High Energy Theory

Universidad de Granada

Effective Field Theory

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University of Granada

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Don’t hesitate to write or pass by my office
if you need anything!

12th International IDPASC school and workshop, 18-28 September 2023

Presentation

Lectures:

➡ 2h theory + 1h tutorial

Assumptions:

➡ Basic knowledge of Quantum Field Theory (QFT)

Scope:

➡ Not a complete EFT course: focus on particle physics (although most concepts are applicable elsewhere)

➡ Emphasis on key concepts with specific examples

➡ Most calculations will be done in the tutorial (by hand or with computer tools)

Don't be shy! Stop me and ask if there is ANYTHING you don't understand

Further material

Some EFT courses/lecture notes:

As Scales Become Separated: Lectures on Effective Field Theory

A. V. Manohar, "Introduction to Effective Field Theories", Les Houches 2017

M. Neubert, "Renormalization Theory and Effective Field Theories", Les Houches 2017

I. Z. Rothstein, "TASI lectures on Effective field Theories", TASI 2002

A. Pich, "Effective field theory"

José Santiago, Lectures on "Effective field theory in particle physics"

Online courses:

Link to video lectures on EFTs, by Toni Pich

Link to MIT online course on Effective Field Theories, by I. Stewart

Why Effective (Field) Theories?

The concept is very general: consider an apple falling from a tree. If you want to know its falling velocity, you will probably use

$$mgh = \frac{mv^2}{2} \quad \Longrightarrow \quad v = \sqrt{2gh}$$

But

… the gravitational potential is not linear in h

[Corrections of $\mathcal{O}(h/R) \sim 10^{-6}$]

… Newtonian gravity is itself an effective theory of General Relativity

Physics decouples!

No need to know all details to describe a system at a given precision

Why Effective (Field) Theories?

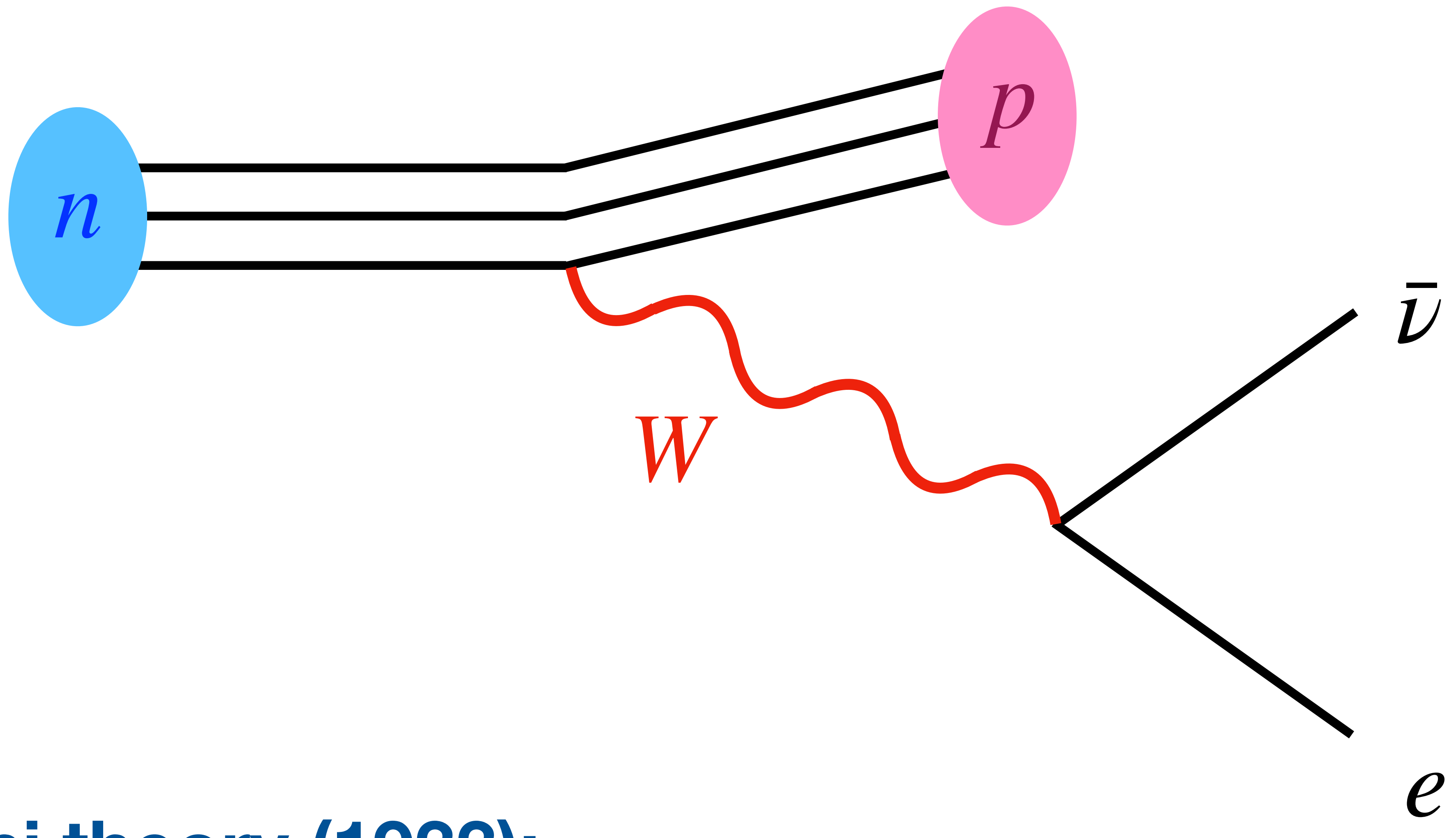
Effective Theories (ET) are ubiquitous in Physics:

- GR → Newtonian gravity
- Charge distribution → Multipolar expansion
- QED → Hidrogen atom
- QCD → Nuclear Physics
- …

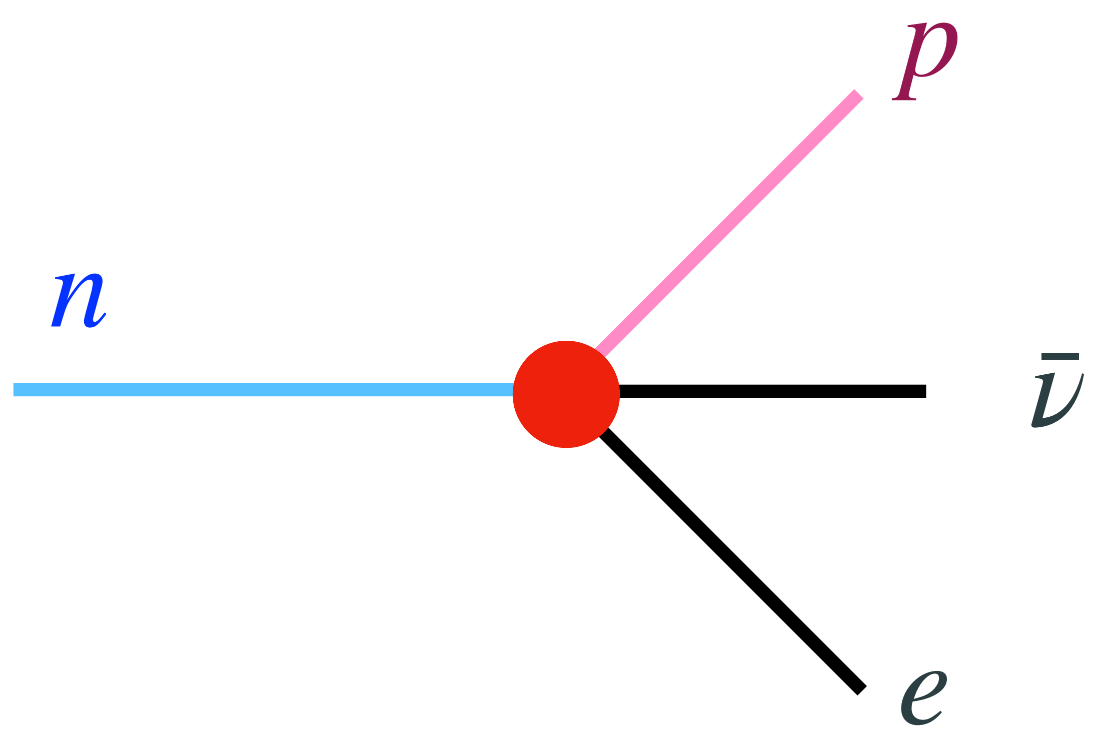
They efficiently separate energy scales:

- ETs are simpler (and more powerful)
- Can be formulated without knowing the full theory
- All theories break down eventually, so they can all be regarded as ETs

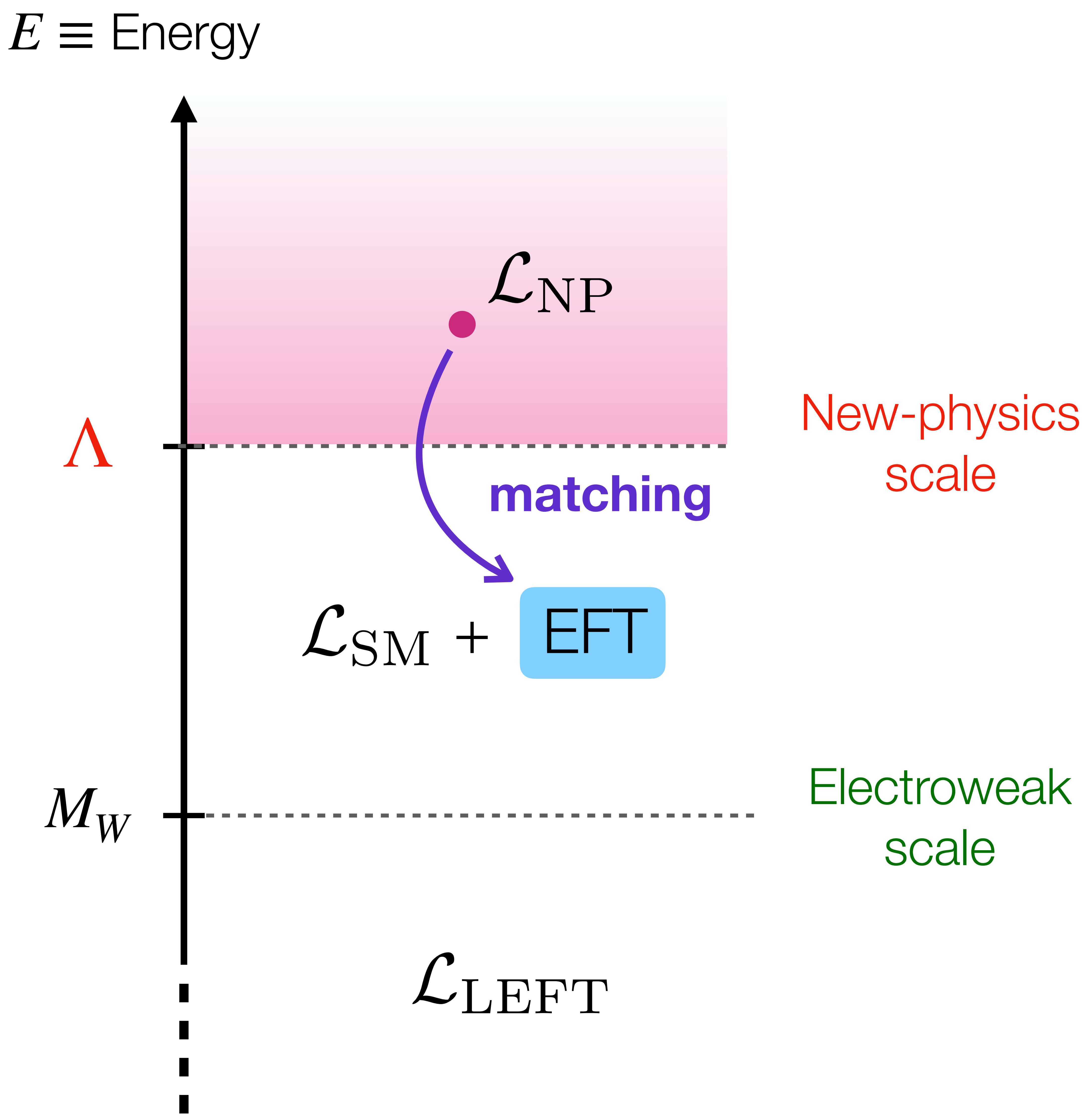
Electroweak theory + QCD (1983):



Fermi theory (1933):



Effective Field Theories (EFT): top-down



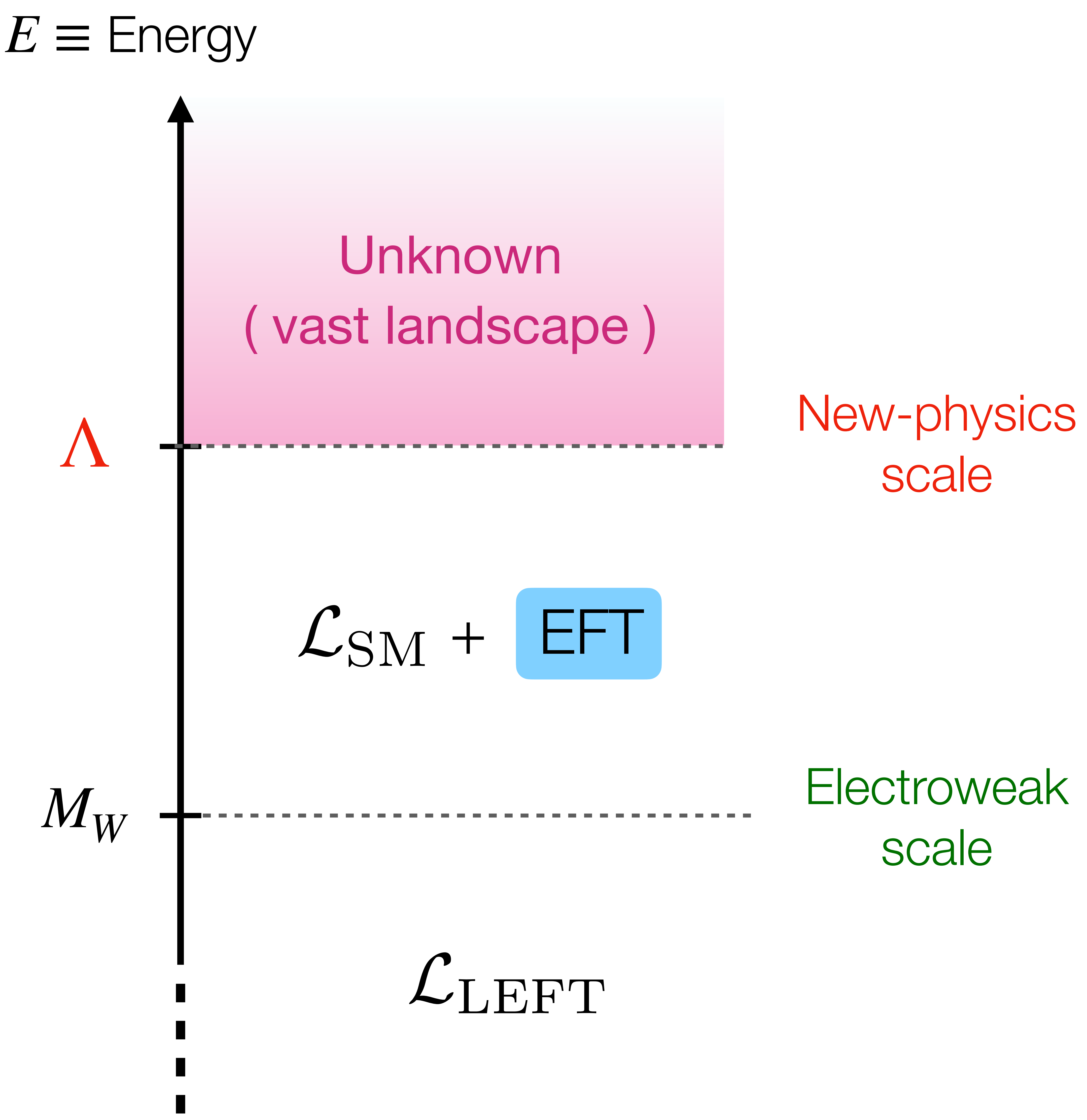
Given a specific new-physics idea:

- Many models share the same EFT, providing a universal framework to connect models with data
- Precision necessitates EFTs: summation of (large) logarithms of E/Λ arising from the quantum corrections

The step to build an EFT from a model is called **matching**

EFTs can even be used when you do not know the exact matching with its UV theory

Effective Field Theories (EFT): bottom-up



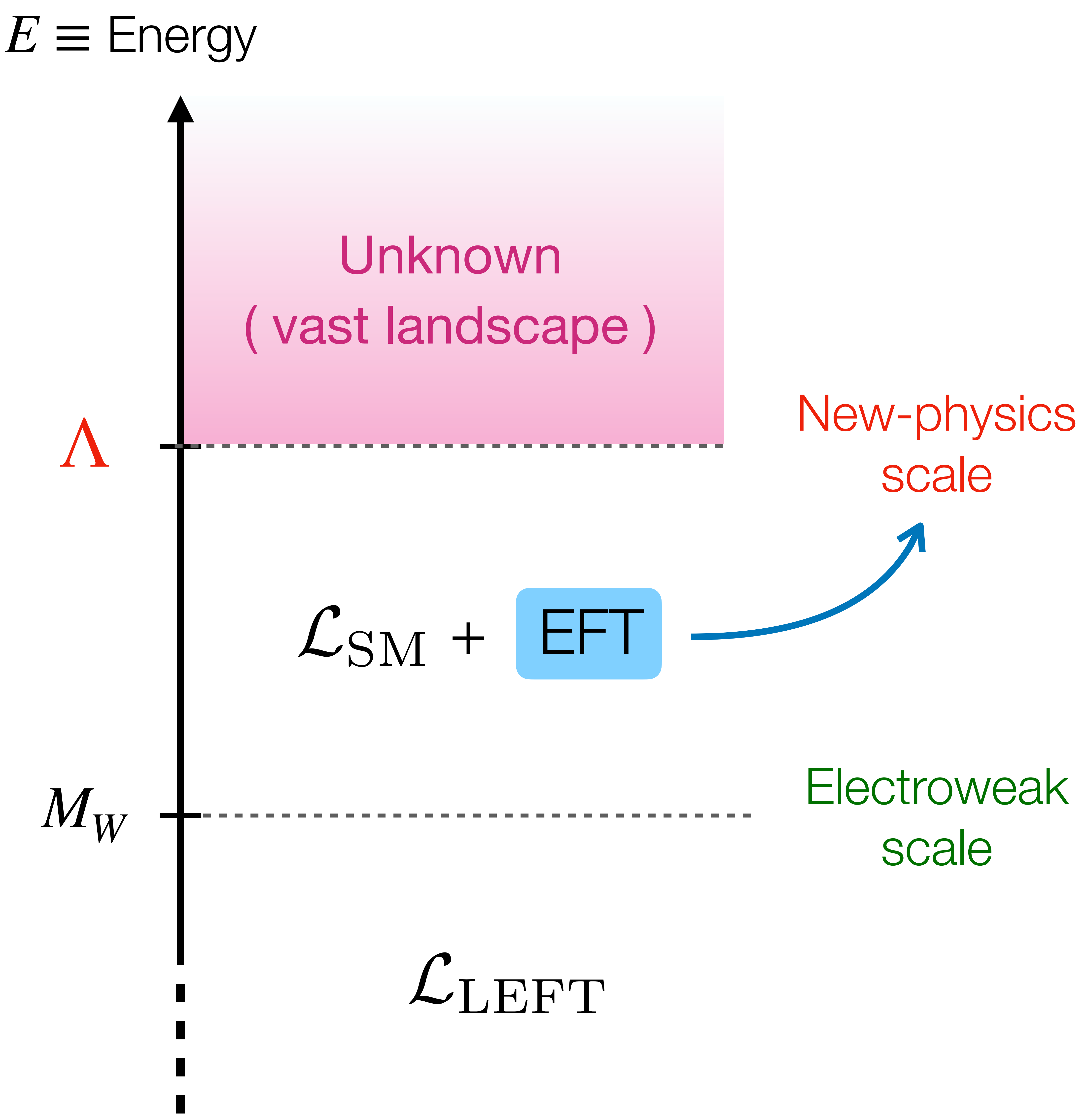
EFTs are great for parametrizing the unknown:

- Can be formulated without knowing the full theory
- Systematically improvable by adding extra terms in a double expansion in quantum corrections and E/Λ

UV physics

$$\mathscr{L}_{\rm EFT}(\eta_L) = \mathscr{L}_{d=4}(\eta_L) + \sum_{\ell=0}^{\infty}\sum_{n=5}^{\infty}\sum_{k} \frac{C^{(\ell)}_{n,k}}{(16\pi^2)^{\ell}\,\Lambda^{n-4}}\, O_{n,k}(\eta_L)$$

Effective Field Theories (EFT): bottom-up



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- Can be formulated without knowing the full theory
- Systematically improvable by adding extra terms in a double expansion in quantum corrections and E/Λ

They give an indication on new-physics scales where a new fundamental theory has to be formulated

For example,

LEFT [Fermi Theory] $\rightarrow$ Electroweak scale $\rightarrow$ Standard Model (SM)

Basic principles of Effective Theories

Degrees of freedom: Building blocks for our theory construction

E.g. fields in a Lagrangian describing light particles

Power counting: Parametric limit that we are considering: what is considered as small?

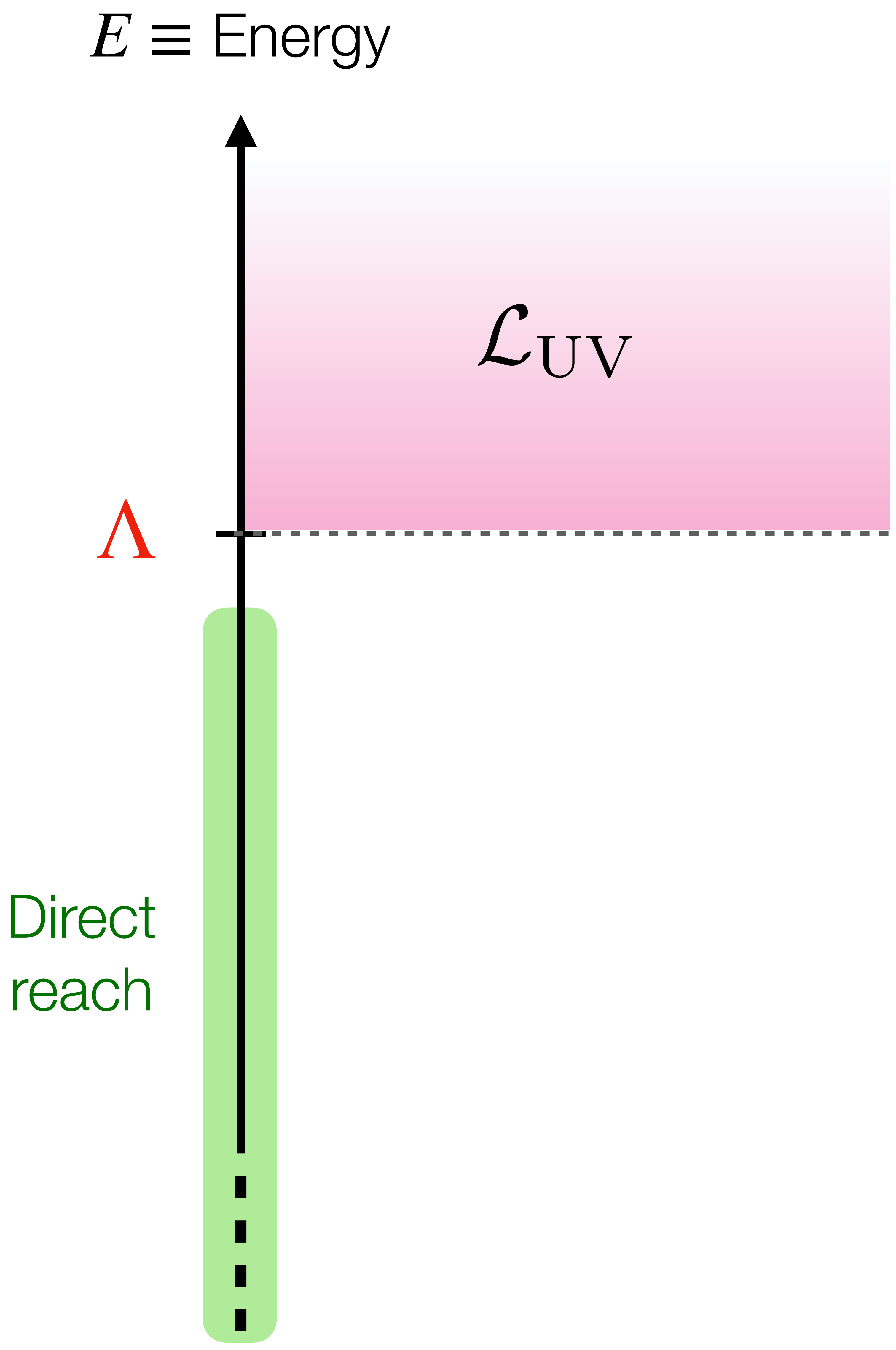
E.g. inverse powers in masses of heavy particles that cannot be directly produced

Symmetries of the system, which constrain possible interactions

Many different types may occur: global, gauged, or accidental symmetries. They could also be broken spontaneously or anomalously.

EFTs are fully-consistent QFTs that incorporate the basic principles of ETs

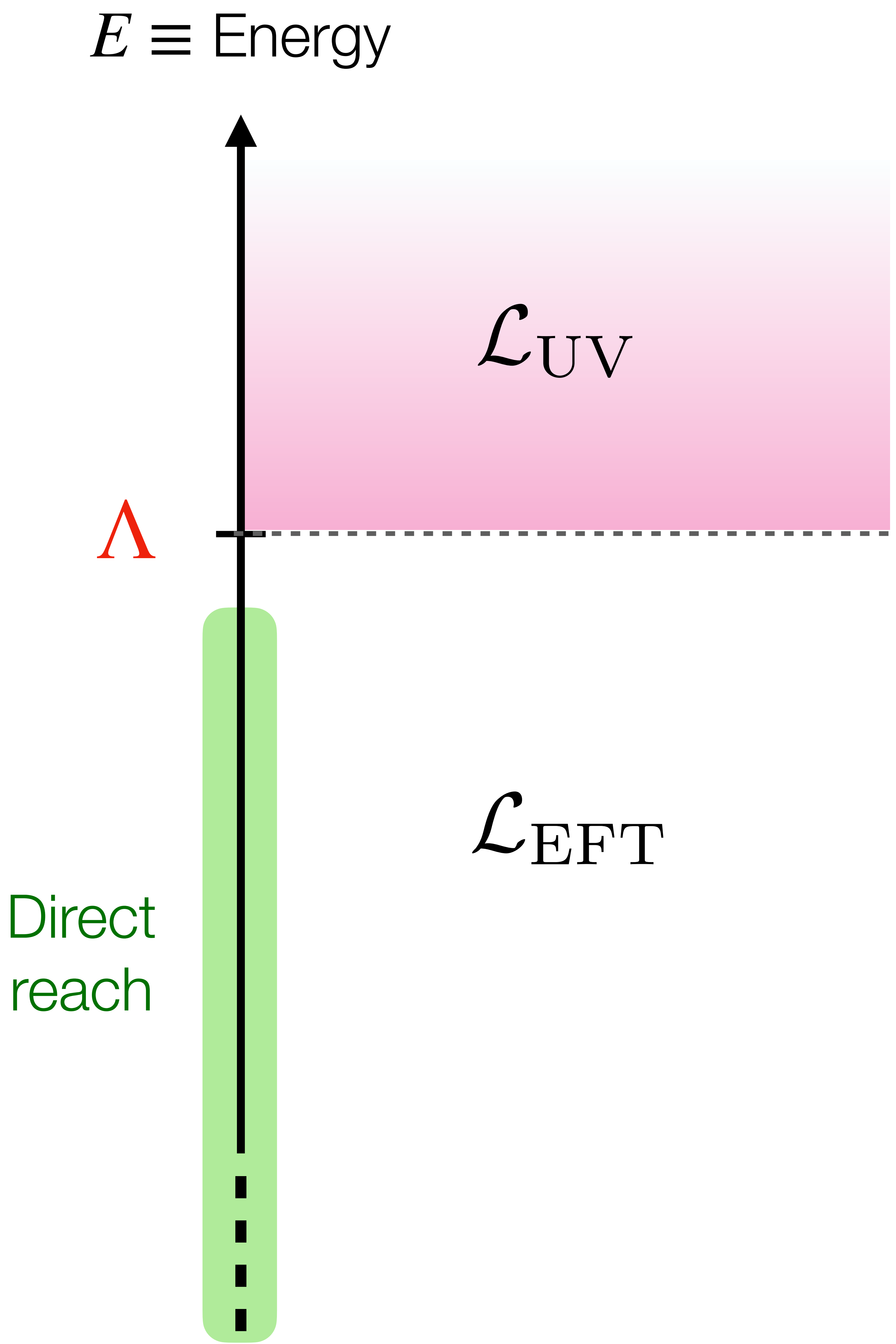
A toy model example



$$\mathscr{L}_{\mathrm{UV}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{4!}\lambda_\phi \phi^4 + \bar{\psi}\, i\partial\!\!\!/ \psi$$
$$+ \bar{\chi}\, i\partial\!\!\!/ \chi - M\bar{\chi}\chi + (y\,\phi\bar{\psi}\chi + \mathrm{h.c.})$$

Let's assume $m_\phi \ll M$, and $p^2 \ll M^2$ so χ cannot be directly produced.

A toy model example



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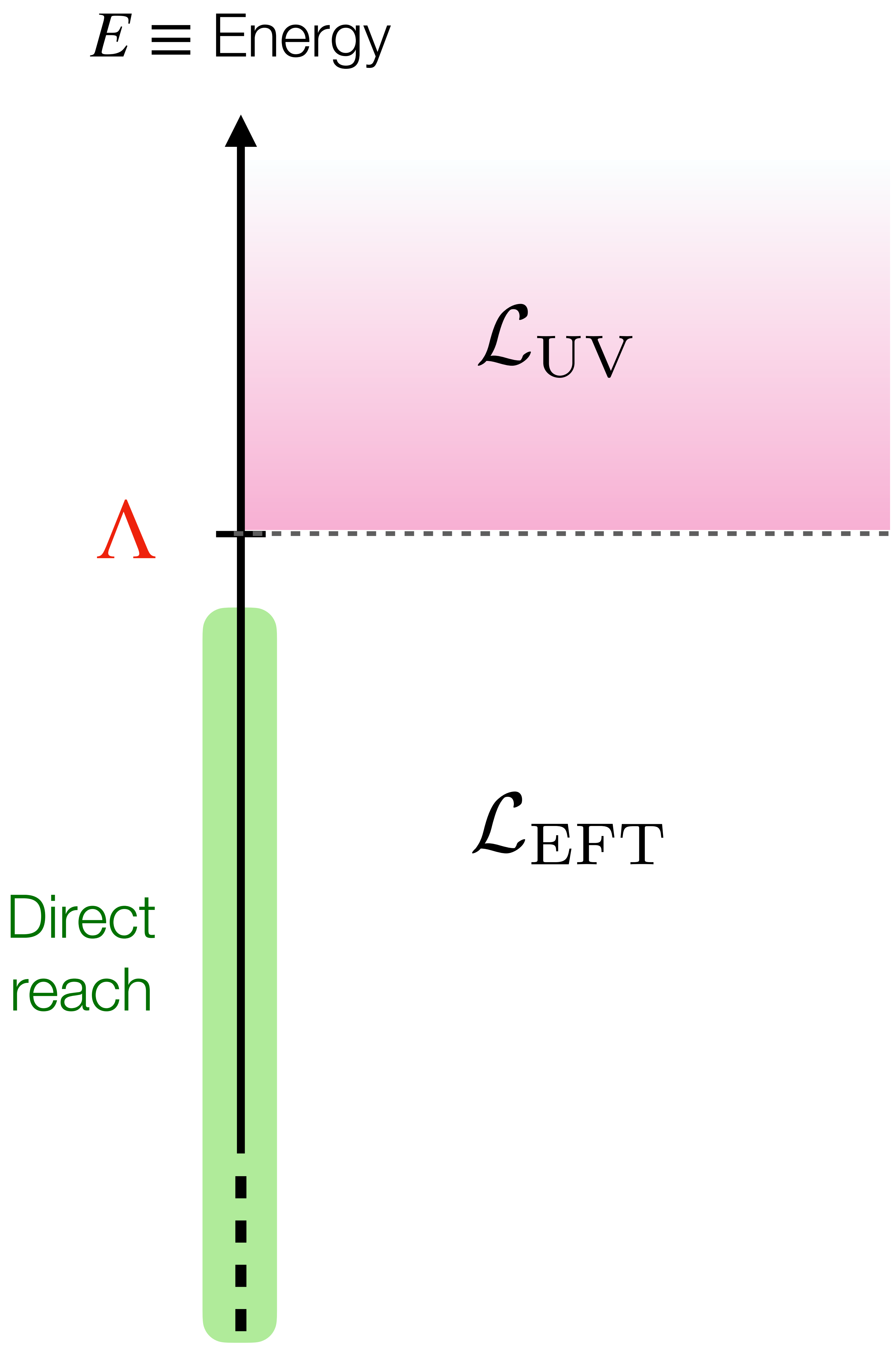
Which is the correct EFT?

A toy model example



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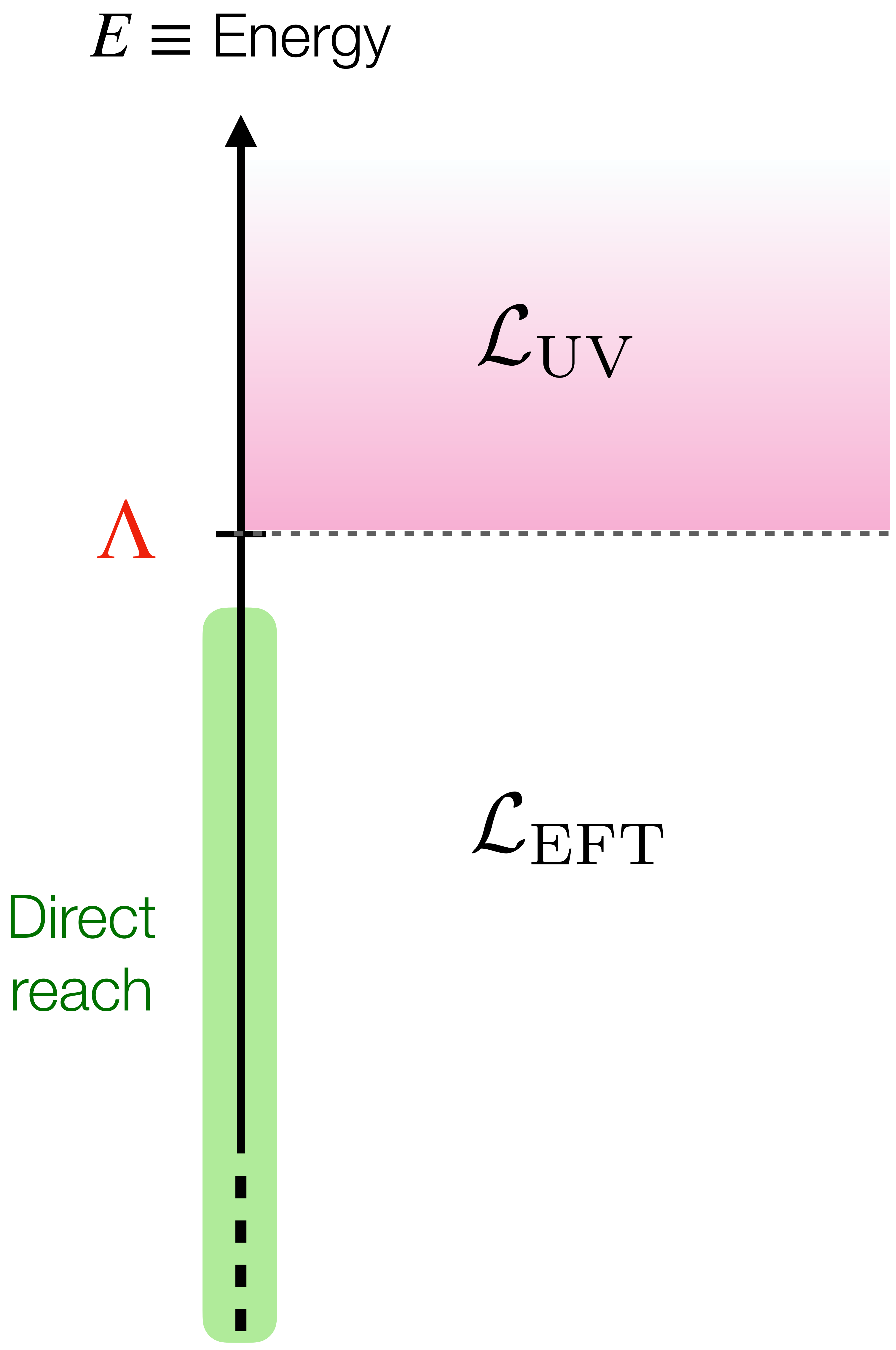

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Degrees of freedom: $\phi\,,\,\psi$

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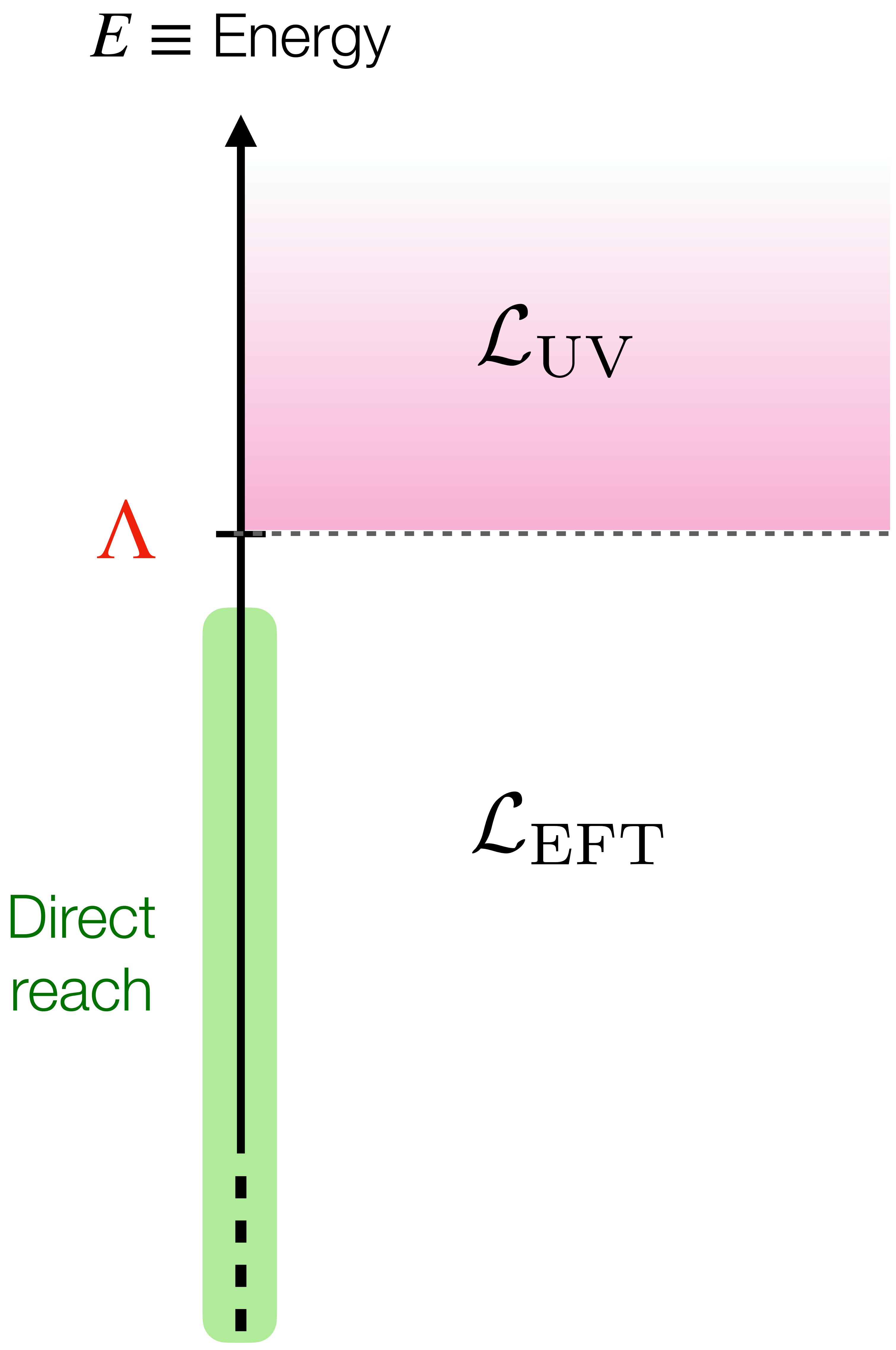

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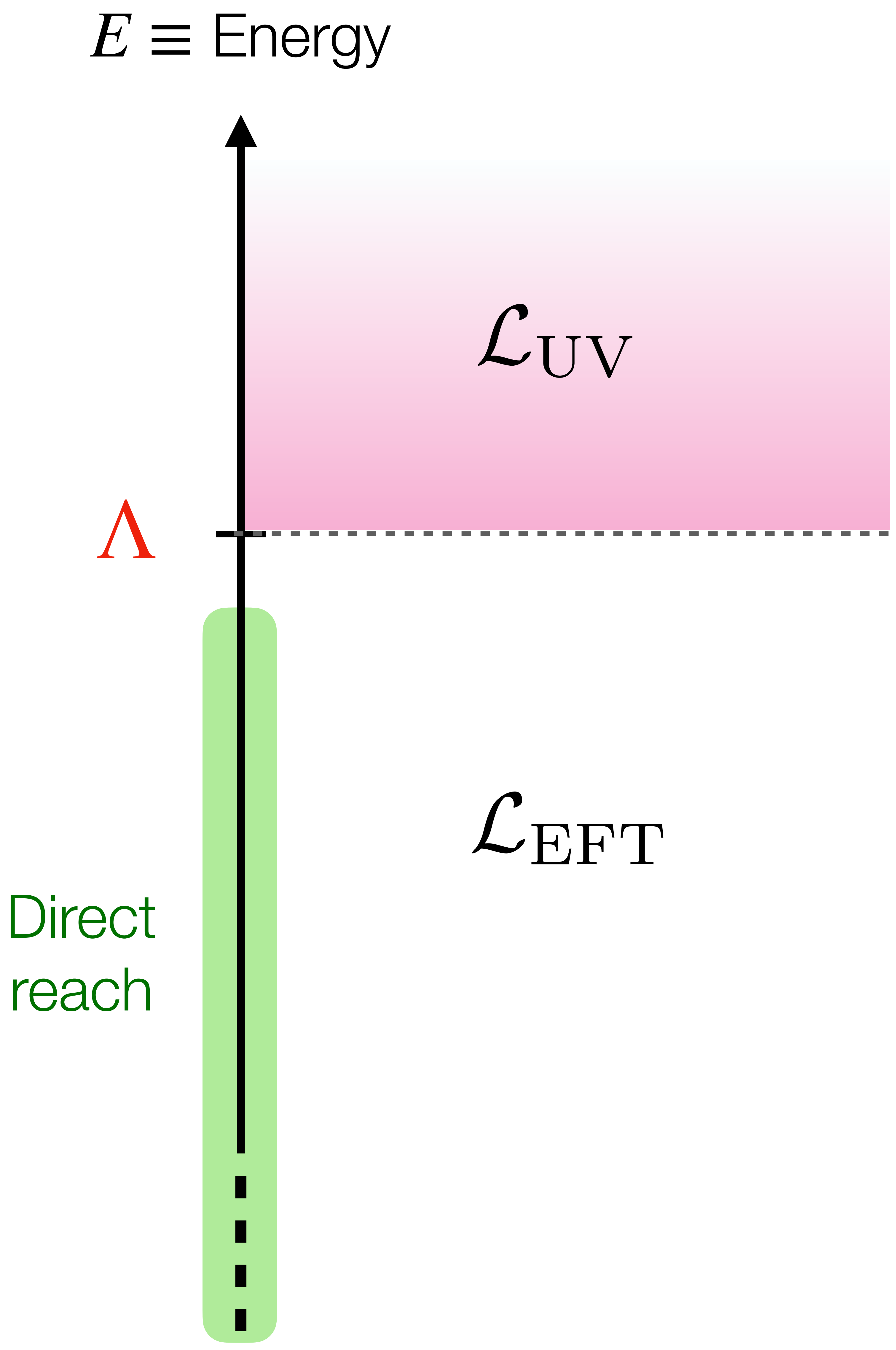
A toy model example



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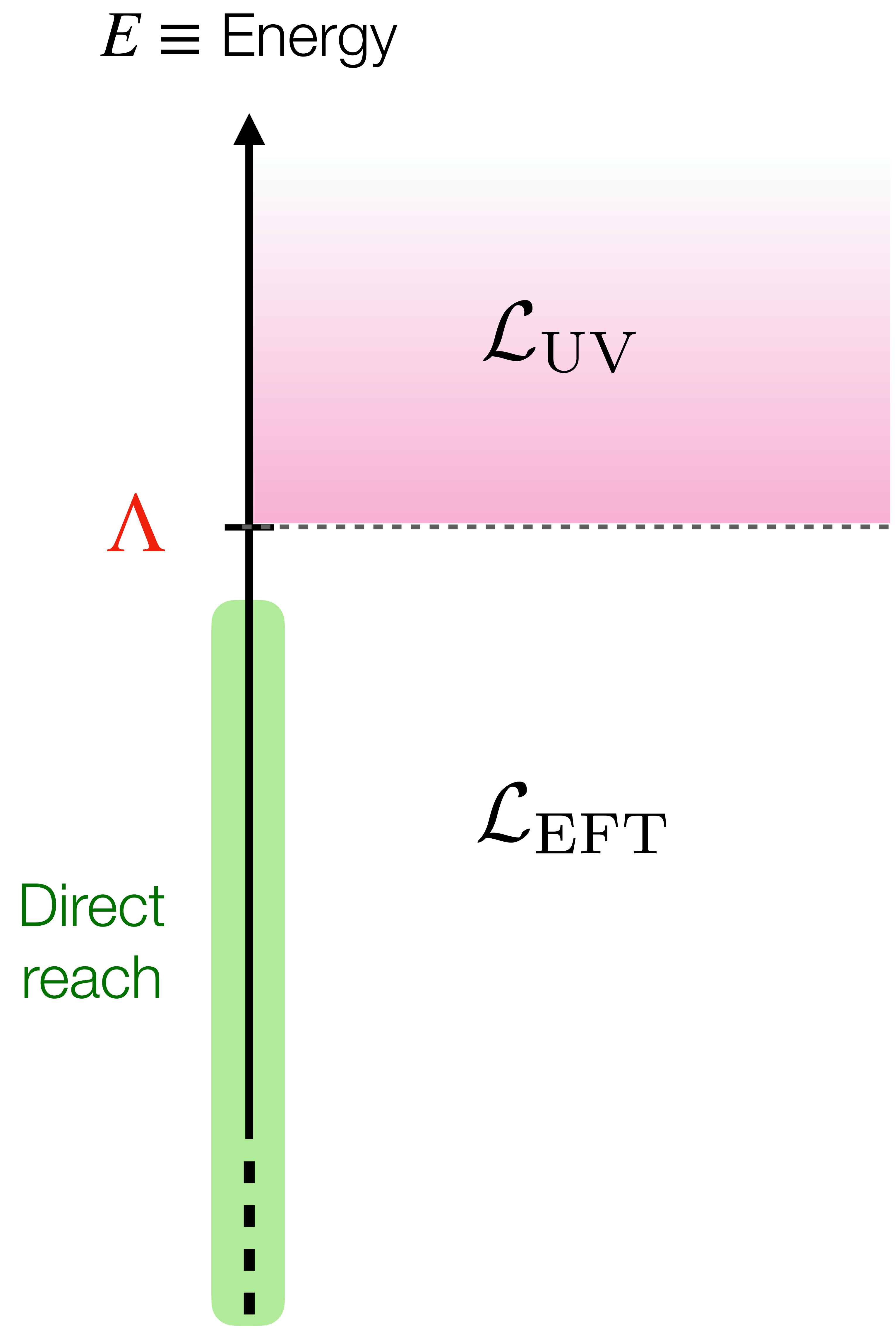
Which is the correct EFT?

Degrees of freedom: $\phi\,,\,\psi$

Power counting: $m^2_{\phi,\psi}/M^2\,,\; p^2/M^2 \sim \mathcal{O}(\Lambda^{-2}) \ll 1$
(in energy dim.)

A toy model example



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Degrees of freedom: $\phi\,,\, \psi$

Power counting:
(in energy dim.)

$$m_{\phi,\psi}^2 / M^2 \,,\; p^2/M^2 \sim \mathcal{O}(\Lambda^{-2}) \ll 1$$

… but also $\phi/M \sim \mathcal{O}(\Lambda^{-1})$

$$\psi/M^{3/2} \sim \mathcal{O}(\Lambda^{-3/2})$$

A toy model example

Energy dimensions:

$$[x] = -1 \qquad [\partial_\mu] = [p_\mu] = [m] = 1$$

$$\mathscr{L} \supset \frac{1}{2}(\partial_\mu \phi)^2 \Rightarrow [\phi] = 1$$

$$[S] = \left[\int d^4x \mathscr{L}\right] = 0 \Rightarrow [\mathscr{L}] = 4$$

$$\mathscr{L} \supset \bar{\psi}\, i\partial\!\!\!/\, \psi \Rightarrow [\psi] = 3/2$$

Direct reach

$\mathcal{L}_{\text{EFT}}$

Degrees of freedom: $\phi\,,\,\psi$

Power counting:
(in energy dim.)

$$m^2_{\phi\,,\psi}/M^2\,,\; p^2/M^2 \sim \mathcal{O}(\Lambda^{-2}) \ll 1$$

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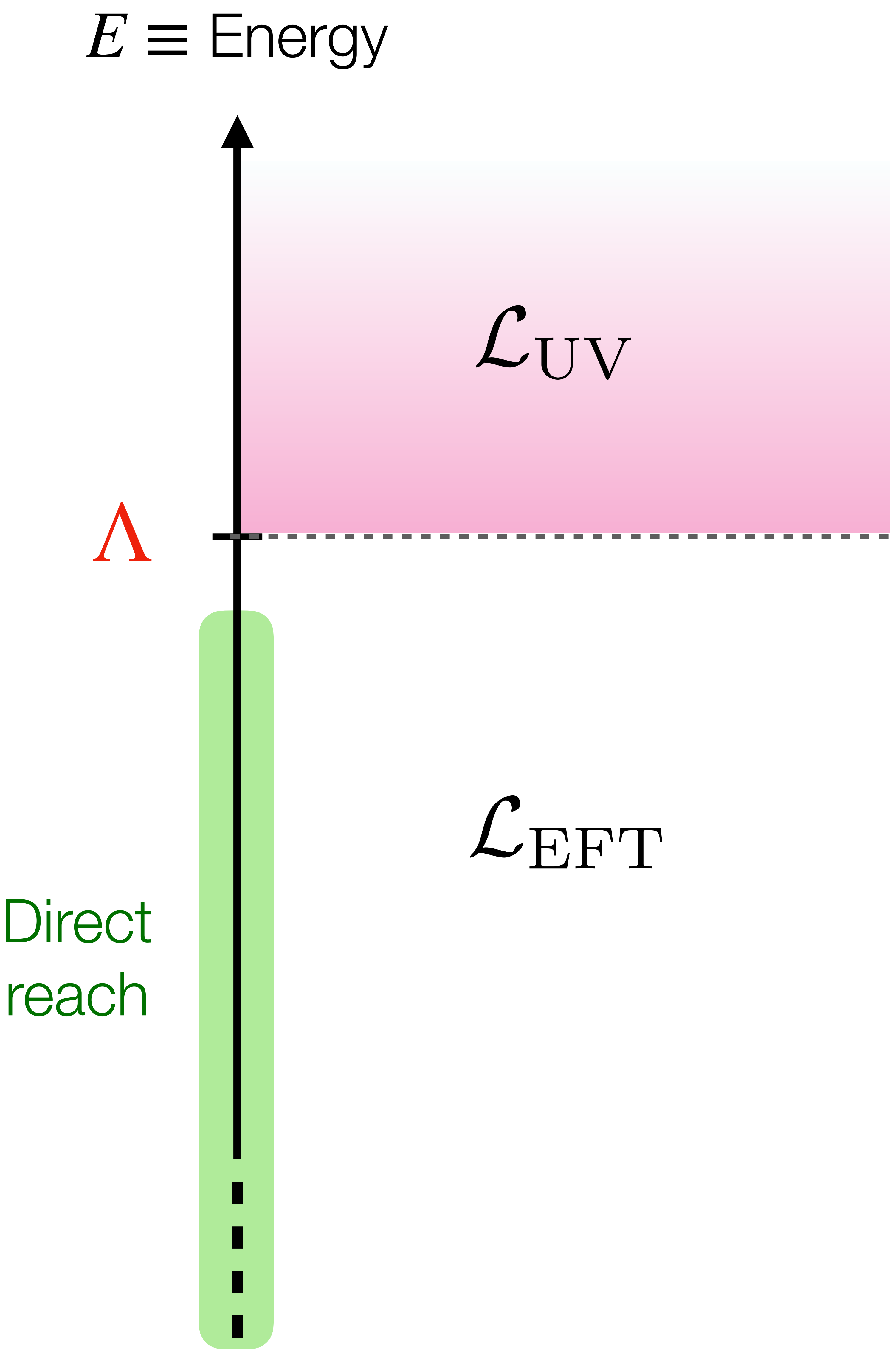

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Which is the correct EFT?

Degrees of freedom: $\phi\,,\,\psi$

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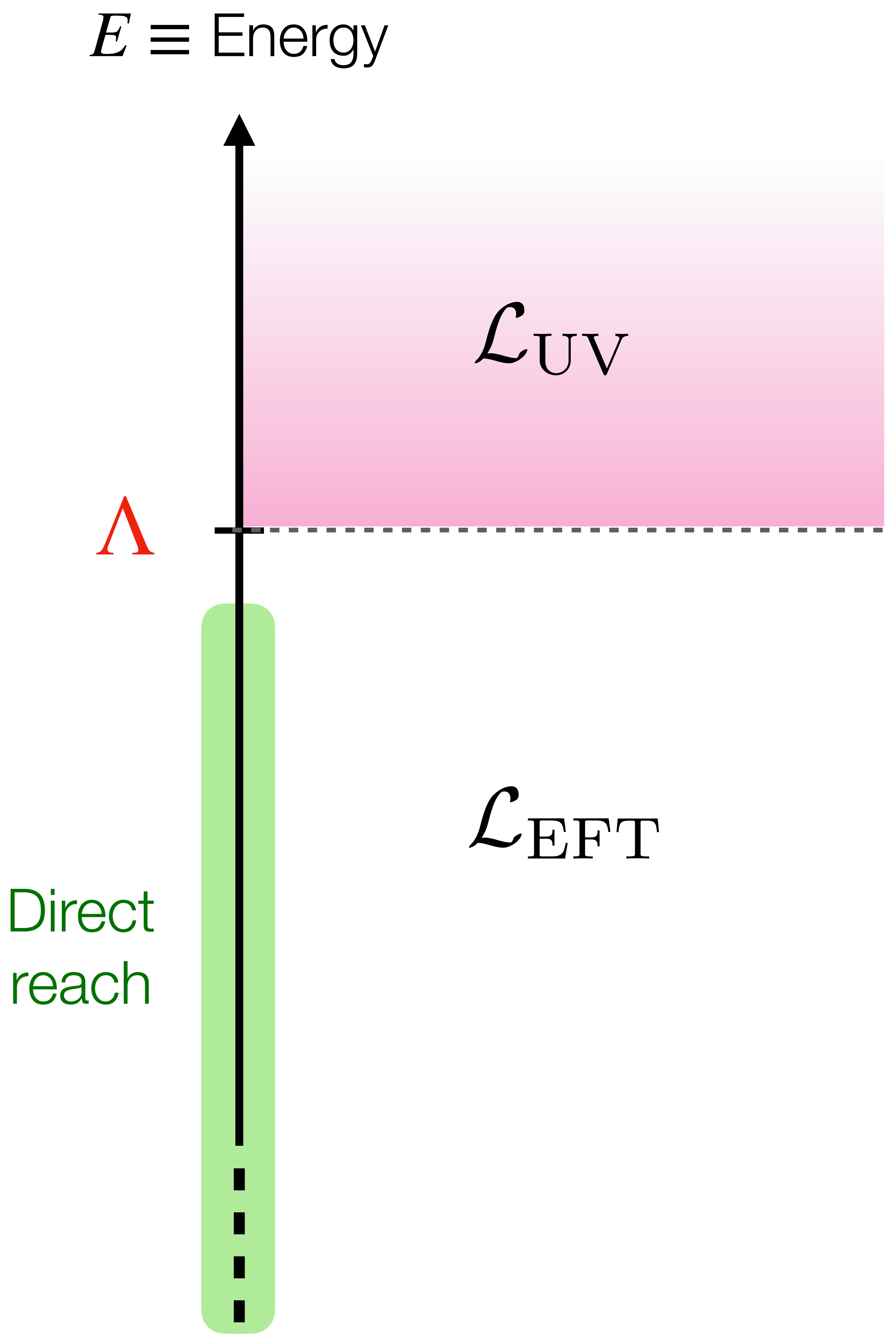
Symmetries:

A toy model example



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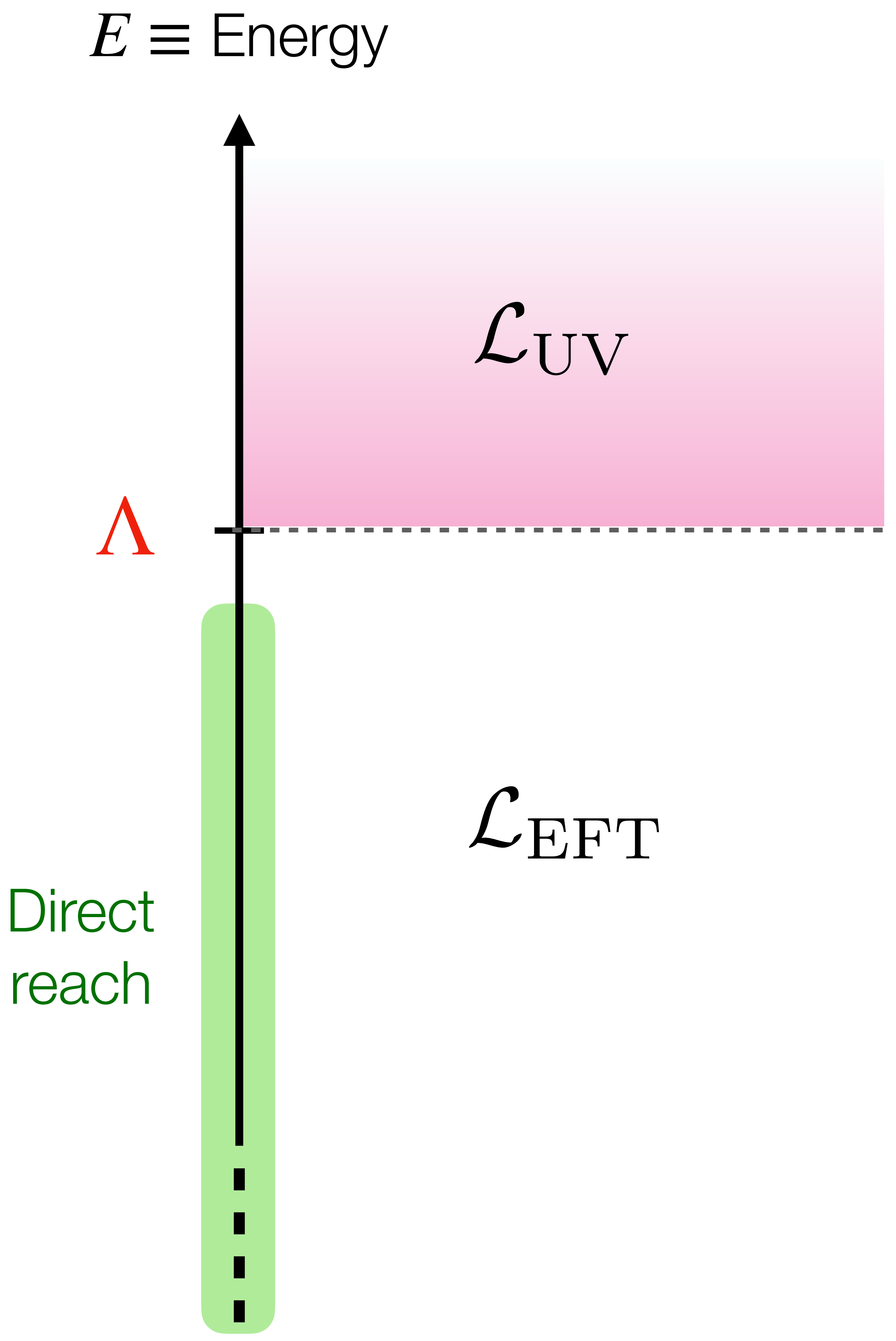
Symmetries: $\phi \to -\phi \qquad \psi \to -\psi$

A toy model example



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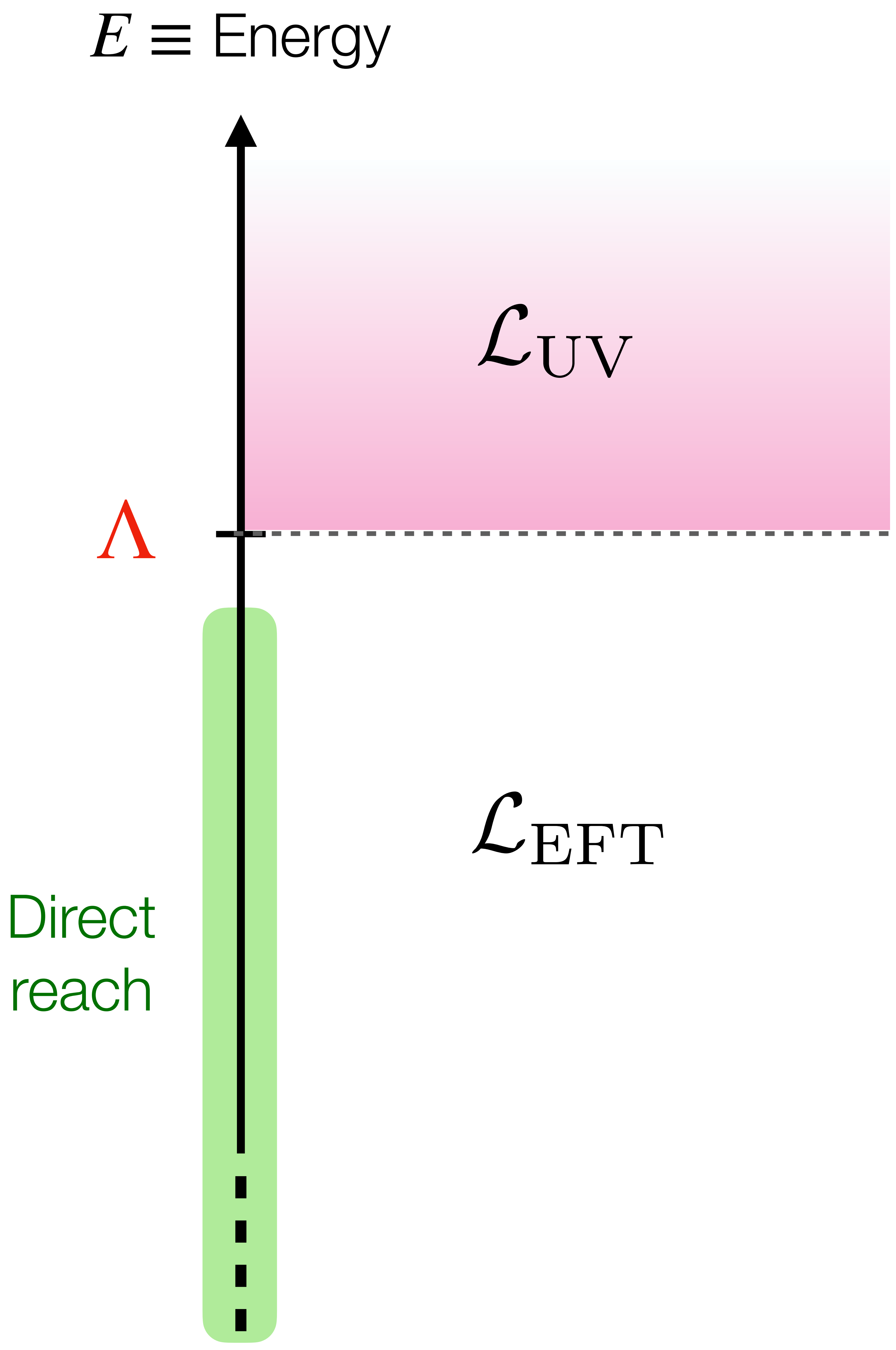

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$$+ \mathscr{O}(\Lambda^{-1})$$

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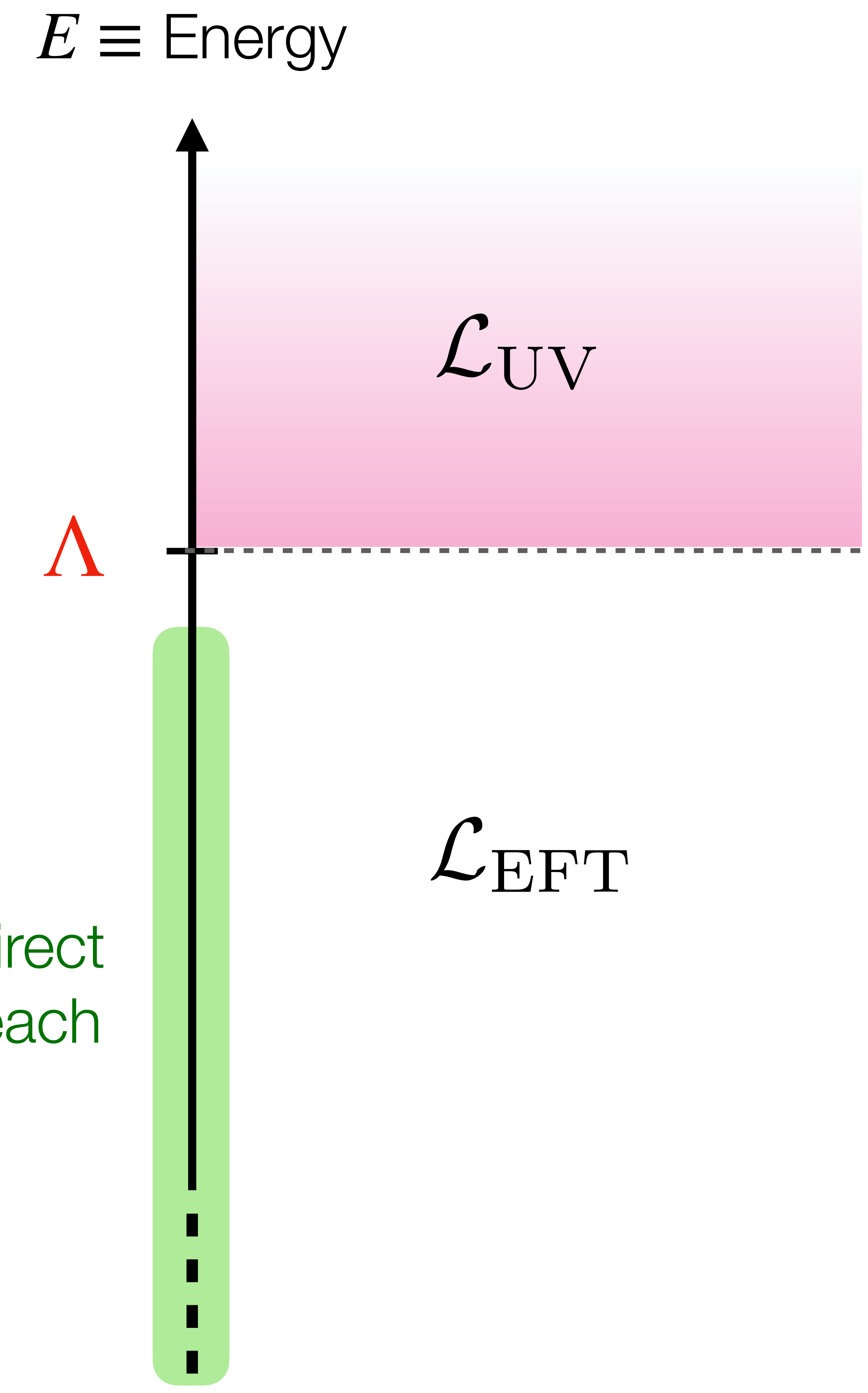

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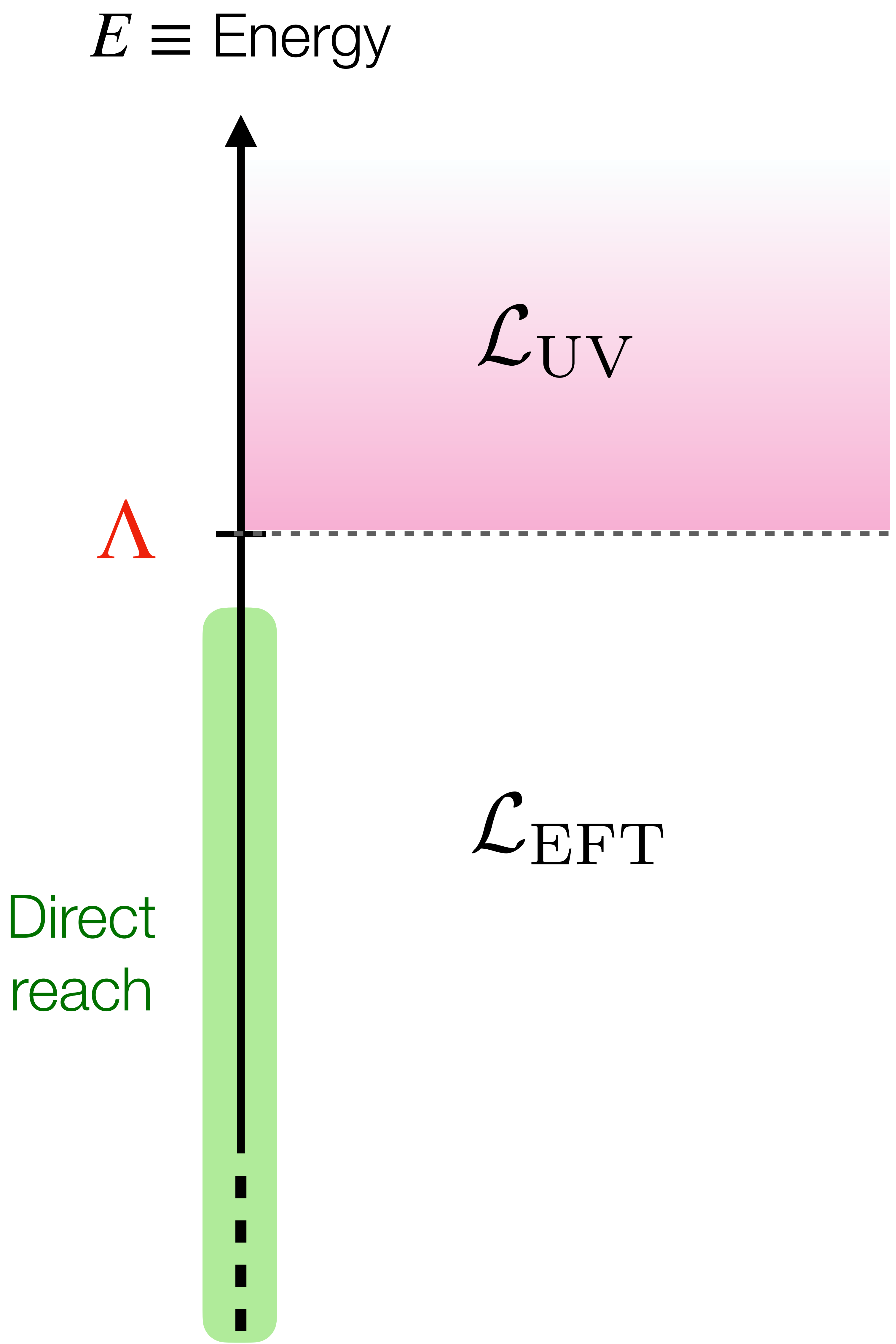
$$+ \mathscr{O}(\Lambda^{-1})$$

May not be the same as in $\mathscr{L}_{\text{UV}}$!

No symmetry forbids this!

A toy model example



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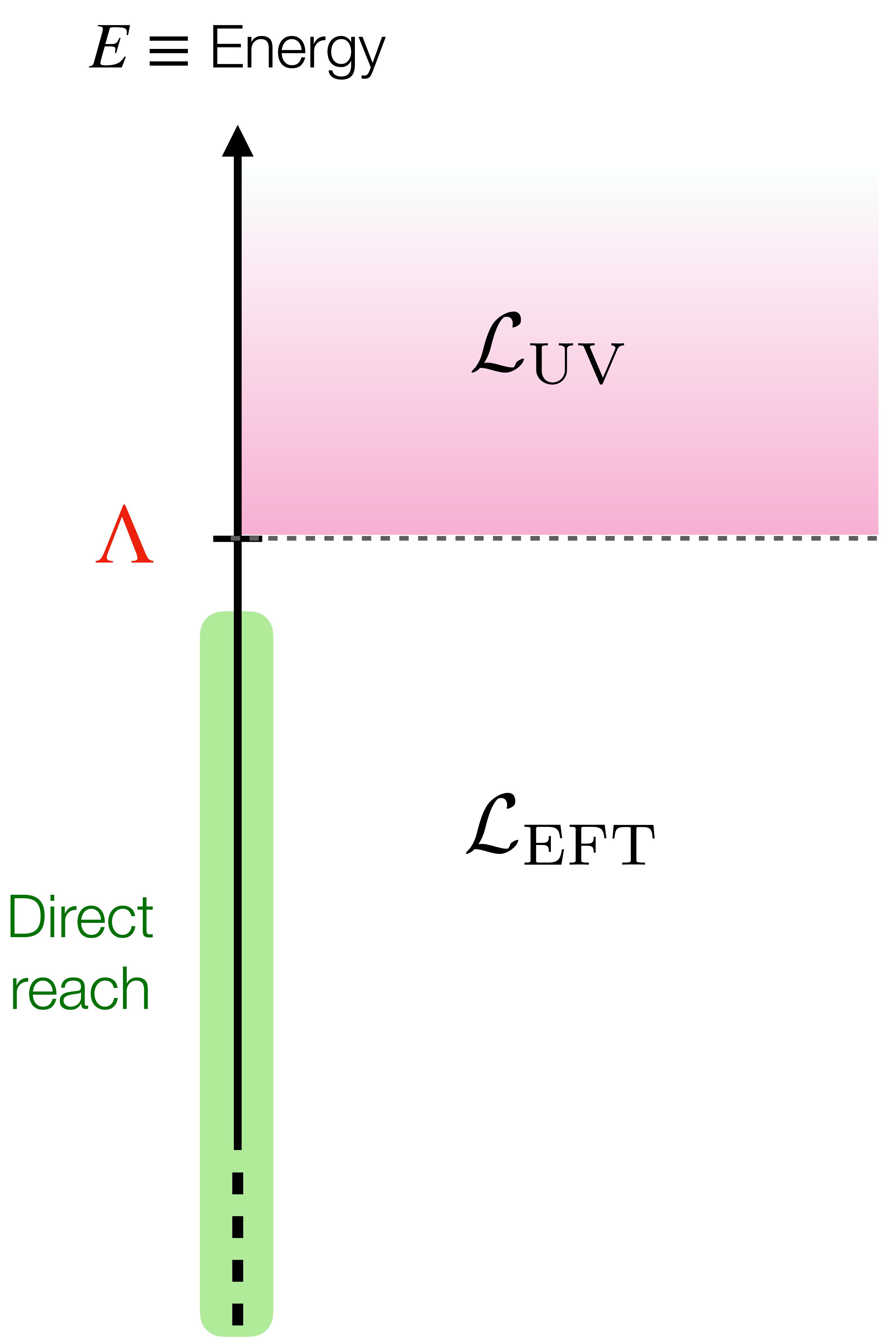

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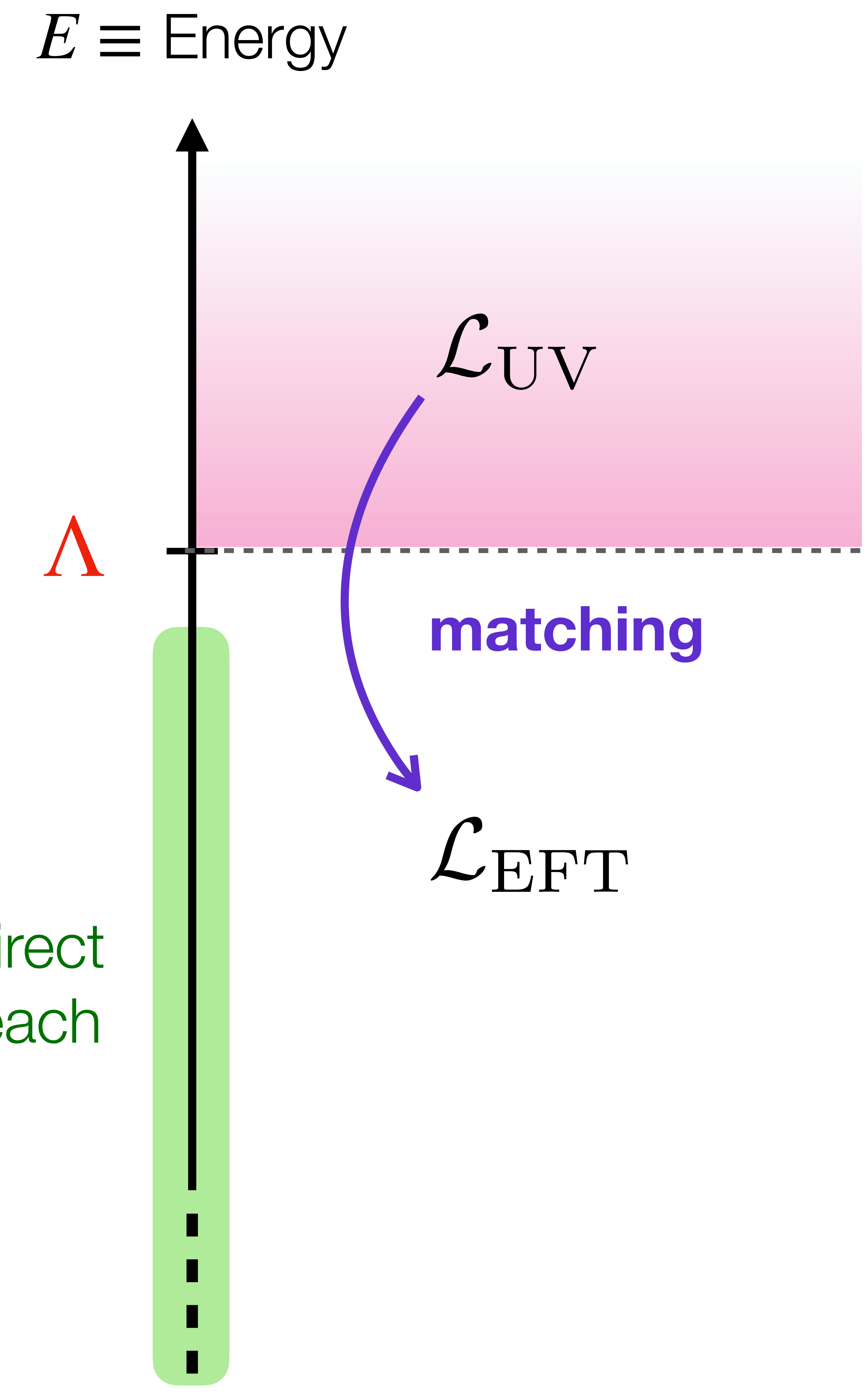
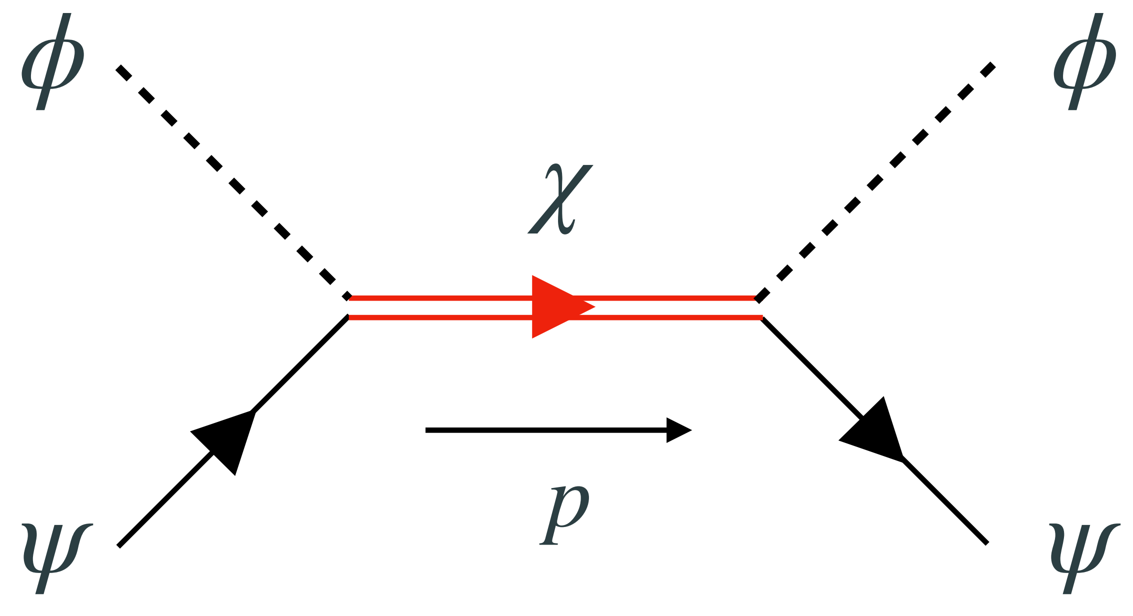
$$- \frac{C_1}{2M}\phi^2 \bar{\psi}\psi \;\; + \mathcal{O}(\Lambda^{-2})$$

A toy model example: amplitude matching

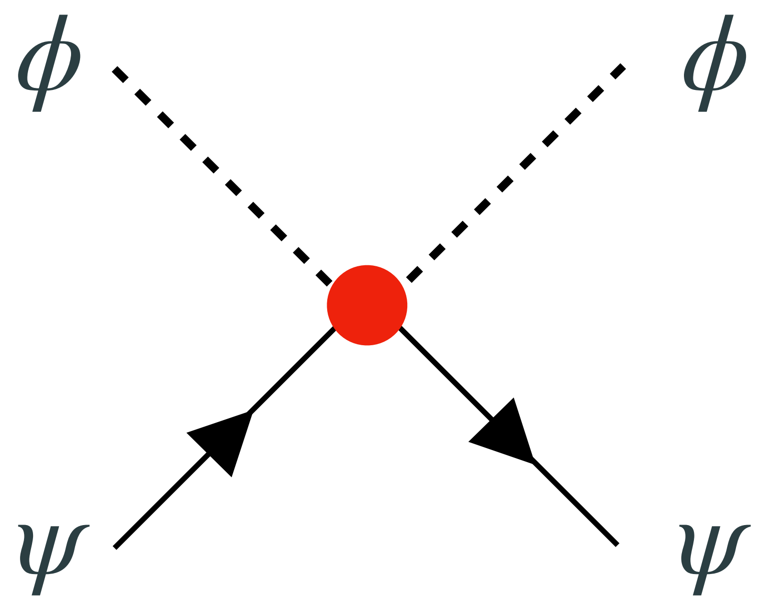




$$= yy^\dagger \cdot \frac{-i}{\cancel{p} - M} = i\,\frac{yy^\dagger}{M} \cdot \left(1 + \frac{\cancel{p}}{M} + \dots\right)$$

$+\,$crossed ϕ

matching



$$= -\,i\,\frac{C_1^{(0)}}{M}$$

$$C_1^{(0)} = -\,2\,yy^\dagger$$

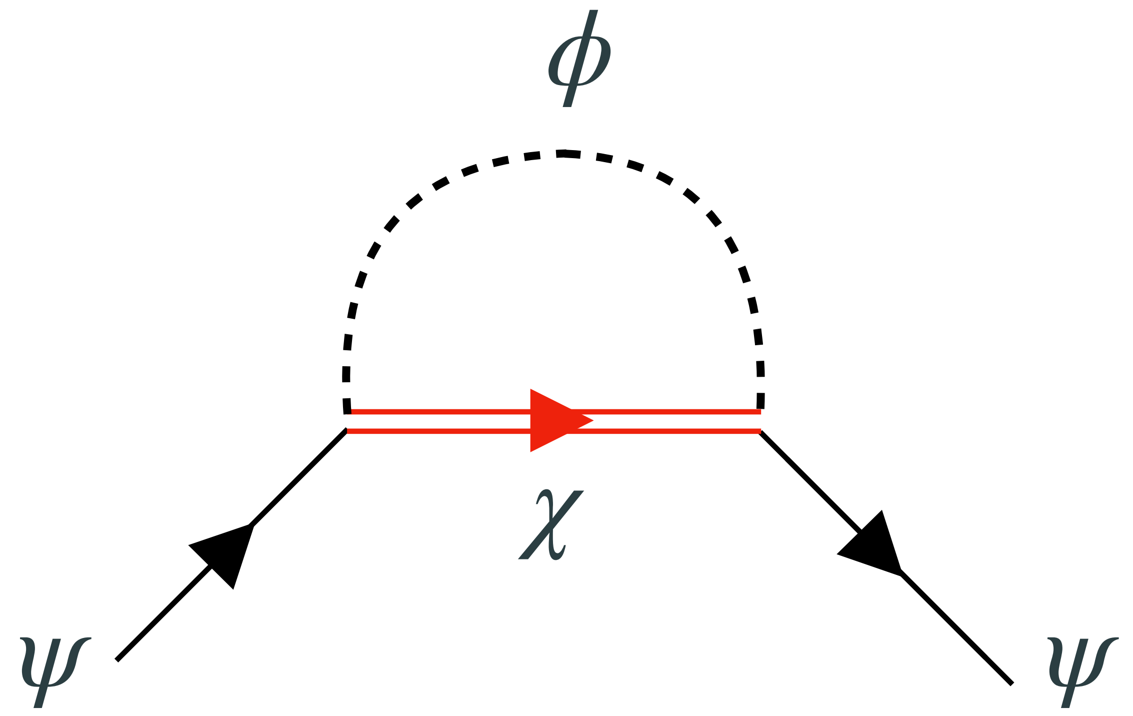
$$m = m_\phi \quad \lambda = \lambda_\phi$$

$$m_\psi = 0$$

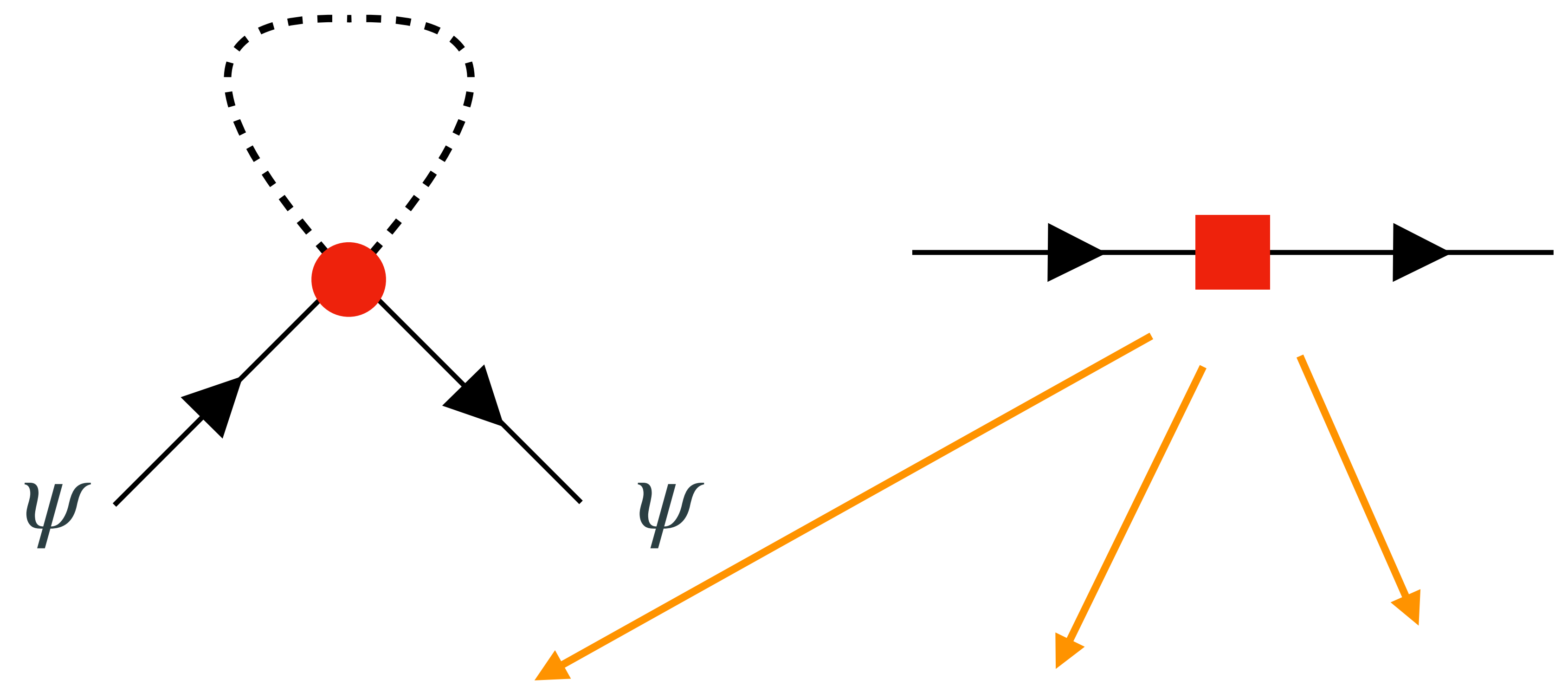
A toy model example: one-loop matching (off-shell)

We are trying to reproduce *all low-energy effects* of the original **Q**FT up to $\mathcal{O}(\Lambda^{-2})$

UV theory



EFT



$$\mathscr{L}_{\mathrm{EFT}} \supset (1+\delta Z_\psi)\,\bar{\psi}\, i\partial\!\!\!/\,\psi - m_\psi\,\bar{\psi}\psi + \frac{C_2}{M}\,\bar{\psi}\,\partial\!\!\!/\,\partial\!\!\!/\,\psi$$

The difference between UV and EFT is always local!!

N.B. 1: I will assume small couplings (perturbativity)

N.B. 2: Similarly with only ϕ in external legs (more in tutorial)

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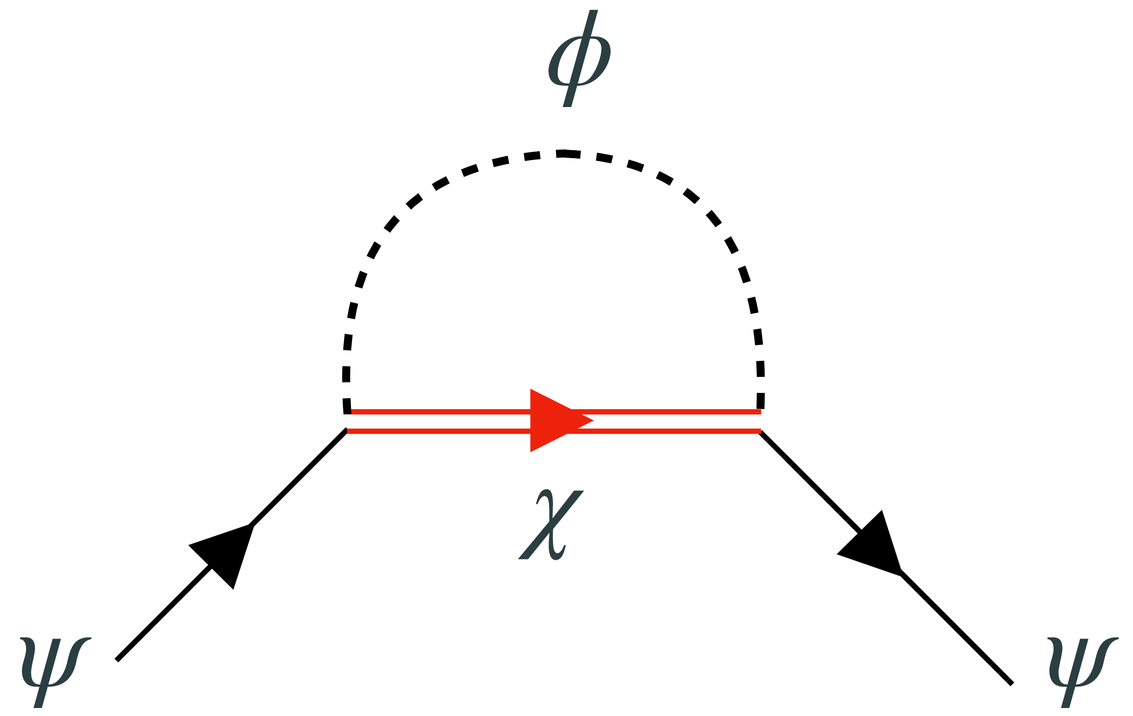
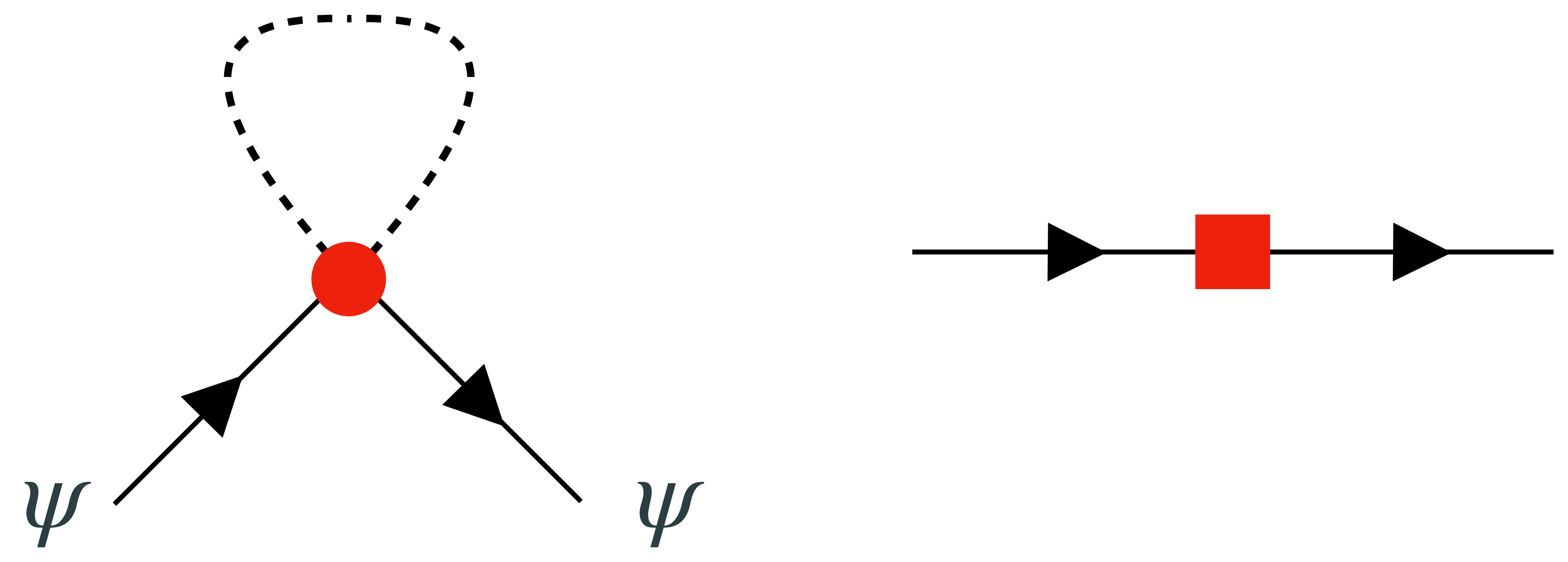


EFT



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Can be shifted to other interactions by field redefinitions:

e.g. $\psi \to \left(1 - \frac{1}{2}\,\delta Z_\psi\right)\psi$

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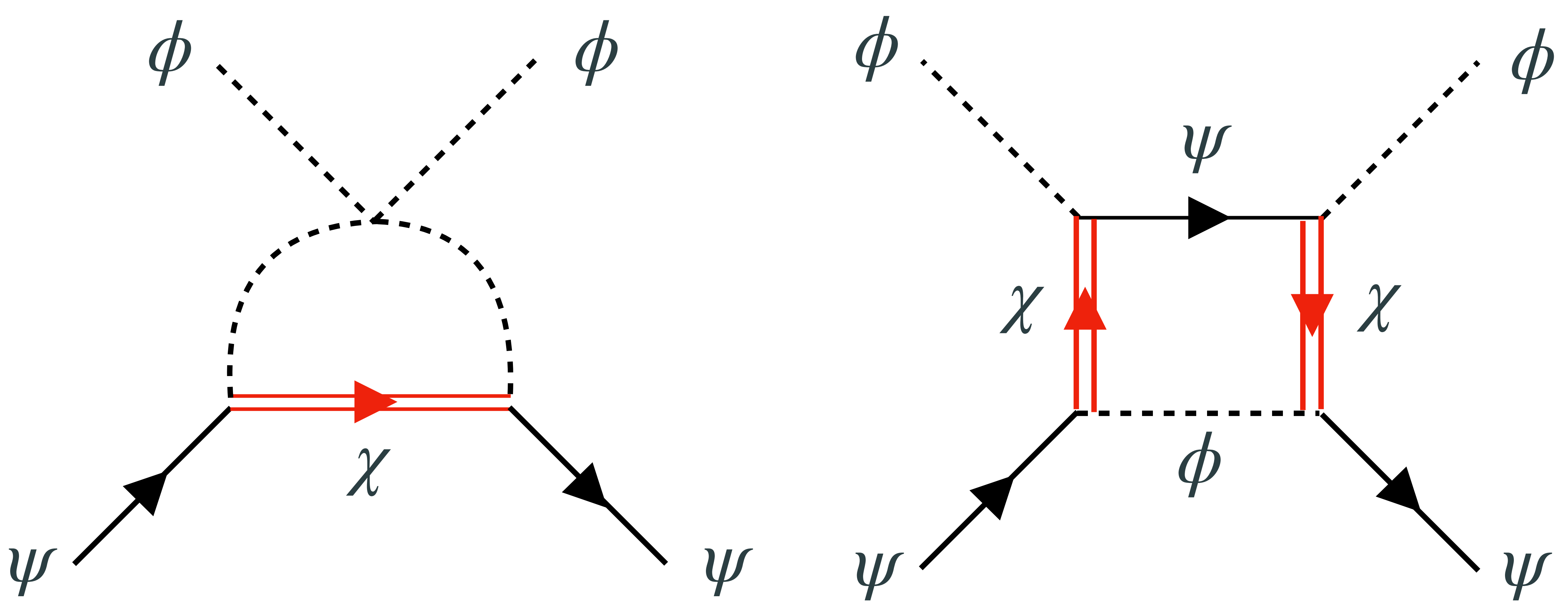
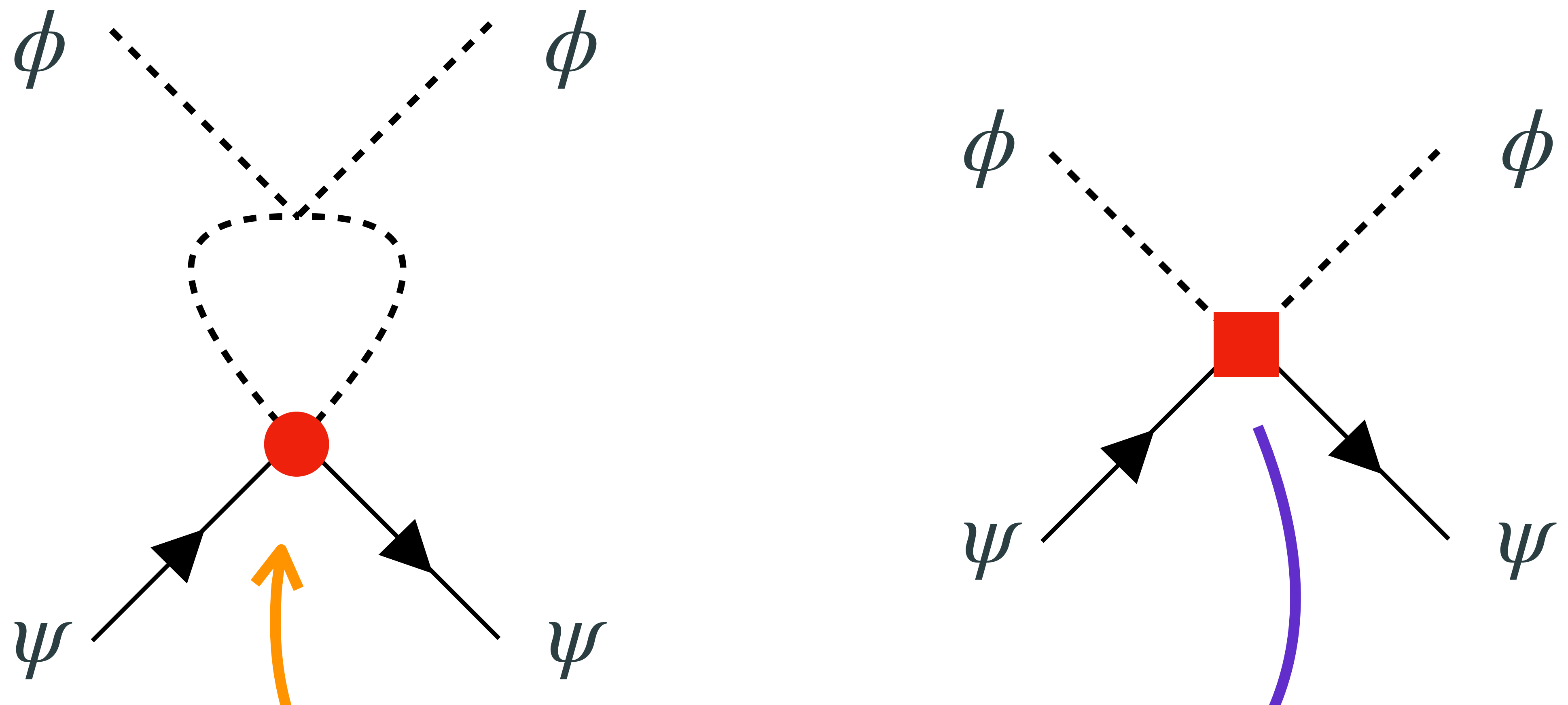
UV theory



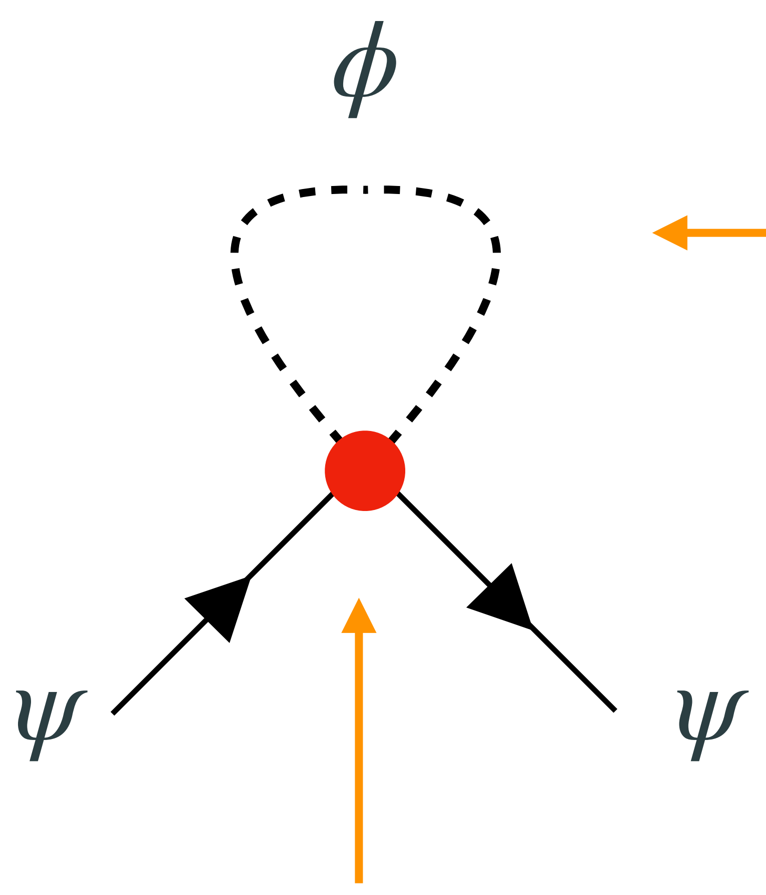
EFT



$$\mathscr{L}_{\mathrm{EFT}} \supset -\frac{C_1^{(0)} + \hbar\, C_1^{(1)}}{2M}\phi^2\bar{\psi}\psi$$

Loop effects: regularization and power counting

Including quantum corrections (loop graphs) in a way that is consistent with the power counting is non-trivial!



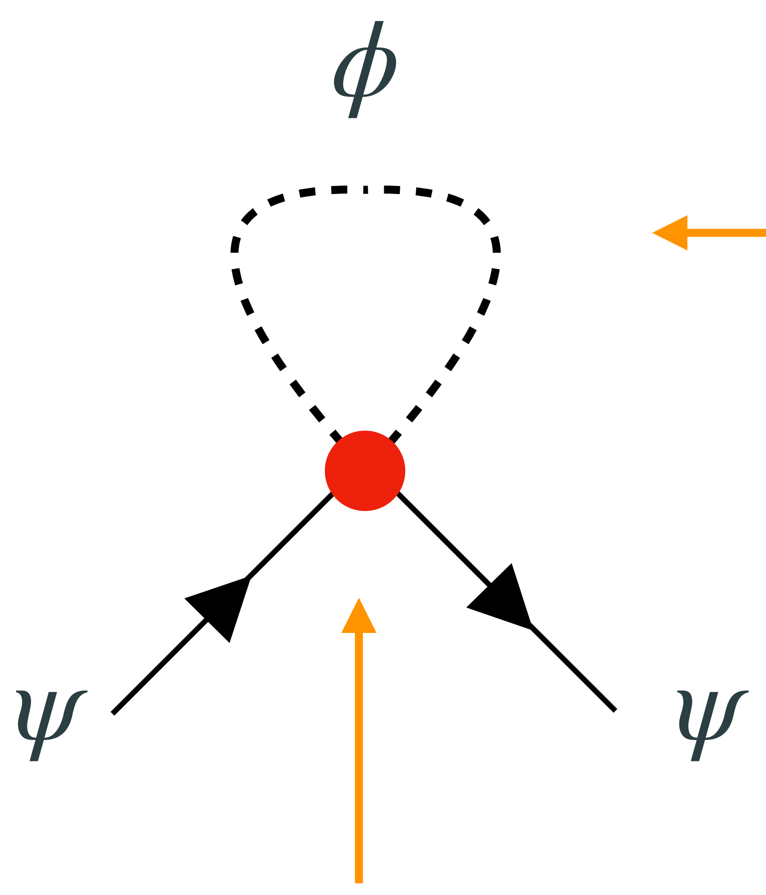
$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m^2}$$

… but we have to integrate over *all loop momenta,* including regions where l/Λ is not small

Valid for $E \ll \Lambda \sim M$

Loop effects: regularization and power counting

Including quantum corrections (loop graphs) in a way that is consistent with the power counting is non-trivial!



$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m^2}$$

… but we have to integrate over *all loop momenta,* including regions where l/Λ is not small

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Furthermore, loop integrals are divergent. They need to be regularized:

<u>Cutoff regularization:</u>

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m^2} = \frac{-i}{4\pi^2} \lim_{\Lambda_c \to \infty} \int_0^{\Lambda_c} \frac{p^2\, d|p|}{\sqrt{p^2 + m^2}}$$

$$= \frac{-im^2}{(4\pi)^2} \left[\frac{2\Lambda_c^2}{m^2} + 1 - \ln \frac{4\Lambda_c^2}{m^2} + \mathcal{O}\left(\frac{m^2}{\Lambda_c^2} \right) \right]$$

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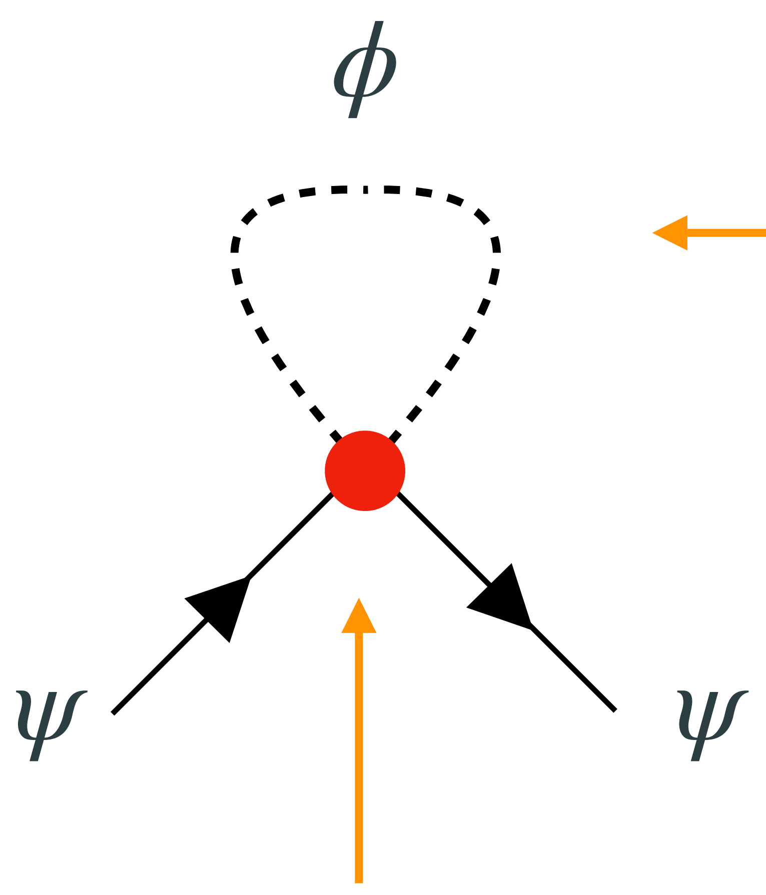

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$$i\mathcal{A} \approx \frac{C_1}{M} \frac{\Lambda_c^2}{(4\pi)^2}$$

Breaks EFT power counting!

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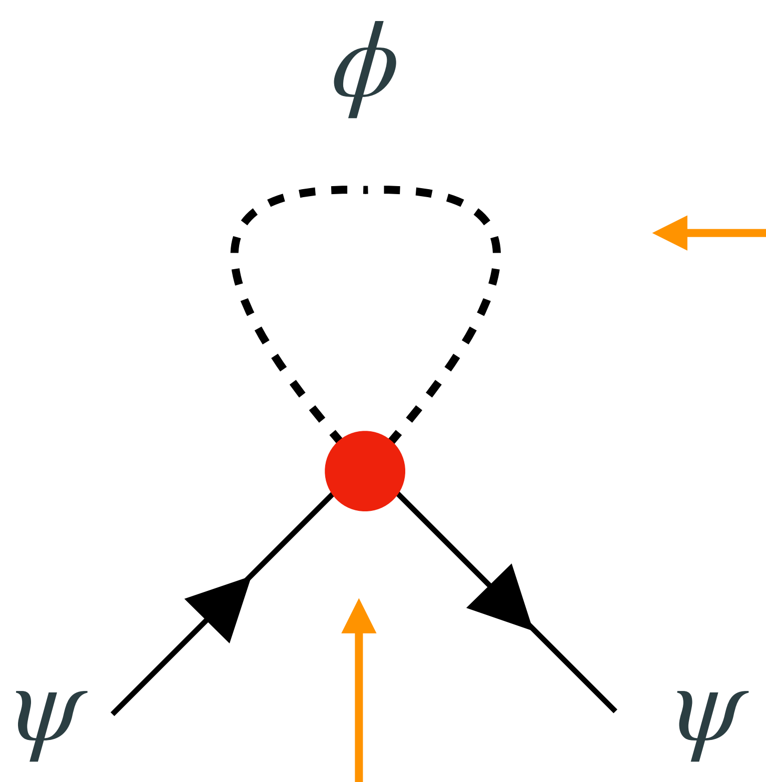

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<u>Dimensional regularization</u> (DimReg) ($d = 4 - 2\epsilon$):

To keep dimensions unchanged

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m^2} = \lim_{\epsilon \to 0} \mu^{2\epsilon} \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2 - m^2}$$

$$= \frac{im^2}{(4\pi)^2} \left[\frac{1}{\bar{\epsilon}} - \ln \frac{m^2}{\mu^2} + 1 + \mathcal{O}(\epsilon) \right]$$

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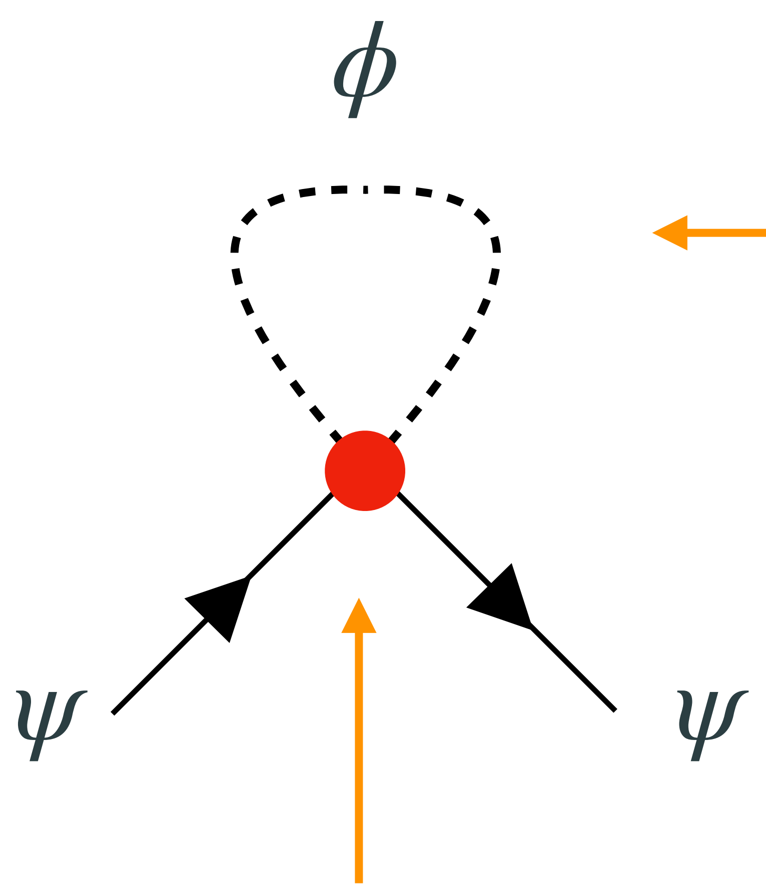

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Preserves EFT power counting!

Dimensional regularization and the method of regions

Method of regions: Loop integrals can be divided in regions by applying the following recipe

[Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589]

1. Divide the space of the loop momenta into regions and, in every region, expand the integrand in a Taylor series with respect to the parameters that are considered small there.

2. Integrate the expanded integrand over the whole integration domain of the loop momenta.

3. Set to zero any scaleless integral (i.e. no scales in propagators). ← **Natural in DimReg**

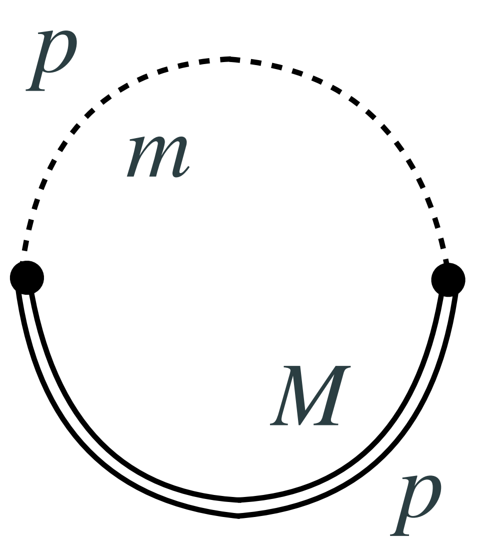
$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{l^{2n}} = 0$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{l^4} = \underset{p \to \infty}{\frac{1}{\epsilon_{UV}}} - \underset{p \to 0}{\frac{1}{\epsilon_{IR}}}$$

The sum of all regions yields the full loop integral result in an expanded form.

N.B.: This method also works for other types of integrals!!

Example of the method of regions



$$I = \int Dp \, \frac{1}{(p^2 - M^2)\,(p^2 - m^2)} \qquad Dp \equiv -\, i\,(4\pi)^2\,\mu^{2\epsilon}\,\frac{d^d p}{(2\pi)^d}$$

$E \equiv$ Energy

Say $m^2 \ll M^2$, we thus have 5 momentum regions

$R_1:\ p \gg M \gg m$

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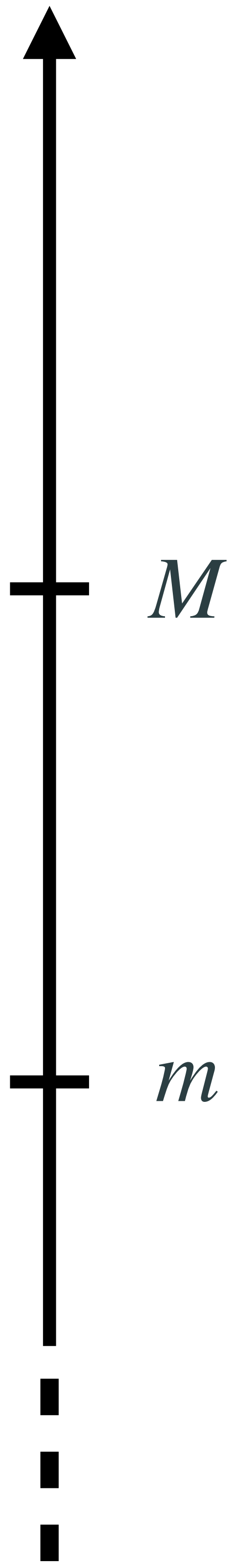

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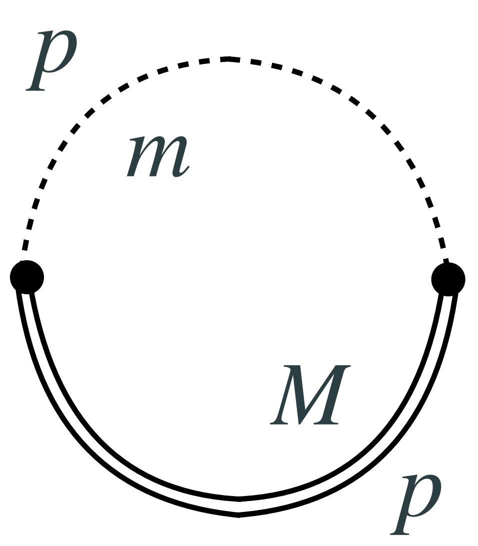

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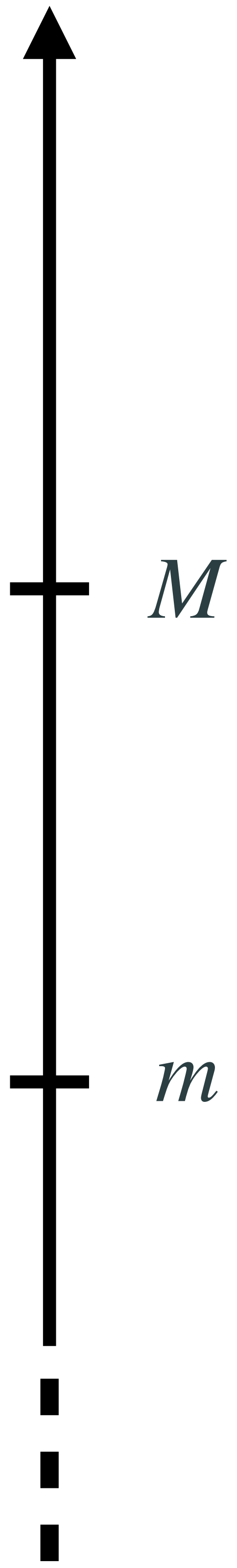

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Partial fraction decomposition

Full integral:

$$I = \frac{1}{M^2 - m^2} \int Dp \left[\frac{1}{p^2 - M^2} - \frac{1}{p^2 - m^2} \right]$$

$$= \frac{1}{M^2 - m^2} \left[M^2 \left(\frac{1}{\bar{\epsilon}} + 1 - \ln \frac{M^2}{\mu^2} \right) - m^2 \left(\frac{1}{\bar{\epsilon}} + 1 - \ln \frac{m^2}{\mu^2} \right) \right]$$

Simple example, integrals get considerably more complicated with increasing number of scales

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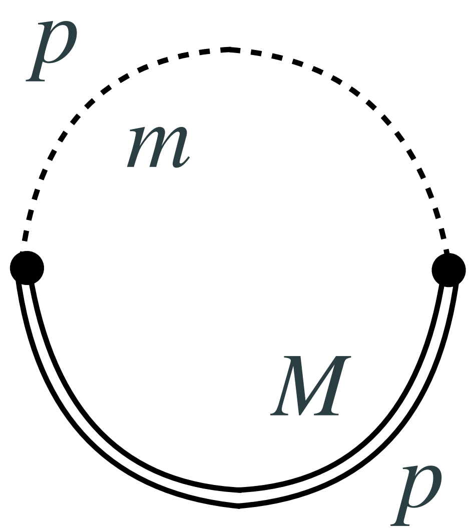

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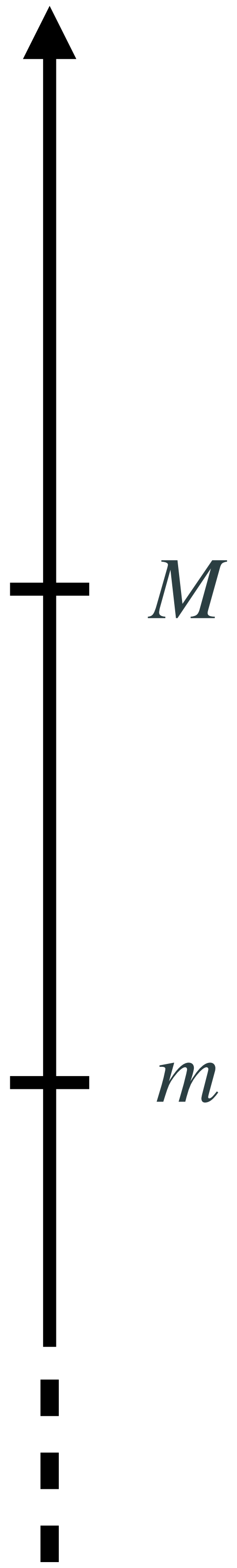


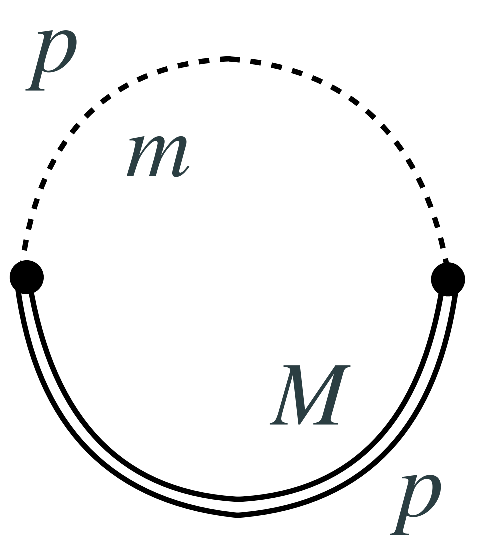
Region 1 expansion:

$$I_1 = \int Dp \, \frac{1}{p^2}\left[1 + \frac{M^2}{p^2} + \ldots\right]\frac{1}{p^2}\left[1 + \frac{m^2}{p^2} + \ldots\right] = 0$$

All integrals are scaleless!!

This solves the issue of loop integration above the domain of EFT validity

Example of the method of regions



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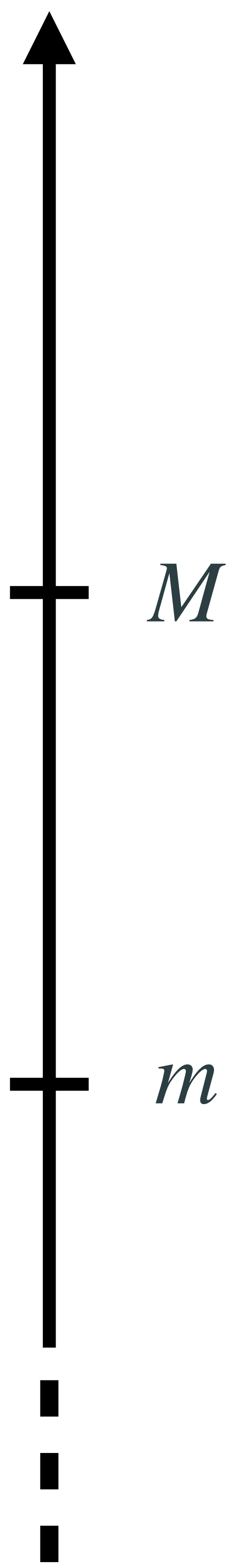

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Region 2 expansion:

$$I_2 = \int Dp \, \frac{1}{p^2 - M^2} \frac{1}{p^2} \left[1 + \frac{m^2}{p^2} + \dots \right] = \left(\frac{1}{\bar{\epsilon}} - \ln \frac{M^2}{\mu^2} + 1 \right) \left[1 + \frac{m^2}{M^2} + \dots \right]$$

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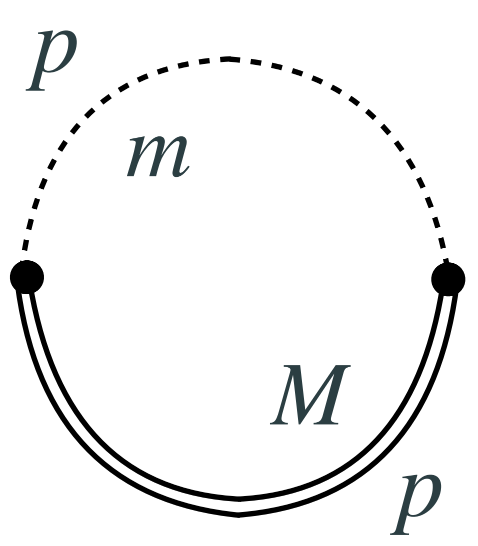

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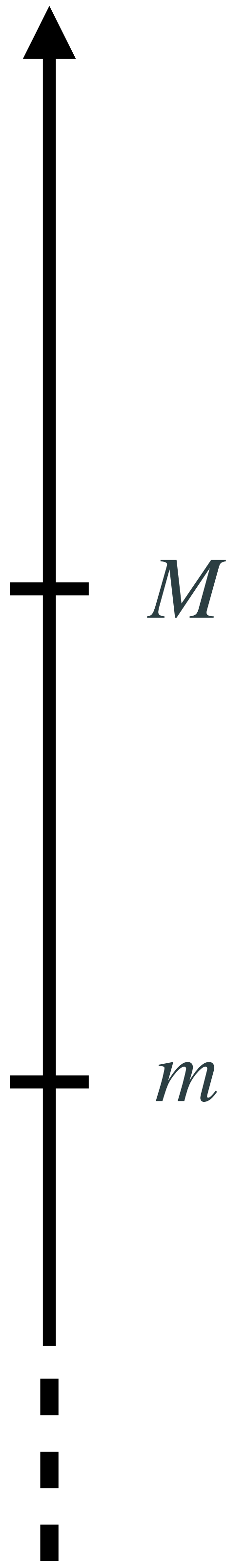

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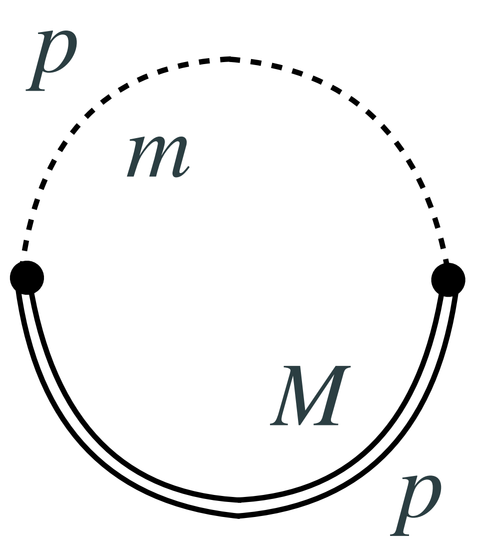

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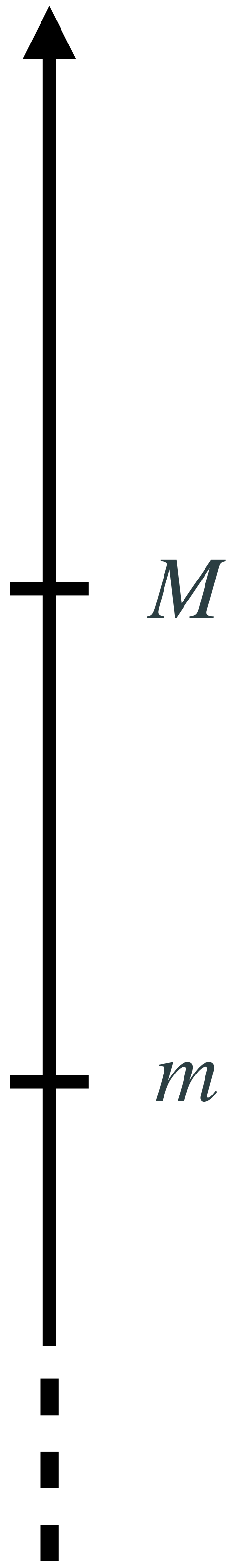

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$$I_3 = \int Dp \, \frac{-1}{M^2}\left[1 + \frac{p^2}{M^2} + \ldots\right]\frac{1}{p^2}\left[1 + \frac{m^2}{p^2} + \ldots\right] = 0$$

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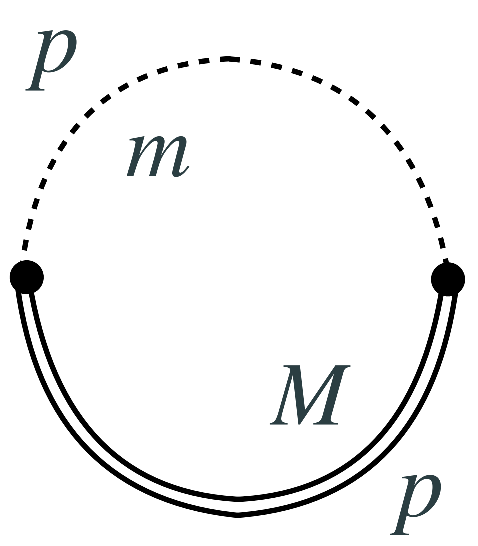

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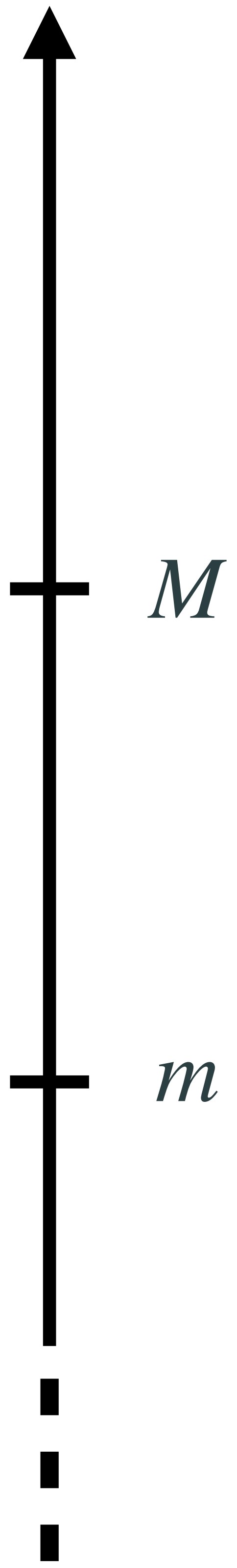

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Region 4 expansion:

$$I_4 = \int Dp \, \frac{-1}{M^2}\left[1 + \frac{p^2}{M^2} + \ldots\right]\frac{1}{p^2 - m^2} = -\left(\frac{1}{\bar{\epsilon}} - \ln\frac{m^2}{\mu^2} + 1\right)\left[1 + \frac{m^2}{M^2} + \ldots\right]$$

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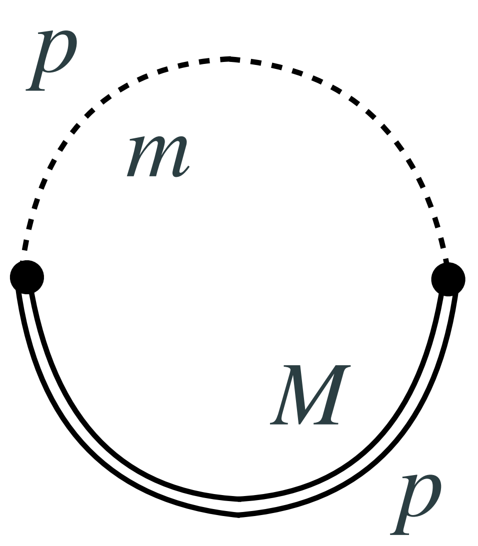

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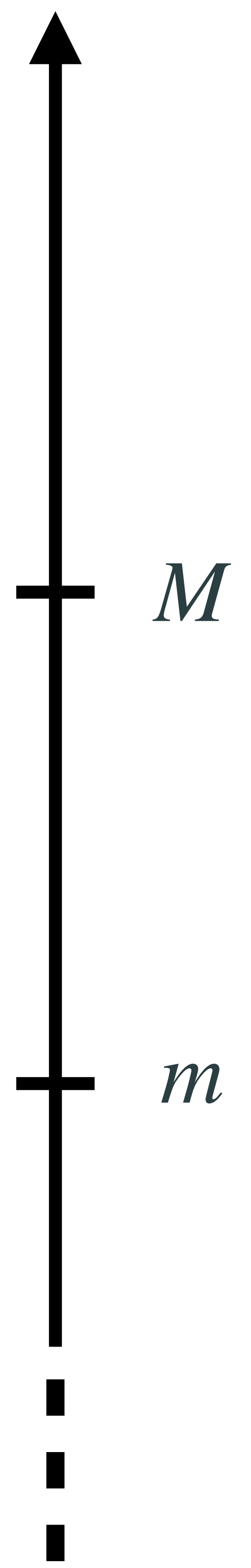

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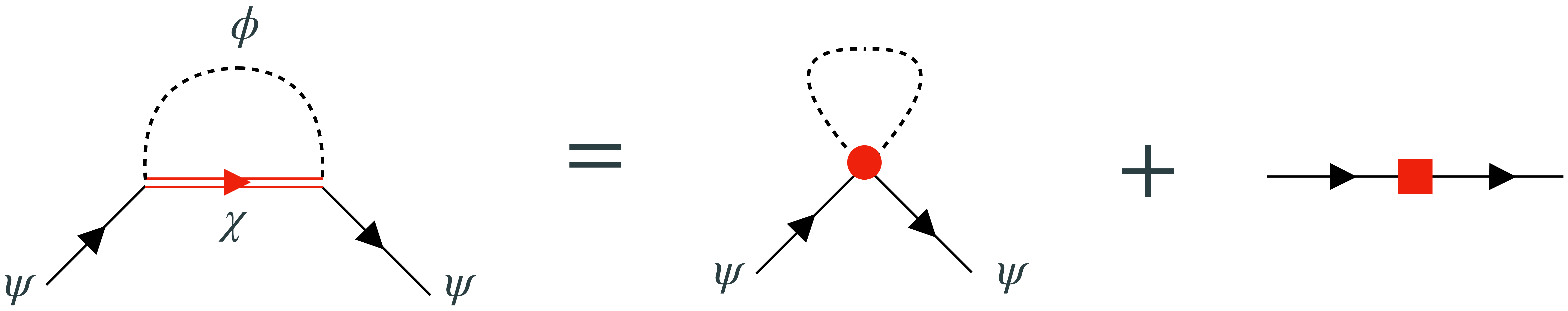

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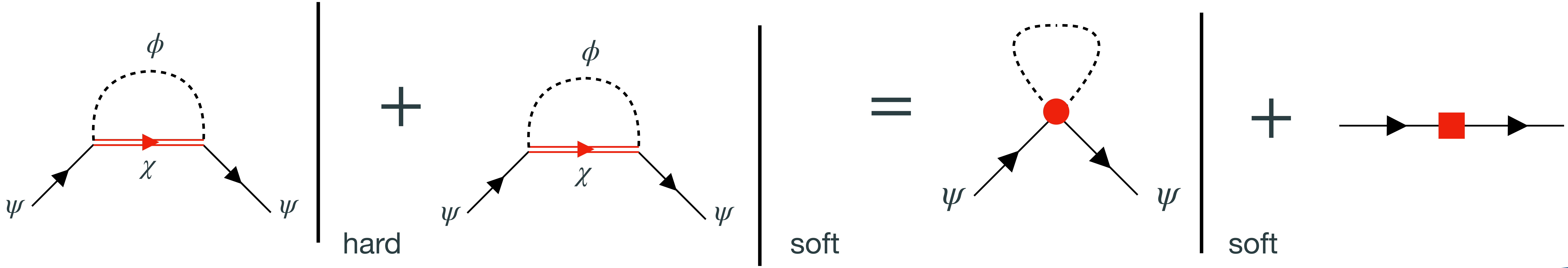

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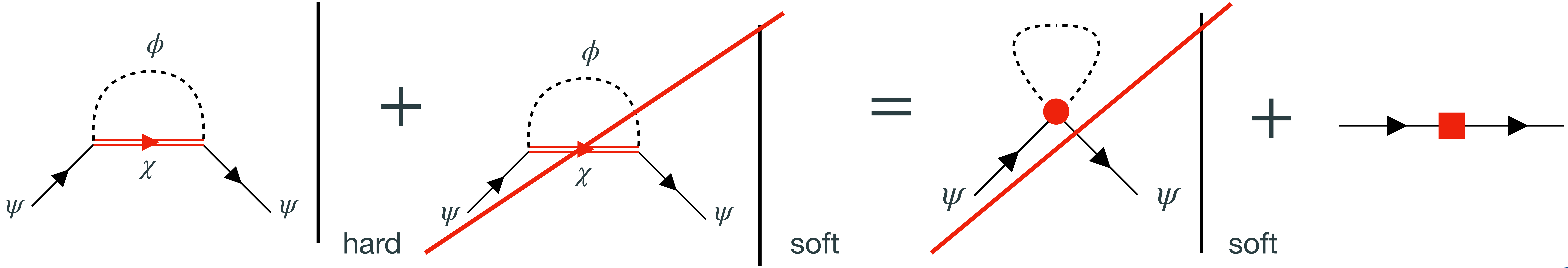

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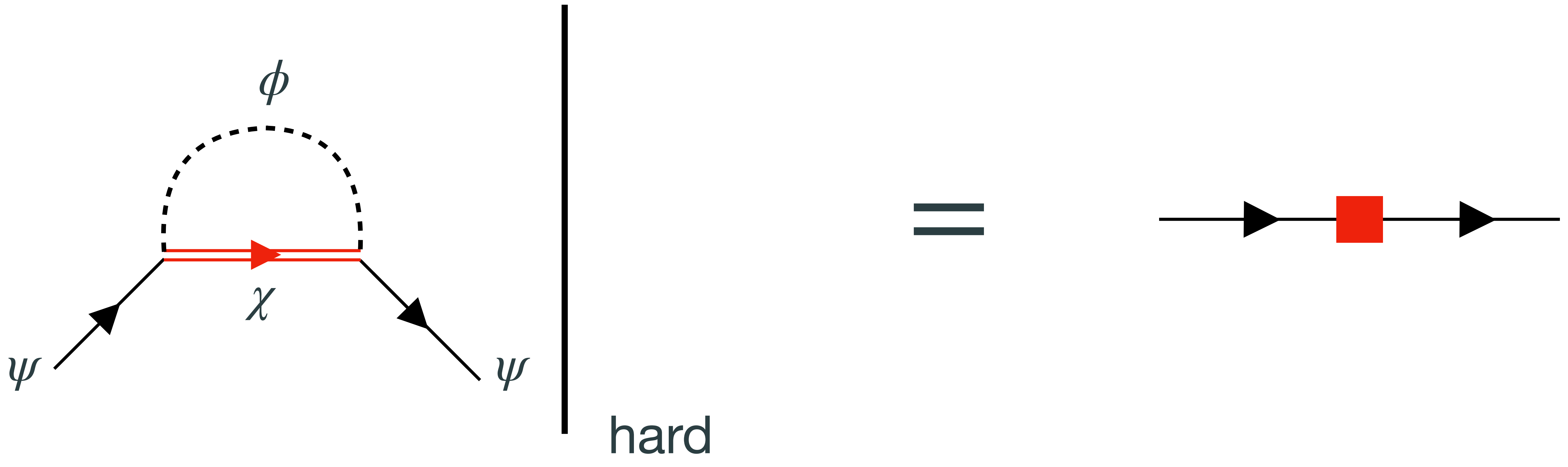

Regularization and renormalization

We have seen that loop integrals contain $1/\epsilon$ poles when regularizing in DimReg. How to make sense of these infinities?

Practical idea: Couplings in a Lagrangian are not observable, so they can take any value (even infinite!).
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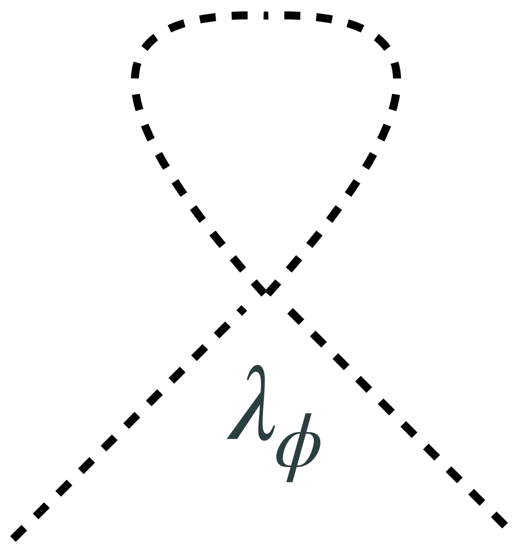
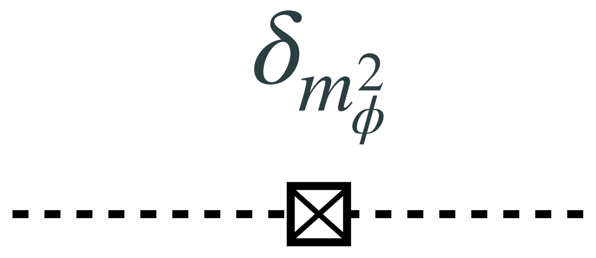
$$\mu^{-2\epsilon}\mathscr{L}_{\rm UV} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{4!}\lambda_\phi\phi^4 - \frac{1}{2}\delta_{m_\phi^2}\phi^2 - \frac{1}{4!}\delta_{\lambda_\phi}\phi^4$$

counterterms



$$= \frac{\lambda_\phi m_\phi^2}{32\pi^2}\left(\frac{1}{\bar\epsilon} - \ln\frac{m^2}{\mu^2} + 1\right) +$$



$$= -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\frac{1}{\bar\epsilon} \equiv -\frac{\delta^{(1)}_{m_\phi^2}}{\bar\epsilon}$$

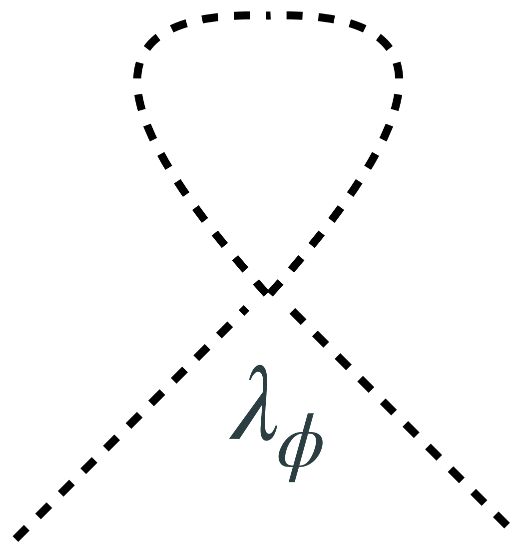
$\overline{\rm MS}$ renormalization

Regularization and renormalization

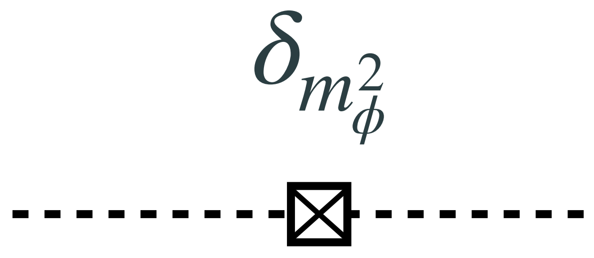
We have seen that loop integrals contain $1/\epsilon$ poles when regularizing in DimReg. How to make sense of these infinities?

Practical idea: Couplings in a Lagrangian are not observable, so they can take any value (even infinite!).
We can subtract the “infinite parts” if we do it consistently

$$\mu^{-2\epsilon}\,\mathscr{L}_{\rm UV} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\, m_\phi^2\, \phi^2 - \frac{1}{4!}\,\lambda_\phi\, \phi^4 - \frac{1}{2}\,\delta_{m_\phi^2}\, \phi^2 - \frac{1}{4!}\,\delta_{\lambda_\phi}\, \phi^4$$



$$= \frac{\lambda_\phi\, m_\phi^2}{32\pi^2}\left(\frac{1}{\bar\epsilon} - \ln\frac{m^2}{\mu^2} + 1\right) \; +$$



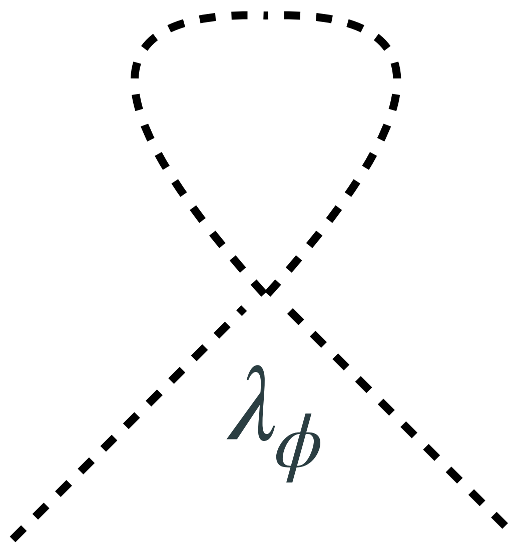
$$= -\frac{\lambda_\phi\, m_\phi^2}{32\pi^2}\,\frac{1}{\bar\epsilon} \equiv -\frac{\delta^{(1)}_{m_\phi^2}}{\bar\epsilon}$$

$\overline{\rm MS}$ renormalization

This approach can be trivially extended to our EFT Lagrangian

Renormalization and renormalization group (RG) equations

As a result of the renormalization procedure, couplings acquire a dependence on the artificial scale μ



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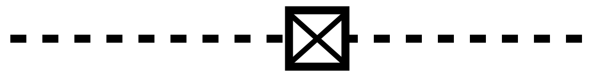


$$= -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\frac{1}{\bar{\epsilon}} \equiv -\frac{\delta^{(1)}_{m_\phi^2}}{\bar{\epsilon}}$$

Can be very large!

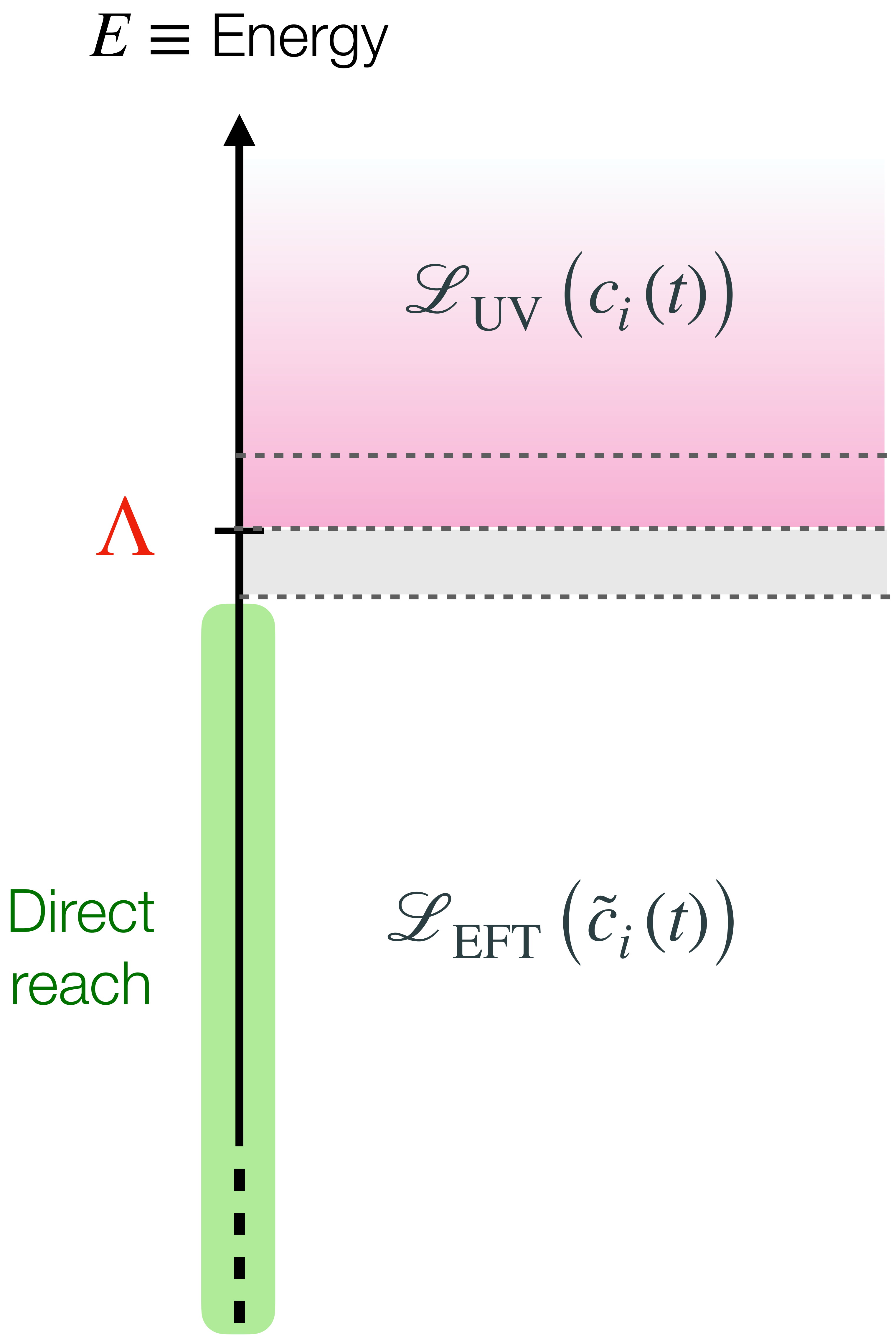
Observables quantities cannot depend on μ, so the log-dependence has to be compensated by the couplings

This μ-dependence is called (renormalization group) running and it can be determined from the counterterms

$$\frac{d}{dt}\left[\mu^{2\epsilon}(c_i + \delta c_i)\right] = 0 \qquad \Longrightarrow \qquad \frac{dc_i^{(0)}(\mu)}{dt} = 2\delta^{(1)}_{c_i} \qquad\qquad t \equiv \ln\mu$$

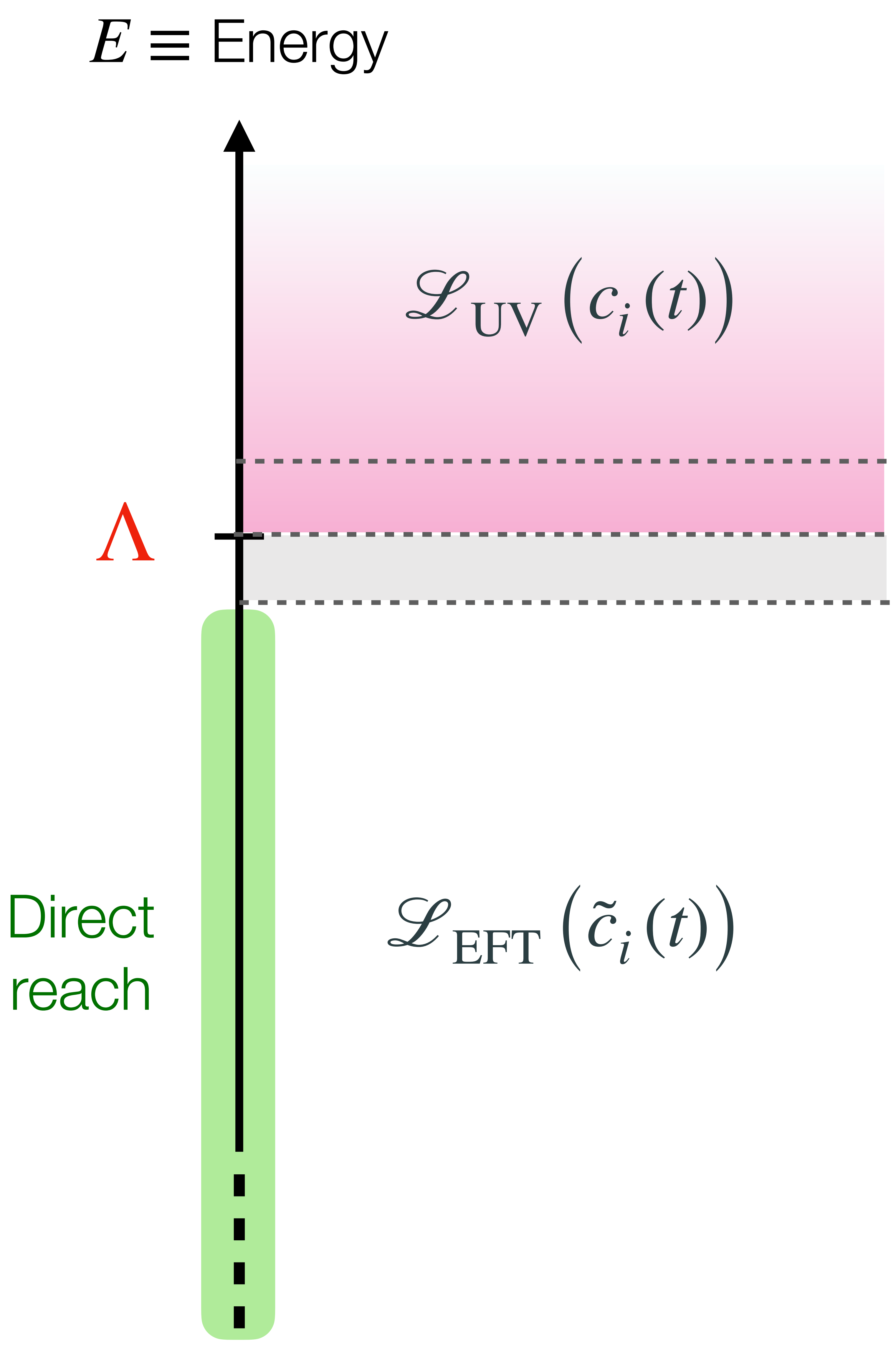
Solving these equations gives an RG-improved perturbation theory (re-summation of large logs)

Renormalization group flow and scheme independence



If couplings evolve with energy, when should we change between our UV theory and its EFT description?

Renormalization group flow and scheme independence



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➡ At the mass of the heavy particle?

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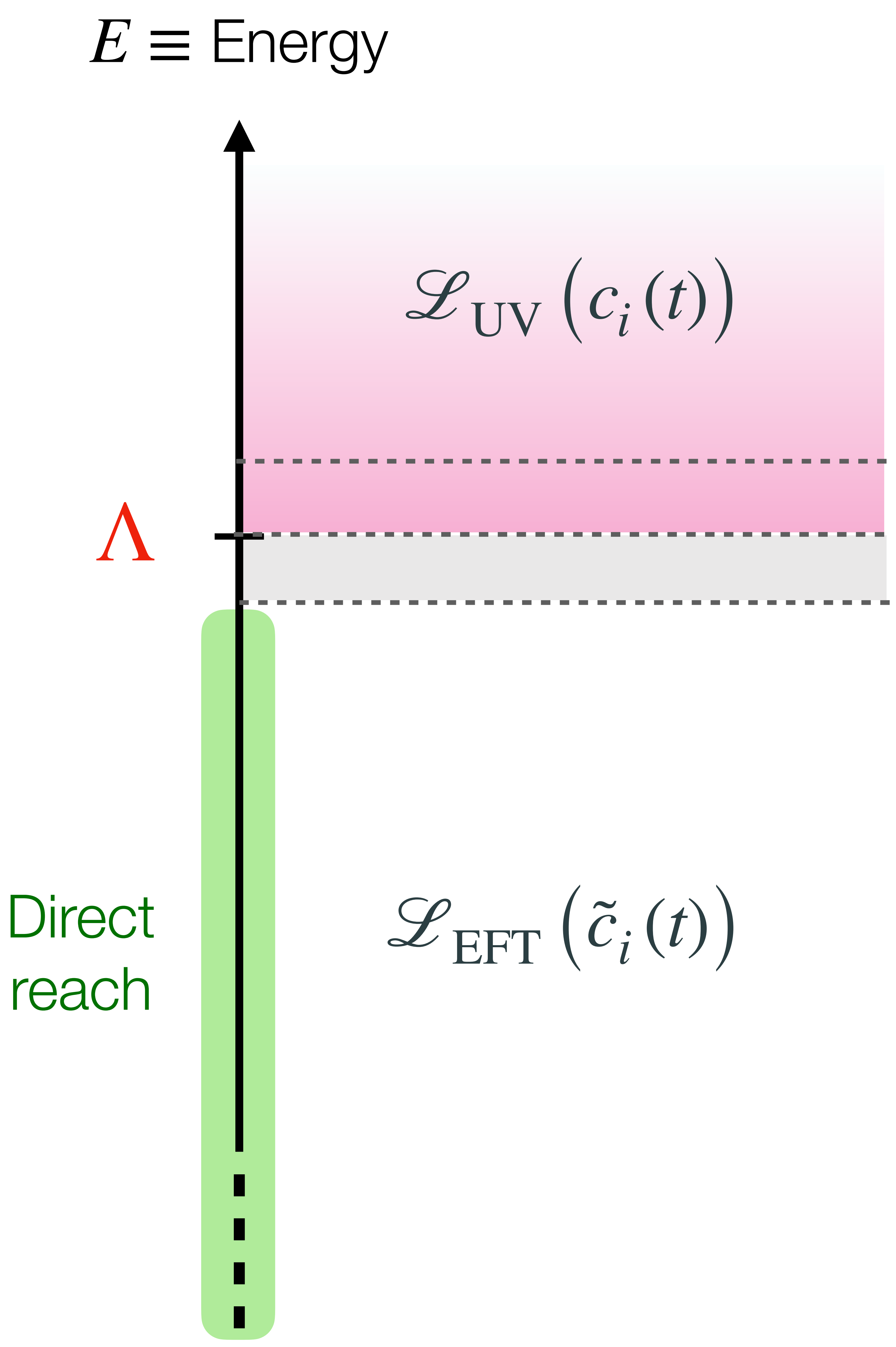

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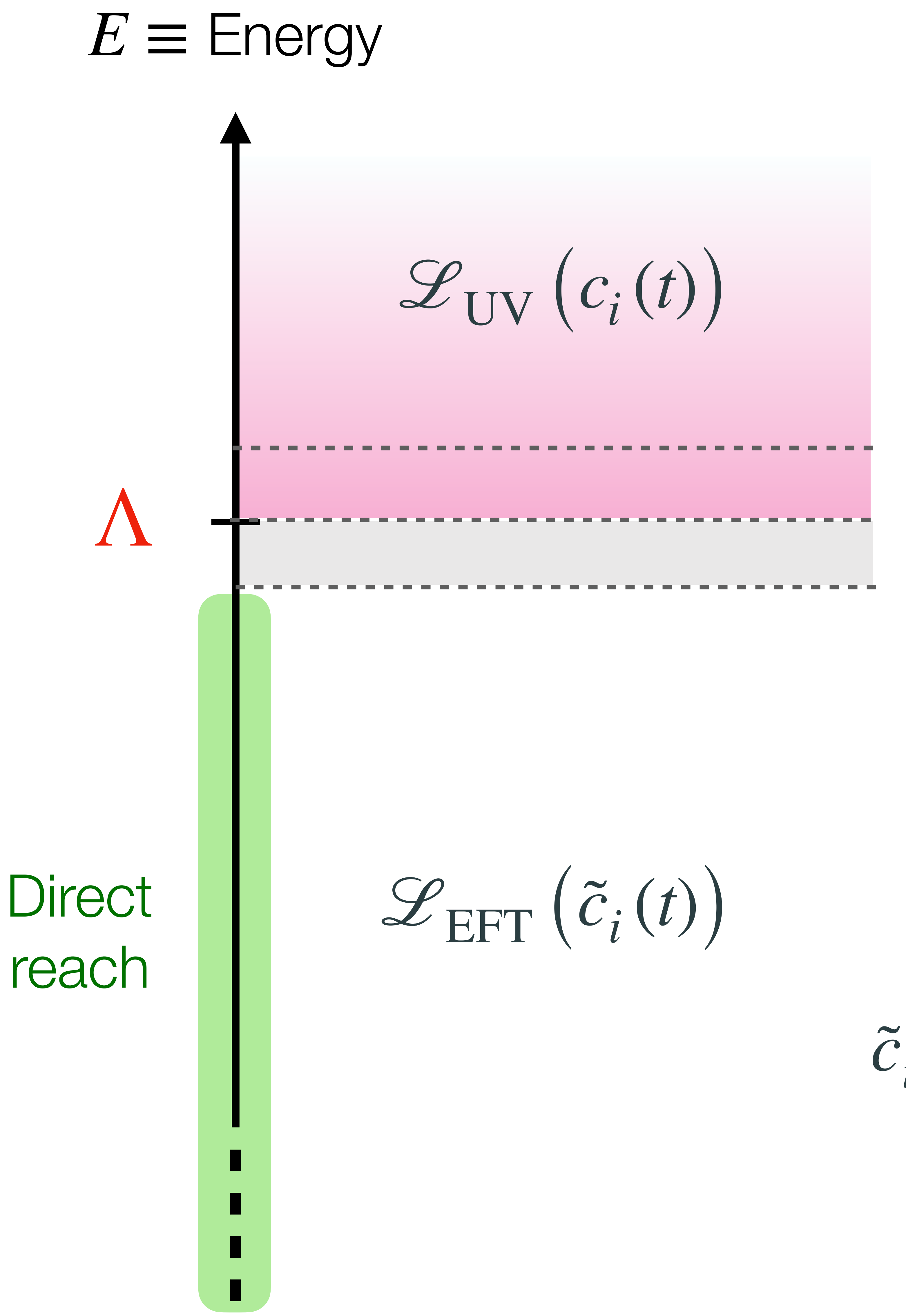

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➡ At the mass of the heavy particle?

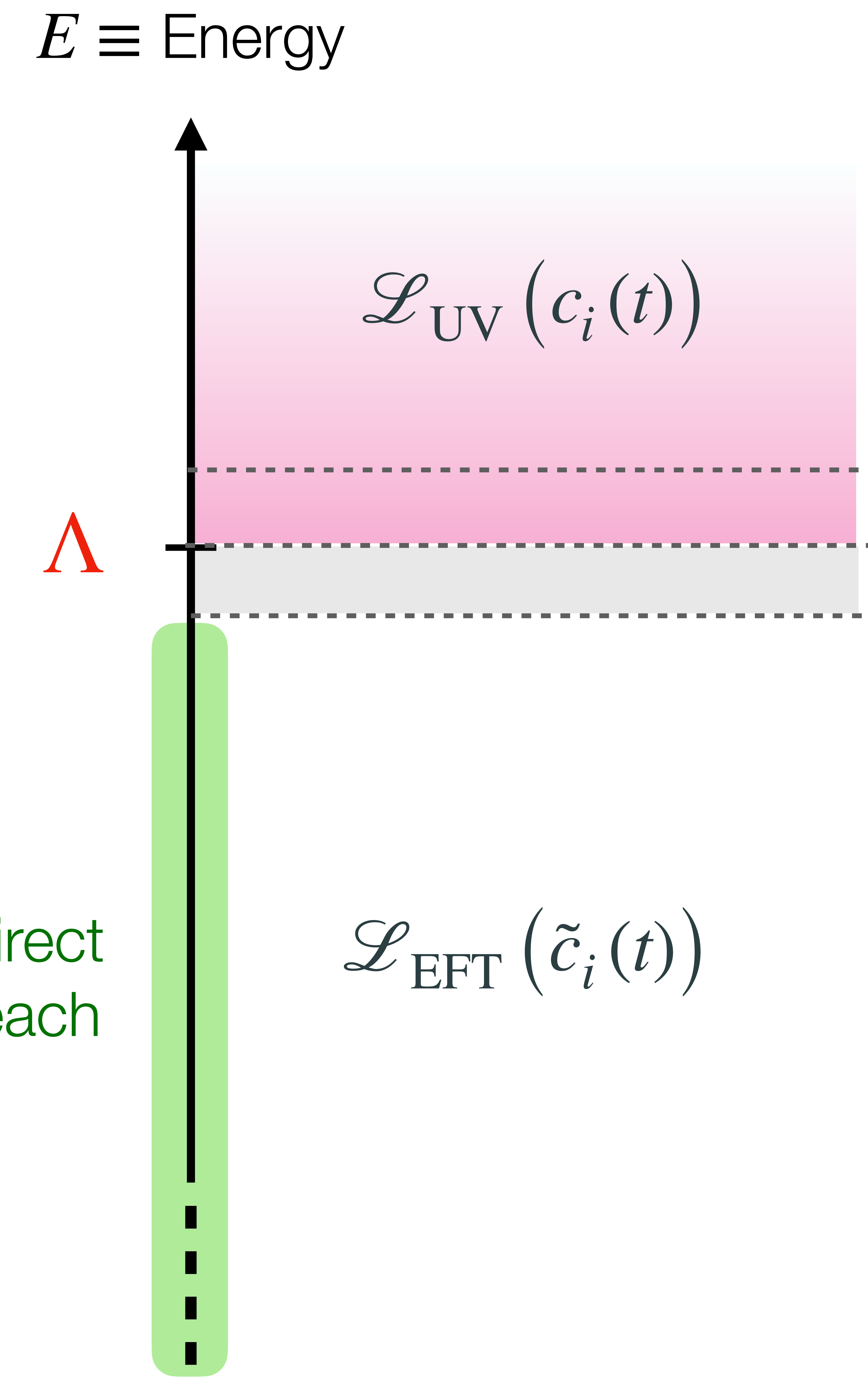
➡ But what if there are several of them?

➡ Or what if is it the situation is more complicated and the two theories have different degrees of freedom?

Actually, it does not matter if the matching is done consistently (scheme independence)

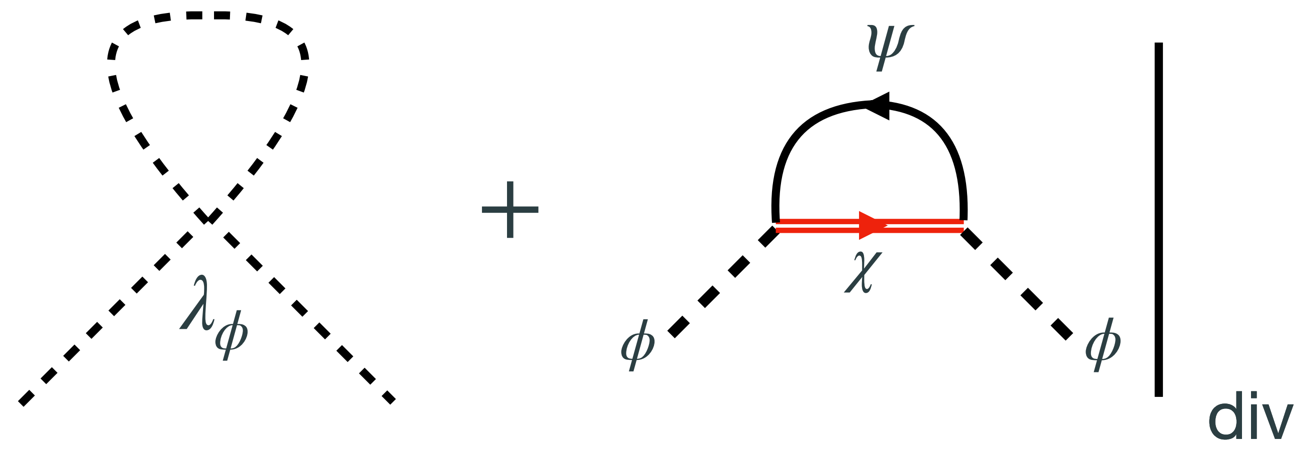
$$\tilde{c}_i = \tilde{c}_i\left(c_i(t), t\right) \ : \quad \underbrace{\frac{d\tilde{c}_i}{dt}}_{\substack{\text{(1-loop)} \\ \text{EFT running}}} = \underbrace{\frac{\partial \tilde{c}_i}{\partial t}}_{\substack{\text{(1-loop)} \\ \text{matching logs}}} + \underbrace{\frac{dc_j}{dt}}_{\substack{\text{(1-loop)} \\ \text{UV running}}} \times \underbrace{\frac{\partial \tilde{c}_i}{\partial c_j}}_{\substack{\text{(tree)} \\ \text{matching}}}$$

Renormalization group flow and scheme independence: an example



$$\mathscr{L}_{\mathrm{UV}} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{4!}\lambda_\phi\phi^4 + \bar{\chi}\, i\partial\!\!\!/\chi - M\bar{\chi}\chi + (y\,\phi\bar{\psi}\chi + \mathrm{h.c.}) + \mathscr{L}_{\mathrm{UV}}^{\mathrm{ct}}$$

$$\frac{dm_\phi^2}{dt} = \frac{\lambda_\phi}{16\pi^2}m_\phi^2 - \frac{y^\dagger y}{\pi^2}M^2 + \mathcal{O}(\hbar)$$



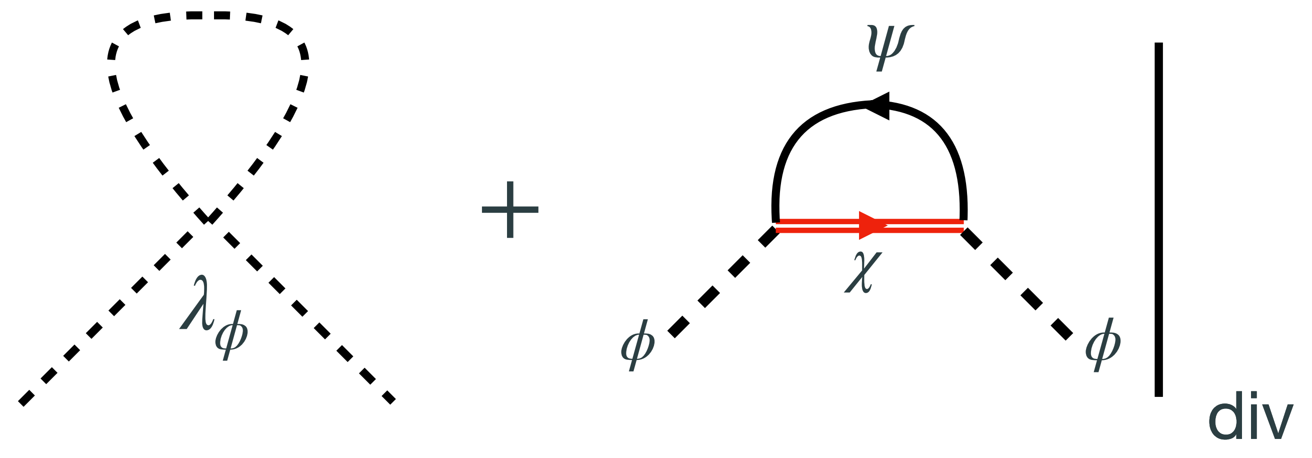
Renormalization group flow and scheme independence: an example

$E \equiv$ Energy

$$\mathscr{L}_{\rm UV} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{4!}\lambda_\phi \phi^4 + \bar\chi i \partial\!\!\!/ \chi - M \bar\chi \chi + (y\,\phi \bar\psi \chi + \mathrm{h.c.}) + \mathscr{L}_{\rm UV}^{\rm ct}$$

$\mathscr{L}_{\rm UV}\left(c_i(t)\right)$

$$\frac{dm_\phi^2}{dt} = \frac{\lambda_\phi}{16\pi^2} m_\phi^2 - \frac{y^\dagger y}{\pi^2} M^2 + \mathscr{O}(\hbar)$$



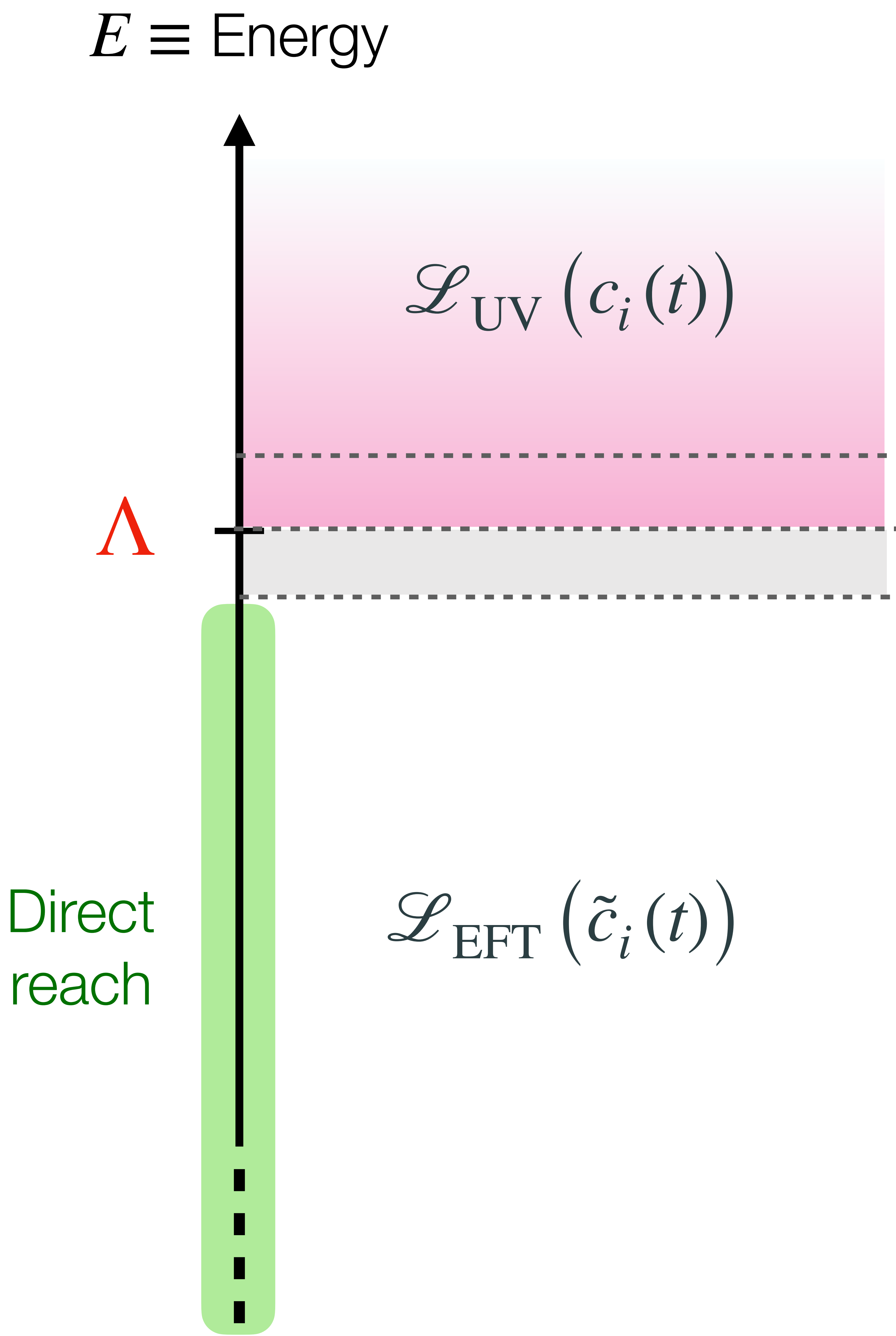
Λ

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{1}{4!}\lambda \phi^4 + \bar\psi i \partial \psi - m_\psi \bar\psi \psi + \mathscr{L}_{\rm EFT}^{\rm ct} + \mathscr{O}(\Lambda^{-1})$$

Direct reach

$\mathscr{L}_{\rm EFT}\left(\tilde c_i(t)\right)$

Renormalization group flow and scheme independence: an example



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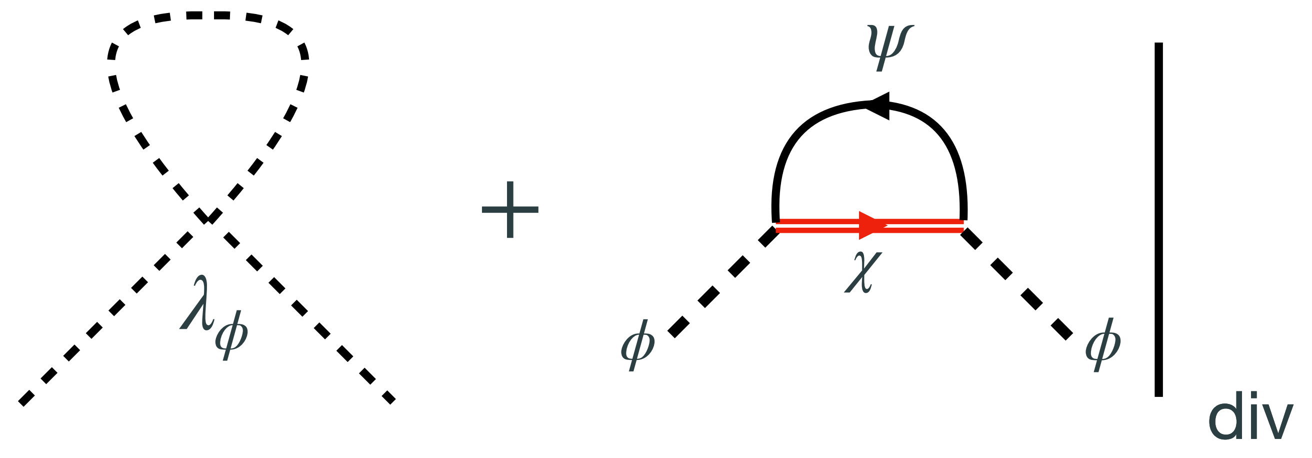

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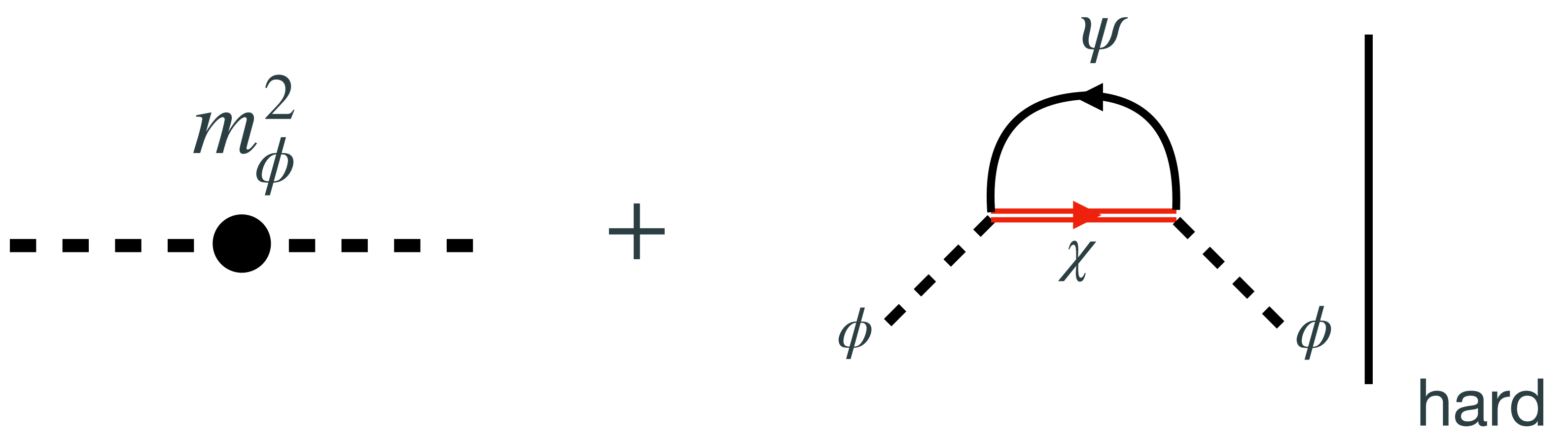
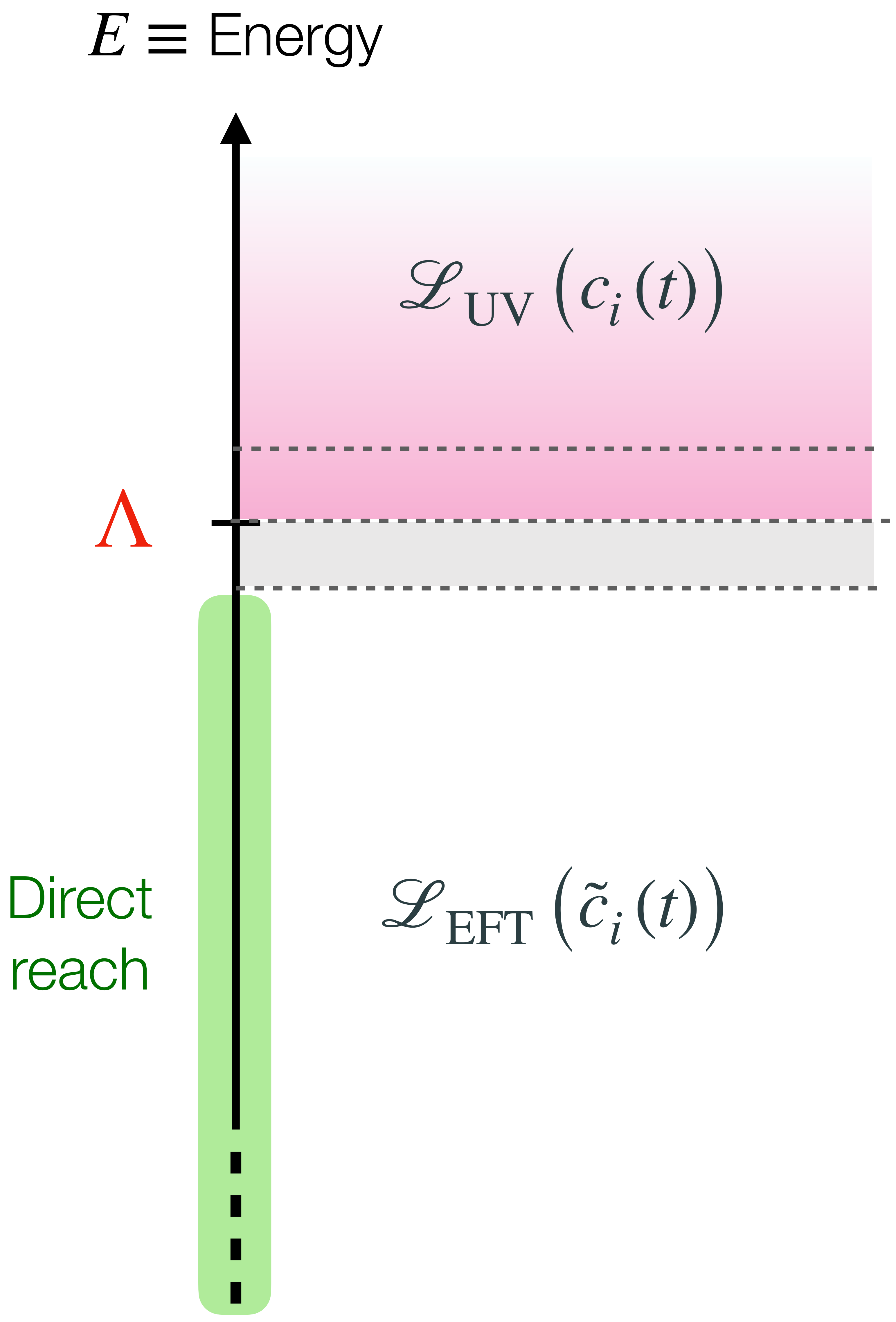


Renormalization group flow and scheme independence: an example



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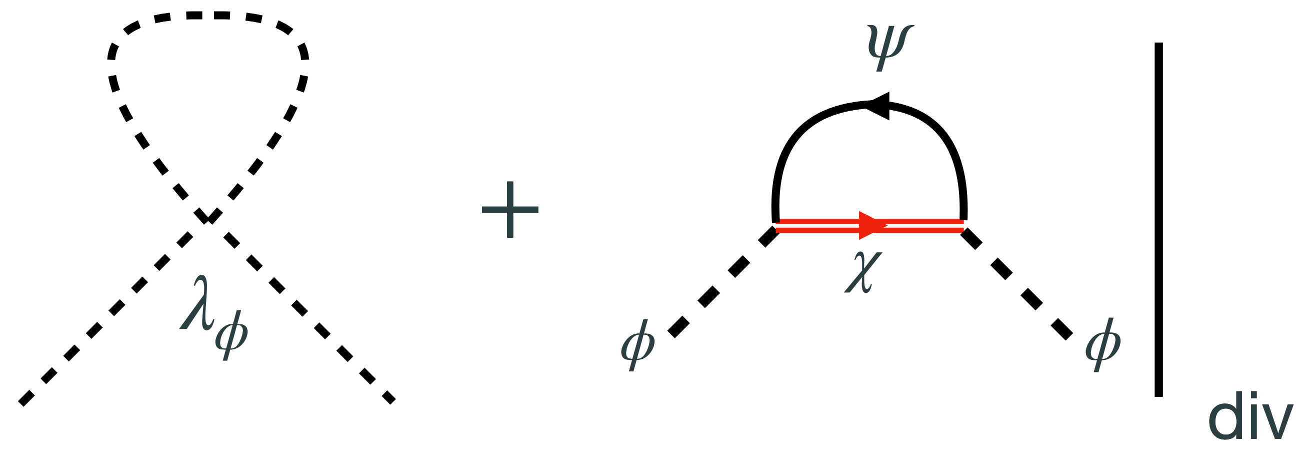

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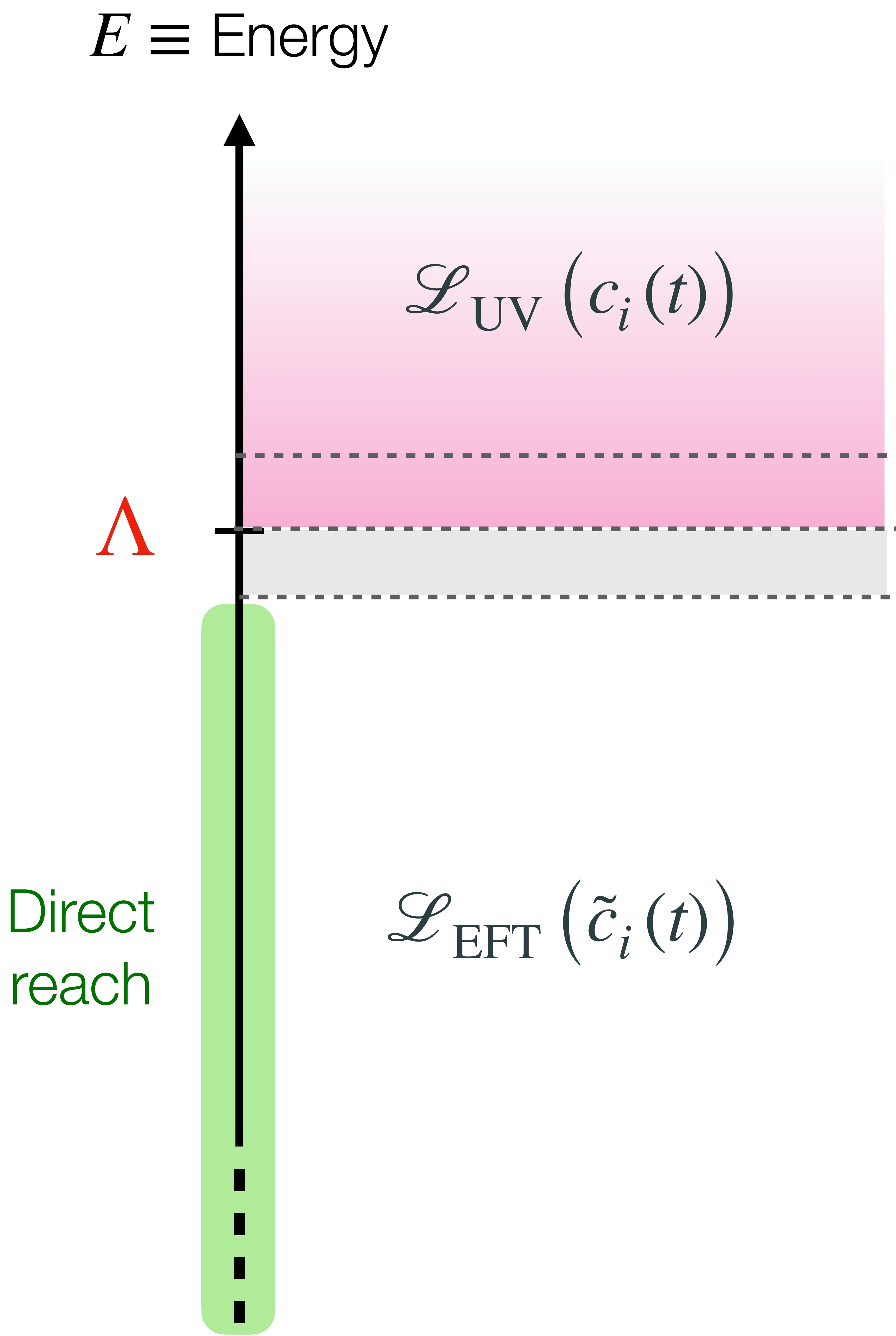

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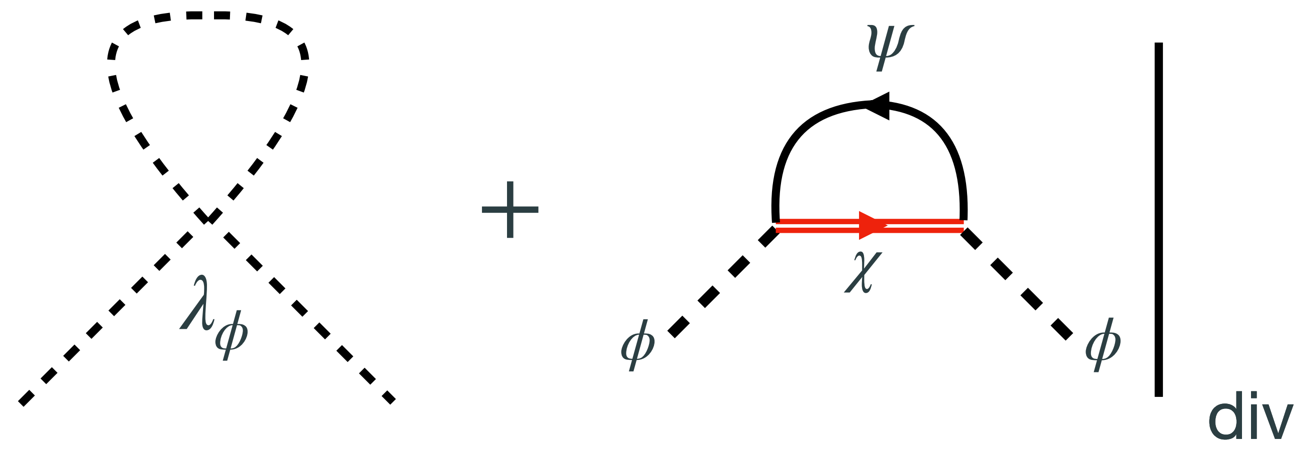

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EFTs bases, redundancies, and field redefinitions

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2$$

Exact simplifications (linear): Integration-by-parts, Dirac and group identities, commutation relations…

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{3C_2 - C_3}{3\Lambda^2}\phi^3\partial^2\phi$$

On-shell equivalence (non-linear): Field redefinitions (sometimes equivalent to using of EOMs)

$$\phi \to \phi + \frac{3\,C_2 - C_3}{3\,\Lambda^2}\phi^3 \qquad \left[\partial^2\phi = -\,m^2\phi - \frac{\lambda}{3!}\phi^3 + \mathcal{O}(\Lambda^{-2})\right]$$

$$\mathscr{L} \to \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \left(\frac{\lambda}{4!} + \frac{m^2(3C_2 - C_3)}{3\,\Lambda^2}\right)\phi^4 + \frac{18\,C_1 - \lambda\,(3C_2 - C_3)}{18\,\Lambda^2}\phi^6$$

Operators that can be eliminated via field redefinitions are not necessary to compute physical observables (but are necessary to compute off-shell quantities).

[For details see Criado, Pérez-Victoria, 1811.09413]

EFTs bases, redundancies, and field redefinitions

In $d = 4$, we can use the Fierz identity $R_{\ell e} = -\frac{1}{2} Q_{\ell e}$

$$\mathscr{L}_{\rm EFT} \supset C_{\ell e}^{prst} R_{\ell e}^{prst} \qquad\qquad R_{\ell e}^{prst} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

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so that $\mathscr{L}_{\rm EFT} = \mathscr{L}'_{\rm EFT}$ at tree level.

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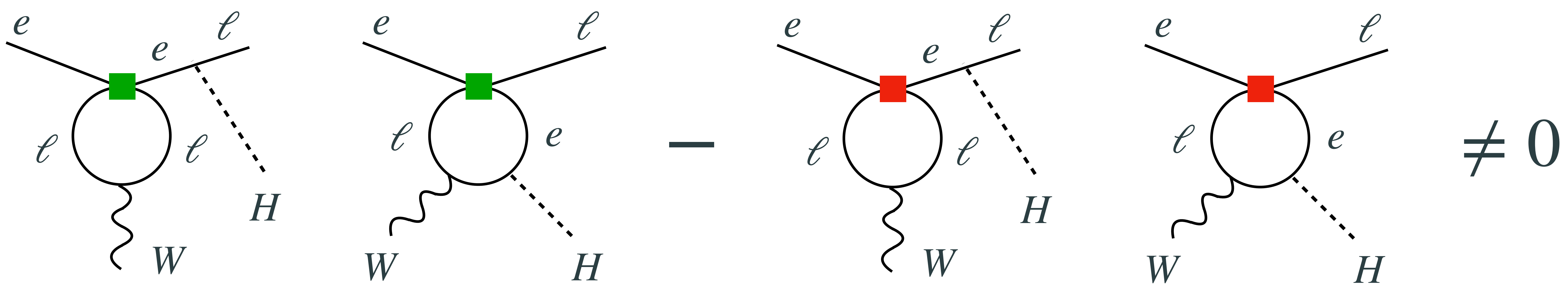

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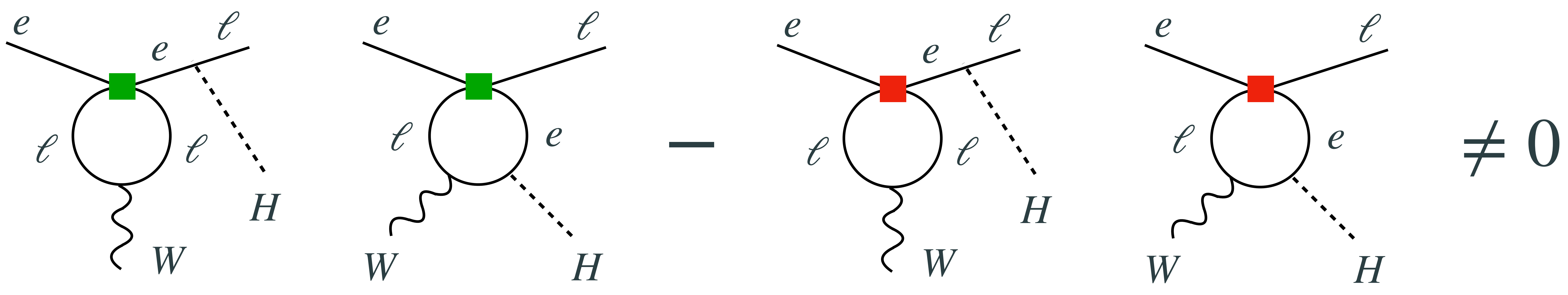

In $d = 4 - 2\epsilon$, there is an evanescent operator that also contributes to the amplitude

$$R^{prst}_{\ell e} = -\frac{1}{2} Q^{prst}_{\ell e} + E^{prst}_{\ell e} \qquad E^{prst}_{\ell e} \xrightarrow{\epsilon \to 0} 0 \qquad E^{prst}_{\ell e} \to -\frac{g_L\, y_e^{ts}}{128\pi^2} Q^{pr}_{eW} + \text{[many other contributions]}$$

The SMEFT Lagrangian

The SM is a very successful theory, as it correctly describes many phenomena over a wide range of energies.
On the other hand, we know that the SM cannot be the ultimate theory…

… unfortunately we do not know how this theory will look like or have any experimental evidence for it so far.

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‘If all you have is a hammer, everything looks like a nail'

— Abraham H. Maslow (1962), Toward a Psychology of Being

Let’s treat the Standard Model as the leading-order approximation of an EFT, the **SMEFT** !

Basic principles of the SMEFT

N.B.: This assumes no undiscovered light particles or different EWSB mechanisms

Degrees of freedom: The SM particles: $q\,,u\,,d\,,l\,,e\,,H$

Power counting: In inverse powers of an unknown heavy scale where the SM will "break"

Symmetries of the system: The SM gauge symmetry: $SU(3)_c \times SU(2)_L \times U(1)_Y$

… but (a priori) not baryon or lepton numbers, or $U(1)_{B-L}$

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)}$$

New physics to be discovered

Any new heavy physics that extend the SM is fully covered by this EFT Lagrangian

The operators of the SMEFT: dimension five

At dimension five, there is only one term

$$\mathscr{L}^{(5)} = \frac{C_{pr}^{(5)}}{\Lambda} Q_{pr}^{(5)} \qquad Q_{pr}^{(5)} = \epsilon^{ij}\epsilon^{kl}\,(l_{ip}^{\intercal} C l_{kr})\, H_j H_l$$

N.B.: Lepton number broken in two units

$$\Big\downarrow\; |\langle H \rangle| = \frac{v}{\sqrt{2}}$$

$$\frac{C_{pr}^{(5)}\, v^2}{2\Lambda}\,(\nu_p^{\intercal} C \nu_r)$$

(Majorana) neutrino masses!

If $C^{(5)} \sim \mathscr{O}(1)$, we can infer a new-physics scale from $m_\nu \leq 0.01$ eV : $\Lambda \approx 10^{15}$ GeV (ballpark of the GUT scale)

The operators of the SMEFT: dimension six

At dimension six, there are 59 + 5 terms

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\,\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\,\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

The operators of the SMEFT: dimension six

At dimension six, there are 59 + 5 terms

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

The modern approach to interpret experimental data

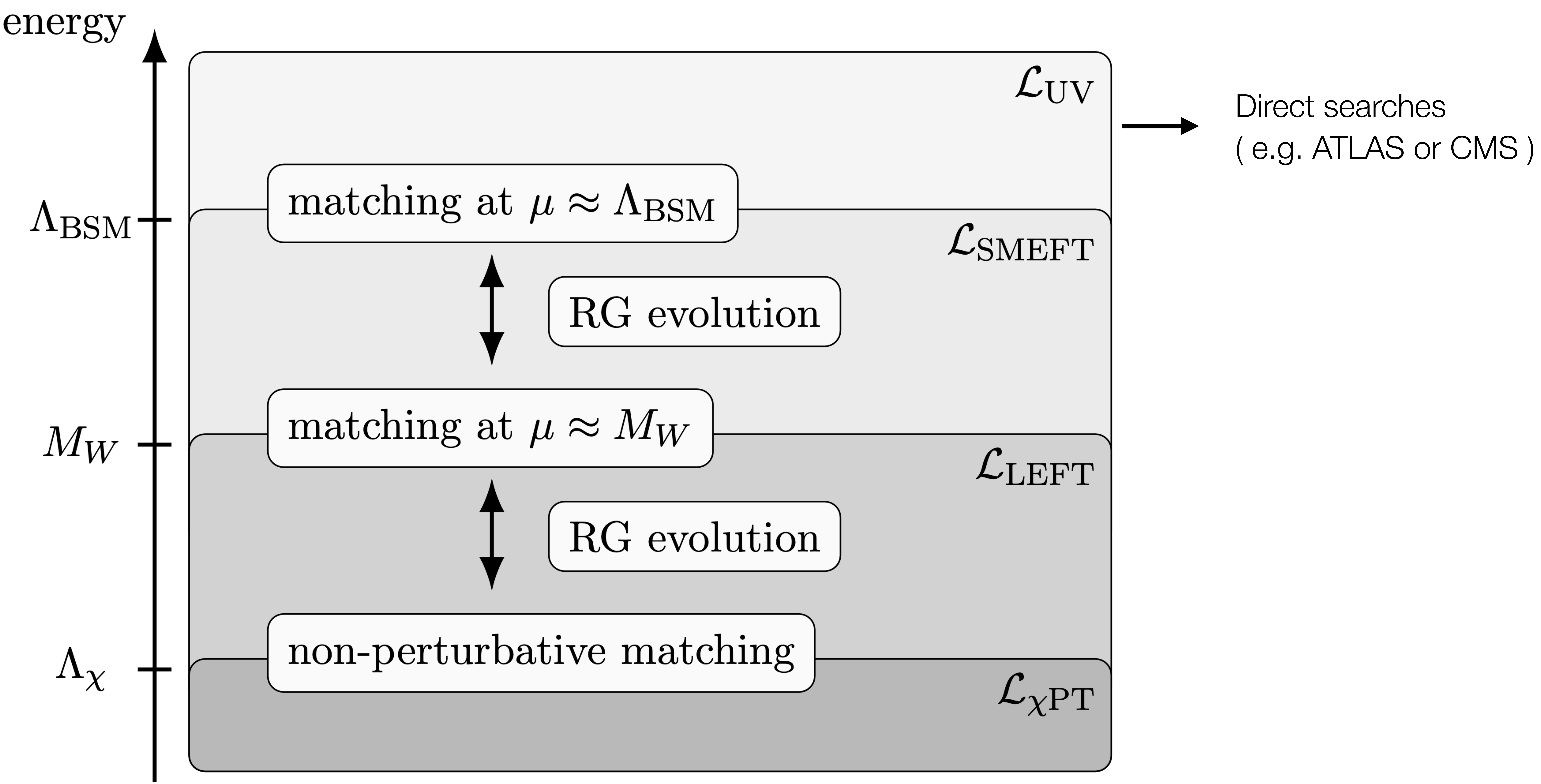
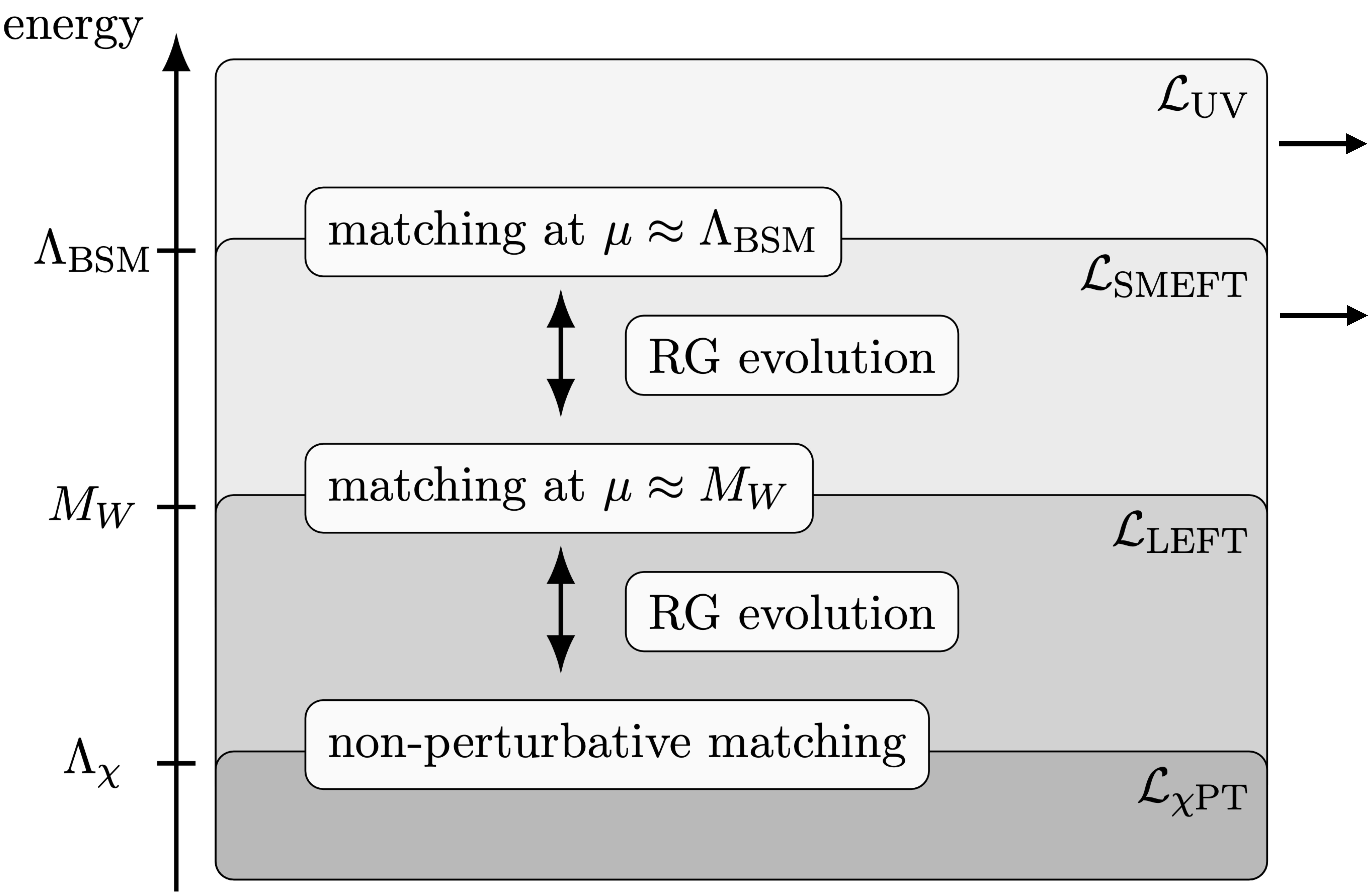


Figure from P. Stoffer

The modern approach to interpret experimental data



Direct searches
(e.g. ATLAS or CMS)

Indirect high-energy searches
(e.g. ATLAS or CMS)

Figure from P. Stoffer

The modern approach to interpret experimental data

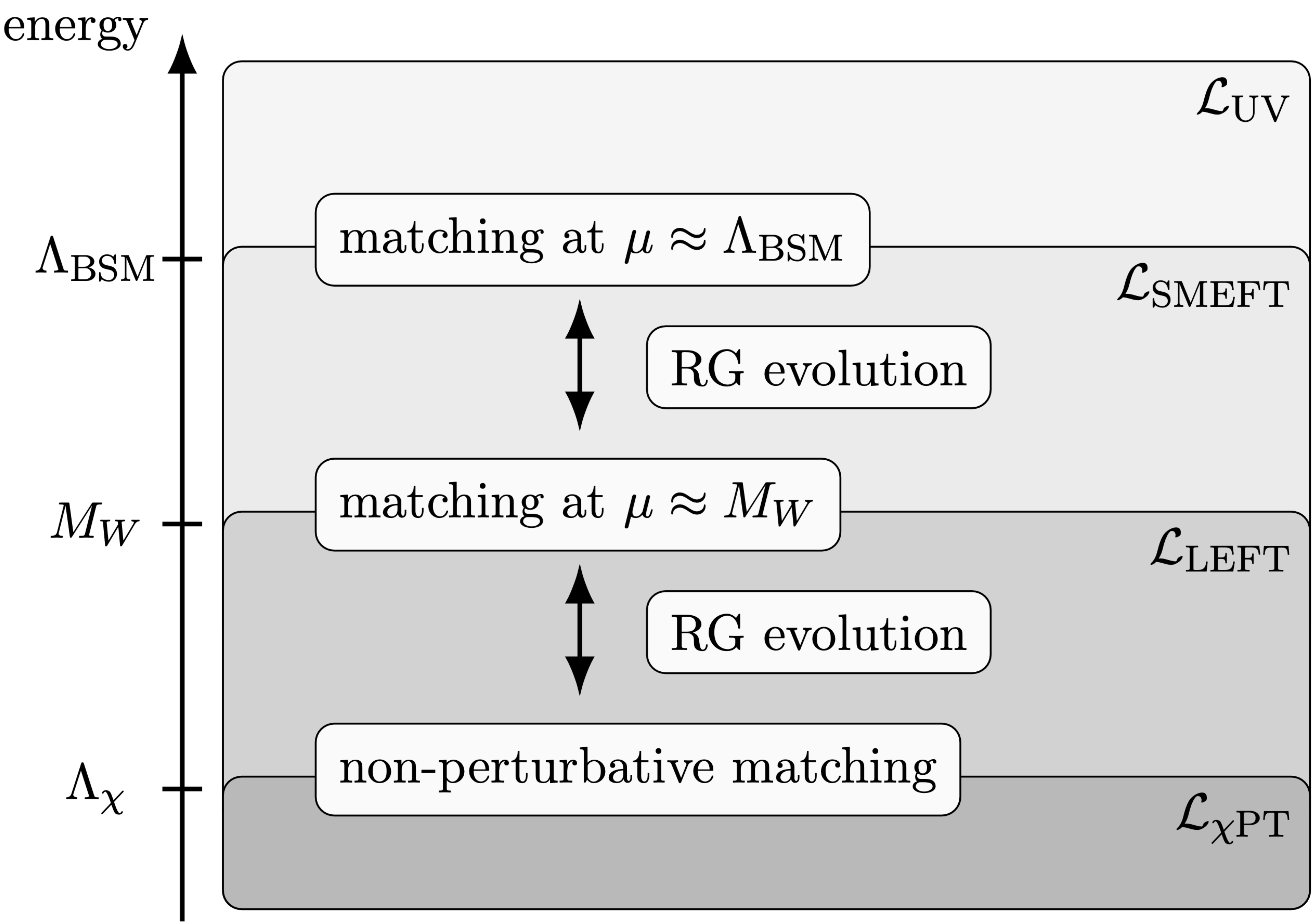


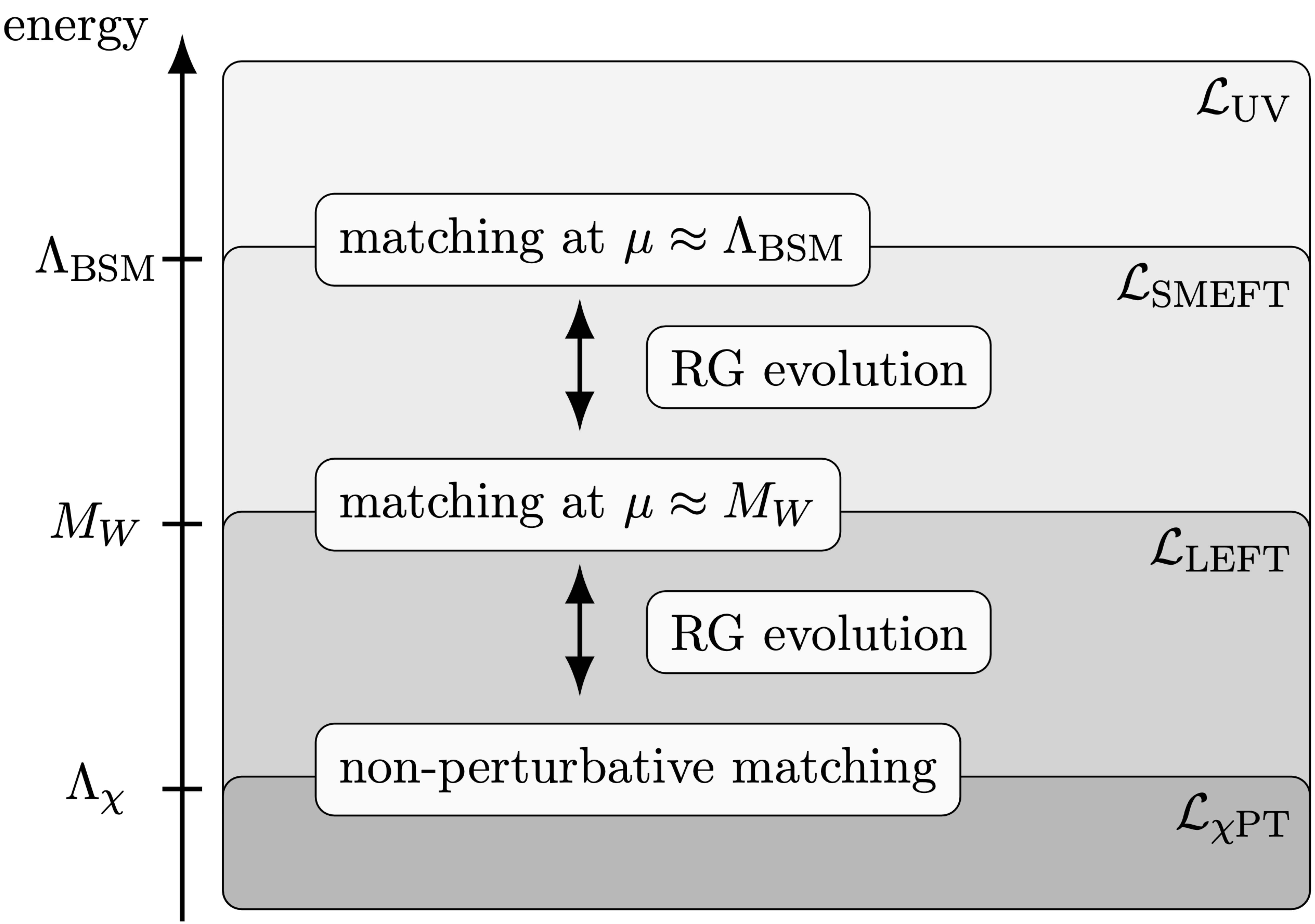
Figure from P. Stoffer

→ Direct searches
(e.g. ATLAS or CMS)

→ Indirect high-energy searches
(e.g. ATLAS or CMS)

→ $Z-$pole observables
(e.g. LEP)

The modern approach to interpret experimental data



Direct searches
(e.g. ATLAS or CMS)

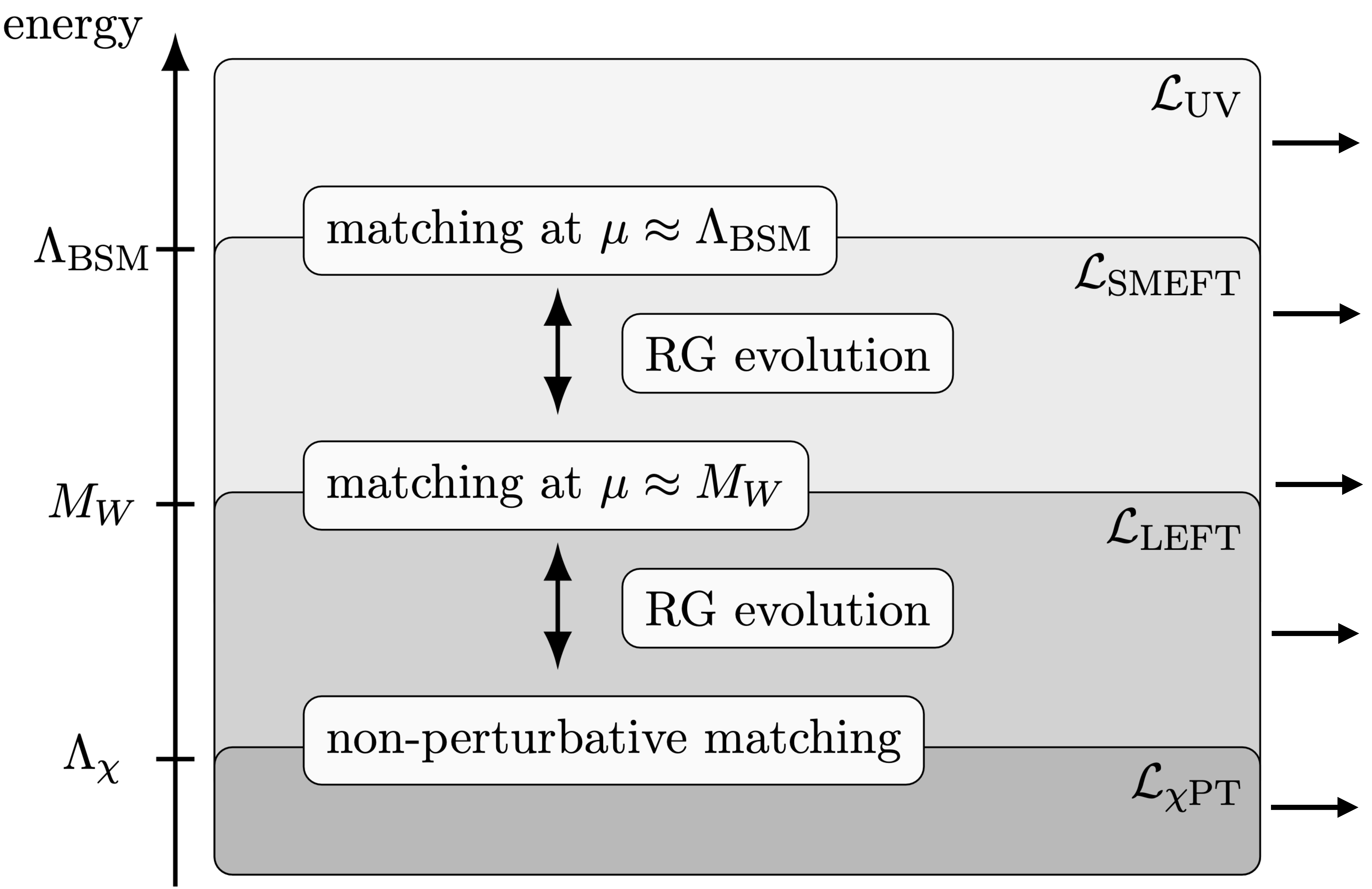
Indirect high-energy searches
(e.g. ATLAS or CMS)

$Z-$pole observables
(e.g. LEP)

Low-energy experiments
(e.g. $g-2$, LHCb, Belle II)

Figure from P. Stoffer

The modern approach to interpret experimental data



Direct searches
(e.g. ATLAS or CMS)

Indirect high-energy searches
(e.g. ATLAS or CMS)

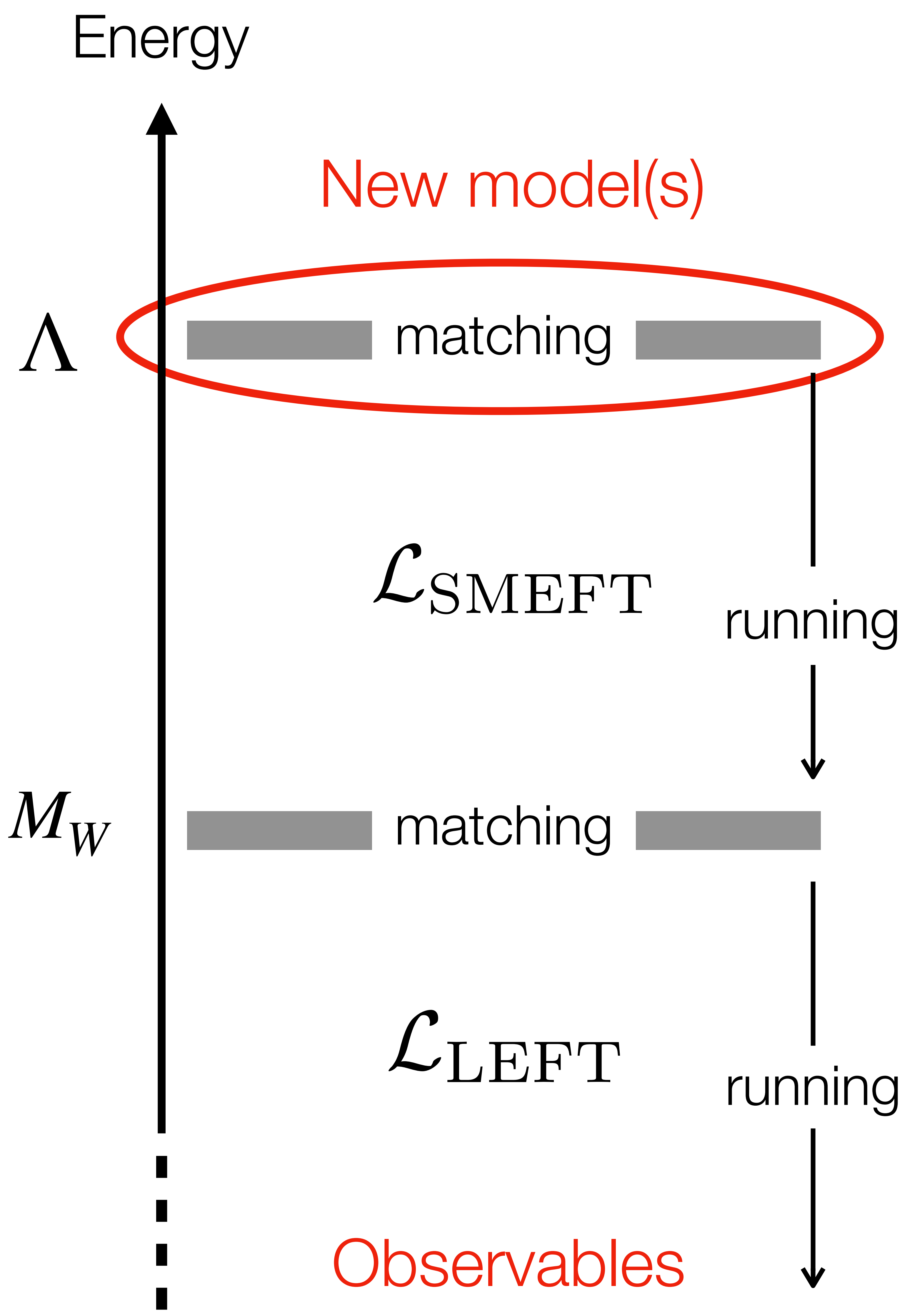
$Z-$pole observables
(e.g. LEP)

Low-energy experiments
(e.g. $g-2$, LHCb, Belle II)

Lower-energy experiments
(e.g. neutron dipole moments)

Figure from P. Stoffer

The rise of automation



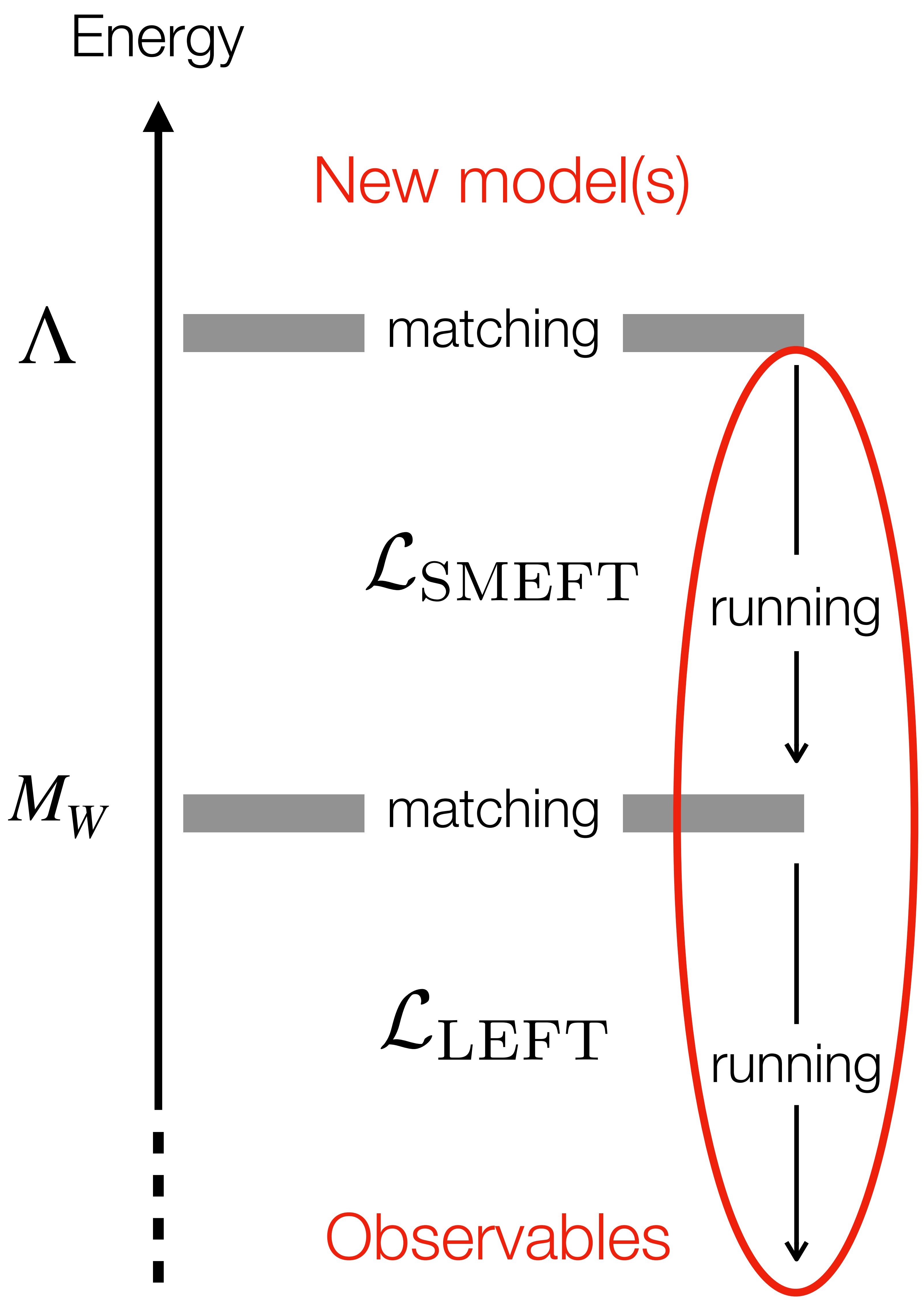
matchmakereft

Carmona et al. '22

JFM et al. '23

Automated one-loop matching of many models

The rise of automation



DsixTools

JFM et al. '17 & '21

wilson

Aebischer et al. '18

"Hard-coded" one-loop results based on:

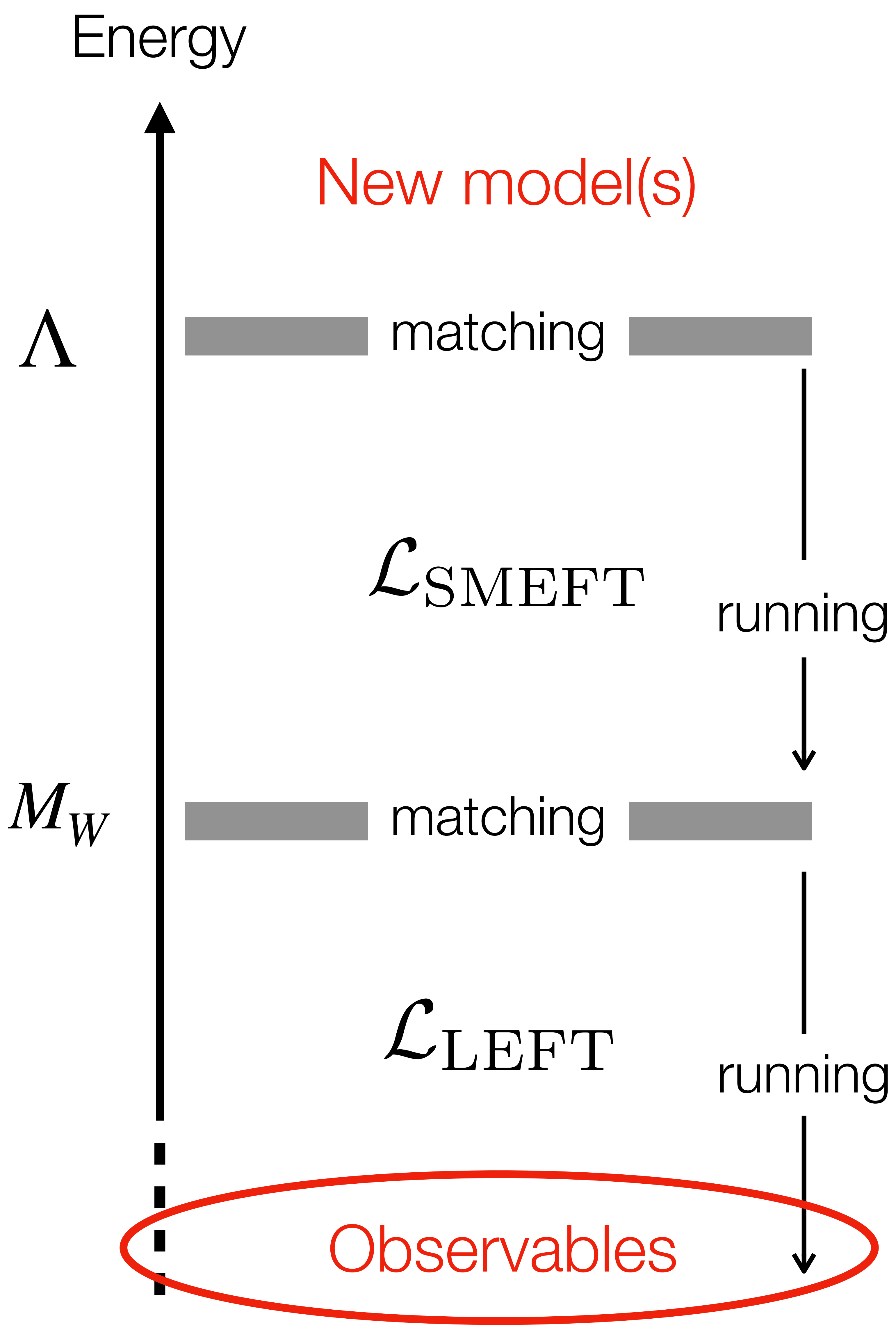
SMEFT running: Jenkins et al. '13, '14; Alonso et al. '14

LEFT basis: Jenkins et al. '18

SMEFT-LEFT matching: Dekens, Stoffer '19

LEFT running: Jenkins et al. '18

The rise of automation



SMEFT likelihood (smelli)

Aebischer et al. '18

flavio

Straub '16

HEPfit

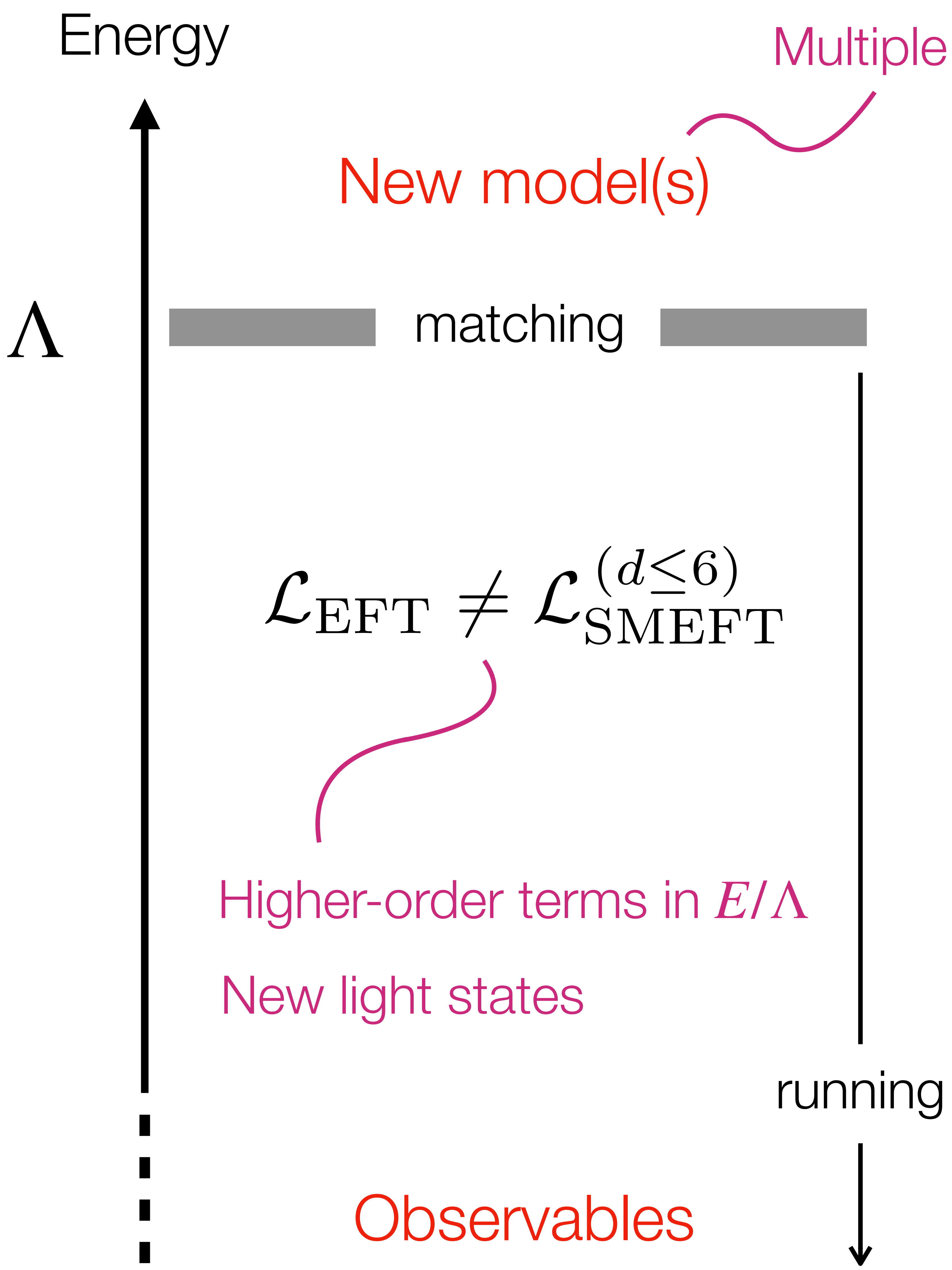
De Blas et al. '19

SMEFiT

Giani et al. '23

Involvement of experimental collaborations into this program is crucial

The rise of automation



Present limitations

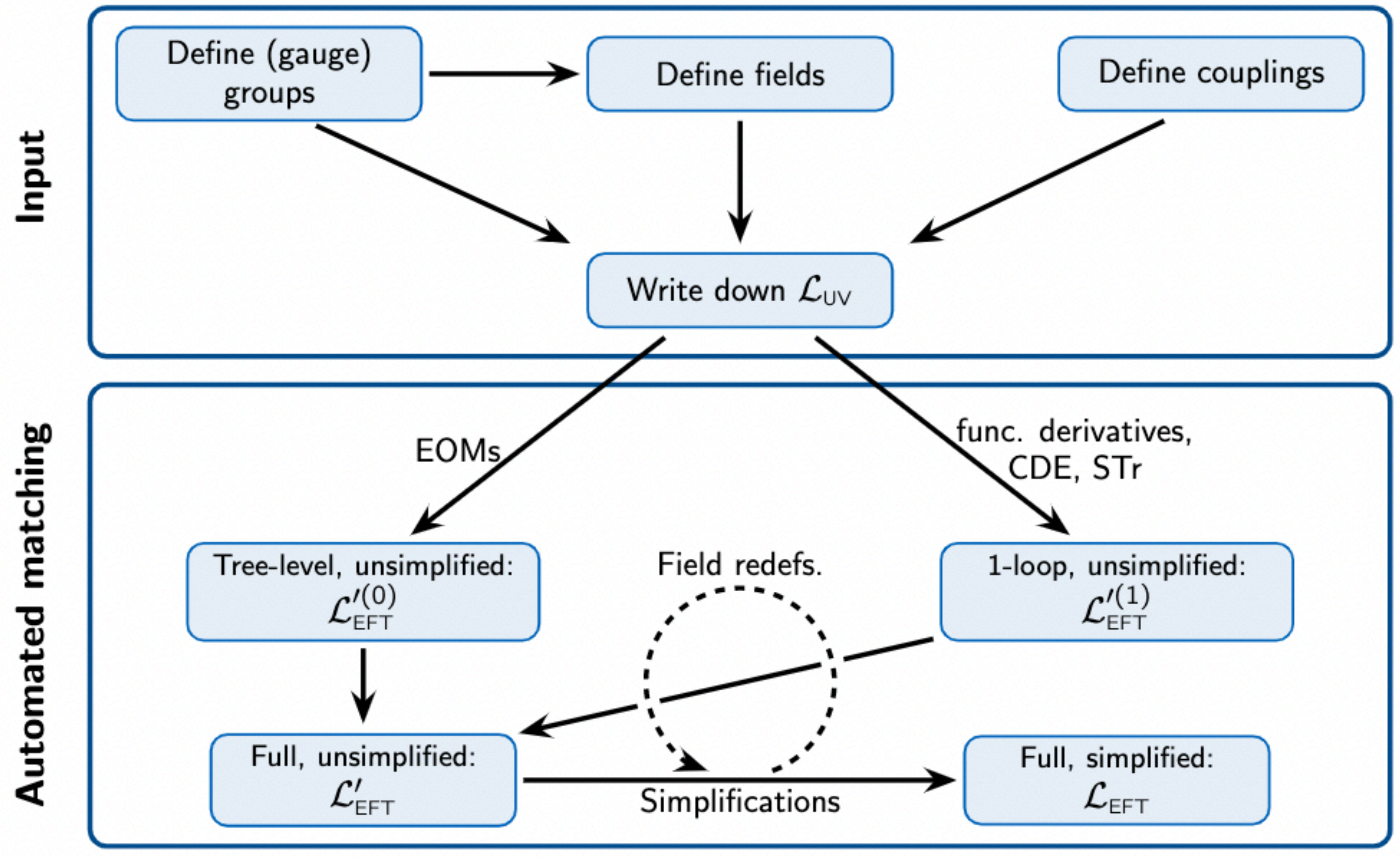
Full automation only for the simplest scenarios

Some steps/approaches require prior knowledge of the target EFT

(Automated) inclusion of higher-loop orders is (so far) non-trivial

The Matchete package

MATCHETE is a **Mathematica package** aimed at fully automating EFT matching and RG evolution of arbitrary weakly-coupled UV theories using functional methods



[JFM, König, Pagès, Thomsen, Wilsch, 2212.04510]

Proof-of-concept version (Matchete v0.1) now publicly available:

- One-loop matching of *any* model with heavy scalars and/or fermions
- Simple and intuitive input/output
- Handles *all* group theory (any group and reps)
- Partial simplifications of the resulting EFT Lagrangian (IBP, field redefinitions,…)
- SSB and heavy vectors not yet supported
  [w.i.p with Olgoso, Santiago, Thomsen]
- Computation of the RGE not yet available

Bonus: The path-integral formulation of EFTs

The quantum effective action is the generating functional of 1PI functions

$$e^{i\,\Gamma_{\rm UV}[\hat{\eta}]} = \int \mathscr{D}\eta\ \exp\big(i\,S_{\rm UV}[\eta + \hat{\eta}]\big)$$

η : Quantum fields (loop lines)

$\hat{\eta}$: Background fields (tree lines)

Bonus: The path-integral formulation of EFTs

The quantum effective action is the generating functional of 1PI functions

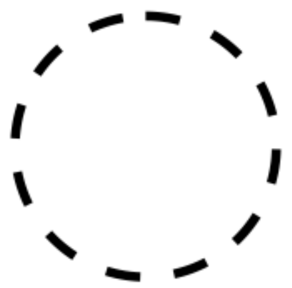
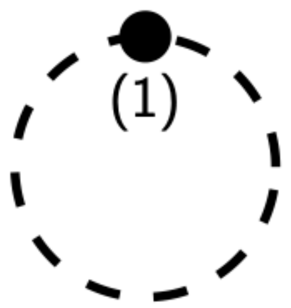
$$e^{i\,\Gamma_{\rm UV}[\hat\eta]} = \int \mathscr{D}\eta \;\exp\!\big(i\,S_{\rm UV}[\eta+\hat\eta]\big)$$

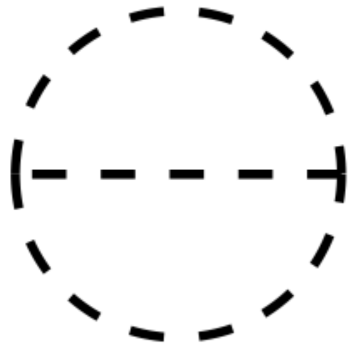
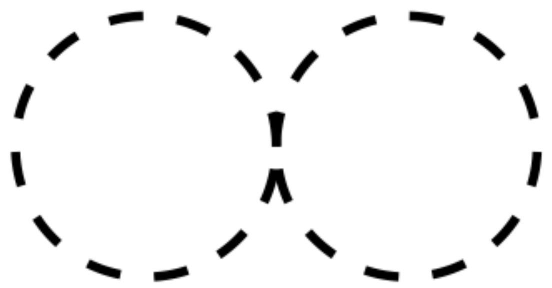
η : Quantum fields (loop lines)

$\hat\eta$: Background fields (tree lines)

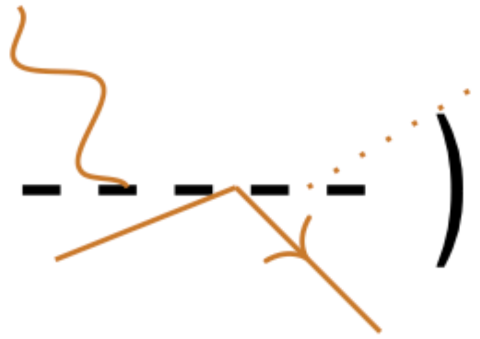
The loop expansion is obtained via the saddlepoint approximation (Taylor expansion around $\hat\eta$):

$$\Gamma_{\rm UV}[\hat\eta] = S_{\rm UV}[\hat\eta] - i\ln\int \mathscr{D}\eta\,\exp\left[i\left(\frac{1}{2}\,Q_{IJ}[\hat\eta]\,\eta_I\eta_J + \frac{1}{3!}\,B_{IJK}[\hat\eta]\,\eta_I\eta_J\eta_K + \frac{1}{4!}\,D_{IJKL}[\hat\eta]\,\eta_I\eta_J\eta_K\eta_L + \dots\right)\right]$$

$= S_{\rm UV}[\hat\eta] + \frac{i}{2}\log$  $+ \frac{i}{2}$ 

$+ \frac{1}{12}$  $- \frac{1}{8}$  $+ \mathcal{O}(\hbar^3)$

All propagators are dressed with arbitrary background field insertions

$$\mathcal{Q}_{IJ}[\hat\eta] = \frac{\delta^2 S}{\delta\eta^I\delta\eta^J}[\hat\eta] = \Big(\;$$  $$\Big)^{-1}$$

Bonus: The path-integral formulation of EFTs

The EFT action is given by

$$S_{\rm EFT}[\phi] = \Gamma_{\rm UV}[\hat{\Phi}, \phi]\Big|_{\rm hard} \qquad \frac{\delta\Gamma_{\rm UV}\big|_{\rm hard}}{\delta\Phi}[\hat{\Phi}, \phi] = 0$$

Φ : Heavy

ϕ : Light

“hard” denotes the part where all loop momenta are $p \sim \Lambda$ (incl. tree-level contributions)

- Already used at one loop order [JFM, Portolés, Ruiz-Femenía, 1607.02142; Z. Zhang 1610.00710]
- Explicit proof to two-loop order [JFM, Thomsen, Palavic, w.i.p]
- The hard region is by far the easiest to compute (only vacuum diagrams at zero external momenta)
- Considerably simplifies (functional) matching and running at any loop order

Tutorial

1. Scaleless integrals vanish in DimReg. Given one can write

$$\int \frac{d^d l}{(2\pi)^l} \frac{1}{k^4} = \int \frac{d^d l}{(2\pi)^l} \frac{1}{k^2(k^2 - M^2)} - \int \frac{d^d l}{(2\pi)^l} \frac{M^2}{k^4(k^2 - M^2)} \equiv I_1 - I_2$$

compute I_1 and I_2 and show that they are identical in DimReg. Discuss whether they are UV or IR divergent. You can use the following general expression for the loop integrals:

$$\int \frac{d^d l}{(2\pi)^l} \frac{1}{(k^2)^n (k^2 - M^2)^m} = \frac{(-1)^{n+m}\, i}{(4\pi)^{2-\epsilon} (M^2)^{n+m-2+\epsilon}} \frac{\Gamma(n+m-2+\epsilon)\,\Gamma(2-n-\epsilon)}{\Gamma(m)\,\Gamma(2-\epsilon)}$$

Tutorial

2. Given the EFT Lagrangian

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{C_0}{\Lambda}\phi^2\partial^2\phi + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2 + \frac{C_4}{\Lambda^2}\phi^2\partial^2\phi^2$$

Use integration-by-part identities to reduce the Lagrangian to an off-shelf basis. Compare your result using Matchete.

Now use field redefinitions to reduce the Lagrangian to an on-shell basis. Once more, compare your result using Matchete. Instead of field redefinitions, use the equations of motion for ϕ. Discuss your result.

Tutorial

3. Given the UV Langrangian

$$\mathscr{L}_{\text{UV}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{4!}\lambda_\phi \phi^4 + \bar{\psi} i\partial\!\!\!/ \psi$$
$$+ \bar{\chi} i\partial\!\!\!/ \chi - M\bar{\chi}\chi + (y\,\phi\bar{\psi}\chi + \text{h.c.})$$

with $m_\phi^2 \ll M^2$. Compute the matching to the corresponding EFT (both at tree-level and one-loop), neglecting corrections of $\mathcal{O}(M^{-2})$. Compute the matching using Matchete and compare the results.

Tutorial

4. Given the UV Langrangian

$$\mathcal{L}_{\mathrm{UV}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{4!}\lambda_\phi \phi^4 + \bar{\psi}\, i\not{\partial}\psi$$
$$+ \bar{\chi}\, i\not{\partial}\chi - M\bar{\chi}\chi + (y\,\phi\bar{\psi}\chi + \mathrm{h.c.})$$

Renormalize the theory and obtain the Renormalization Group equations. What happens with the mass of ψ as the Lagrangian evolves in energy? What would happen if we replace the Yukawa interaction by $(y\,\phi\bar{\psi}_L\chi_R + \mathrm{h.c.})$? Discuss the results in terms of the symmetries of the Lagrangian.

Include the UV counterterms into the matching calculation from exercise 3. What do you observe?

Tutorial

5. Explore and have fun using Matchete!
(see notebook examples attached)