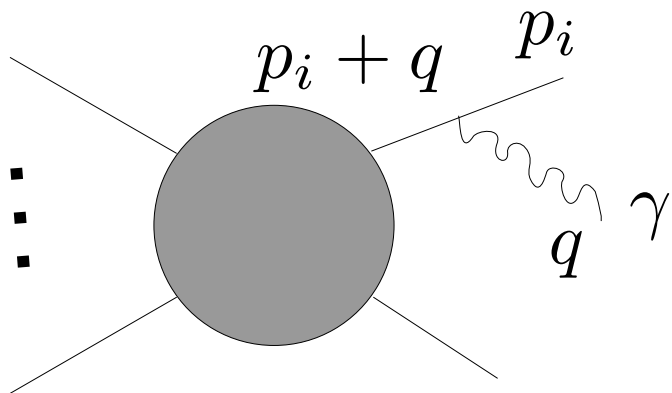
$$\mathcal{A} = \sum_i \mathcal{A}_0 \times Q_i \frac{\{p_i \cdot \epsilon, q \cdot \epsilon, \dots\}}{(q + p_i)^2}$$

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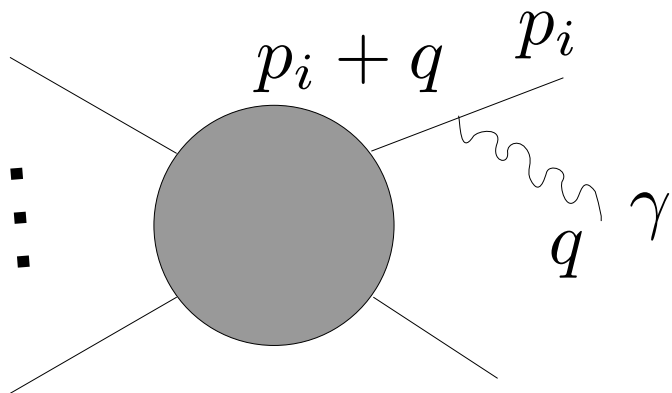
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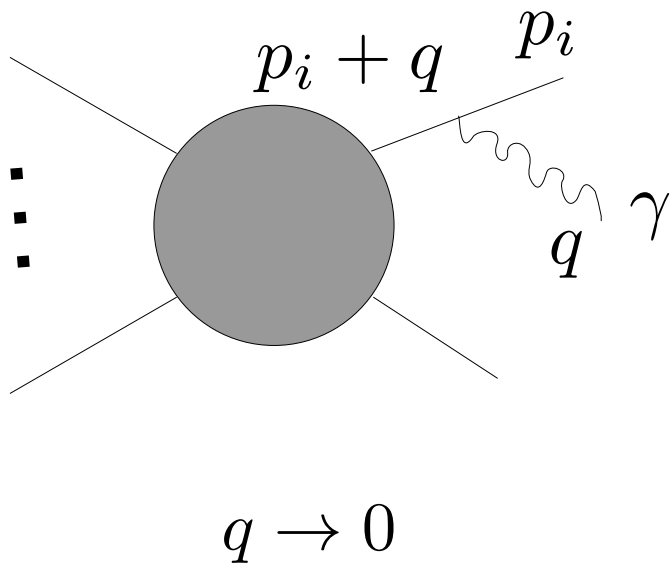
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$$\begin{aligned}
 \mathcal{A} &= \sum_i \mathcal{A}_0 \times Q_i \frac{\{p_i \cdot \epsilon, q \cdot \epsilon, \dots\}}{(q + p_i)^2} \\
 &= \mathcal{A}_0 \times \sum_i Q_i \frac{p_i \cdot \epsilon}{p_i \cdot q} \\
 &= \mathcal{A}_0 \sum_i Q_i \frac{p_i \cdot (\epsilon + \alpha q)}{p_i \cdot q} \\
 &\Rightarrow \alpha \sum_i Q_i \frac{p_i \cdot q}{p_i \cdot q} = 0 \rightarrow \sum_i Q_i = 0
 \end{aligned}$$

# Equivalence principle from fields

The universal coupling of gravity can be deduced from standard quantum field theory by analysing amplitudes in the soft limit

Following Weinberg, let us first prove charge conservation

Repeat the previous exercise but for gravitons instead of gluons, taking into account that, in this case

$$\mathcal{A} = \mathcal{A}_0 \times \sum_i \kappa_i \frac{p_i^\mu p_i^\nu \epsilon_{\mu\nu}}{p_i \cdot q}$$

and gauge redundancy reads

$$\epsilon_{\mu\nu} \rightarrow \epsilon_{\mu\nu} + \alpha_\mu q_\nu + \alpha_\nu q_\mu + \alpha q_\mu q_\nu$$

# Observations

Photons must be non-interacting

All particles have spin smaller than or equal to 2

Charge is conserved

Forces are described by Yang-Mills theory

Gravity couples universally to all particles

There can be only one type of graviton

Existence of spin  $3/2$  particles implies supersymmetry

# Things that we do not explain

There is the same number of leptons and quarks

There are three families of fermions

Values of particle masses seemingly unrelated (neutrino masses are tiny)

Many free parameters in the SM

Matter-antimatter asymmetry in the Universe

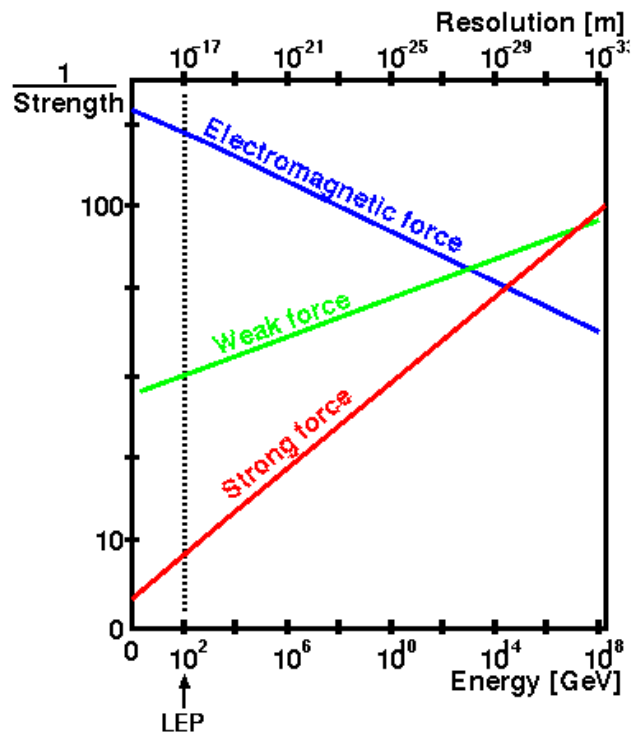
The experimental evidence of dark matter

The Higgs mass is too small?

# Gauge coupling unification

Within the SM, gauge couplings do not fully unify at high energy

Supersymmetrising the spectrum adds new contributions to the running of gauge couplings

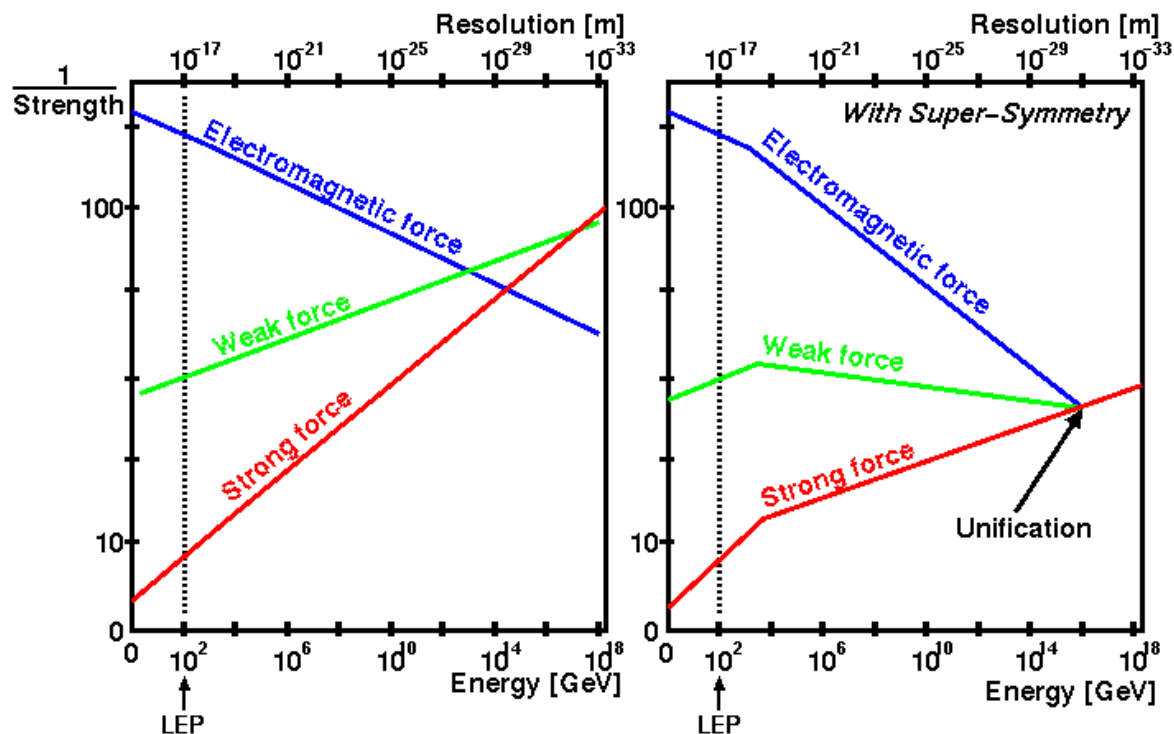




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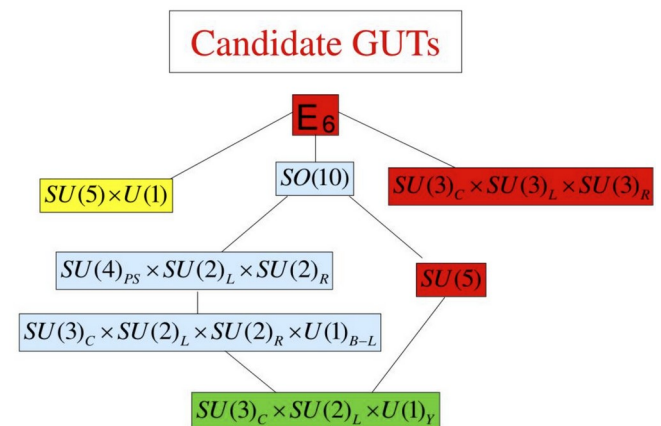
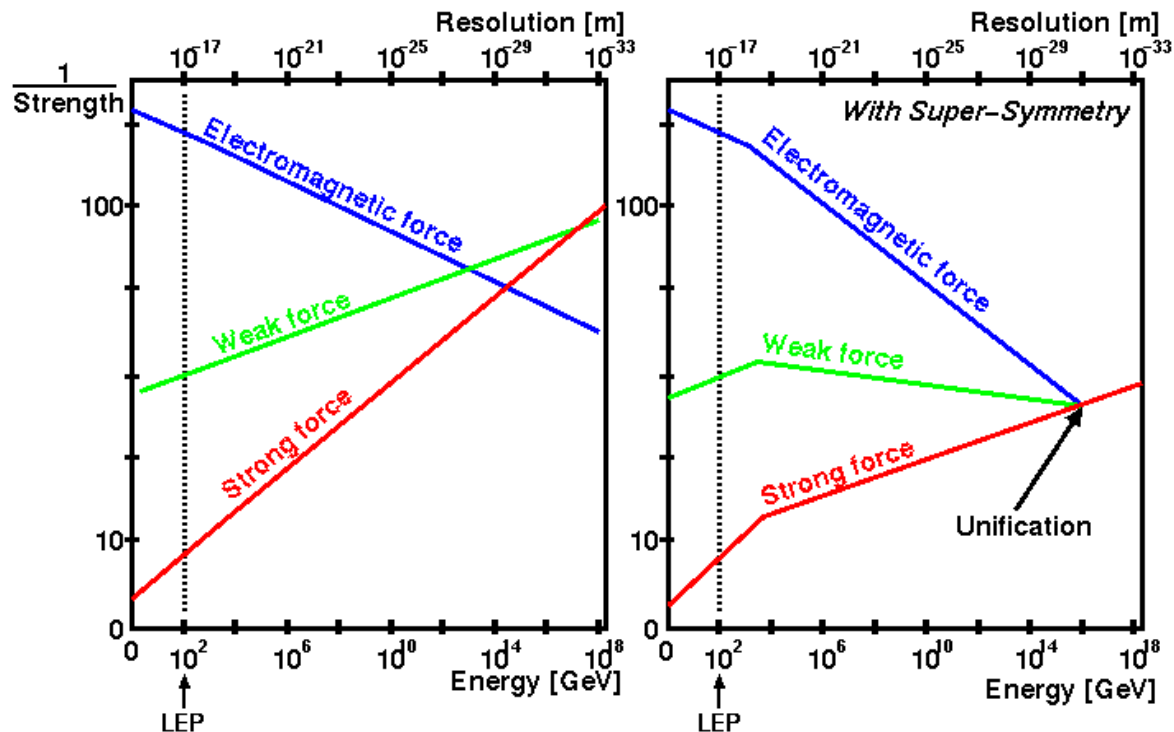
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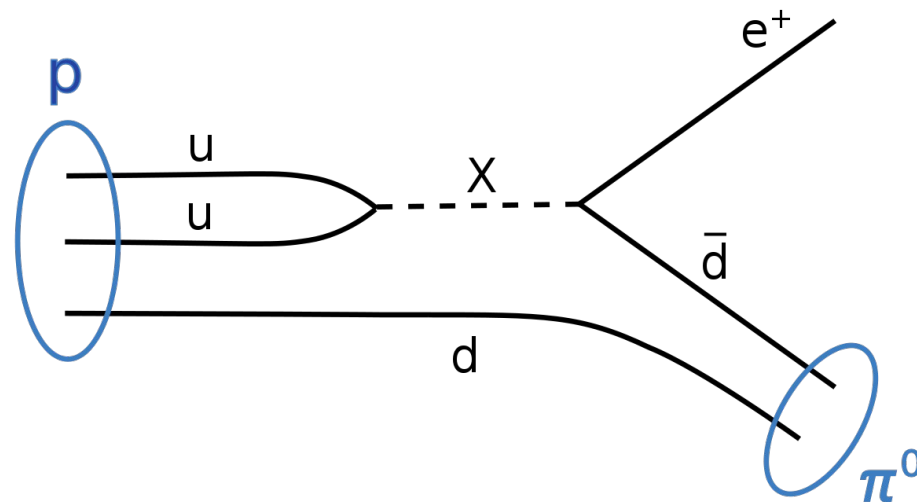


# Gauge coupling unification

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Supersymmetrising the spectrum adds new contributions to the running of gauge couplings

This has implications (or problems), e.g. proton decay



# Baryogenesis

There is substantially more matter than antimatter in the Universe

If one wants to explain this asymmetry assuming there was no asymmetry in the early Universe, then three (Sakharov) conditions are needed (together with thermal non-equilibrium)

baryon number violation

C violation

CP violation

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$$\Gamma(p_L^+ \rightarrow e_R^+ \gamma_L) + \Gamma(p_R^+ \rightarrow e_L^+ \gamma_R)$$

$\parallel$  because C is conserved  $\parallel$

$$\Gamma(p_L^- \rightarrow e_R^- \gamma_L) \quad \Gamma(p_R^- \rightarrow e_L^- \gamma_R)$$

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Show that, even if both B and C are broken, but CP is conserved, then there is no net baryon asymmetry.

Use the fact that under CP, left (right) particles transform into right (left) antiparticles

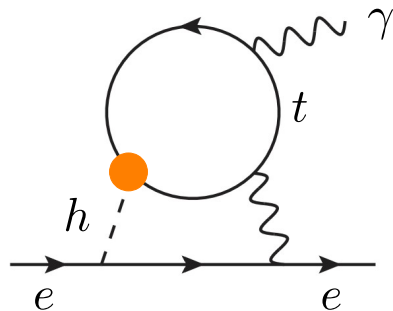


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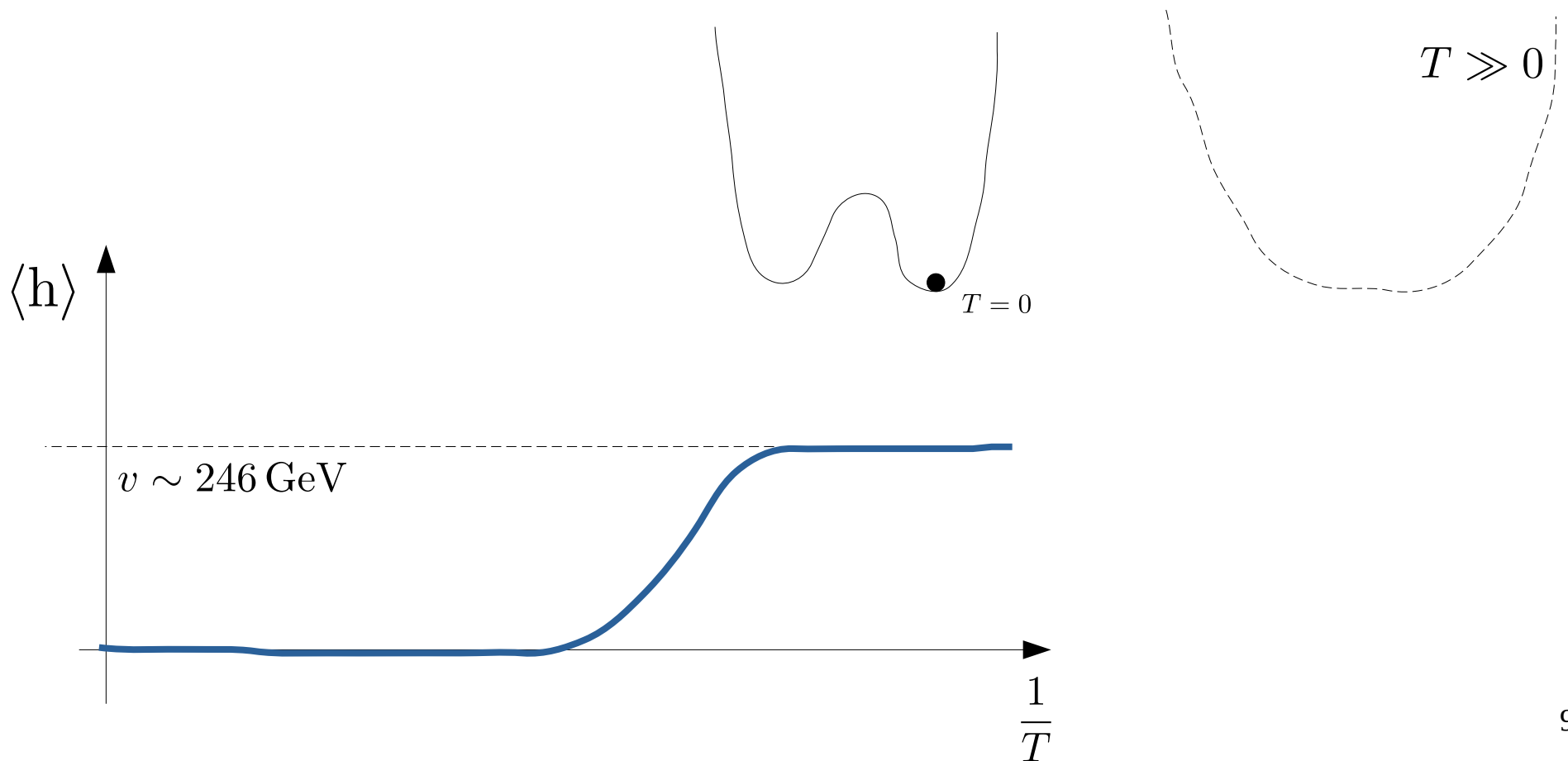
Within the SM, the amount of CP violation (one phase in the CKM matrix) is not sufficient. In SUSY many more CP-violating terms. But this is also a problem



$$|d_e| < 1.1 \cdot 10^{-29} \text{ e} \cdot \text{cm}$$

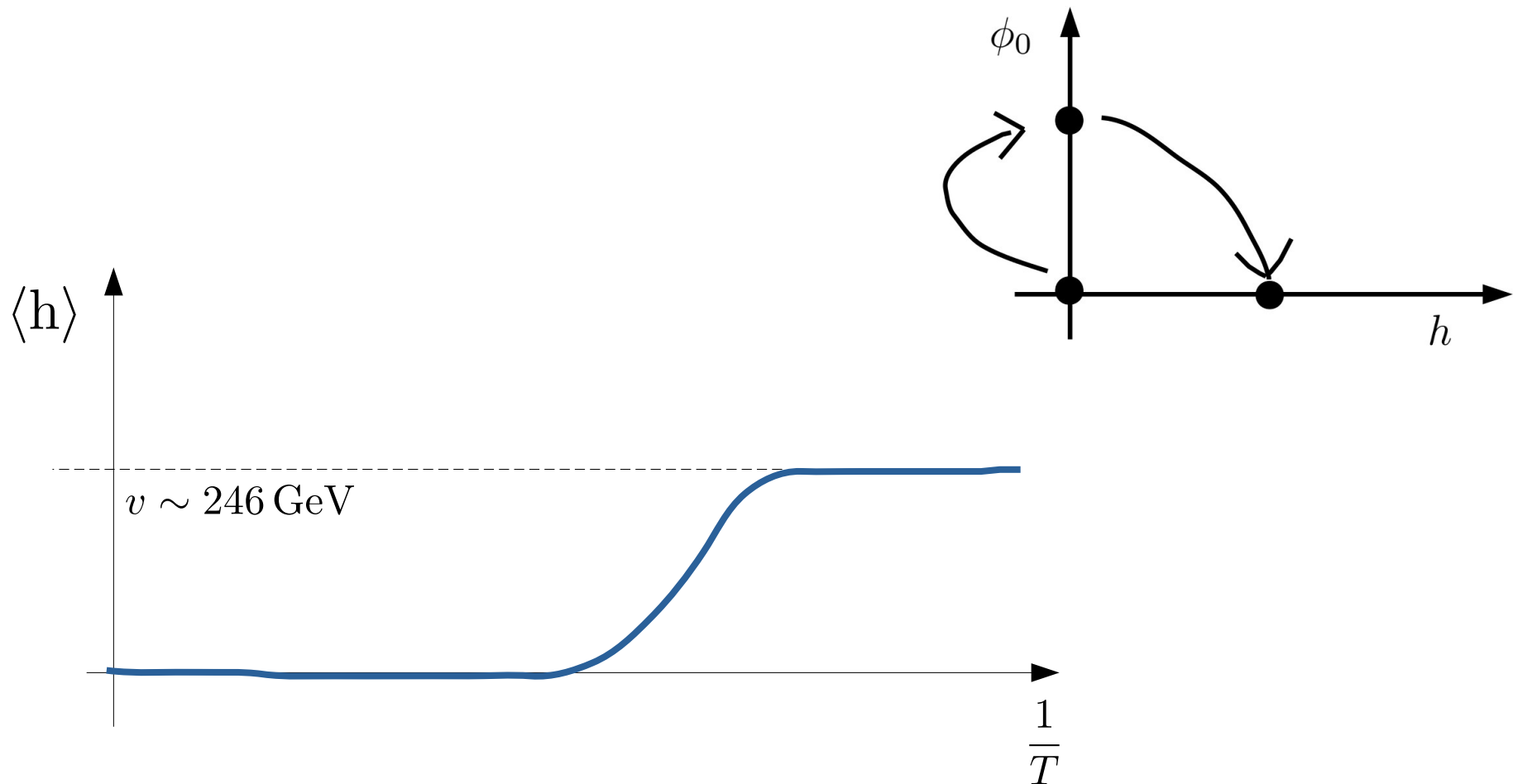
# Electroweak baryogenesis

The Higgs potential in the SM has two minima, but only at zero temperature. The transition from high to low temperatures is a cross-over



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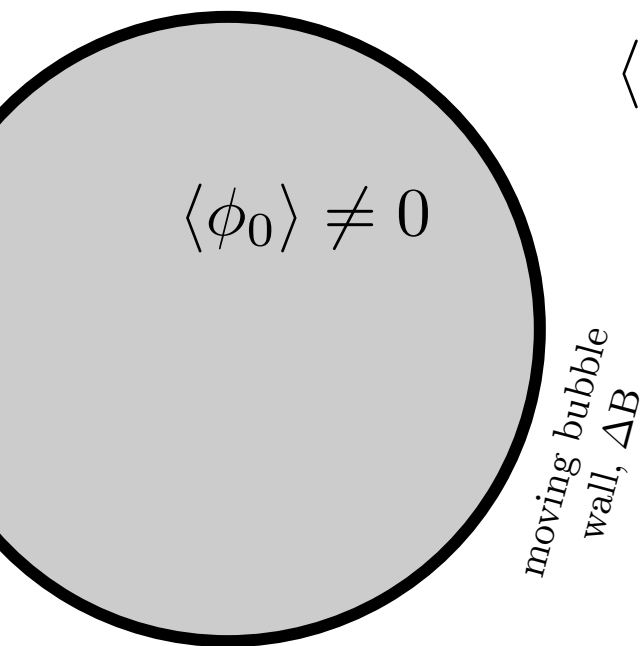
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$$\langle \phi_0 \rangle = 0$$

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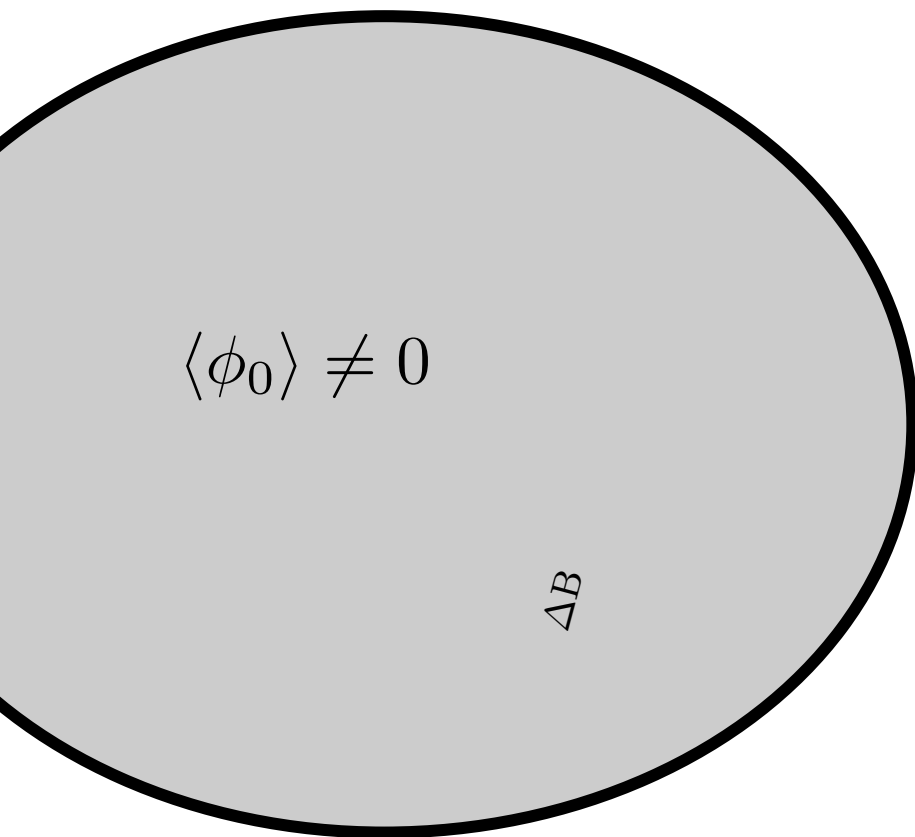
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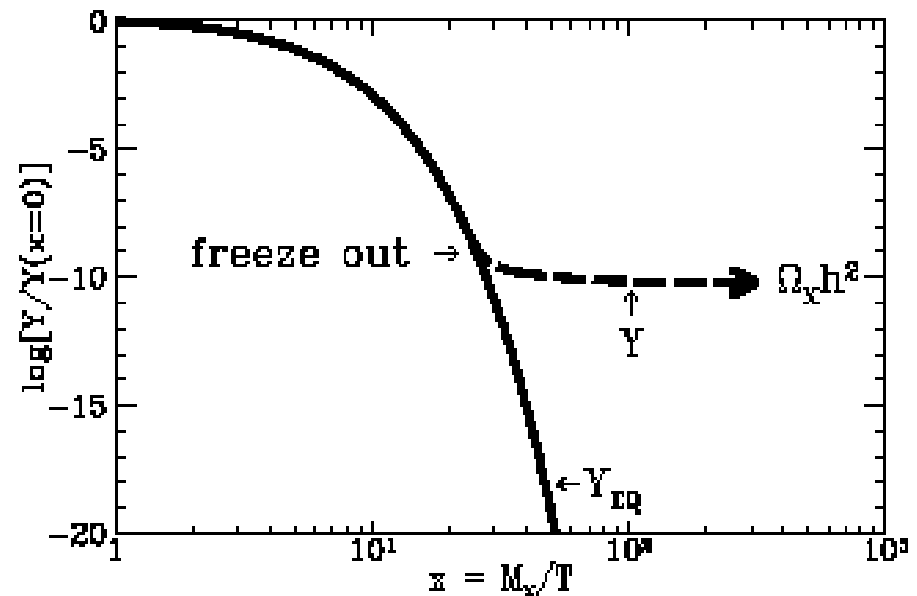
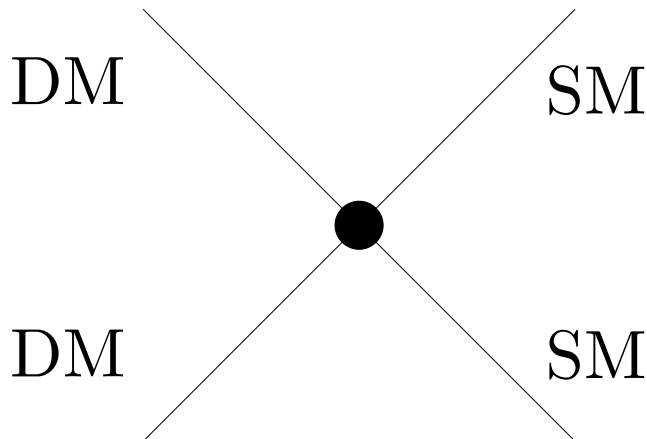
$$\Gamma_{ws} \sim T^4 \exp \left\{ -\frac{E_{sph}}{T} \right\}$$

# Dark matter

There is overwhelming evidence of some unidentified form of matter in galaxies

A very plausible explanation is that it consists of non-relativistic (cold) neutral particles (dark)

Neutrinos are too light to be cold, but neutralinos might



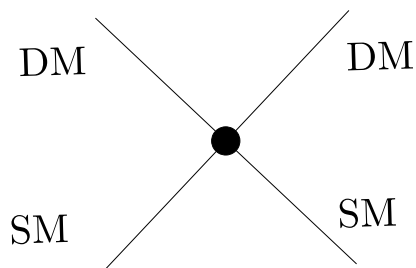
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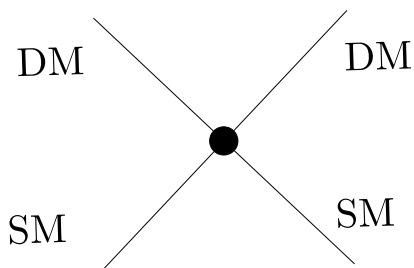
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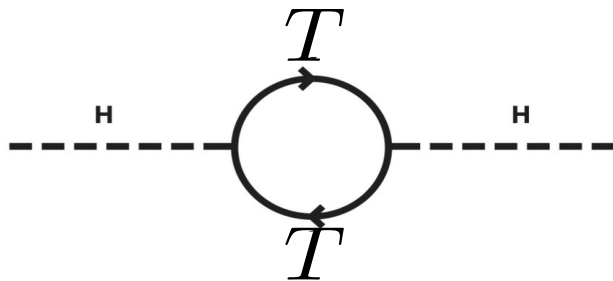
$$A \sim p_{\text{transferred}}^2$$

# The naturalness problem

The Higgs is the only (seemingly) elementary scalar that we know of

Quantum corrections to the Higgs mass scale with the mass of particles in loop

Traditional solutions involve Supersymmetry (heavy particle loops canceled by heavy sparticle loops) and composite Higgs models (at high energies we no longer see the Higgs but its constituents)



$$m_H^2 \sim m_{H^0}^2 - m_T^2$$

# Other problems

Neutrinos are massless within the SM, but we know they have mass

We do not understand why the SM particles have the masses (and charges) they have, nor why there are three families

Strong CP problem

Certain experimental observations (muon magnetic moment, some meson decays, ...)